

# Distributed Algorithms For Swarm Robots in $R^2$

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# Outline of the lechture

Distributed  
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- Leader Election
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- Circle Formation under Limited Visibility
- Circle Formation under Unlimited Visibility
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# Swarm Robots

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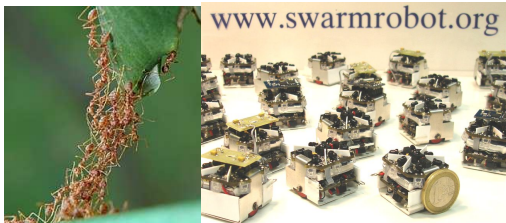
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- Group of small, inexpensive, identical, autonomous, mobile robots.
- **Collaboratively executing work**
  - moving large objects, cleaning big surface.
- **Geometric point of view:** points moving on the 2D plane.
- **Objective:** Forming geometric patterns like point, circle, straight line etc.
- Distributed in nature.

# General Characteristics of Swarm Robots

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- Point/Unit Disc
- Autonomous
- Identical
- No message passing
- Sense surroundings
- Move on the 2D plane
- Limited computational power
- Oblivious

# Computational Model of Swarm Robots

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- Execute **wait-look-compute-move** cycle.

In wait state robots do nothing.

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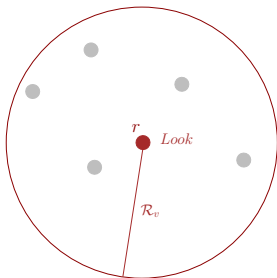
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- Execute **wait-look-compute-move** cycle.



- $\mathcal{R}_v$  (**visibility range**) can be **limited** or **unlimited**

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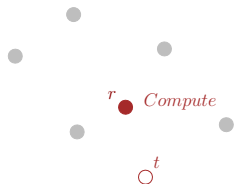
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- Execute **wait-look-compute-move** cycle.



- $r$  computes its destination  $t$

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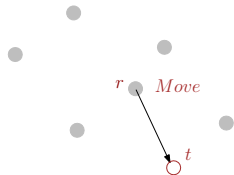
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- Execute **wait-look-compute-move** cycle.



- $r$  moves to  $t$ 
  - SYm: Rigid motion.
  - CORDA: Non-rigid motion.



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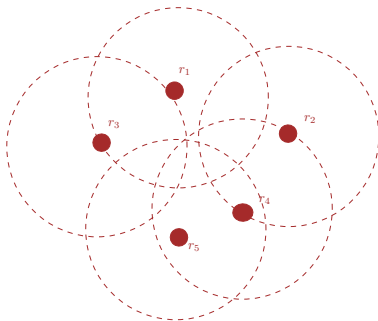
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- Execute **wait-lock-compute-move** cycle **synchronously**.



- All robots look at the same time

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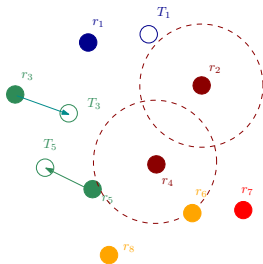
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- Execute **wait-look-compute-move** cycle **asynchronously**.



- Different robots look, compute and move at different times.

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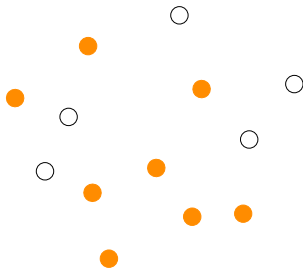
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- Execute **wait-look-compute-move** cycle **semi-synchronously**.



- A arbitrary set of robots looks at same time.

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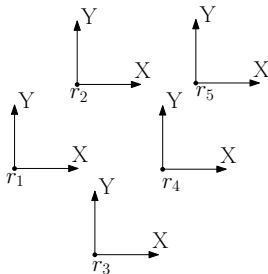
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- Agreement on co-ordinate system.



- Robots having same Sense of Directions (SoD) and same chirality

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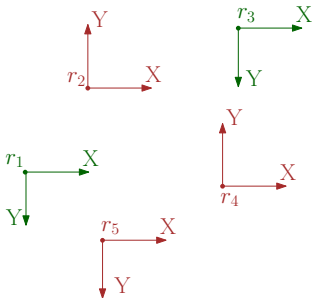
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- Agreement on co-ordinate system.



- Robots having same SoD but different chirality

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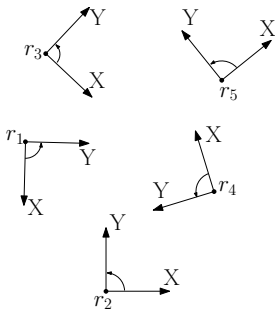
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- Agreement on co-ordinate system.



