Coverage in WSNs
(Sweep Coverage on Graph)

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Outline

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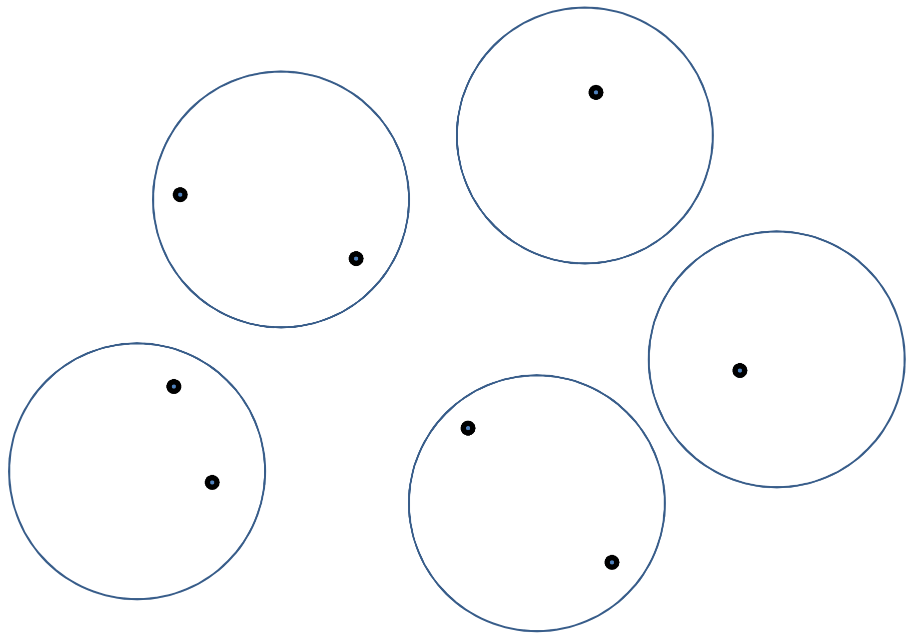
- Coverage
- Sweep coverage problems
- Previous Results and its Flaw
- Our Solution
- Sweep coverage with different sweep periods and processing time
- Sweep coverage with mobile sensors having different speeds
- Conclusion

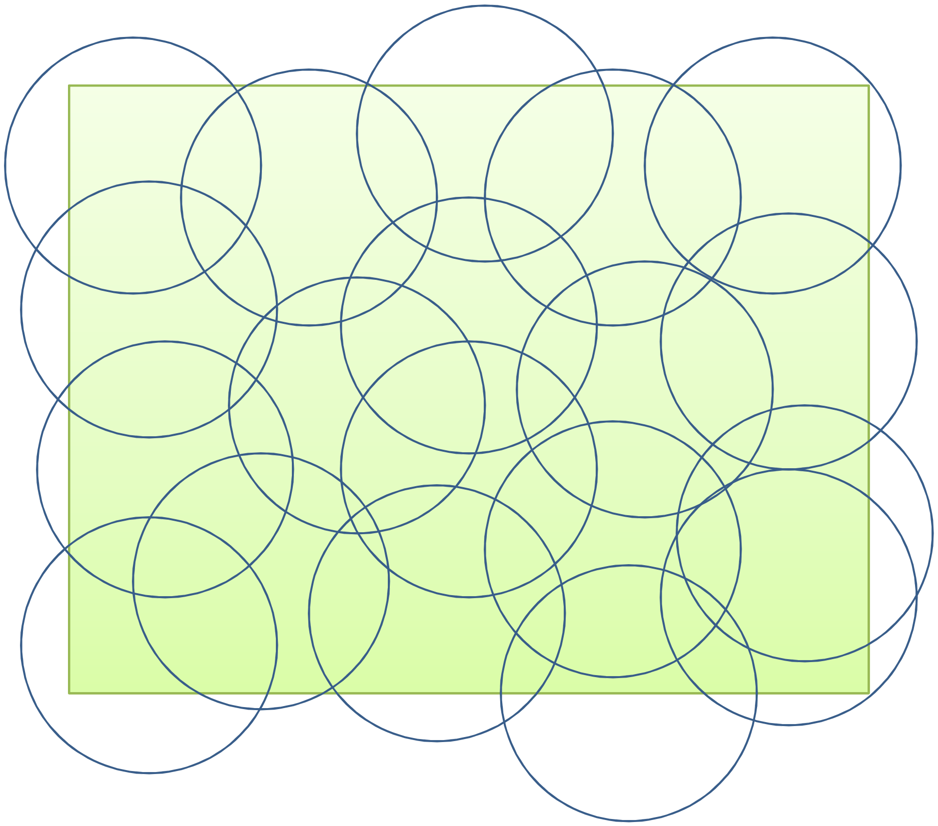
Introduction

- Coverage it is defined as quality of surveillance of a sensing function in WSNs.
- Is is a widely studied research area, many efforts have been made for addressing coverage problems in sensor networks, which are:
  - Point coverage
  - Area coverage,
  - Barrier coverage,
  - k-coverage, etc.

