Chomsky hierarchy

Context-Sensitive Languages (CSLs)

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A CSL is generated by a *context-sensitive grammar (CSG)*: CSG is same as unrestricted grammar except that for each production $P$, the length of the right side of $P$ is at least the length of the left side of $P$.

- CSG is a.k.a. a *type-1 grammar*
- $\epsilon \not\in$ any CSL
- ex. $L' = \{a^ib^ic^i | i > 0\}$
Linear Bounded Automata

- LBA is a restricted TM in which the tape-head is not permitted to move off a constant multiple of portion of the tape containing input.

- If $L$ is a CSL then $L$ is accepted by some LBA.
  
  use a two-track LBA, wherein each track length is bounded by input length
  encode productions as states and transitions of an automata
  exploit non-determinism

- If $L$ is the language of a LBA then $L - \{\epsilon\}$ is a CSL.

- $\{< M, w > | M$ is an LBA that accepts string $w\}$ is decidable.
Closure Properties

- Closed under union, concatenation, positive closure, intersection, and substitution.

- Unknown (open problem) whether every CSL has an equivalent DLBA even though it is known that CSLs are closed under complement (latter will be proved later).
CSLs in Chomsky hierarchy

- Every CSL is recursive.
  - construct a two-tape NDTM (not necessarily a NDLBA) that halts for every input
  - terminate a derivation whenever either the string length exceeds input length or the derivation loops.

- ∃ a recursive language, say $L_{d'}$, that is not context-sensitive.

where $L_{d'}$ is not a language of any halting TM.

here, $L' = \{a^ib^ic^i|i \geq 0\}$. 

(Context-Sensitive Languages)