Multi-objective optimization

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Multi-objective optimization
Two objectives are

• Minimize weight
• Minimize deflection
Multi-objective optimization

- More than one objectives
- Objectives are conflicting in nature
- Dealing with two search space
  - Decision variable space
  - Objective space
- Unique mapping between the objectives and often the mapping is non-linear
- Properties of the two search space are not similar
- Proximity of two solutions in one search space does not mean a proximity in other search space
Multi-objective optimization
Vector Evaluated Genetic Algorithm (VEGA)

Propose by Schaffer (1984)
Give more emphasize on the non-dominated solutions of the population

This can be implemented by subtracting $\epsilon$ from the dominated solution fitness value

Suppose $N'$ is the number of sub-population and $n'$ is the non-dominated solutions. Then total reduction is $(N' - n') \epsilon$.

The total reduction is then redistributed among the non-dominated solution by adding an amount $(N' - n') \epsilon / n'$

This method has two main implications

Non-dominated solutions are given more importance

Additional equal emphasis has been given to all the non-dominated solution
Weighted based genetic algorithm (WBGA)

- The fitness is calculated

\[ F = \sum_{j=1}^{M} w_j \frac{f_i - f_j \min}{f_{\max} - f_{\min}} \]

- The spread is maintained using the sharing function approach

Sharing function

\[ Sh(d_{ij}) = \begin{cases} 1 - (d_{ij}/\sigma), & \text{if } d_{ij} < \sigma; \\ 0, & \text{otherwise.} \end{cases} \]

Niche count

\[ nc_i = \sum_{j=1}^{N} Sh(d_{ij}) \]

Modified fitness

\[ F' = \frac{F}{nc} \]

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Multiple objective genetic algorithm (MOGA)

Maximize $f_1 = 1.1 - x_1$
Maximize $f_2 = 60 - \frac{1 + x_1}{x_2}$
Subject to $0.1 \leq x_1 \leq 1$
$0 \leq x_2 \leq 5$

Solution space

Objective space
Multiple objective genetic algorithm (MOGA)

Maximize $f_1$

Maximize $f_2$

First goal: To push all the solutions toward Pareto optimal front.

Second goal: To maintain good diversity.

Maximize $f_1$

The assigned fitness value based on the non-dominated ranking.

The rank is assigned as \( r_i = 1 + n_i \) where \( r_i \) is the ranking of the \( i^{th} \) solution and \( n_i \) is the number of solutions that dominate the solution.
Fonseca and Fleming (1993) maintain the diversity among the non-dominated solution using niching among the solution of same rank.

The normalize distance was calculated as,

$$d_{i,j} = \sqrt{\sum_{k=1}^{M} \left( \frac{f_k^i - f_k^j}{f_{\text{max}}^k - f_{\text{min}}^k} \right)^2}$$

The niche count was calculated as,

$$nc_i = \sum_{j=1}^{\mu(r_i)} Sh(d_{ij})$$
Srinivas and Deb (1994) proposed NSGA

The algorithm is based on the non-dominated sorting.

The spread on the Pareto optimal front is maintained using sharing function

\[ d_{i,j} = \sqrt{\sum_{k=1}^{P_1} \left( \frac{x_k^i - x_k^j}{x_{k_{max}} - x_{k_{min}}} \right)^2} \]
Non-dominated Sorting Genetic Algorithms

- NSGA II is an elitist non-dominated sorting Genetic Algorithm to solve multi-objective optimization problem developed by Prof. K. Deb and his student at IIT Kanpur.
- It has been reported that NSGA II can converge to the global Pareto-optimal front and can maintain the diversity of population on the Pareto-optimal front.
Non-dominated sorting

Objective 1 (Minimize)
Objective 2 (Minimize)

Infeasible Region
Feasible Region
Calculation crowding distance

Cd, the crowded distance is the perimeter of the rectangle constituted by the two neighboring solutions.

Cd value more means that the solution is less crowded.

Cd value less means that the solution is more crowded.
Crowded tournament operator

- A solution $i$ wins a tournament with another solution $j$, if:
  - If the solution $i$ has better rank than $j$, i.e. $r_i < r_j$
  - If they have the same rank, but $i$ has a better crowding distance than $j$, i.e. $r_i = r_j$ and $d_i > d_j$. 

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Replacement scheme of NSGA II

Non dominated sorting

Selected

Rejected

Rejected based on crowding distance

P

Q

$r_1$

$r_2$

$r_3$

$r_4$

$r_5$
Initialize population of size N

Calculate all the objective functions

Rank the population according to non-dominating criteria

Selection

Crossover

Mutation

Calculate objective function of the new population

Combine old and new population

Non-dominating ranking on the combined population

Calculate crowding distance of all the solutions

Get the N member from the combined population on the basis of rank and crowding distance

Replace parent population by the better members of the combined population

Pareto-optimal solution

Termination Criteria?

No

Yes

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THANKS