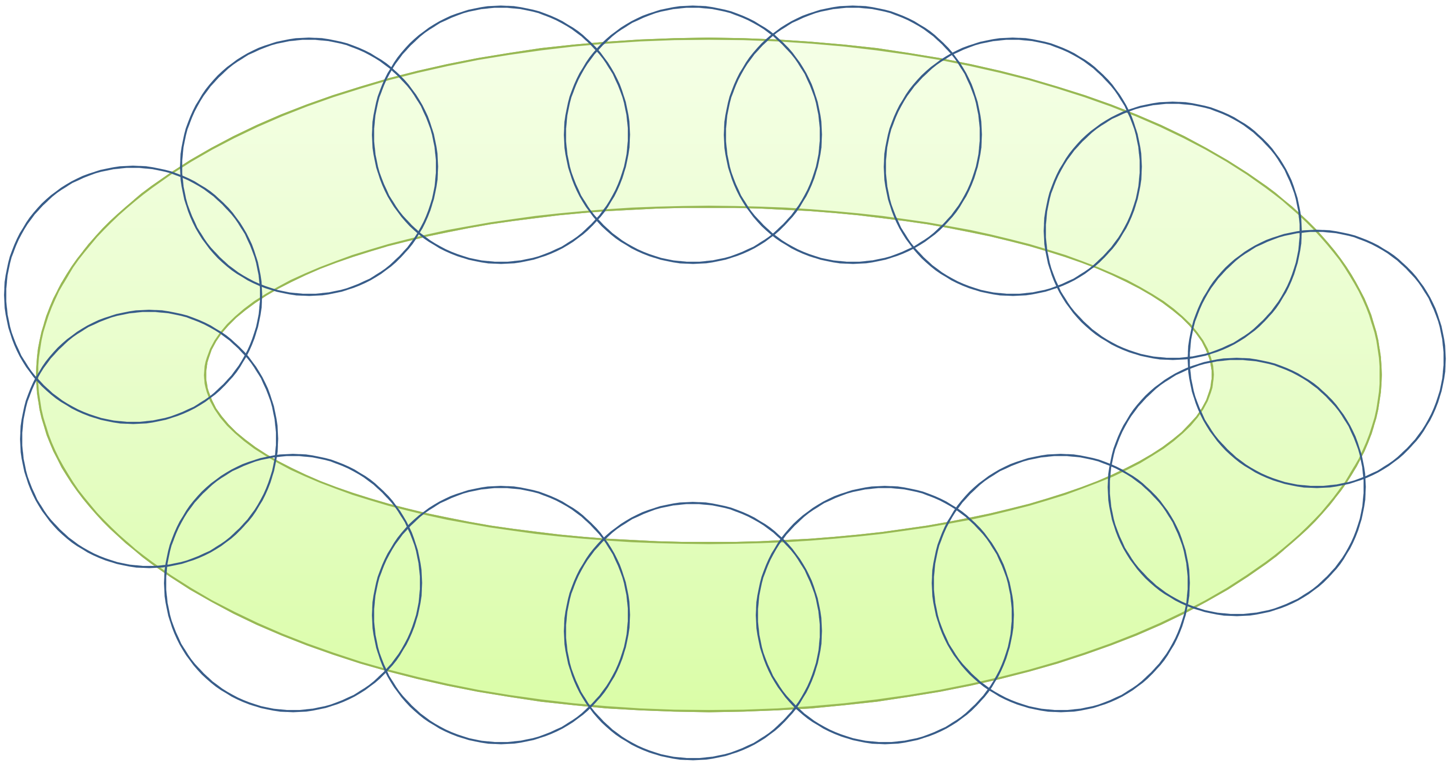
Point Coverage



Area Coverage



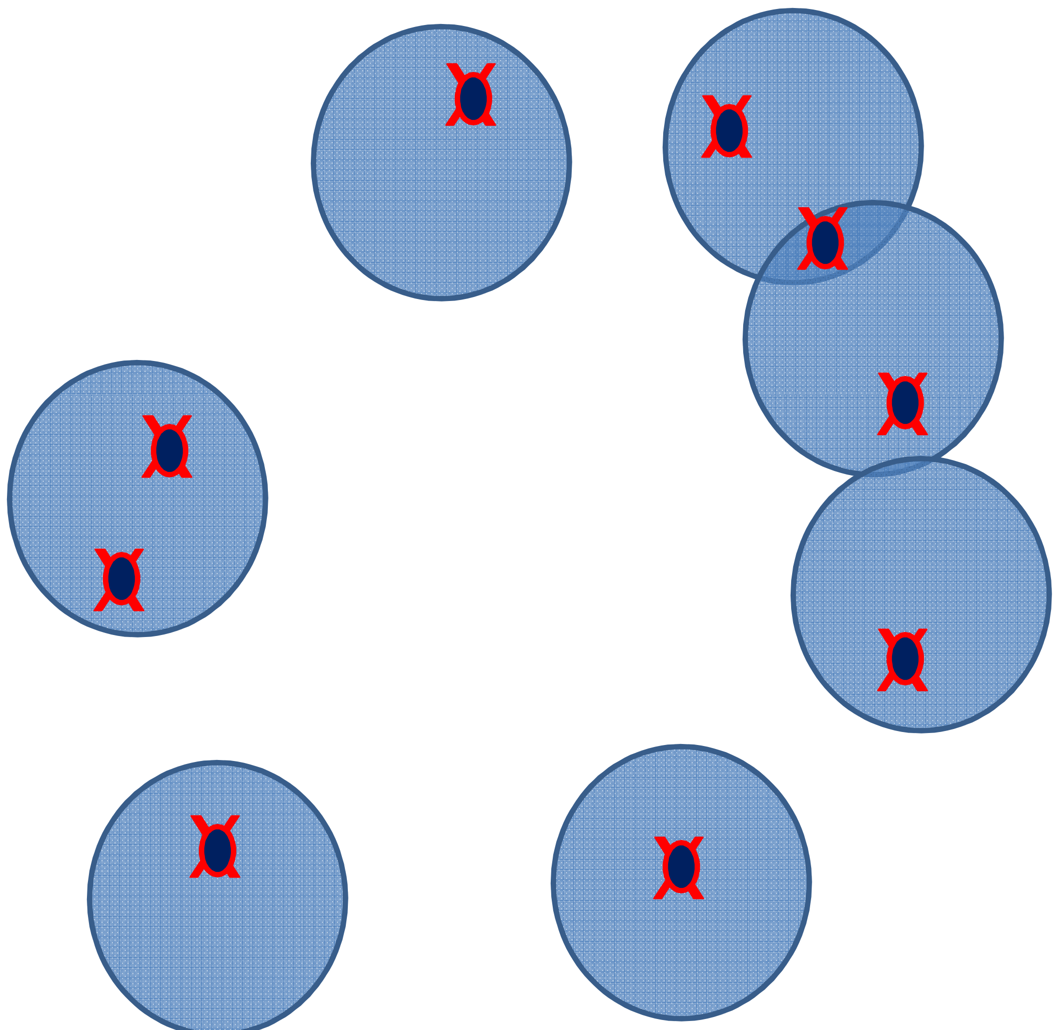
Barrier Coverage



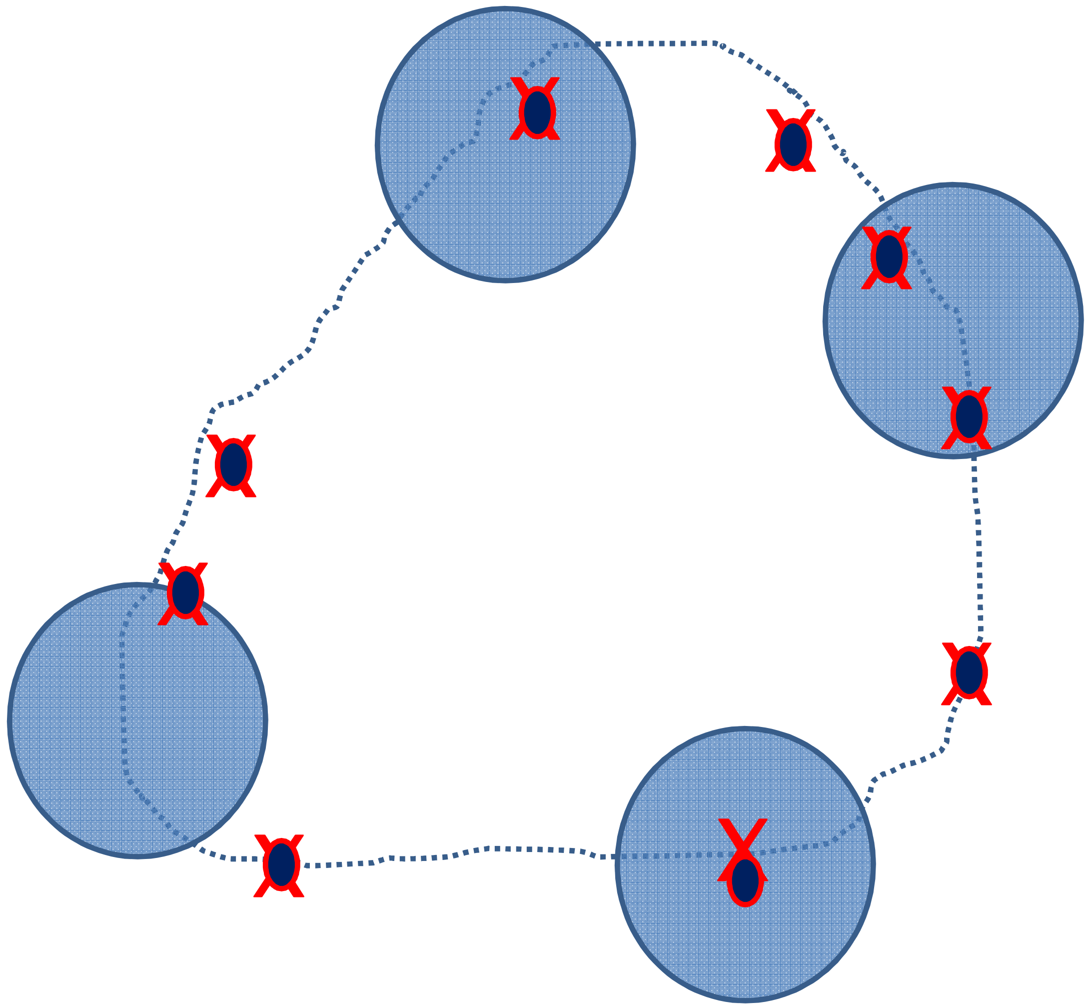
Sweep Coverage

- For those coverage scenarios, the monitored objective being **covered all time**, featured as **static coverage** or **full coverage**.
- In some applications, **patrol inspection/ periodic monitoring** are sufficient instead of continuous monitoring, which is featured as a **sweep coverage.**
- For such applications a **small number** of **mobile sensor nodes** can guarantee **sweep coverage** with a given **sweep period**.

