Unit Disk Cover Problem in 2D

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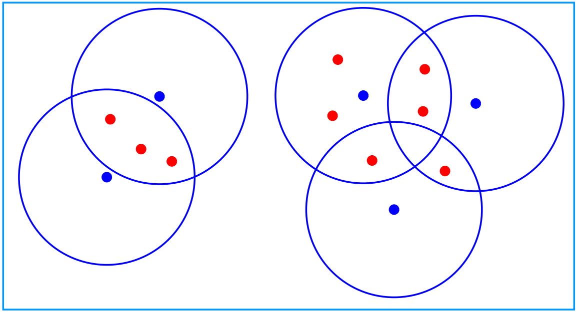
Organization of Talk

1. **Introduction**
2. **Discrete Unit Disk Cover(DUDC)**
   - Restricted Discrete Unit Disk Cover
     - Within Strip DUDC(WSDUDC)
     - Restricted Line-Separable DUDC(RLSDUDC)
     - Line-Separable DUDC(LSDUDC)
   - Solution of Discrete Unit Disk Cover (DUDC) Problem
3. **Rectangular Region Cover(RRC)**
   - Approximation Algorithm for RRC Problem
   - RRC Problem in Reduce Radius Setup
4. **Conclusion**

Discrete Unit Disk Cover(DUDC)

• Given *m* unit disks *D* and *n* points *Q* in the plane, the discrete unit disk cover problem is to select a minimum subset of the disks to cover the points.



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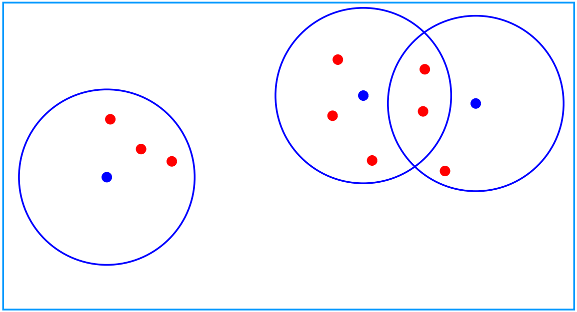

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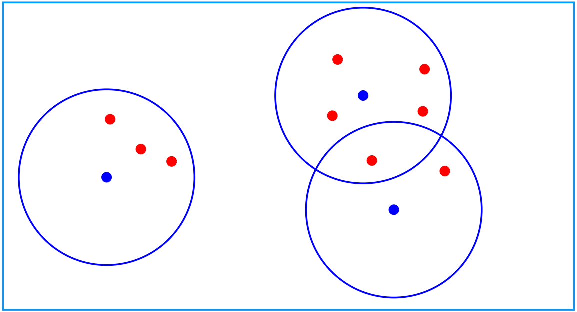
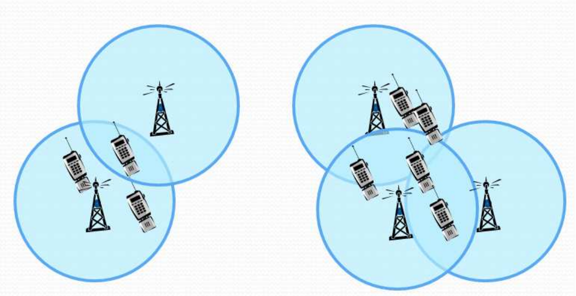


Discrete Unit Disk Cover(DUDC)

Applications



Discrete Unit Disk Cover(DUDC)

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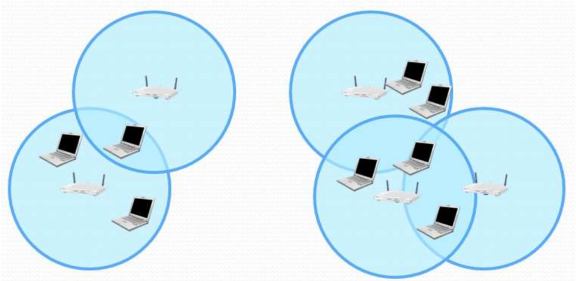


Discrete Unit Disk Cover(DUDC)

