ME 101: Engineering Mechanics

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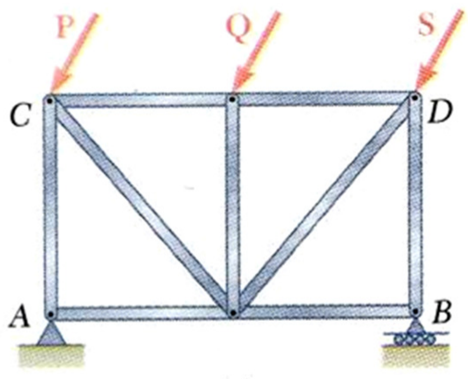
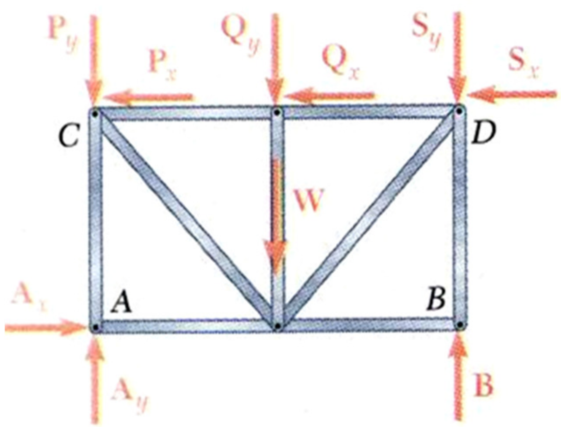
Equilibrium of rigid body



Equations of equilibrium become

$$\sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_A = 0$$

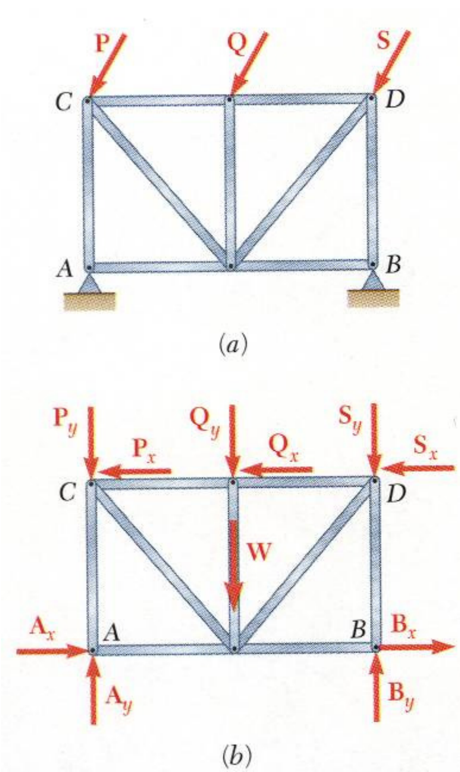
There are three unknowns and number of equation is three.



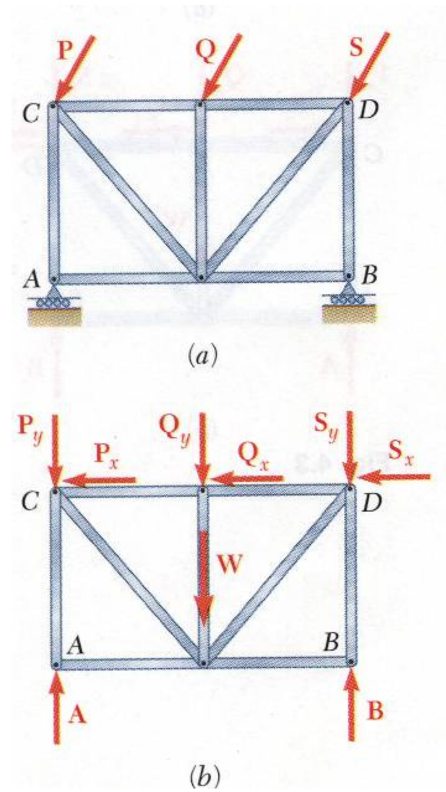
Therefore, the structure is statically determinate

The rigid body is completely constrained

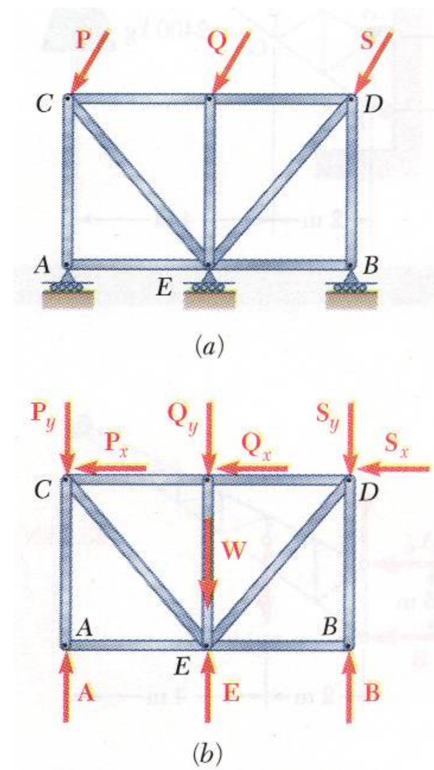
Equilibrium of rigid body



More unknowns than equations

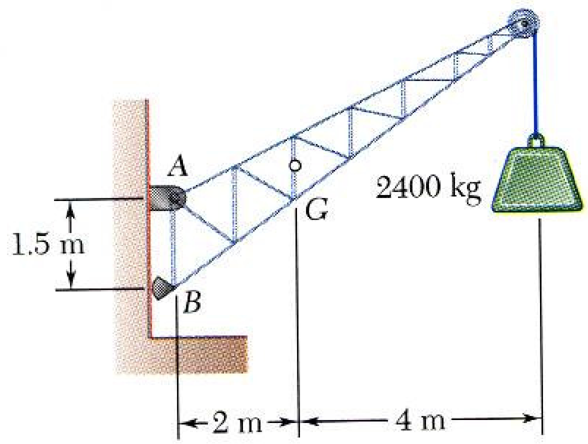


Fewer unknowns than equations, partially constrained



Equal number unknowns and equations but improperly constrained

Example problem 1



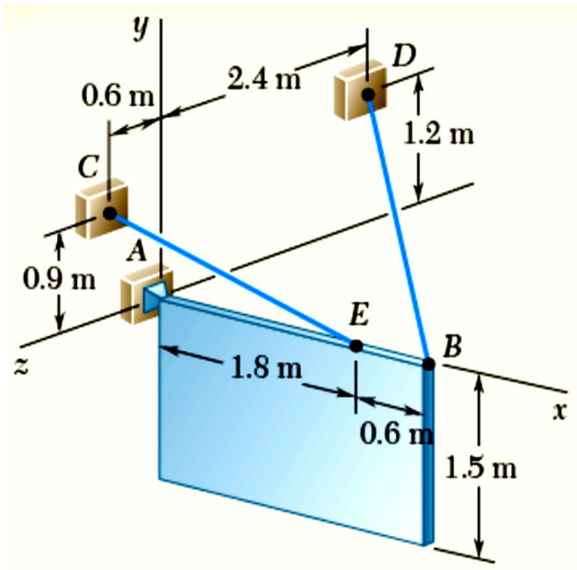
A fixed crane has a mass of 1000 kg and is used to lift a 2400 kg crate. It is held in place by a pin at A and a rocker at B. The center of gravity of the crane is located at G.

Determine the components of the reactions at A and B.

SOLUTION:

- Create a free-body diagram for the crane.
- Determine B by solving the equation for the sum of the moments of all forces about A. Note there will be no contribution from the unknown reactions at A.
- Determine the reactions at A by solving the equations for the sum of all horizontal force components and all vertical force components.
- Check the values obtained for the reactions by verifying that the sum of the moments about B of all forces is zero.

Example problem 4



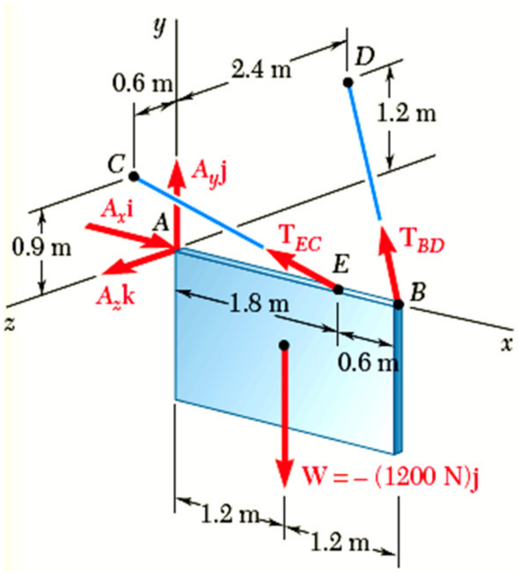
A sign of uniform density weighs 1200-N and is supported by a ball-and-socket joint at A and by two cables.

Determine the tension in each cable and the reaction at A.

SOLUTION:

- Create a free-body diagram for the sign.
- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

Example problem 4



- Create a free-body diagram for the sign.

  Since there are only 5 unknowns, the sign is partially constrain. It is free to rotate about the x axis. It is, however, in equilibrium for the given loading.

$$\vec{T}_{BD} = T_{BD}\frac{\vec{r}_D - \vec{r}_B}{\left|\vec{r}_D - \vec{r}_B\right|}$$

$$= T_{BD}\frac{-2.4\vec{i} + 1.2\vec{j} - 2.4\vec{k}}{3.6}$$

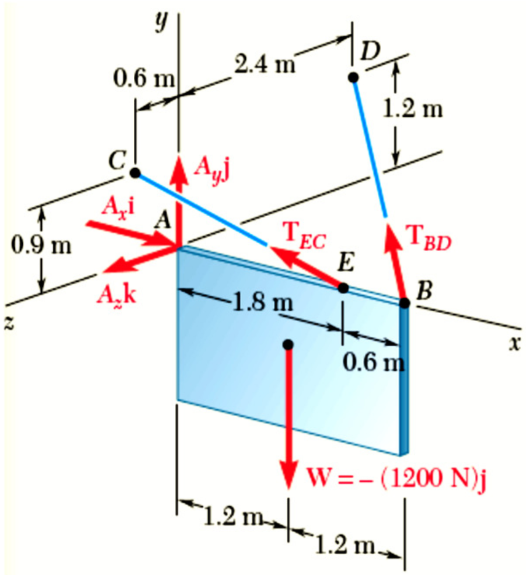
$$= T_{BD}\left(-\tfrac{2}{3}\vec{i} + \tfrac{1}{3}\vec{j} - \tfrac{2}{3}\vec{k}\right)$$

$$\vec{T}_{EC} = T_{EC}\frac{\vec{r}_C - \vec{r}_E}{\left|\vec{r}_C - \vec{r}_E\right|}$$

$$= T_{EC}\frac{-1.8\vec{i} + 0.9\vec{j} + 0.6\vec{k}}{2.1}$$

$$= T_{EC}\left(-0.85\vec{i} + 0.428\vec{j} + 0.285\vec{k}\right)$$

Example problem 4



- Apply the conditions for static equilibrium to develop equations for the unknown reactions.