Static coverage/Full coverage



Sweep Coverage



Sweep Coverage

Sweep Coverage

Let $U = \{u_1, u_2, \ldots, u_n\}$ be a set of points on a two dimensional plane and $M = \{m_1, m_2, \ldots m_p\}$ be a set of mobile sensor nodes. A point u_i is said to be **t-sweep covered** if and only if at least one mobile sensor node visits u_i within every **t time period**.

- The set U is said to be **globally sweep** covered by the mobile sensor nodes of M if all u_i are **t-sweep covered**.
- The **time period t** is called the **sweep period** of the points in U.

[1] **Sweep coverage with mobile sensors**, Mo Li, Wei-Fang Cheng, Kebin Liu, Yunhao Liu, Xiang-Yang Li, and Xiangke Liao, ***IEEE Trans. Mob. Comput.,*** **10(11):1534–1545, 2011.**

Previous Results

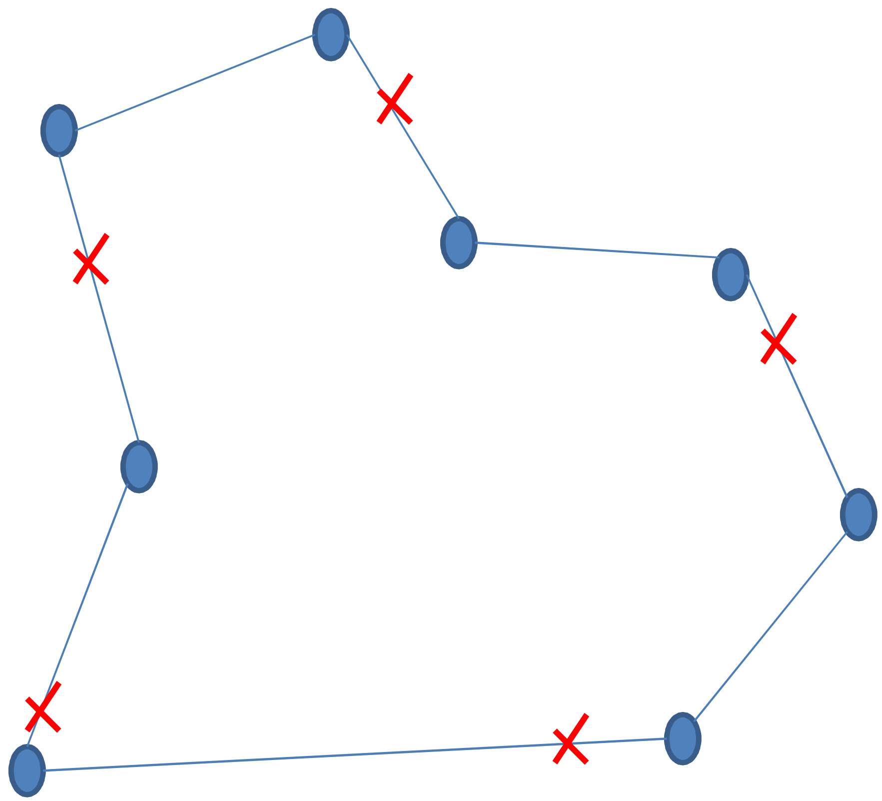
- The contribution of Li et al. in [1] are the following:
  – The sweep coverage problem is NP-hard.
  – It is not possible to approximate sweep coverage problem with a factor less than 2 unless P =NP.
  – Proposed a 3-approximation algorithm for sweep coverage problem.

Previous Results

Basic idea of the proposed algorithm [1]

- Find an approximated TSP tour among the set of points. (1.5 factor)
- Divide the tour into parts of length (vt/2).
- One mobile sensor is deployed at each of the parts.
- Mobile sensors moves **back-and-forth** to cover the points belonging to the corresponding parts.

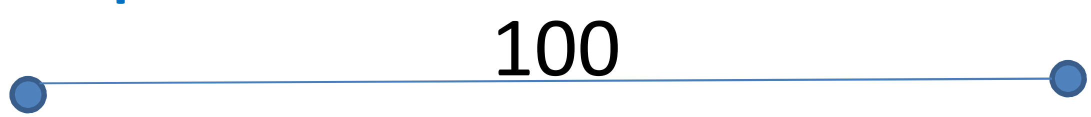
Previous Results



Correctness

- Li et al. proved that the proposed algorithm is a 3 factor approximation algorithm.
- **But the statement is wrong**

Counter Example:



v=20, t=1

- Number of mobile sensors needed according to [1] is 20 Optimal solution is 2 as two sensor at two points will be sufficient

Sweep Coverage Problem

- We introduce a variation of sweep coverage named as GSweep coverage problem, where the PoIs are represented by vertices of a weighted graph.
- We propose a 3−*approximation algorithm* to guarantee sweep coverage of all vertices of the graph.
- We generalize the above algorithm to solve the problem with approximation factor $O(\log \rho)$ when vertices of the graph have different sweep periods and processing times, where ρ is the ratio of the max and min sweep periods

GSweep coverage problem

- Let G = (U,E) be a weighted graph, where weight of an edge (ui, uj) for (ui, uj) ∈U is denoted by |(ui, uj)|. Let n be the total number of vertices in G. For any subgraph H of G, we denote |H| as the sum of the edge weights of H.
- **Definition (GSweep coverage):** Let $U = \{u1, u2, \cdots, un\}$ be the vertices of a weighted graph $G = (U, E, w)$ and $M = \{m1, m2, \cdots, mn\}$ be the set of mobile sensors. The mobile sensors move with a uniform speed v along the edges of the graph. For given $t > 0$, find the minimum number of mobile sensors such that each vertex of G is *t*-sweep covered.
- The problem is NP hard, follows from the hardness proof given in [1].

Algorithm

Algorithm 1: GSweepCoverage

for $k = 1$ *to* n **do**

  Find the minimum spanning forest F_k on G with $n - k$ edges. Let $C_1, C_2, \cdots, C_k$ be the connected components of F_k.

  $N_k = 0$

  for $j = 1$ *to* k **do**

    if C_j is a component having more than one vertex **then**

      $N_k = N_k + \left\lceil \frac{2w(C_i)}{vt} \right\rceil$

    else

      $N_k = N_k + 1$

    end if

  end for

end for

Let J be the index $\in \{1, 2, \cdots, n\}$ such that $N_J = \min\{N_1, N_2, \cdots, N_n\}$

Let $C_1, C_2, \cdots, C_J$ be the connected components of F_J.

for $i = 1$ to J **do**

  if C_i is a component having more than one vertex **then**

    Find a tour T_i on C_i by doubling each edge of C_i. Partition the tour into $\left\lceil \frac{w(T_i)}{vt} \right\rceil$ parts and deploy one mobile sensor at each of the partitioning points.

