Leader Election and Gathering Transparent Fat Robots

Definition of the Problem

Model

- $R = n$ ($n \geq 5$) asynchronous transparent fat robots.
- Infinite visibility range.
- No agreement in coordinate system.

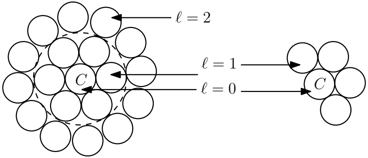
Goal

- The robots in R have to gather
- Characterization of all geometric configurations where gathering is not possible

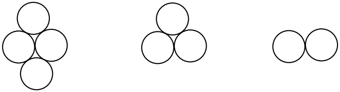
Features of our algorithm

- Agreement in coordinate system.
- Collision free paths.
- Finite time termination.

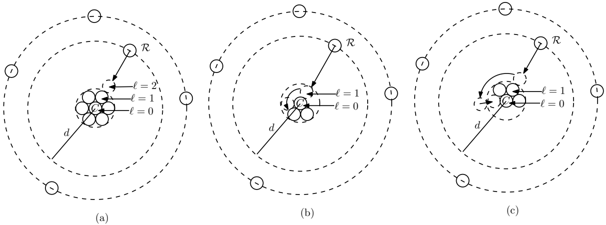
Gathering Pattern



(a) (b)



Outline of Gathering Algorithm



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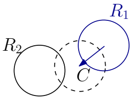
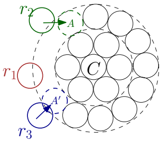
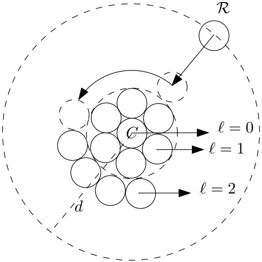




Lemma

If gathering area is initially occupied by other robots, then also gathering will be completed successfully.

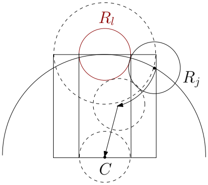
Correctness of Gathering Algorithm



Lemma

When a robot is moving towards C (the center of SEC) or sliding around C, its distance from C is still minimum among all the robots in set R.

Correctness of Gathering Algorithm



Lemma

R_j has enough place to move for touching R_l and slide on R_l.

Lemma

When R_j is moving, no other robot will be selected to move.

Lemma

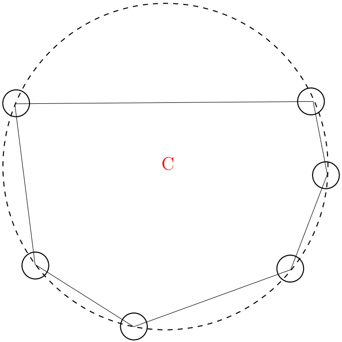
No obstacle appears in the path of a mobile robot during its motion.

Condition for Gathering

Theorem

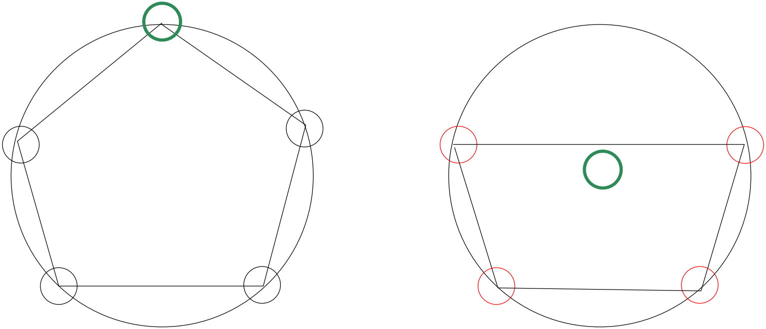
If leader election is possible for a set of transparent fat robots, then formation of gathering pattern is also possible by the robots.

If Multiple Robots are at equidistant from C.



Only Leader Election is not sufficient-Why

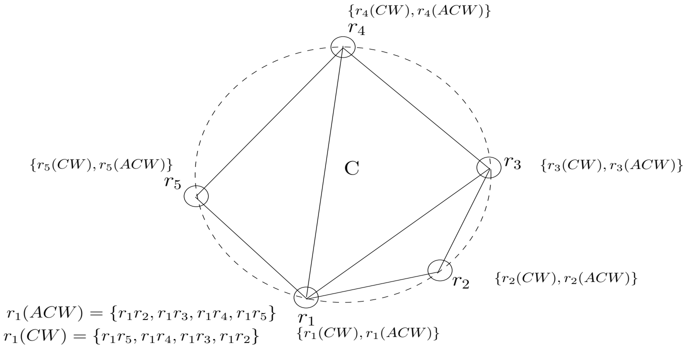
Leader is elected.