$$\sum \vec{F} = \vec{A} + \vec{T}_{BD} + \vec{T}_{EC} - (1200\ \mathrm{N})\vec{j} = 0$$

$$\vec{i}: \quad A_x - 0.67T_{BD} - 0.86T_{EC} = 0$$

$$\vec{j}: \quad A_y + 0.33T_{BD} + 0.43T_{EC} - 1200\ \mathrm{N} = 0$$

$$\vec{k}: \quad A_z - 0.67T_{BD} + 0.29T_{EC} = 0$$

$$\sum \vec{M}_A = \vec{r}_B \times \vec{T}_{BD} + \vec{r}_E \times \vec{T}_{EC} + (1.2\ \mathrm{m})\vec{i} \times (-1200\ \mathrm{N})\vec{j} = 0$$

$$\vec{j}: \quad 1.6T_{BD} - 0.514T_{EC} = 0$$

$$\vec{k}: \quad 0.8T_{BD} + 0.771T_{EC} - 1440\ \mathrm{N\,.\,m} = 0$$

Solve the 5 equations for the 5 unknowns,

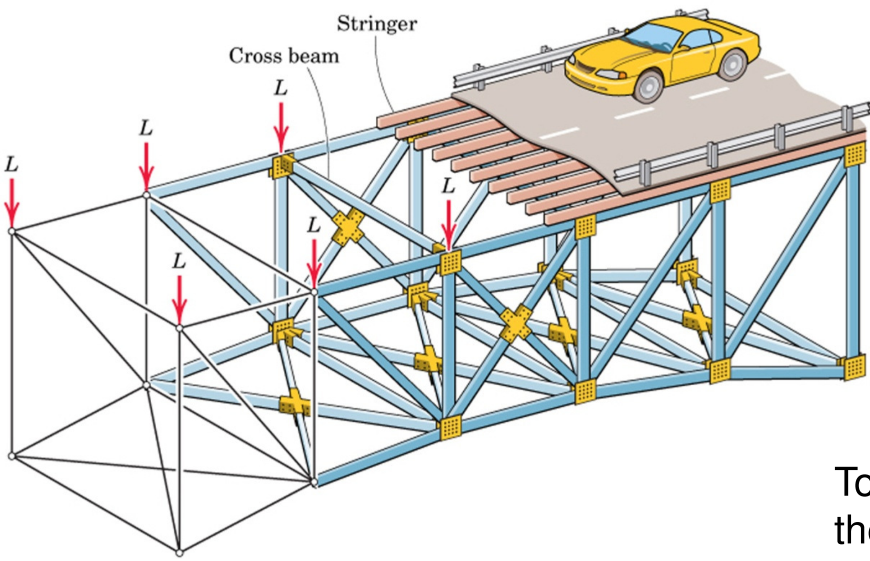
$$\boxed{\begin{array}{l} T_{BD} = 451\ \mathrm{N} \quad T_{EC} = 1402\ \mathrm{N} \\ \vec{A} = (1502\ \mathrm{N})\vec{i} + (419\ \mathrm{N})\vec{j} - (100.1\ \mathrm{N})\vec{k} \end{array}}$$

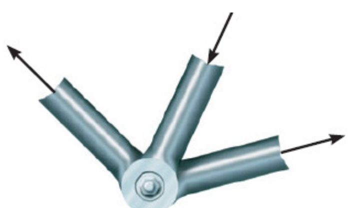
Structural Analysis

Engineering Structure

An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

Structural Analysis





Statically Determinate Structures

To determine the internal forces in the structure, dismember the structure and analyze separate free body diagrams of individual members or combination of members.

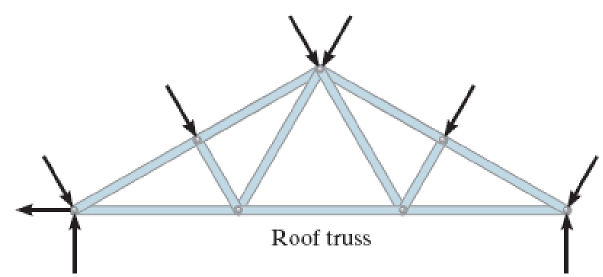
Structural Analysis: Plane Truss

Truss: A framework composed of members joined at their ends to form a rigid structure

Joints (Connections): Welded, Riveted, Bolted, Pinned

Plane Truss: Members lie in a single plane

(a)

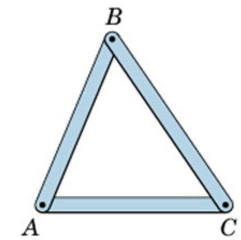


(b)

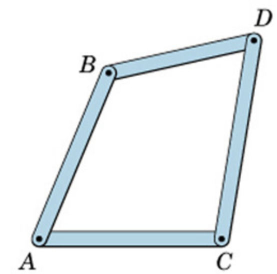
Structural Analysis: Plane Truss

Simple Trusses

Basic Element of a Plane Truss is the Triangle



- Three bars joined by pins at their ends → Rigid Frame
  - Non-collapsible and deformation of members due to induced internal strains is negligible

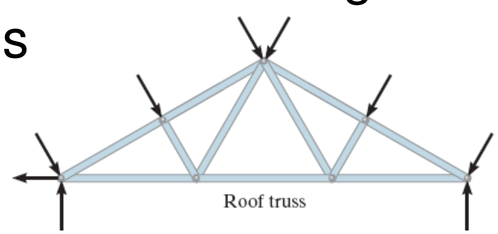


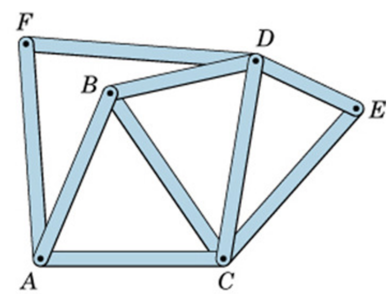
- Four or more bars polygon → Non-Rigid Frame

  How to make it rigid or stable?

  by forming more triangles!

Structures built from basic triangles
→Simple Trusses

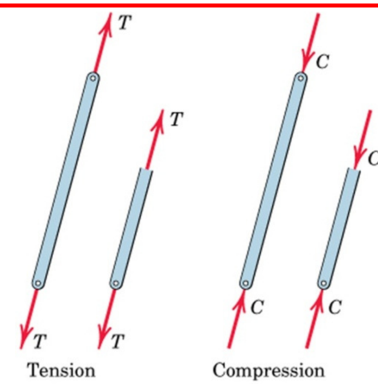




Structural Analysis: Plane Truss

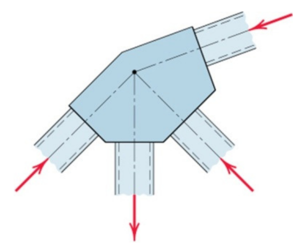
Basic Assumptions in Truss Analysis

- All members are two-force members.
- Weight of the members is small compared with the force it supports (weight may be considered at joints)
- No effect of bending on members even if weight is considered



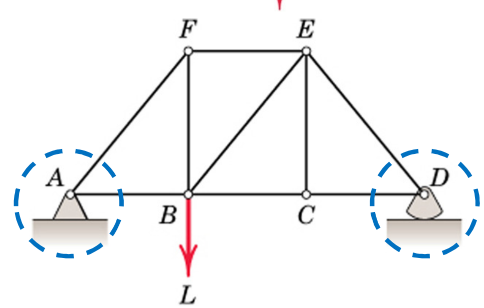
Two-Force Members

- External forces are applied at the pin connections
- Welded or riveted connections
- Pin Joint if the member centerlines are concurrent at the joint



Common Practice in Large Trusses

- Roller/Rocker at one end. Why?
  - to accommodate deformations due to temperature changes and applied loads.
  - otherwise it will be a statically indeterminate truss



Structural Analysis: Plane Truss

Truss Analysis: Method of Joints

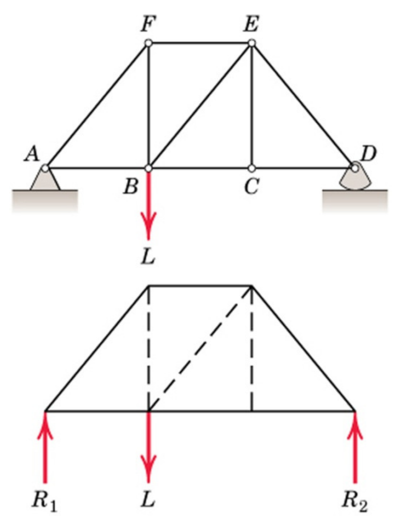
- Finding forces in members

Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint

- Equilibrium of concurrent forces at each joint
- only two independent equilibrium equations are involved

Steps of Analysis

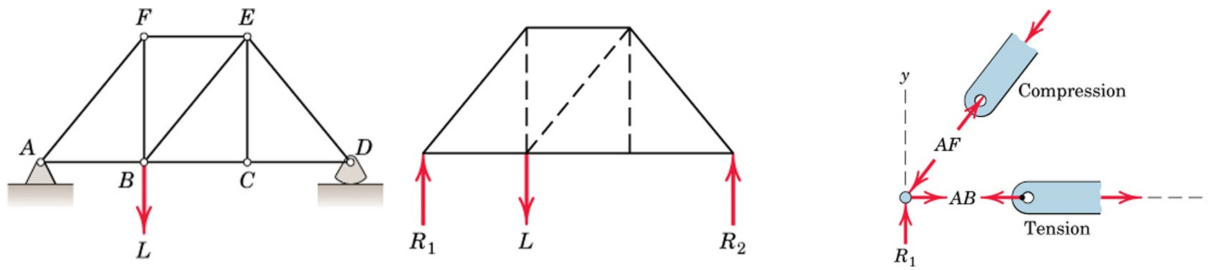
1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints



Structural Analysis: Plane Truss

Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.



FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

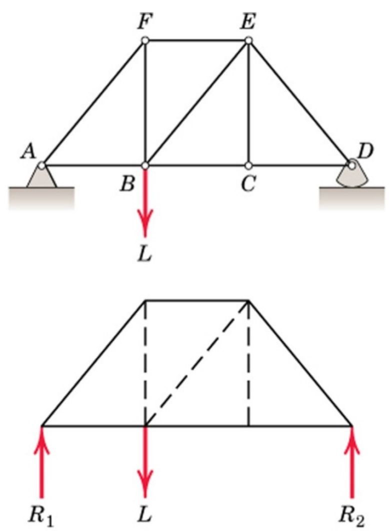
Magnitude of AF from $\sum F_y = 0$

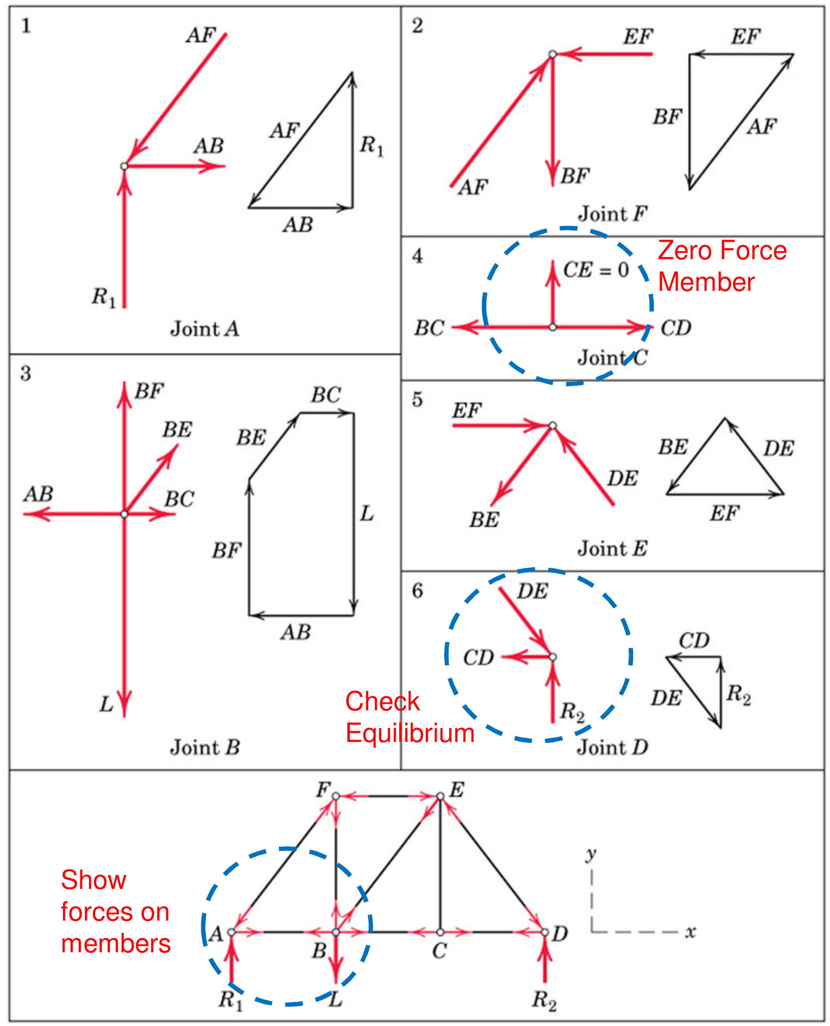
Magnitude of AB from $\sum F_x = 0$

Analyze joints F, B, C, E, & D in that order to complete the analysis

Structural Analysis: Plane Truss

Method of Joints

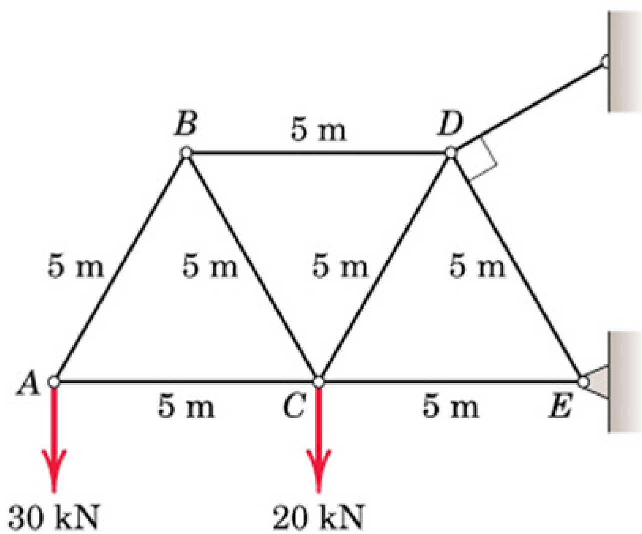




- Negative force if assumed sense is incorrect

Method of Joints: Example

Determine the force in each member of the loaded truss by Method of Joints



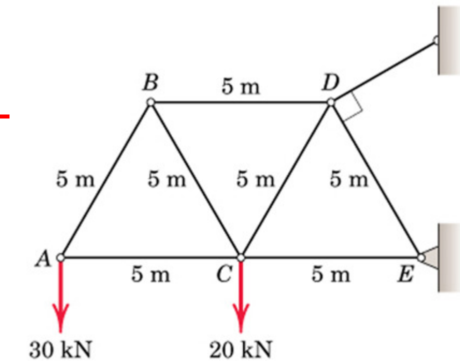
Method of Joints: Example

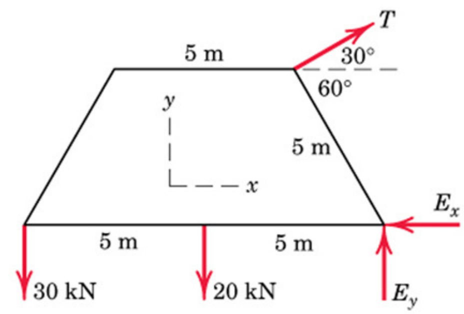
Solution

$[\Sigma M_E = 0] \qquad 5T - 20(5) - 30(10) = 0 \qquad T = 80\ \text{kN}$

$[\Sigma F_x = 0] \qquad 80 \cos 30° - E_x = 0 \qquad E_x = 69.3\ \text{kN}$

$[\Sigma F_y = 0] \qquad 80 \sin 30° + E_y - 20 - 30 = 0 \qquad E_y = 10\ \text{kN}$



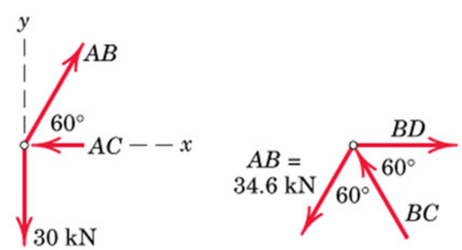


$[\Sigma F_y = 0] \qquad 0.866AB - 30 = 0 \qquad AB = 34.6\ \text{kN}\ T$

$[\Sigma F_x = 0] \qquad AC - 0.5(34.6) = 0 \qquad AC = 17.32\ \text{kN}\ C$

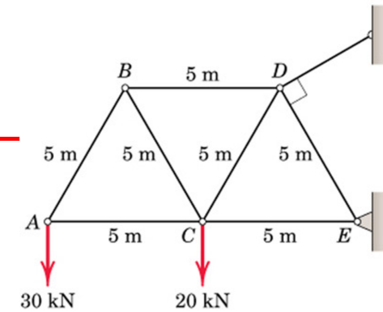
$[\Sigma F_y = 0] \qquad 0.866BC - 0.866(34.6) = 0 \qquad BC = 34.6\ \text{kN}\ C$

$[\Sigma F_x = 0] \qquad BD - 2(0.5)(34.6) = 0 \qquad BD = 34.6\ \text{kN}\ T$

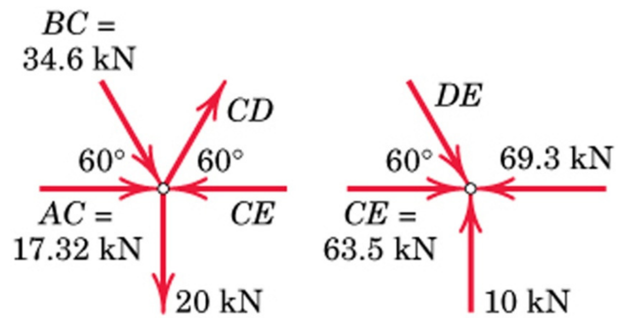


Joint *A* Joint *B*

Method of Joints: Example



Solution



Joint C   Joint E

$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \quad DE = 11.55 \text{ kN } C$

and the equation $\Sigma F_x = 0$ checks.

Structural Analysis: Plane Truss

When more number of members/supports are present than are needed to prevent collapse/stability

→ Statically Indeterminate Truss

- cannot be analyzed using equations of equilibrium alone!
- additional members or supports which are not necessary for maintaining the equilibrium configuration → Redundant

Internal and External Redundancy

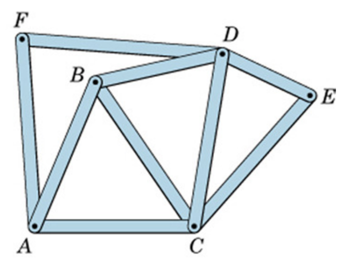
Extra Supports than required → External Redundancy

– Degree of indeterminacy from available equilibrium equations

Extra Members than required → Internal Redundancy

(truss must be removed from the supports to calculate internal redundancy)

– Is this truss statically determinate internally?



Truss is statically determinate internally if $m + 3 = 2j$

$m = 2j - 3$ m is number of members, and j is number of joints in truss

Structural Analysis: Plane Truss

Internal Redundancy or
Degree of Internal Static Indeterminacy

Extra Members than required → Internal Redundancy

Equilibrium of each joint can be specified by two scalar force equations → 2j equations for a truss with "j" number of joints

→ Known Quantities

For a truss with "m" number of two force members, and maximum 3 unknown support reactions → Total Unknowns = m + 3

("m" member forces and 3 reactions for externally determinate truss)

Therefore:

$m + 3 = 2j$ → Statically Determinate Internally

$m + 3 > 2j$ → Statically Indeterminate Internally

$m + 3 < 2j$ → Unstable Truss

A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss

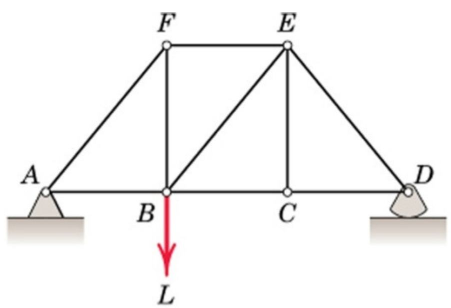
Structural Analysis: Plane Truss

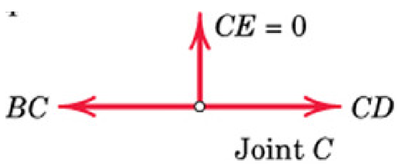
Why to Provide Redundant Members?