  else

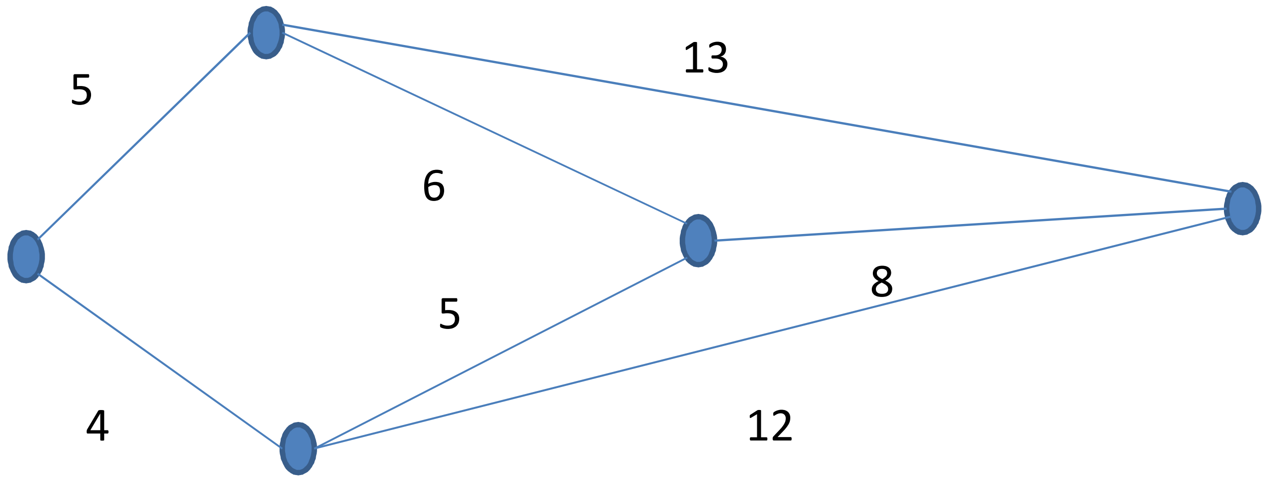
    Deploy one mobile sensor at the vertex of C_i.

  end if

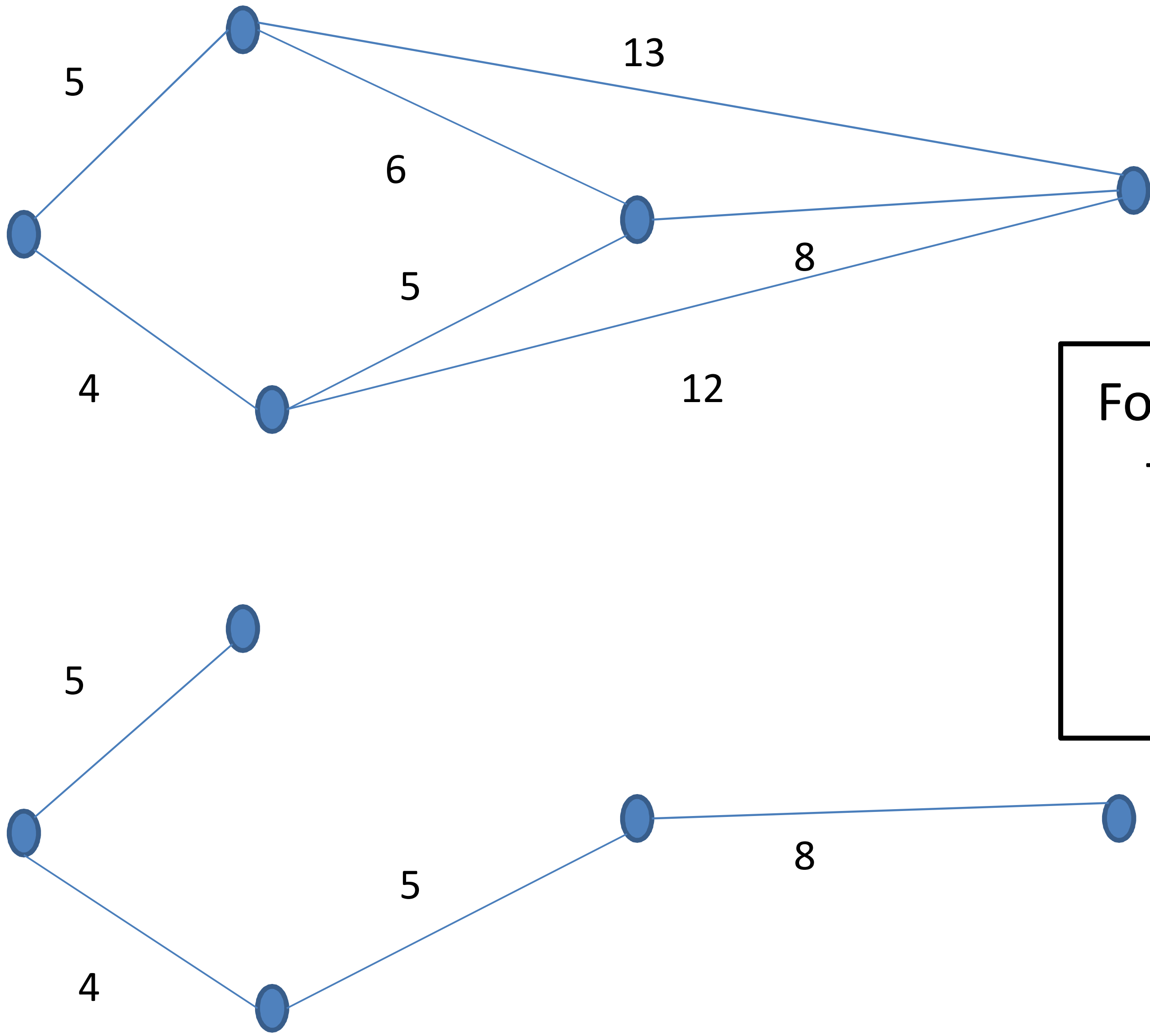
end for

All mobile sensors start moving at the same time along the respective tours having more than one vertex in same direction. If a mobile sensor is deployed on a tour containing only one vertex then it periodically monitors the vertex like a static sensor.