About DUDC

- NP-Hard (Johnson, 1982)
- Approximation algorithms:
  - 108-approximate (Călinescu et al., 2004)
  - 72-approximate (Narayanappa and Voytechovsky, 2006)
  - 38-approximate (Carmi et al., 2007)
  - 22-approximate, $O(m^2 n^4)$ algorithm (Claude et al., 2010)
  - 18-approximate, $O(mn + n \log n + m \log m)$ algorithm (Das et al., 2012)
  - 15-approximate, $O(m^6 n)$ algorithm (Fraser & López-Ortiz, 2012)
  - $(1+\epsilon)$-approximate (Mustafa and Ray, 2009)
    - Uses ϵ-net based local improvement approach
    - $O(m^{257} n)$time for a 2-approximation solution

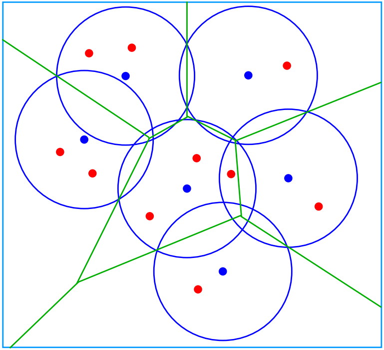
Discrete Unit Disk Cover(DUDC)

Our Results:

- $(1+\mu)$-approximate(PTAS) for *line separable discrete unit disk cover*(LSDUDC) problem in $O((m^{3(1+\frac{1}{\mu})} n \log n)$ time $(0 < \mu \leq 1)$.
- $(9+\epsilon)$-approximate, $O(\max(m^{3(1+\frac{6}{\epsilon})} n \log n, m^6 n))$ algorithm for DUDC problem using the above PTAS for LSDUDC, where $0 < \epsilon \leq 6$.
- $(9+\epsilon)$-approximate, $O(\max(m^{3(1+\frac{6}{\epsilon})} n \log n, m^6 n))$ algorithm for *rectangular region cover*(RRC) problem using the above algorithm for DUDC problem, where $0 < \epsilon \leq 6$.
- 2.25-approximate result for RRC problem in reduced radius setup.

Discrete Unit Disk Cover(DUDC)

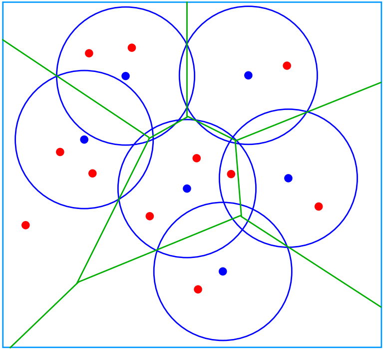
Feasibility Test



- Feasibility test can be done in $O(m \log m + n \log m)$ time.

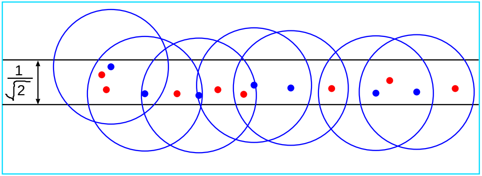
Discrete Unit Disk Cover(DUDC)

Feasibility Test



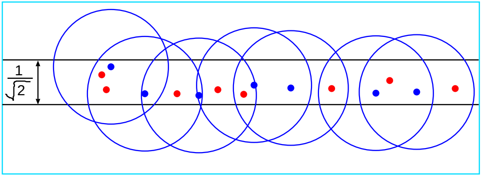
- No feasible solution.

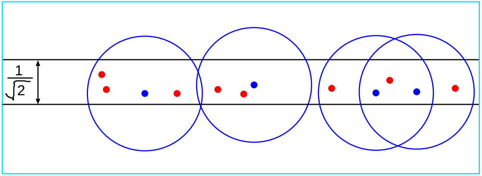
Within Strip DUDC(WSDUDC)



- All points in Q and center of the disks in D are within a horizontal strip of width $\frac{1}{\sqrt{2}}$
- Want $D \subseteq D$ of minimum cardinality which covers point in Q

Within Strip DUDC(WSDUDC)





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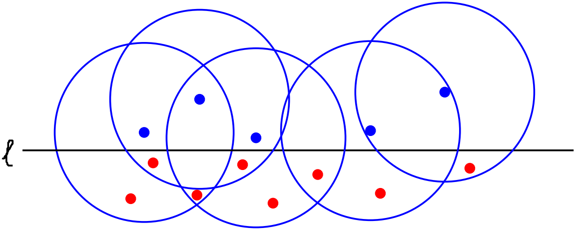
3-factor approximation algorithm in $O(m^6 n)$ time [Fraser and López-Ortiz, 2012]

Restricted Line-Separable DUDC(RLSDUDC)

- Disk centers and points in Q are separable by a line
- Formally, given sets of points $P=\{p_1,p_2,\cdots,p_m\}$ and $Q=\{q_1,q_2,\cdots,q_n\}$, where $D=\{d_1,d_2,\cdots,d_m\}$ is the set of unit disks centered at the points in P, find $D' \subseteq D$ of minimum cardinality such that all points in Q are covered by unit disks in D'.