- Robots having different SoD but same chirality

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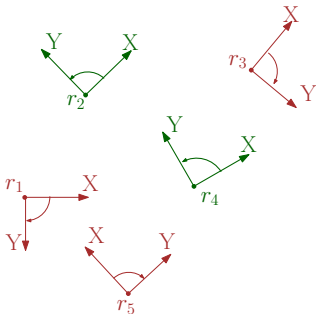
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- Agreement on co-ordinate system.



- Robots having different SoD and different chirality

# Example of Some Geometric Problems on Swarm Robots

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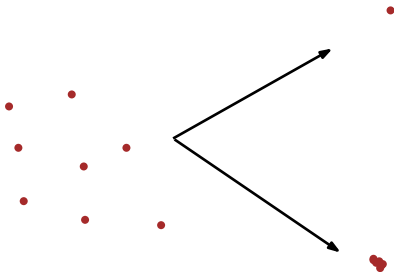
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- Gathering [Prencipe2007] or Convergence [Cohen2006]





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## ■ Scattering [Lali2011]



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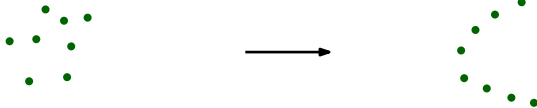
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## ■ Pattern formation [Flocchini2008]



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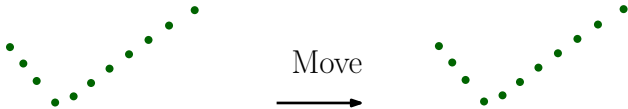
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## ■ Flocking [Canepa2007]



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- Leader Election
- Gathering
- Arbitrary Pattern Formation
- Circle Formation

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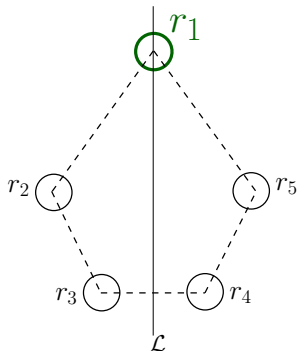
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## ■ Leader Election.



- The robots elect  $r_1$  as their leader.

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## Some Works for Leader Election

| <i>SoD</i> | <i>Chirality</i> | <i># of robots</i> | <b>Earlier results</b>   |
|------------|------------------|--------------------|--|
| Yes        | Yes              | Any                | Leader election possible [Flocchini1999].  |
| Yes        | No               | Odd                | Leader election possible [Flocchini2001].  |
| No         | Yes/No           | Any                | Leader election not possible [Flocchini2001].  |
| <b>No</b>  | <b>Yes</b>       | <b>Any</b>         | <b>characterization of all<br/>geometric positions [DieudonneL2007].</b>   |
| <b>No</b>  | <b>No</b>        | <b>Odd</b>         | <b>characterization of all<br/>geometric positions [DieudonneL2007].</b>   |
| <b>No</b>  | <b>No</b>        | <b>Any</b>         | <b>Characterization of all<br/>geometric positions where<br/>iterative leader election<br/>(total ordering of robots) is possible.<br/>[Gan Chaudhuri2010]</b> |

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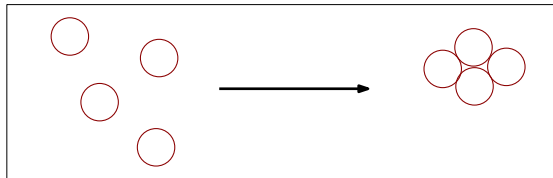
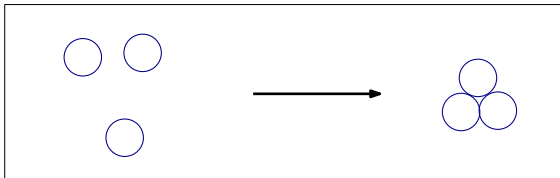
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## ■ Gathering Fat Robots [Czyzowicz2009].



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## Earlier Works on Gathering Point Robots

| <i>Scheduling</i> | <i>Visibility range</i> | <i>Agreement in co-ordinate</i> | <i>Multiplicity detection</i> | <i>Earlier results</i>  |
|-------------------|-------------------------|---------------------------------|-------------------------------|---|
| Sync              | unlimited               | No                              | No                            | Solved [Ando1999].  |
| ASync             | Any                     | Yes                             | No                            | Solved[Flocchini2001].  |
| ASync             | unlimited               | No                              | Yes                           | Not solvable for two robots [Prencipe2007].   |
| ASync             | unlimited               | No                              | Yes                           | Solved for three and four robots [Cieliebak2002].<br>Solved for more than four robots initially<br>(a) in bi-angular configuration.<br>(b) not in any regular $n$ gon |
| ASync             | unlimited               | No                              | No                            | Not solvable [Prencipe2007].  |