Example

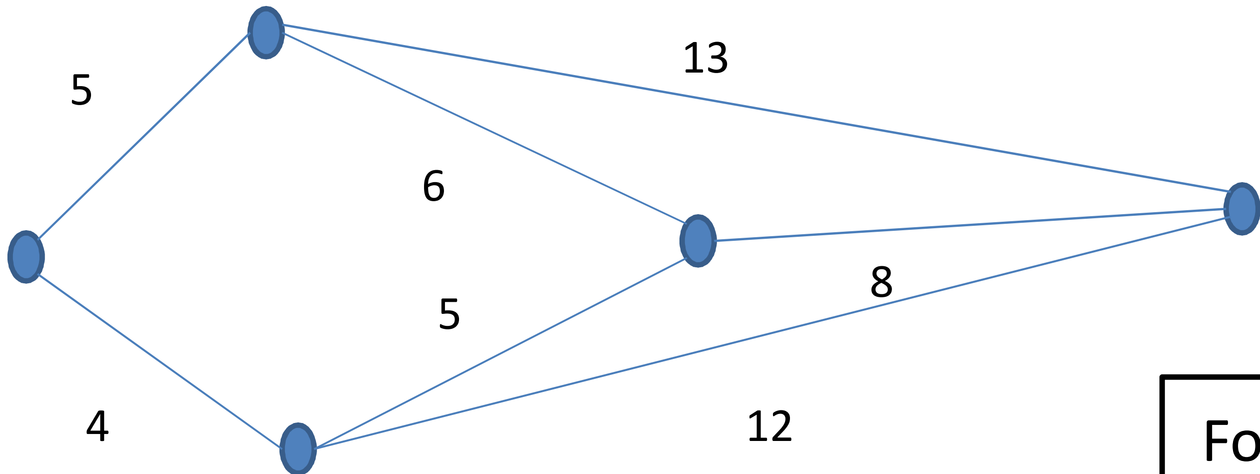


1^{st} Iteration, vt=8

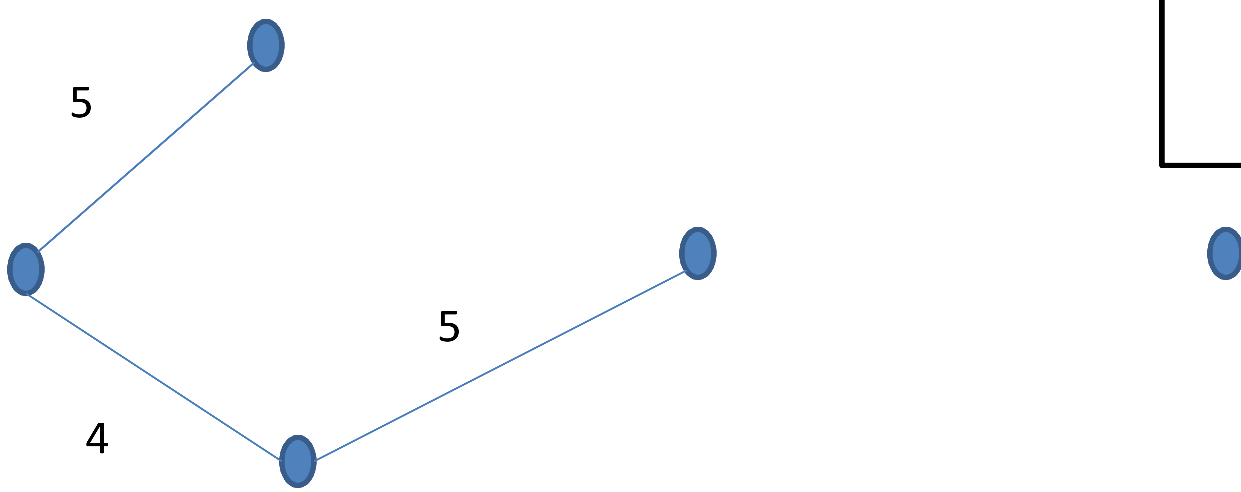


For k=1, The weight of the Forest F_1 is 22
Number of sensor nodes needed is $\lceil 44/8 \rceil = 6$

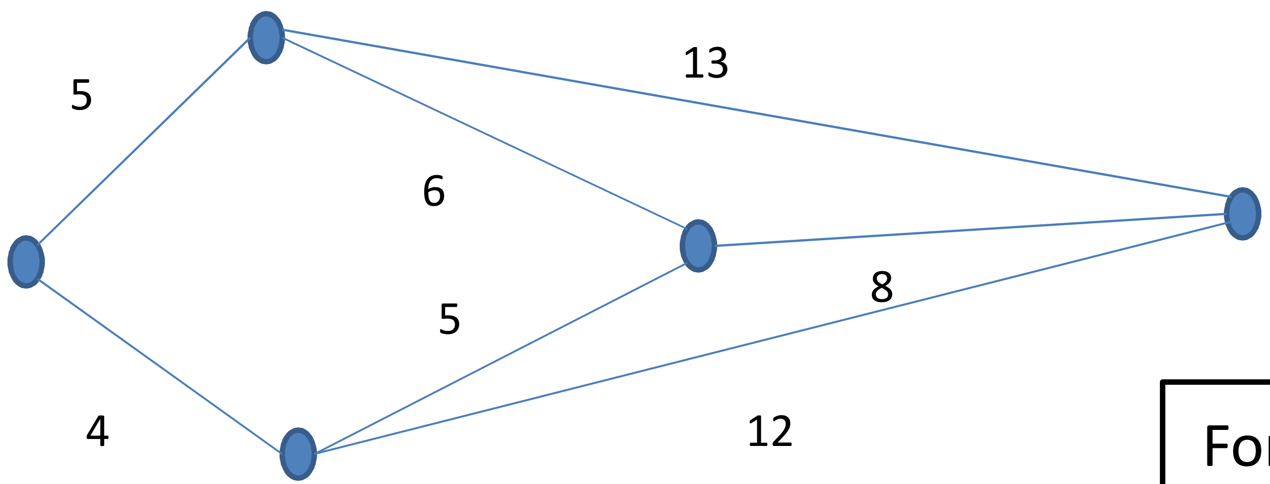
2^{nd} Iteration, vt=8



For k=2, The weight of the Forest F_2 is 14
Number of sensor nodes needed is $\lceil 28/8 \rceil + 1 = 5$