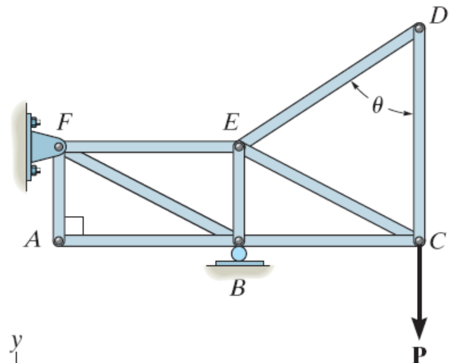
- To maintain alignment of two members during construction
- To increase stability during construction
- To maintain stability during loading (Ex: to prevent buckling of compression members)
- To provide support if the applied loading is changed
- To act as backup members in case some members fail or require strengthening
- Analysis is difficult but possible

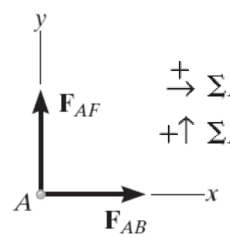
Structural Analysis: Plane Truss

Zero Force Members



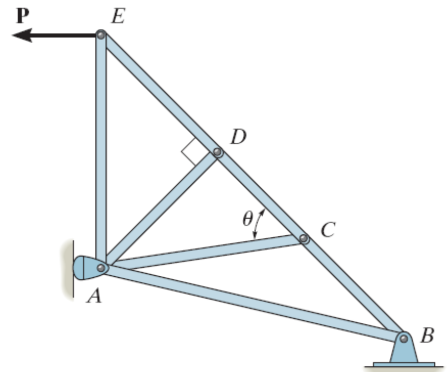


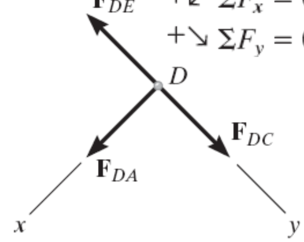




$\xrightarrow{+} \Sigma F_x = 0;\ \boxed{F_{AB} = 0}$

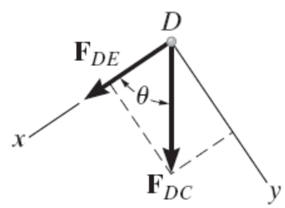
$+\uparrow \Sigma F_y = 0;\ \boxed{F_{AF} = 0}$





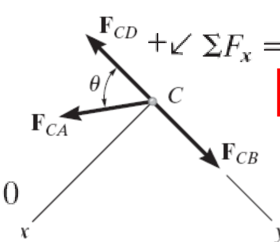
$+\swarrow \Sigma F_x = 0;\ \boxed{F_{DA} = 0}$

$+\searrow \Sigma F_y = 0;\ F_{DC} = F_{DE}$



$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ \boxed{F_{DC} = 0}$ since $\sin\theta \neq 0$

$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ \boxed{F_{DE} = 0}$



$+\swarrow \Sigma F_x = 0;\ F_{CA} \sin\theta = 0$

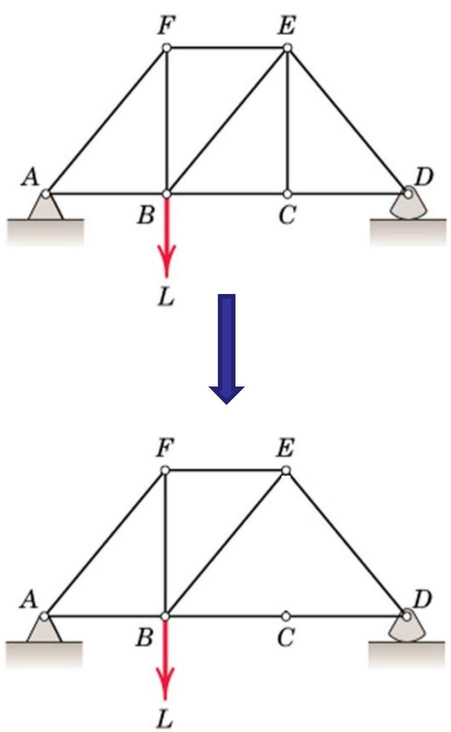
$\boxed{F_{CA} = 0}$

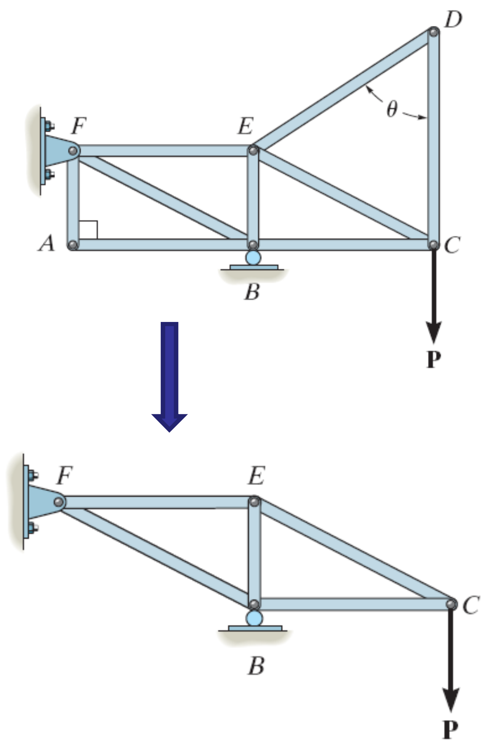
since $\sin\theta \neq 0$

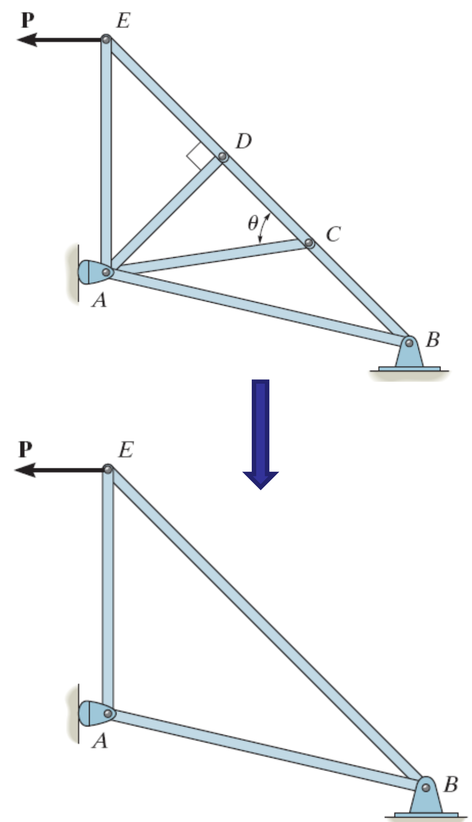
$+\searrow \Sigma F_y = 0;$

$F_{CB} = F_{CD}$

Structural Analysis: Plane Truss





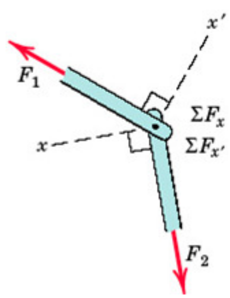


Structural Analysis: Plane Truss

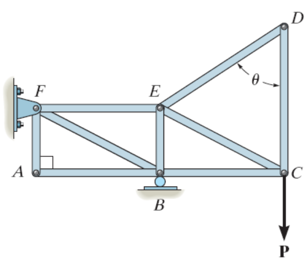
Zero Force Members: Conditions

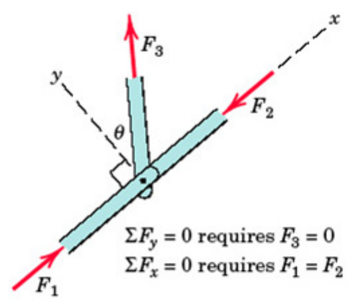
If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members

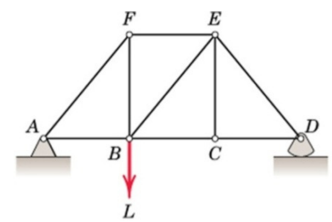
If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint

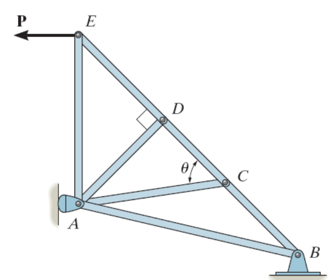


$\Sigma F_x = 0$ requires $F_1 = 0$
$\Sigma F_{x'} = 0$ requires $F_2 = 0$





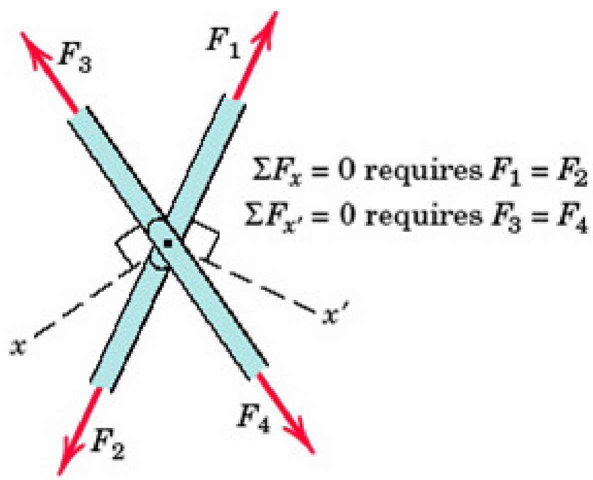




Structural Analysis: Plane Truss

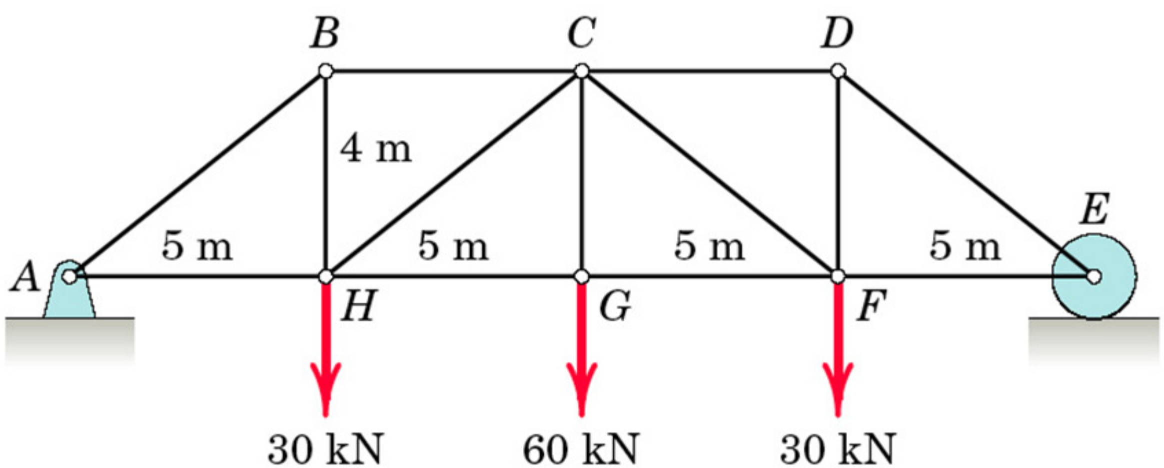
Special Condition

When two pairs of collinear members are joined as shown in figure, the forces in each pair must be equal and opposite.



Method of Joints: Example

Determine the force in each member of the loaded truss by Method of Joints.



Is the truss statically determinant externally?

Is the truss statically determinant internally?

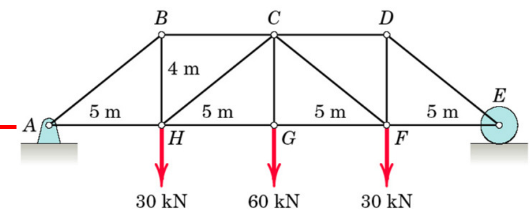
Are there any Zero Force Members in the truss?

