Introduction to Differential Evolution

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Differential Evolution

It is a stochastic, population-based optimization algorithm for solving nonlinear optimization problem

The algorithm was introduced by Storn and Price in 1996

Consider an optimization problem

Minimize $f(X)$

Where $X = [x_1, x_2, x_3, ..., x_D]$, $D$ is the number of variables
Evolutionary algorithms

This is a population based algorithm

Consider a population size of $N$

The population matrix can be shown as

$$x_{n,i}^g = [x_{n,1}^g, x_{n,2}^g, x_{n,3}^g, \ldots, x_{n,D}^g]$$

Where, $g$ is the Generation and $n = 1,2,3, \ldots N$
Initial population

Initial population is generated randomly between upper lower and upper bound

\[ x_{n,i} = x_{n,i}^L + \text{rand}(\text{)} \ast (x_{n,i}^U - x_{n,i}^L) \quad i = 1,2,3, \ldots D \text{ and } n = 1,2,3, \ldots N \]

Where \( x_{i}^L \) is the lower bound of the variable \( x_i \)

Where \( x_{i}^U \) is the upper bound of the variable \( x_i \)
Mutation

From each parameter vector, select three other vectors $x_{r1n}^g$, $x_{r2n}^g$ and $x_{r3n}^g$ randomly.

Add the weighted difference of two of the vectors to the third

$$v_{n}^{g+1} = x_{r1n}^g + F(x_{r2n}^g - x_{r3n}^g) \quad n = 1,2,3,\ldots,N$$

$v_{n}^{g+1}$ is called donor vector

$F$ is generally taken between 0 and 1
Recombination

A trial vector \( u_{n,i}^{g+1} \) is developed from the target vector, \( x_{n,i}^{g} \), and the donor vector, \( v_{n,i}^{g+1} \)

\[
\begin{align*}
  u_{n,i}^{g+1} &= \begin{cases} 
    v_{n,i}^{g+1} & \text{if } \text{rand}() \leq C_p \text{ or } i = I_{\text{rand}} \\
    x_{n,i}^{g} & \text{if } \text{rand}() > C_p \text{ and } i \neq I_{\text{rand}}
  \end{cases} \\
  i &= 1,2,3,...D \quad \text{and} \quad n = 1,2,3,...N
\end{align*}
\]

\( I_{\text{rand}} \) is a integer random number between \([1,D]\)

\( C_p \) is the recombination probability
Selection

The target vector $x_{n,i}^g$ is compared with the trial vector $u_{n,i}^{g+1}$ and the one with the lowest function value is selected for the next generation

$$
x_{n}^{g+1} = \begin{cases} 
  u_{n,i}^{g+1} & \text{if } f(u_{n,i}^{g+1}) < f(x_{n}^{g}) \\
  x_{n}^{g} & \text{Otherwise}
\end{cases}
$$

$$n = 1,2,3, \ldots N$$
THANKS