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## Some Works for Gathering Fat Robots

| <i>Scheduling</i> | <i>Agreement in<br/>co-ordinate</i> | <b>Earlier results</b>   |
|-------------------|-------------------------------------|--|
| ASync             | No                                  | Solved for up to<br>four robots [Czyzowicz2009].   |
| Sync              | No                                  | Solved for any number<br>of robots [CordLandwehr2011].<br>(randomized / considering<br>robots with identification<br>and communication power). |
| Sync              | No                                  | Solved by simulation [Bolla2012].  |
| <b>ASync</b>      | <b>Chirality</b>                    | <b>Solved for any<br/>number of robots [Agathangelou2012].</b>   |
| <b>Async</b>      | <b>No</b>                           | <b>Gathering any number of<br/>transparent fat robots.<br/>without collision.<br/>[Gan Chaudhuri2010].</b>                                     |

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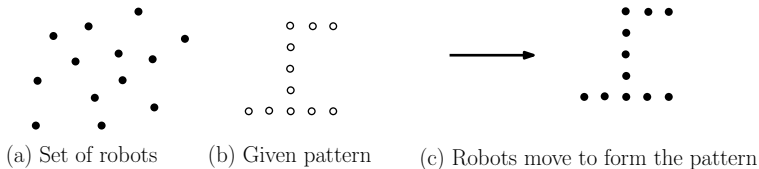
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## ■ Arbitrary Pattern formation[Flocchini2008]



■ A set of robots form a given pattern

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## Some Works for Arbitrary Pattern Formation for Point Robots

| <i>Scheduling</i> | SoD       | Chirality  | <b>Earlier results</b>   |
|-------------------|-----------|------------|--|
| Async             | Yes       | No         | Any pattern formable with odd no. of robots and <i>Symmetric pattern</i> is formable for even no. of robots [Flocchini1999]. |
| Async             | Yes       | Yes        | Arbitrary pattern is formable for any no. of robots [Flocchini2001].   |
| <b>ASync</b>      | <b>No</b> | <b>Yes</b> | <b>Arbitrary pattern formation not possible [Flocchini2008].</b>   |
| <b>ASync</b>      | <b>No</b> | <b>No</b>  | <b>Asymmetric pattern formation is possible without collision [Ghike2010].</b>   |

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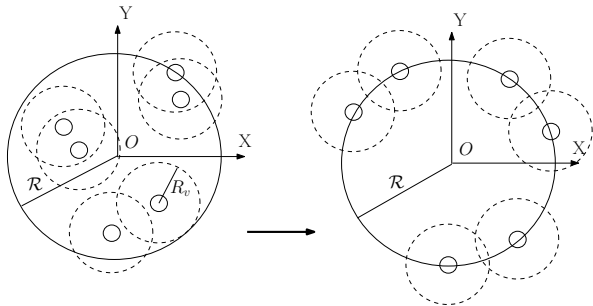
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- Circle formation [Defago2008].



- Circle formation in limited visibility range ( $R_v$ ) and agreement in co-ordinate system

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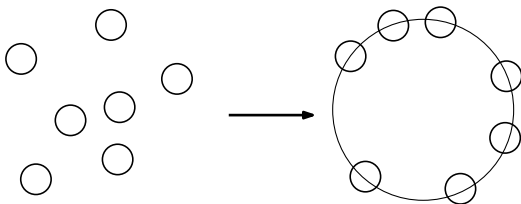
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## ■ Circle formation [Defago2008]



- Circle formation in unlimited visibility range and no agreement in co-ordinate system

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## Some Works on Circle Formation for Fat Robots

| <i>Scheduling</i> | <i>Visibility range</i> | <i>Agreement in co-ordinate</i> | <i>Reported results</i>                                   |
|-------------------|-------------------------|---------------------------------|---|
| Sync              | Limited                 | No                              | Heuristic of approximate circle formation [Sugihara1990]. |
| Ssync             | Unlimited               | No                              | Circle formation [Defago2002].                            |
| ASync             | Unlimited               | No                              | Bi-angular Circle formation [Katreniak2005].              |
| ASync             | Unlimited               | No                              | Circle formation [Defago2008].                            |

No reported result on **fat robots**.

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| Problem           | Shape                           | Visibility range           | Co-ordinate system                          |
|-------------------|---------------------------------|----------------------------|---|
| Leader election   | Point or transparent fat robots | Unlimited visibility range | Local co-ordinate systems with no chirality |
| Gathering         | Transparent fat robots          | Unlimited visibility range | Local co-ordinate systems with no chirality |
| Pattern formation | Point robots                    | Unlimited visibility range | Local co-ordinate systems with no chirality |
| Circle formation  | Transparent fat robots          | Limited visibility range   | Global co-ordinate systems                  |
| Circle formation  | Transparent fat robots          | Unlimited visibility range | Local co-ordinate systems with no chirality |

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- Solid Fat Robots: Visibility Block model.
- Unequal visibility range.
- Optimal/efficient Algorithms.