Yes

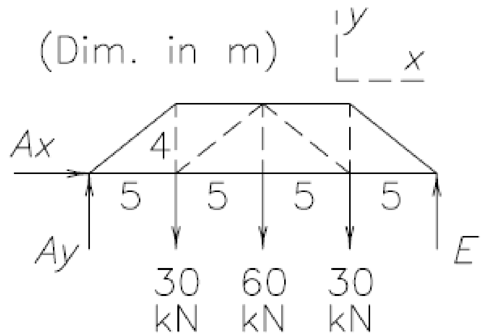
Yes

No

Method of Joints: Example



Solution

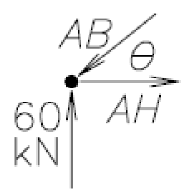


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60\text{ kN}$ by

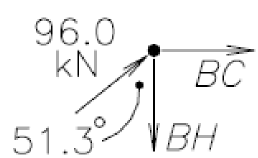
$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$



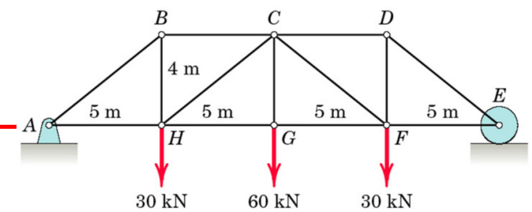
$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0,\ \underline{AB = 96.0\text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta,\ \underline{AH = 75\text{ kN } T} \end{cases}$$

Joint B:

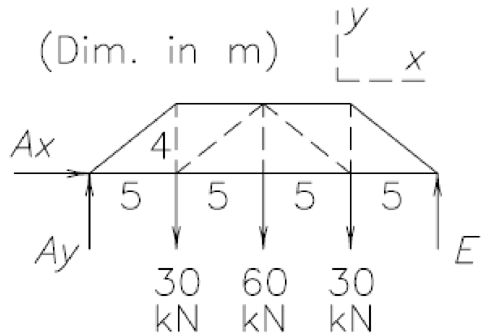


$$\begin{cases} \Sigma F_x = 0: BC + 96.0\sin 51.3° = 0,\ \underline{BC = -75\text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0,\ \underline{BH = 60\text{ kN } T} \end{cases}$$

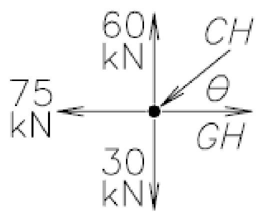
Method of Joints: Example



Solution

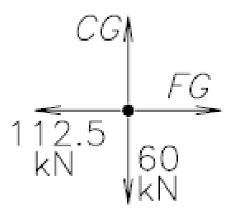


Joint H:



$$\begin{cases} \Sigma F_y = 0: -CH\sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0: 48.0\cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint G:



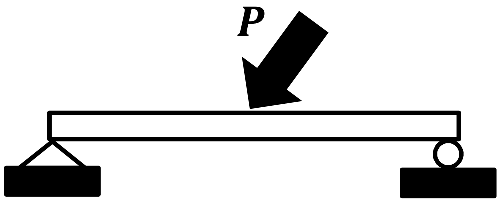
$$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$$

By symmetry:

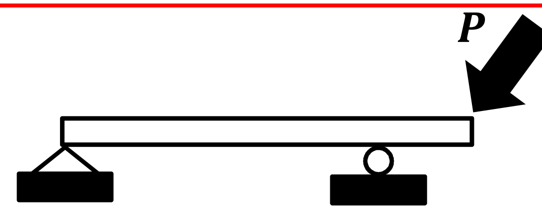
$FG = 112.5$ kN T, $CF = 48.0$ kN C

$CD = 75$ kN C, $DF = 60$ kN T

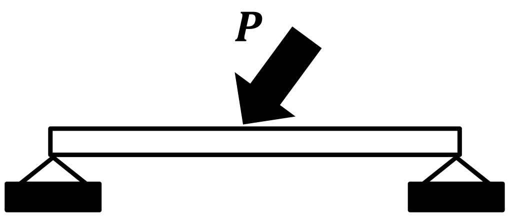
$\underline{EF = 75 \text{ kN } T, \ DE = 96.0 \text{ kN } C}$



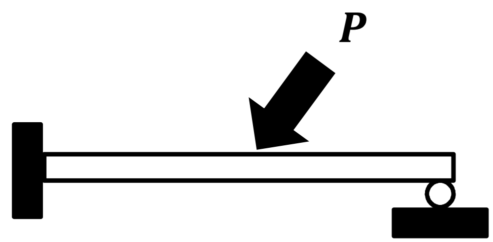
DETERMINATE



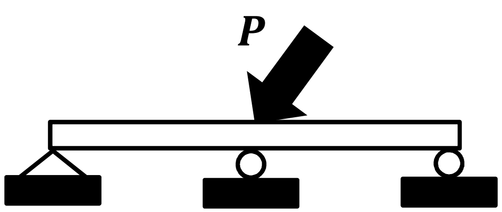
DETERMINATE



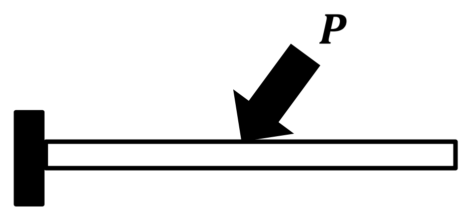
INDETERMINATE



INDETERMINATE

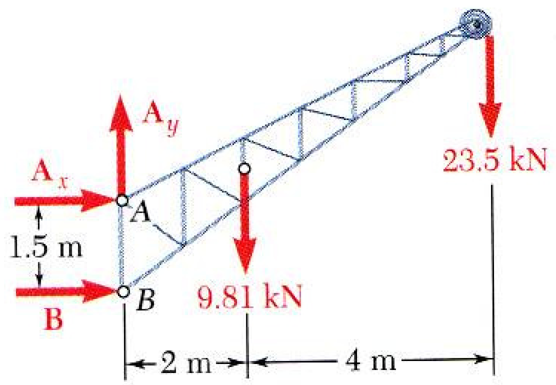


INDETERMINATE



DETERMINATE

Example problem 1



- Create the free-body diagram.

Determine B by solving the equation for the sum of the moments of all forces about A.

$$\sum M_A = 0: \quad +B(1.5\text{m}) - 9.81\,\text{kN}(2\text{m}) - 23.5\,\text{kN}(6\text{m}) = 0$$

$$\boxed{B = +107.1\,\text{kN}}$$

Determine the reactions at A by solving the equations for the sum of all horizontal forces and all vertical forces.

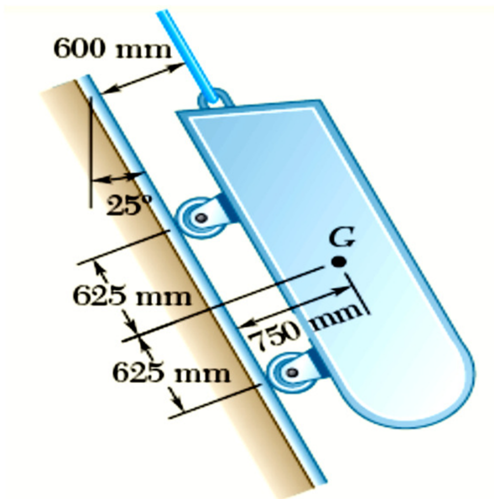
$$\sum F_x = 0: \quad A_x + B = 0$$

$$\boxed{A_x = -107.1\,\text{kN}}$$

$$\sum F_y = 0: \quad A_y - 9.81\,\text{kN} - 23.5\,\text{kN} = 0$$

$$\boxed{A_y = +33.3\,\text{kN}}$$

Example problem 2



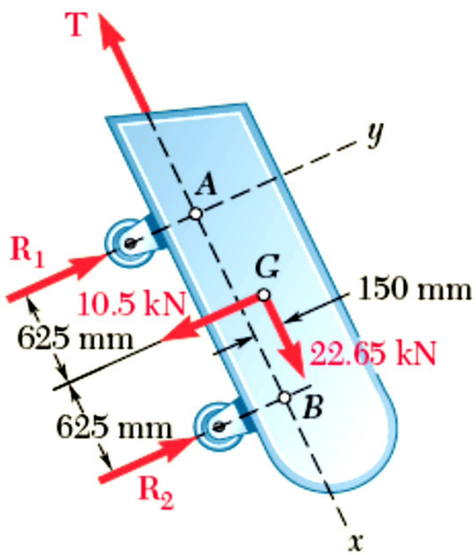
A loading car is at rest on an inclined track. The gross weight of the car and its load is 25 kN, and it is applied at G. The cart is held in position by the cable.

Determine the tension in the cable and the reaction at each pair of wheels.

SOLUTION:

- Create a free-body diagram for the car with the coordinate system aligned with the track.
- Determine the reactions at the wheels by solving equations for the sum of moments about points above each axle.
- Determine the cable tension by solving the equation for the sum of force components parallel to the track.
- Check the values obtained by verifying that the sum of force components perpendicular to the track are zero.

Example problem 2



Create a free-body diagram

$$W_x = +(25\text{ kN})\cos 25^\circ$$
$$= +22.65\text{ kN}$$
$$W_y = -(25\text{ kN})\sin 25^\circ$$
$$= -10.5\text{ kN}$$

Determine the reactions at the wheels.

$$\sum M_A = 0: \quad -(10.5\text{ kN})625\text{ mm} - (22.65\text{ kN})150\text{ mm} + R_2(1250\text{ mm}) = 0$$

$$\boxed{R_2 = +8\text{ kN}}$$

$$\sum M_B = 0: \quad +(10.5\text{ kN})625\text{ mm} - (22.65\text{ kN})150\text{ mm} - R_1(1250\text{ mm}) = 0$$

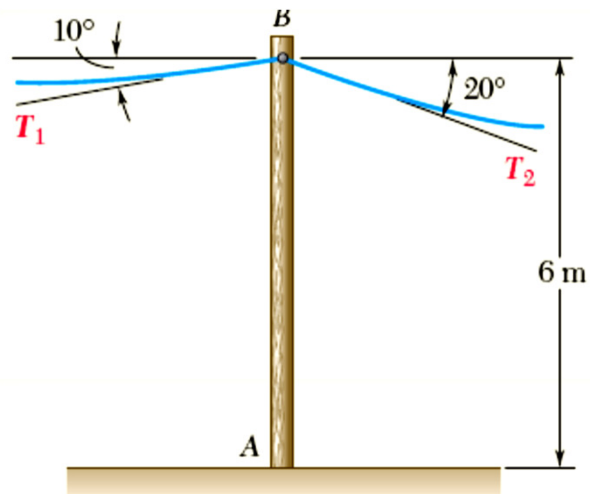
$$\boxed{R_1 = 2.5\text{ kN}}$$

Determine the cable tension.

$$\sum F_x = 0: \quad +22.65\text{ kN} - \text{T} = 0$$

$$\boxed{T = +22.7\text{ kN}}$$

Example problem 3



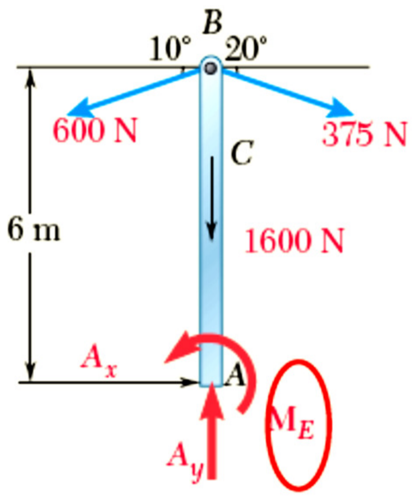
A 6-m telephone pole of 1600-N used to support the wires. Wires $T_1 = 600$ N and $T_2 = 375$ N.