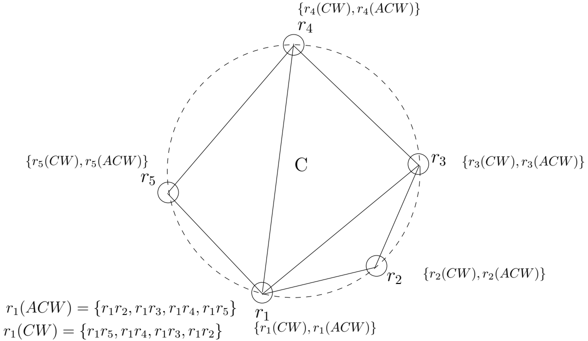
But stuck in next step.

Ordering of Robots Helps

Order the vertices with respect to C.



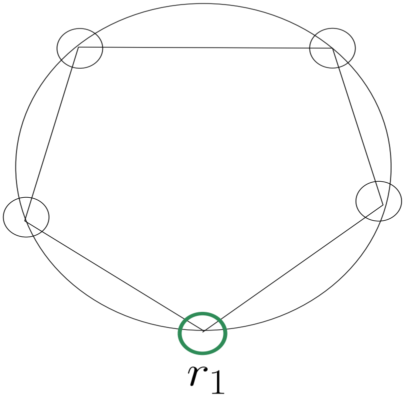
Ordering Algorithm Overview



- For asymmetric polygon $r_i(CW) \neq r_i(ACW)$ for $1 \leq i \leq 5$.(Proved)
- For asymmetric polygon $\{r_i(CW), r_i(ACW)\} \neq \{r_j(CW), r_j(ACW)\}$ for $1 \leq i \neq j \leq 5$.(Proved)
- Compute $min(\{r_i(CW), r_i(ACW)\})$ for $1 \leq i \leq 5$ lexicographically.
- Let $\{r_m(CW), r_m(ACW)\}$ be such minimum (unique).
- If $\{r_m(CW) < r_m(ACW)\}$, take $r_m(CW)$ as ordering. Else take $r_m(ACW)$ as ordering.

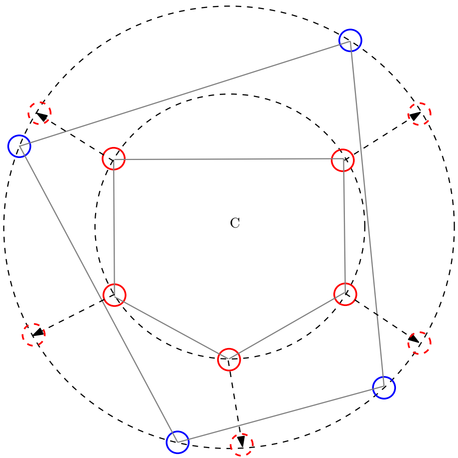
Ordering Algorithm Overview

If the polygon is symmetric i.e., $r_1(CW) = r_1(ACW)$



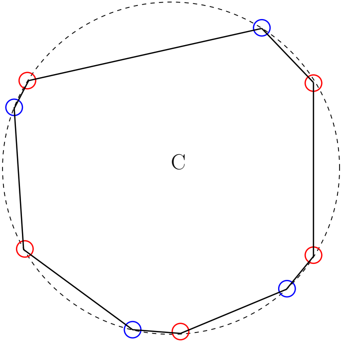
Ordering Algorithm Overview

Find asymmetry from outer layer.



Ordering Algorithm Overview

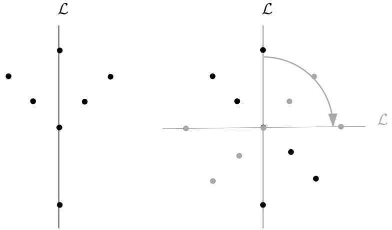
We get an asymmetric polygon.



Theorem

If the union of any two of concentric polygons in P constructs an asymmetric polygon, then P is orderable.

Characterization of the geometric configurations of robots for ordering



Straight Symmetric

Skew Symmetric

Asymmetric Configuration:

If P (set of points) is not in symmetric configuration then it is in asymmetric configuration.

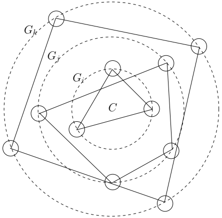
Theorem

Let P be a non-empty, non-singleton set of points. If P is in symmetric configuration, then P is not orderable.

Observation

An asymmetric polygon is orderable.

Characterization of the geometric configurations of robots for ordering



$\mathcal{G} = \{G_1, \ldots, G_t\}$

Definition

G_i and G_j, in $\mathcal{G}$ is called a symmetric pair, if G_i and G_j have a common line of symmetry.
A pair is asymmetric if it is not symmetric.

- An asymmetric pair is orderable.
- If there exists an asymmetric pair in $\mathcal{G}$, then $\mathcal{G}$ is orderable.
- If $\mathcal{G}$ contains at least one asymmetric polygon then $\mathcal{G}$ is orderable.

Theorem

The points in $\mathcal{G}$ are orderable if and only if the points in $\mathcal{G}$ are in asymmetric configuration.