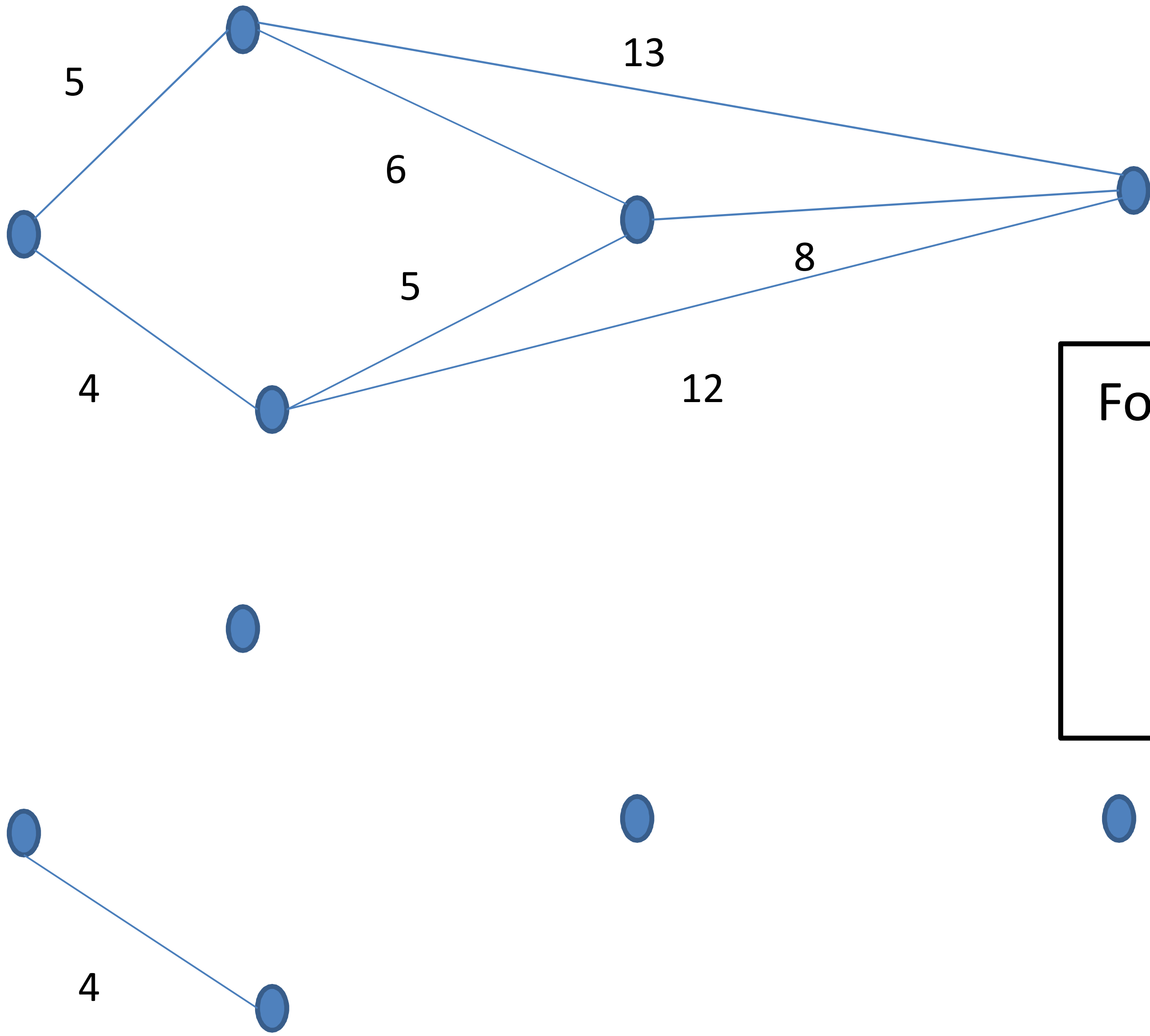


3rd Iteration, vt=8



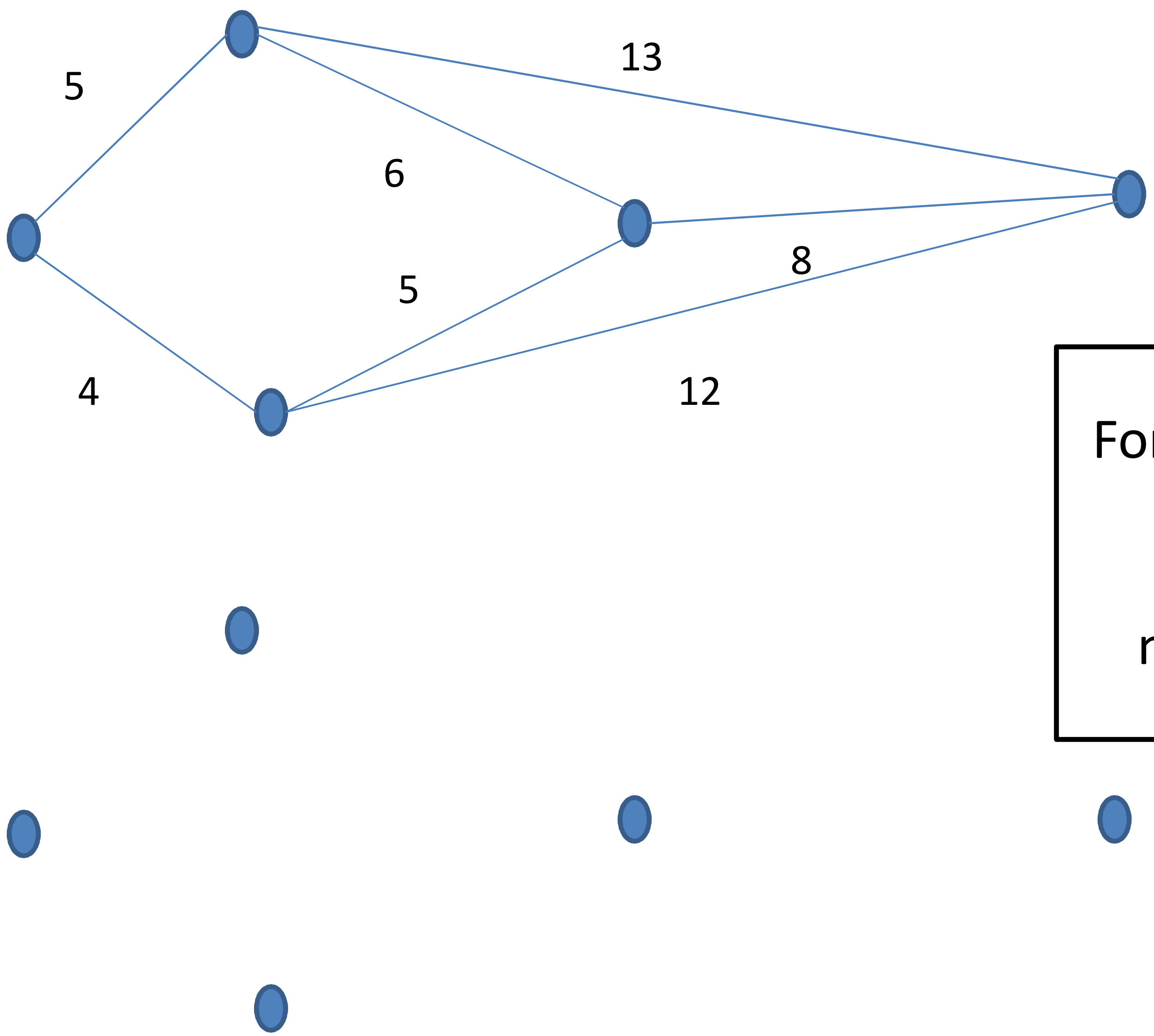
For k=3, The weight of the Forest F_3 is 9
Number of sensor nodes needed is
$\lceil 18/8 \rceil + 2 = 5$

4^{th} Iteration, vt=8



For k=4, The weight of the Forest F_4 is 9
Number of sensor nodes needed is
$\lceil 8/8 \rceil + 3 = 4$

5^{th} Iteration, vt=8



For k=5, The weight of the Forest F_5 is 9
Number of sensor nodes needed is 5

Analysis

Lemma: Let opt be the minimum number of sensors needed in the optimal solution. Let opt' be the minimum number of paths of length $\leq vt$ which span U on G. Then opt $\geq$ opt'.

Proof: Let us assume that opt < opt'.

Let $P_1, P_2, \ldots, P_{opt}$ be the movement paths of the sensors in optimal solution in a time interval $[t_0, t+t_0]$. Then $|P_i| \leq vt$.

- According to the definition of Gsweep coverage, all vertices of G are covered by the set of paths $P_1, P_2, \ldots, P_{opt}$.
- Hence $P_1, P_2, \ldots, P_{opt}$ is a collection of paths with $|P_i| \leq vt$ which spans U on G, which contradicts the fact that opt < opt'.
- Therefore opt $\geq$ opt'.

Analisys

Theorem: The Algorithm is a 3 factor approximation algorithm.

Proof:

- Let opt be the number of mobile sensors needed in optimal solution.
- Let opt' be the minimum number of paths of length $\leq vt$ which span U on G.
- We know opt $\geq$ opt’ (from previous lemma)
- Let Min_path be the sum of the edge weights of opt' number of paths of length $\leq vt$ which span U.

Proof (continue..)

- Algorithm 1 chooses the minimum over all N_k *for k = 1 to n*
- $N \leq N_k$ for k=1 to n.
- Consider the iteration when k=opt'.
- Then $|F_k| \leq$ Min_path and Min_path $\leq$ k.vt
- Total length of the tours after doubling the edges of F_k is $2|F_k|$.
- Since $\lceil |T_i|/vt \rceil \leq (|T_i|/vt)+1$, therefore total number of sensor nodes needed is

$$N_k = 2(|F_k|/vt) + k \leq 2k+k = 3k \leq 3 \text{ opt}.$$

Therefore approximation factor of our algorithm is 3.

Proof: Let opt be the minimum number of mobile sensors required in the optimal solution. Let opt' be the minimum number of paths of weight $\leq vt$ which span U on G and Min_path be sum of the weight of all the paths. Then by Lemma 1,

$$opt' \leq opt \quad (1)$$

and

$$Min\_path \leq opt' \times vt \quad (2)$$

Again, these opt' number of paths of weights $\leq vt$ forms a spanning forest with opt' disjoint connected components and $F_{opt'}$ is the minimum spanning forest with opt' connected components. Therefore,

$$w(F_{opt'}) \leq Min\_path \quad (3)$$

The Algorithm is a 3 factor approximation algorithm.

The Algorithm 1 chooses the minimum over all N_k for $k = 1$ to n. Let us consider the iteration of the algorithm when $k = opt'$.

After doubling edges in step 16 of the algorithm, the total weight of the movement paths of the mobile sensors is $\leq 2w(F_k)$. Since $\left\lceil \frac{T_i}{vt} \right\rceil \leq \frac{T_i}{vt} + 1$, the number of mobile sensors needed in our solution is

$$\begin{aligned} N &\leq \frac{2w(F_k)}{vt} + k \\ &\leq \frac{2Min\_path}{vt} + k && \text{from Equation (3)} \\ &\leq 2k + k && \text{from Equation (2)} \\ &= 3k \\ &\leq 3opt && \text{from Equation (1)} \end{aligned}$$

Sweep Coverage with Different Sweep Periods and Processing Time

- In practice some finite amount of time i.e., processing time is required by a mobile sensor to process some tasks such as monitoring, sampling or exchanging data at each of the vertices during its visits.
- Sweep periods may be different for different vertices as per requirement of applications.