Determine the reaction at the fixed end A.

SOLUTION:

- Create a free-body diagram for the telephone cable.
- Solve 3 equilibrium equations for the reaction force components and couple at A.

Example problem 3



- Create a free-body diagram for the frame and cable.

- Solve 3 equilibrium equations for the reaction force components and couple.

$$\sum F_x = 0: \quad A_x + (375\text{N})\cos 20° - (600\text{N})\cos 10° = 0$$

$$\boxed{A_x = +238.50\text{ N}}$$

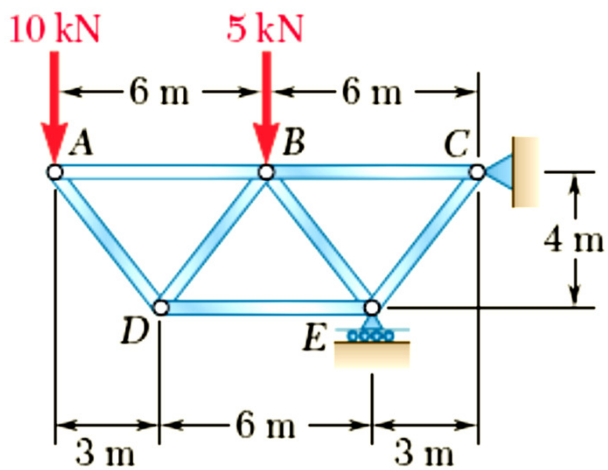
$$\sum F_y = 0: \quad A_y - 1600\text{N} - (600\text{N})\sin 10° - (375\text{ N})\sin 20° = 0$$

$$\boxed{A_y = +1832.45\text{ N}}$$

$$\sum M_A = 0: \quad M_A + (600\text{N})\cos 10°(6m) - (375\text{N})\cos 20° (6\text{m}) = 0$$

$$\boxed{M_A = +1431.00\text{N.m}}$$

Example

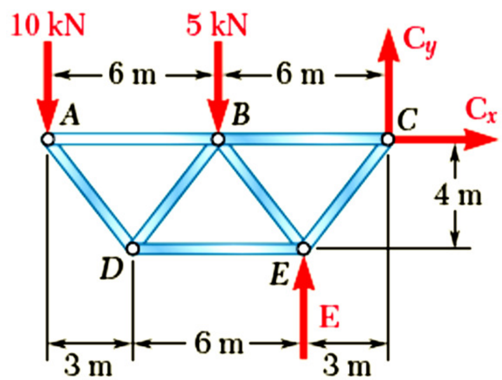


Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.

Example



SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C.

$$\sum M_C = 0$$
$$= (10\text{ kN})(12\text{ m}) + (5\text{ kN})(6\text{ m}) - E(3\text{ m})$$

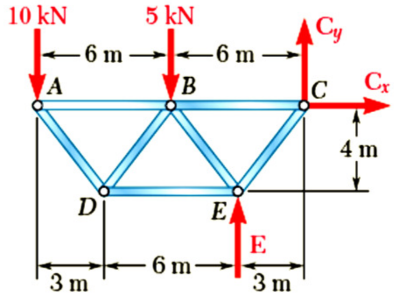
$$\boxed{E = 50\text{kN} \uparrow}$$

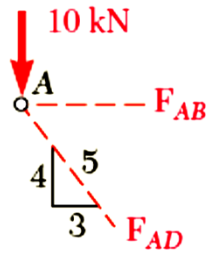
$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

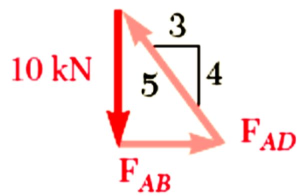
$$\sum F_y = 0 = -10\text{kN} - 5\text{ kN} + 50\text{ kN} + C_y$$

$$\boxed{C_y = 35\text{ kN} \downarrow}$$

Example



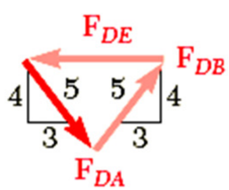




- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{10\text{ kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\boxed{\begin{aligned} F_{AB} &= 7.5\text{ kN } T \\ F_{AD} &= 12.5\text{ kN } C \end{aligned}}$$



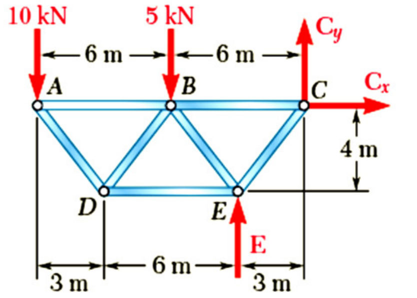
- There are now only two unknown member forces at joint D.

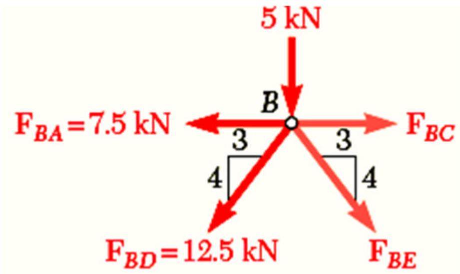
$$F_{DB} = F_{DA}$$

$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

$$\boxed{\begin{aligned} F_{DB} &= 12.5\text{ kN } T \\ F_{DE} &= 15\text{ kN } C \end{aligned}}$$

Example





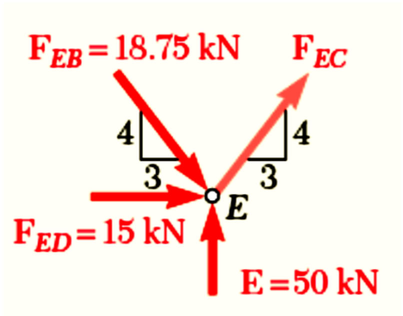
- There are now only two unknown member forces at joint B. Assume both are in tension.

$$\sum F_y = 0 = -5\text{kN} - \tfrac{4}{5}(12\text{kN}) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -18.75\text{ kN} \qquad \boxed{F_{BE} = 18.75\text{ kN } C}$$

$$\sum F_x = 0 = F_{BC} - 7.5\text{kN} - \tfrac{3}{5}(12.5\text{kN}) - \tfrac{3}{5}(18.75)$$

$$F_{BC} = +26.25\text{ kN} \qquad \boxed{F_{BC} = 26.25\text{ kN } T}$$

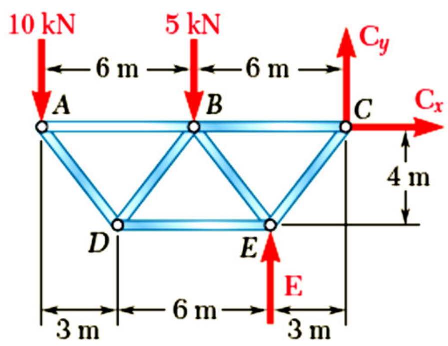


- There is one unknown member force at joint E. Assume the member is in tension.

$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 15\text{kN} + \tfrac{3}{5}(18.75\text{kN})$$

$$F_{EC} = -43.75\text{ kN} \qquad \boxed{F_{EC} = 43.75\text{ kN } C}$$

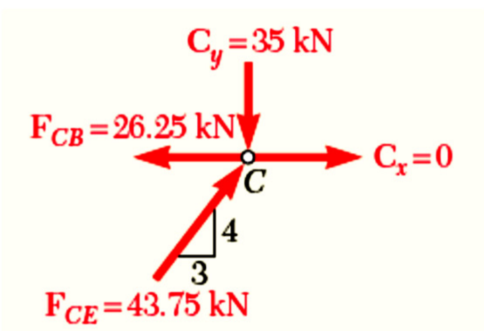
Example



- All member forces and support reactions are known at joint C. However, the joint equilibrium requirements may be applied to check the results.

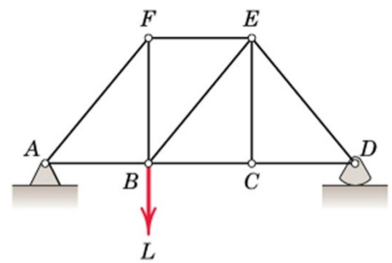
$$\sum F_x = -26.25 + \tfrac{3}{5}(43.75) = 0 \quad \text{(checks)}$$

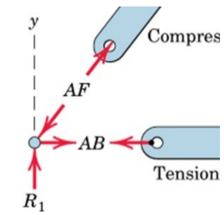
$$\sum F_y = -35 + \tfrac{4}{5}(43.75) = 0 \quad \text{(checks)}$$



L 5

Structural Analysis: Plane Truss

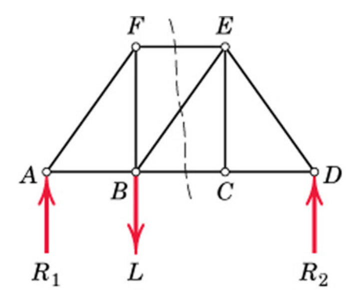




Method of Joints: only two of three equilibrium equations were applied at each joint because the procedures involve concurrent forces at each joint

→Calculations from joint to joint

→More time and effort required

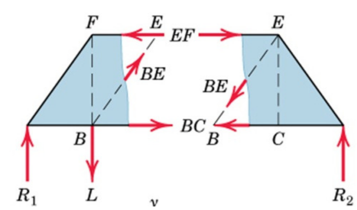


Method of Sections

Take advantage of the 3rd or moment equation of equilibrium by selecting an entire section of truss

→Equilibrium under non-concurrent force system

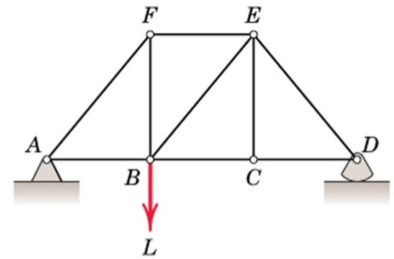
→Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations

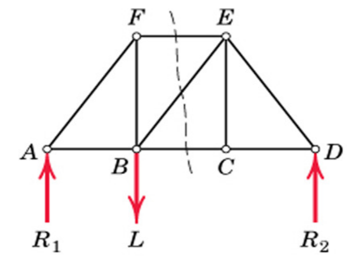


Structural Analysis: Plane Truss

Method of Sections

Find out the reactions from equilibrium of whole truss



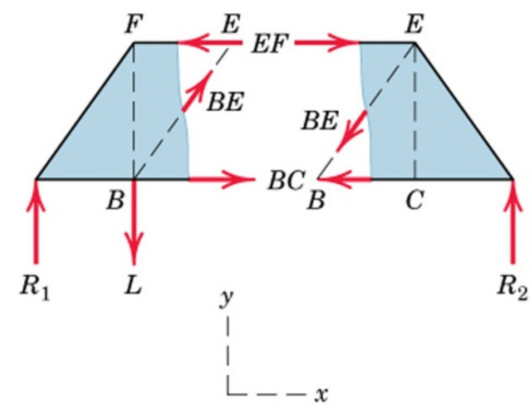


To find force in member BE
Cut an imaginary section (dotted line)
Each side of the truss section should remain in equilibrium

For calculating force on member EF, take moment about B

Take moment about E, for calculating force BC

Now apply $\sum F_y = 0$ to obtain forces on the members BE



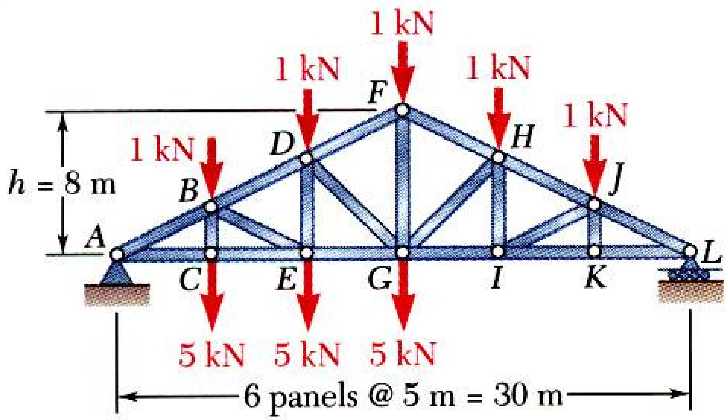
Structural Analysis: Plane Truss

Method of Sections

- Principle: If a body is in equilibrium, then any part of the body is also in equilibrium.
- Forces in few particular member can be directly found out quickly without solving each joint of the truss sequentially
- Method of Sections and Method of Joints can be conveniently combined
- A section need not be straight.
- More than one section can be used to solve a given problem

Structural Analysis: Plane Truss

Method of Sections: Example



Find out the internal forces in members FH, GH, and GI

Find out the reactions

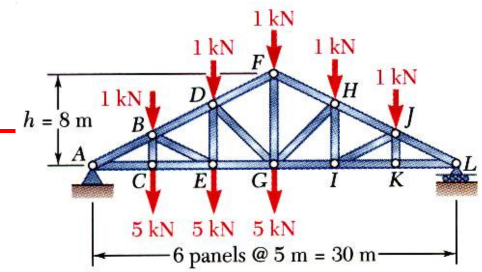
$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$
$$- (20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (30\text{ m})L$$

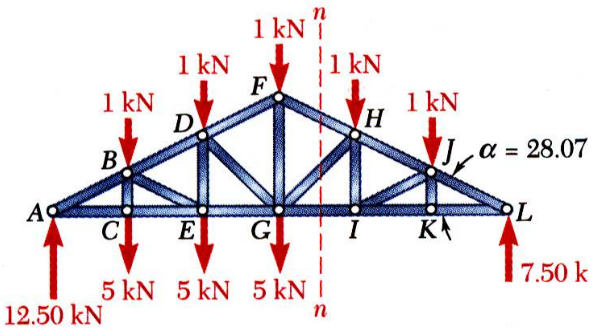
$$L = 7.5\text{ kN} \uparrow$$

$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

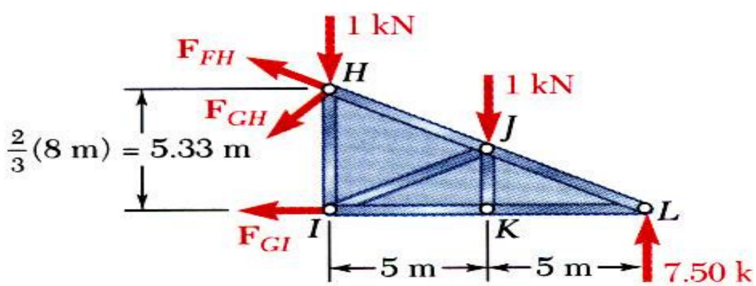
Method of Sections: Example Solution





- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.

$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$



- Apply the conditions for static equilibrium to determine the desired member forces.

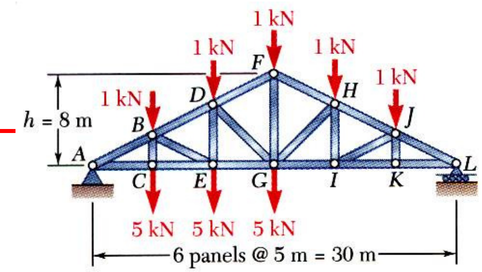
$$\sum M_H = 0$$

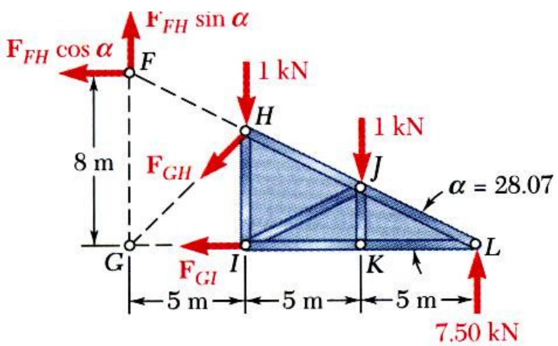
$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

$$F_{GI} = +13.13\text{ kN}$$

$$\boxed{F_{GI} = 13.13\text{ kN} \quad T}$$

Method of Sections: Example Solution





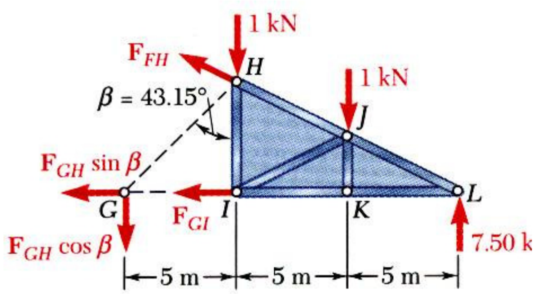
$$\sum M_G = 0$$

$$(7.5\text{ kN})(15\text{ m}) - (1\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m})$$

$$+ (F_{FH}\cos\alpha)(8\text{ m}) = 0$$

$$F_{FH} = -13.82\text{ kN}$$

$$\boxed{F_{FH} = 13.82\text{ kN}\ \ C}$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\text{ m}}{\frac{2}{3}(8\text{ m})} = 0.9375 \qquad \beta = 43.15°$$

$$\sum M_L = 0$$

$$(1\text{ kN})(10\text{ m}) + (1\text{ kN})(5\text{ m}) + (F_{GH}\cos\beta)(15\text{ m}) = 0$$

$$F_{GH} = -1.371\text{ kN}$$

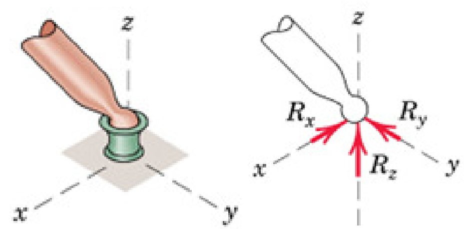
$$\boxed{F_{GH} = 1.371\text{ kN}\ \ C}$$

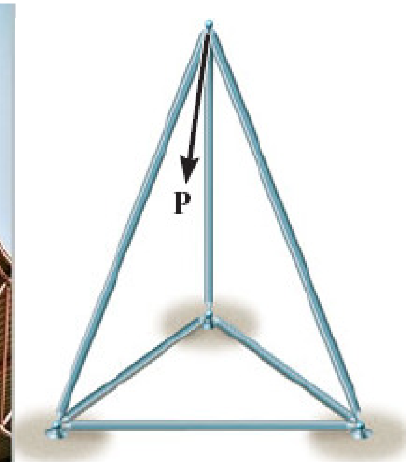
Structural Analysis: Space Truss

Space Truss

3-D counterpart of the Plane Truss

Idealized Space Truss → Rigid links connected at their ends by ball and socket joints

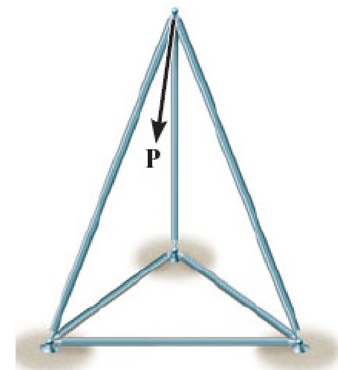


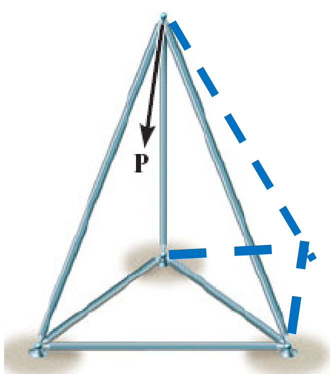


Structural Analysis: Space Truss

Space Truss

- 6 bars joined at their ends to form the edges of a tetrahedron as the basic non-collapsible unit.
- 3 additional concurrent bars whose ends are attached to three joints on the existing structure are required to add a new rigid unit to extend the structure.