Sweep Coverage with Different Sweep Periods and Processing Time

- **Problem 2.** Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertex set of a weighted graph $G = (U, E, w)$, $\{t_1, t_2, \cdots, t_n\}$ and $\{\tau_1, \tau_2, \cdots, \tau_n\}$ be the corresponding set of sweep periods and processing times of the vertices. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors which can move with a uniform speed v along the edges of the graph. Find the minimum number of mobile sensors such that each u_i is t_i-sweep covered.

Proposed Algorithm

- The proposed algorithm execute in two different phases.
- In the 1st phase, we will compute the number of mobile sensors required for sweep coverage.
- In the 2nd phase, initial positions and movement schedules of mobile sensors are computed.

Finding Number of Mobile sensors

- For the given graph $G=(U,E,w)$, compute the complete graph $G'=(U, E',w')$, where

  $w'(u_i,u_j)=d(u_i,u_j)+v(\tau_i+\tau_j)/2$

Where $d(u_i,u_j)$ is the shortest path between u_i,u_j with respect to w.

- Let $\lambda=(t_{max}/t_{min})$, where t_{min} and t_{max} are *the* minimum and maximum sweep periods among the vertices.

Finding Number of Mobile sensors

- Let $U_i = \{u_j \mid 2^{i-1} t_{min} \leq t_j < 2^i t_{min}\}$
- $\{U_i \mid i = 1 \text{ to } \lceil \log \ell \rceil\}$ is a partition of U.
- Let G'i be the induced subgraph of G' for the vertex set Ui.

- For each i, i= 1 to $\lceil \log \ell \rceil$, Apply step 1 to 12 of Algorithm 1 on G'_i to find the number of mobile sensors required for $(2^{i-1} \cdot t_{min})$-sweep coverage for all the vertex in U_i.

Movement strategy of the mobile sensors

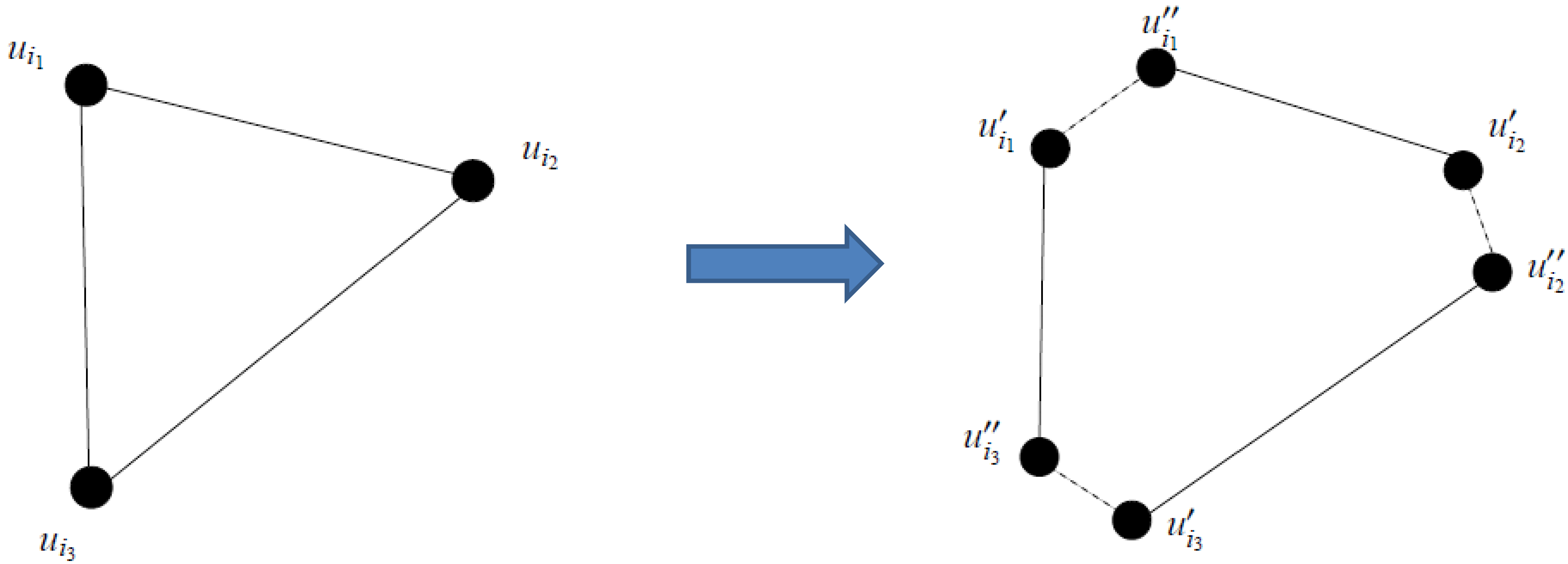
- Let C_1, C_2, …., C_{ji} be the connected components for G'i.
- Find tours T_1, T_2, …., T_{ji} after doubling the edges.
- If T_k contains only one vertex, deploy one mobile sensor which acts like a static sensor.

Movement strategy of the mobile sensors

- If T_k contains more than one vertex, compute T'_k by replacing vertices and edges of T_k.
- Let $u_{i_1}, u_{i_2}, \ldots, u_{i_h}$ be the vertices of T_k in the clockwise direction.
- Replace each u_{i_l} by two vertices u'_{i_l} and u''_{i_l} and introduce an edge (u'_{i_l}, u''_{i_l}) with

  $w'(u'_{i_l}, u''_{i_l}) = v\tau_l$.
- Each edge $(u_{i_l}, u_{i_{l+1}})$ is replaced by $(u''_{i_l}, u'_{i_{l+1}})$

Movement strategy of the mobile sensors



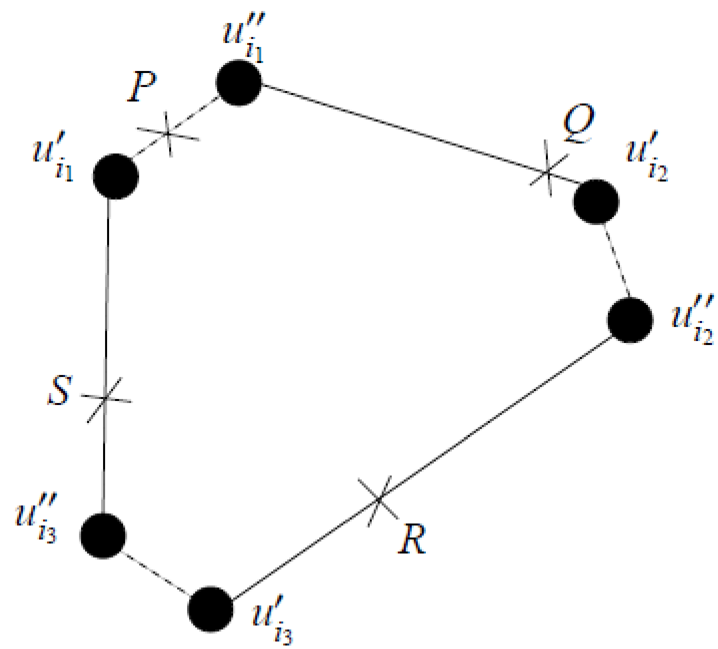
Movement strategy of the mobile sensors

- Partition T'_k into $\lceil w'(T'_k)/vt \rceil$ parts of weight at most vt.
- Deploy one mobile sensor at each of the partitioning points.
- The mobile sensor starts their movement along T'_k at the same time in the same direction.