A space truss formed in this way is called a Simple Space Truss

If center lines of joined members intersect at a point

- Two force members assumption is justified
- Each member under Compression or Tension

Structural Analysis: Space Truss

Static Determinacy of Space Truss

$$\Sigma F_x = 0 \quad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \quad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \quad \Sigma M_z = 0$$

Six equilibrium equations available to find out support reactions

- If these are sufficient to determine all support reactions
- The space truss is Statically Determinate Externally

Equilibrium of each joint can be specified by three scalar force equations

- 3j equations for a truss with "j" number of joints
- Known Quantities

For a truss with "m" number of two force members, and maximum 6 unknown support reactions → Total Unknowns = m + 6

("m" member forces and 6 reactions for externally determinate truss)

Therefore:

m + 6 = 3j→Statically Determinate Internally

m + 6 > 3j→Statically Indeterminate Internally

m + 6 < 3j→Unstable Truss

A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss

Structural Analysis: Space Truss

Method of Joints

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma F_z = 0$$

- If forces in all members are required
- Solve the 3 scalar equilibrium equations at each joint
- Solution of simultaneous equations may be avoided if a joint having at least one known force & at most three unknown forces is analysed first
- Use Cartesian vector analysis if the 3-D geometry is complex ($\sum \mathbf{F}=0$)

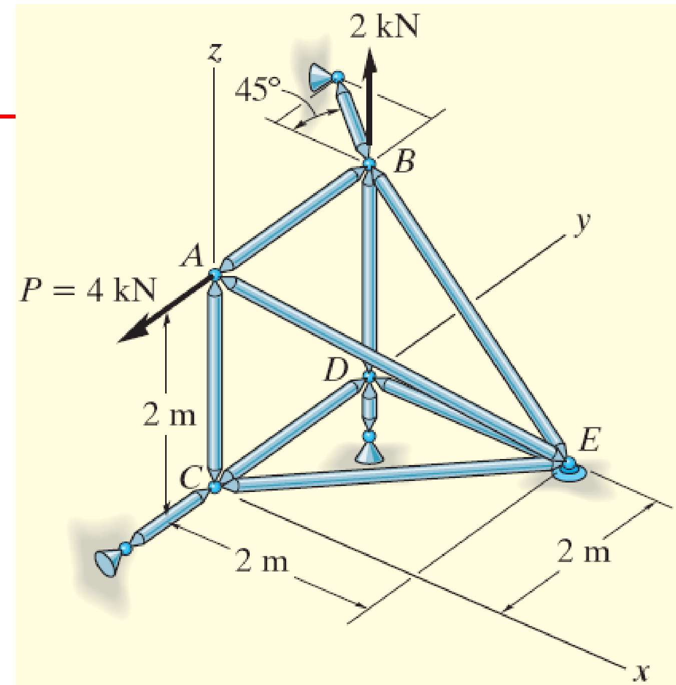
Method of Sections

$$\Sigma F_x = 0 \quad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \quad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \quad \Sigma M_z = 0$$

- If forces in only few members are required
- Solve the 6 scalar equilibrium equations in each cut part of truss
- Section should not pass through more than 6 members whose forces are unknown
- Cartesian vector analysis ($\sum \mathbf{F}=0$, $\sum \mathbf{M}=0$)

Space Truss: Example

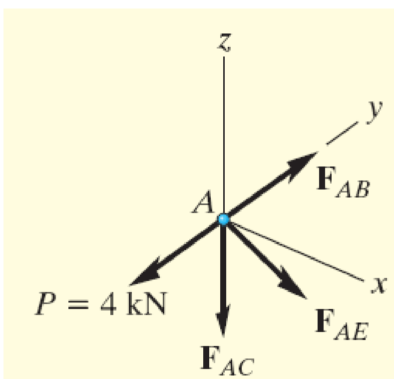
Determine the forces acting in members of the space truss.



Solution:

Start at joint A: Draw free body diagram

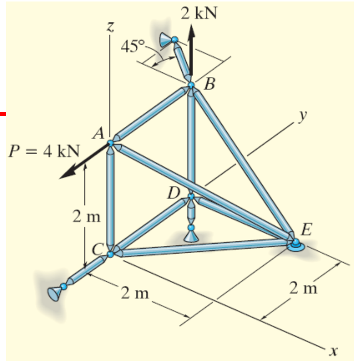
Express each force in vector notation



$$\mathbf{P} = \{-4\mathbf{j}\}\ \text{kN}, \qquad \mathbf{F}_{AB} = F_{AB}\mathbf{j}, \quad \mathbf{F}_{AC} = -F_{AC}\mathbf{k},$$

$$\mathbf{F}_{AE} = F_{AE}\left(\frac{\mathbf{r}_{AE}}{r_{AE}}\right) = F_{AE}(0.577\mathbf{i} + 0.577\mathbf{j} - 0.577\mathbf{k})$$

Space Truss: Example



For equilibrium,

$$\Sigma \mathbf{F} = \mathbf{0}; \qquad \mathbf{P} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AE} = \mathbf{0}$$

$$-4\mathbf{j} + F_{AB}\mathbf{j} - F_{AC}\mathbf{k} + 0.577F_{AE}\mathbf{i} + 0.577F_{AE}\mathbf{j} - 0.577F_{AE}\mathbf{k} = \mathbf{0}$$

Rearranging the terms and equating the coefficients of **i**, **j**, and **k** unit vector to zero will give:

$$\Sigma F_x = 0; \qquad 0.577F_{AE} = 0$$

$$\Sigma F_y = 0; \qquad -4 + F_{AB} + 0.577F_{AE} = 0$$

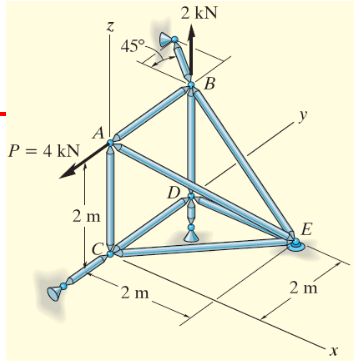
$$\Sigma F_z = 0; \qquad -F_{AC} - 0.577F_{AE} = 0$$

$$F_{AC} = F_{AE} = 0$$

$$F_{AB} = 4 \text{ kN}$$

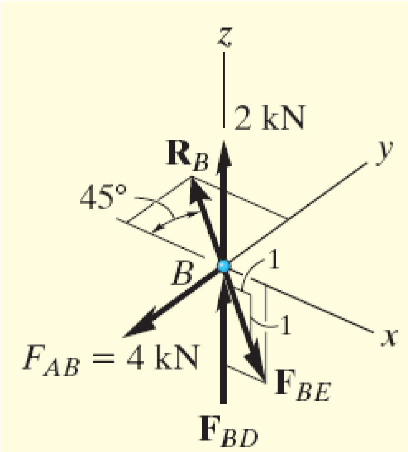
Next Joint B may be analysed.

Space Truss: Example



Joint B: Draw the Free Body Diagram

Scalar equations of equilibrium may be used at joint B



$$\Sigma F_x = 0; \qquad -R_B \cos 45^\circ + 0.707 F_{BE} = 0$$

$$\Sigma F_y = 0; \qquad -4 + R_B \sin 45^\circ = 0$$

$$\Sigma F_z = 0; \qquad 2 + F_{BD} - 0.707 F_{BE} = 0$$

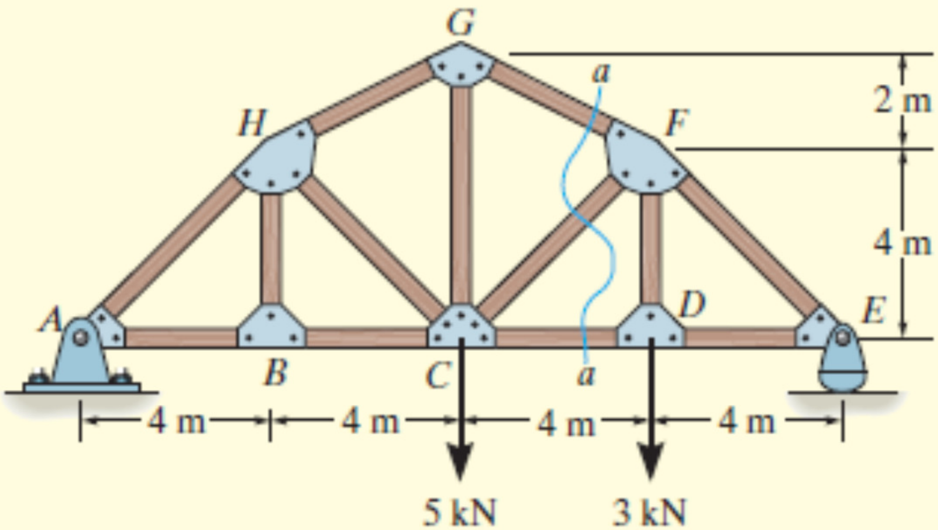
$$R_B = F_{BE} = 5.66 \text{ kN} \quad (\text{T}), \qquad F_{BD} = 2 \text{ kN}$$

Using Scalar equations of equilibrium at joints D and C will give:

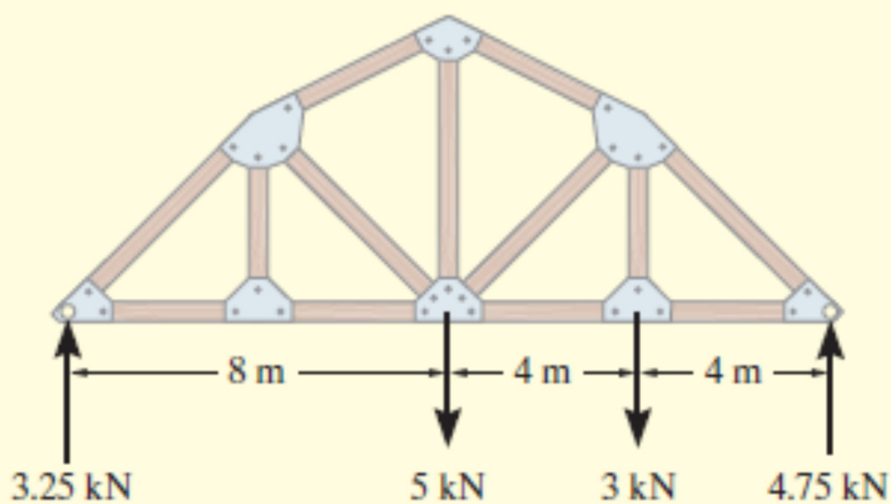
$$F_{DE} = F_{DC} = F_{CE} = 0$$

Example problem

Determine the force in member *CF* of the truss shown in Fig. 6–17*a*. Indicate whether the member is in tension or compression. Assume each member is pin connected.

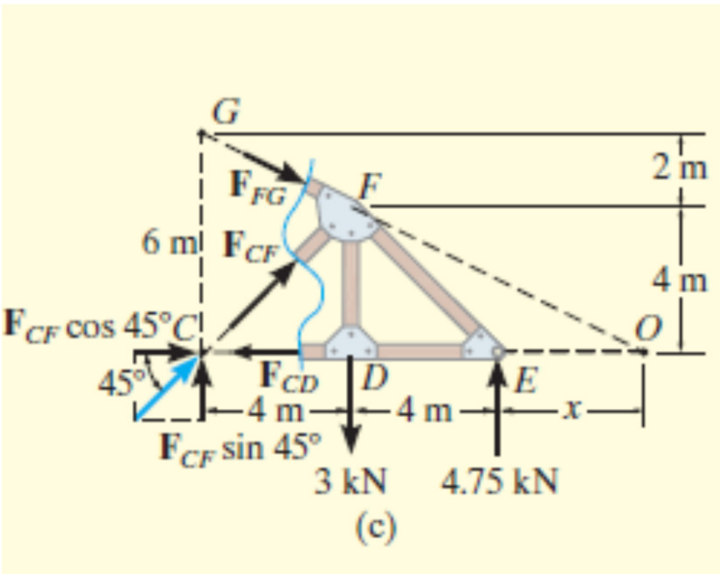


(a)



(b)

Example problem



From similar triangle OFD and OGC, We have

$$\frac{4}{4+x} = \frac{6}{8+x} \qquad x = 4m$$

$$\circlearrowleft + \Sigma M_O = 0;$$

$$-F_{CF}\sin 45°(12\text{ m}) + (3\text{ kN})(8\text{ m}) - (4.75\text{ kN})(4\text{ m}) = 0$$

$$F_{CF} = 0.589\text{ kN} \quad (\text{C}) \qquad \textit{Ans.}$$