Partition on the tour T'_k

- The mobile sensor deployed at P will wait for a time taken to move from P to ui1''. Then it moves with velocity v along the tour.

- The Mobile sensor deployed at Q will start moving along the edges (ui2,ui3) in G'.



Analysis

- Theorem: According to the movement strategy of the mobile sensors each vertex ui in U is ti-GSweep covered with processing time τ_i.

Proof:

- If ui belongs to a component with one vertex, then ti sweep coverage is trivial by the mobile sensor deployed at ui.
- Now if ui belongs to a component with more than one vertex, then according to the proposed algorithm for Problem 2, ui ϵ Uj for some j = 1 to $\lceil \log \eta \rceil$.
- By Theorem 1, ui is sweep covered with sweep *period 2^{j-1} · tmin ≤ ti. Therefore ui is ti sweep covered.*

Analysis

- Theorem: The approximation factor of the proposed algorithm for Problem 2 is $6 \lceil \log \ell \rceil$.

Proof:

- Let OPT be the optimal solution for problem 2.
- Let $OPT_1, OPT_2, \ldots, OPT_{\lceil \log \ell \rceil}$ be the optimal solutions on $G'_1, G'_2, \ldots, G'_{\lceil \log \ell \rceil}$.
- Then $OPT \le OPT_j$, for $j = 1$ to $\lceil \log \ell \rceil$

Proof(cont..)

- Let $OUT_1, OUT_2, \ldots, OUT_{\lceil \log \ell \rceil}$ be the number of mobile sensors required for our algorithm.
- Then, $OUT_j \le 6\ OPT_j$, $j = 1$ to $\lceil \log \ell \rceil$, as as the length of the partition in OPT_j *is* at most twice of the length of the partition in OUT_j and the Algorithm 1 is a 3-approximation algorithm.
- Therefore, total number of mobile sensors required is
- $OUT_1 + OUT_2 + \ldots + OUT_{\lceil \log \ell \rceil} \le 6 \lceil \log \ell \rceil$ OPT.
- Hence the proof follows.

Sweep Coverage with Mobile Sensors Having Different Speeds

- Problem 3: Let $U = \{u_1, u_2, \cdots, u_n\}$ be the vertex set of a weighted graph $G = (U, E, w)$. Let $M = \{m_1, m_2, \cdots, m_n\}$ be the set of mobile sensors with velocities $\{v_1, v_2, \ldots, v_n\}$ respectively. For a given $t>0$, Find the minimum number of mobile sensors such that each u_i is t-sweep covered.

Sweep Coverage with Mobile Sensors Having Different Speeds

- Theorem: No polynomial time constant factor approximation algorithm exists to solve the sweep coverage problem by mobile sensors with different velocities unless P=NP.

Proof :

We will prove this Theorem using reduction from Metric-TSP problem.

Proof(Cont..)

- If possible let there is a k factor approximation algorithm *A*.
- Consider an instance (G1,L) of metric TSP problem, where L is weight of the minimum weight TSP tour in a complete weighted graph G1 = (U1, E1, w1) with n (> k) Vertices which satisfy triangular inequality.

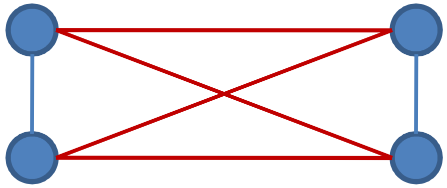
Proof(Cont..)

- Construct a complete graph $G2=(U_2,E_2,w_2)$ with n^2 vertices as follows:

For each vertex ui in U1, consider n vertices $u_{i1},u_{i2},\ldots, u_{in}$ in U_2.

$w_2(u_{ik},u_{jl})=w_1(u_i,u_j)-1/(n+1)^2$

$w_2(u_{ik},u_{il})=1/(n-1)(n+1)^2$



Proof(Cont..)

- Claim: G1 has a tour with weight at most l iff G2 has a tour with weight at most l.

Let T_1: $u_{i1}, u_{i2}, \ldots, u_{in}, u_{i1}$ be a tour of G with weight $\leq$ l.

Construct T2: $u_{i1}^1, u_{i1}^2, \ldots u_{i1}^n, u_{i2}^1, u_{i2}^2, u_{i2}^n, \ldots u_{in}^1, u_{in}^2, \ldots u_{in}^n, u_{i1}^1$.

Note that $w_2(T_2) \leq$ l.

Proof(Cont..)

- Claim: G1 has a tour with weight at most l iff G2 has a tour with weight at most l.

Conversely, let T2’: x1,x2,…,xn2, x1 be a tour in G2 with w2(T2’) ≤ l.

- Construct a tour T1’ from *T2’ as follows:*

Delete all the edges of type (uik,uil) from T2’.

For each of the remaining edges of form (uij,uki), consider the corresponding edge in G1 and construct the sub graph $\mathcal{E}$. *of G1.*

- For example, if edges (*uk i , ul j) and* (*uk2 i , ul2 j) for k1 , k2 and l1 , l2 are in T0* 2, then we consider the edge (*ui, uj) twice in* $\mathcal{E}$.

Proof(Cont..)

- Claim: G1 has a tour with weight at most l iff G2 has a tour with weight at most l.

- Construct a tour T1’ from E by short cutting.
- Note that, w1(T1’) ≤l

Proof(Cont..)

- Hence we can say that if L is the weight of the optimal tour in G1, then the weight of the optimal tour in G2 is also L, otherwise by the above fact, a tour of G1 with weight less than L can be found.

Proof(Cont..)

- We consider an instance of the sweep coverage problem by mobile sensors with different velocities as follows:
- We take G2 as the graph, sweep period t = 1, and a set of n^2 mobile sensors with velocity L for one and zero for remaining $n^2 - 1$ mobile sensors.
- Clearly, the optimal solution of the problem is one, since using the mobile sensor with velocity L, 1-sweep coverage of each of n2 vertices can be guaranteed.

Proof(Cont..)

- As A is a *k factor approximation algorithm, A returns*
- the number of mobile sensors required for sweep coverage of *G2 is k in worst case.*
- *Among these k mobile* sensors, one is with velocity *L and remaining are* with velocities zero.
- Therefore, the mobile sensor with velocity *L moves along a tour covering n2 − k + 1 vertices.*
- Remaining *k − 1 mobile sensors cover remaining k − 1 vertices one for each.*

Proof(Cont..)

- Hence at least one vertex from each of the set {*u1i* , *u2i*, · · · , *uni*} for different *i is visited by the* mobile sensor with velocity *L and weight of the tour is* at most *L.*
- From this tour, a tour *T of weight at most L of G1 can be constructed in the same way by constructing* an Eulerian graph of *G1 and short cutting as explained* before.

Proof(Cont..)

- Since *L is the weight of the optimal tour in G1, w(T) = L.*
- *Hence optimal solution for TSP problem on G1 can be computed in polynomial time by applying algorithm* A on *G2, which is not possible unless P=NP.*
- Hence statement of the theorem follows.

Conclusion

- In this paper we overcome the limitation of a previous study [1] on sweep coverage.
- The key argument is that when the graph is sparse, it is better to provide sweep coverage with a mixture of static and mobile sensors, instead of using only mobile sensors as the previous study did.
- We propose a 3-approximation algorithm to solve this NP hard problem for any positively weighted graph.

Conclusion

- We have generalized the above problem with different sweep periods and introducing different processing times for the vertices of the graph.
- Our proposed algorithm for this generalized problem achieves approximation factor *$O(\log \rho)$,*

  where $\rho = t_{max} / t_{min}$, *t_{min} and t_{max} are the minimum and maximum sweep periods* among the vertices.
- If velocities of the mobile sensors are different, we have proved that it is impossible to design any constant factor approximation algorithm to solve the sweep coverage problem unless P=NP.

THANK YOU