- Optimal solution in $O(mn + n\log n)$ time [Claude et al., 2010].

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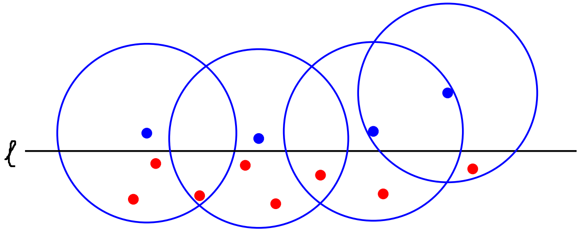
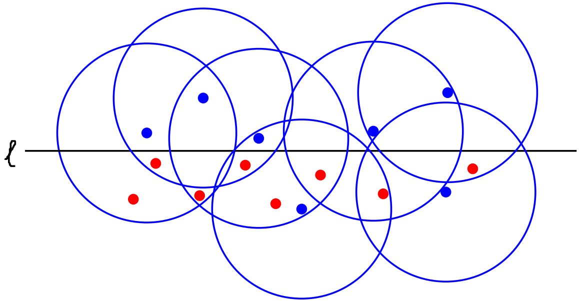


- Optimal solution in $O(mn + n\log n)$ time [Claude et al., 2010].

Line-Separable DUDC(LSDUDC)

- All points Q on one side of the line ℓ, and disks D centered both above and below ℓ.
- Want $D' \subseteq D$ of minimum cardinality.
- A 2-approximation in $O(mn + n \log n)$ time [Claude et al., 2010].



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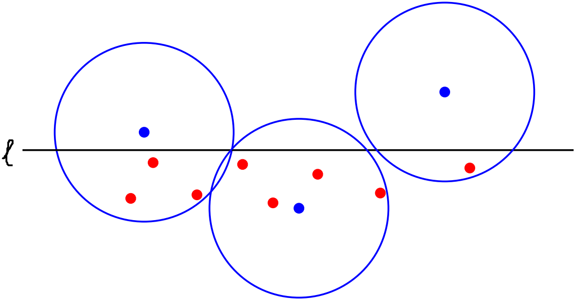
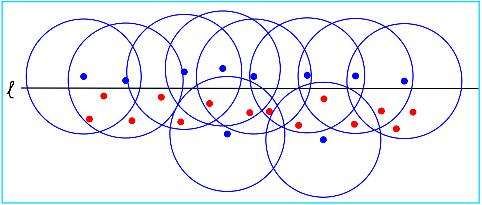


PTAS for Line-Separable DUDC(LSDUDC)



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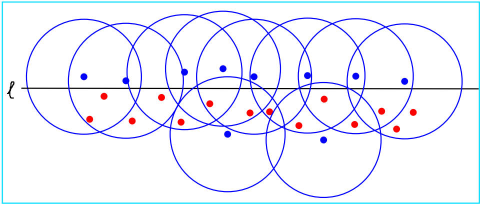

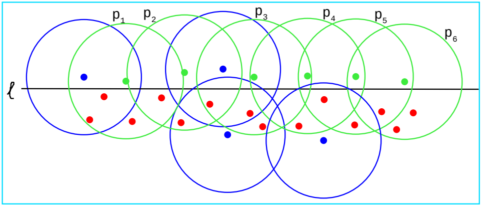

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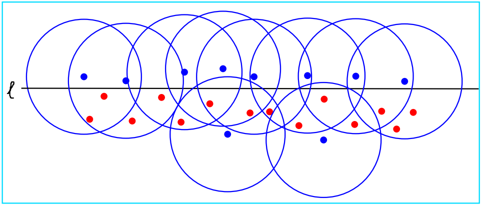

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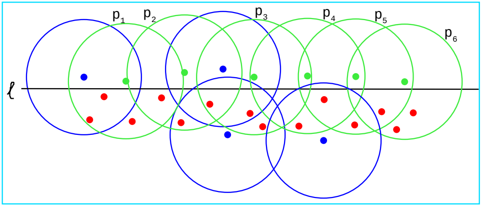

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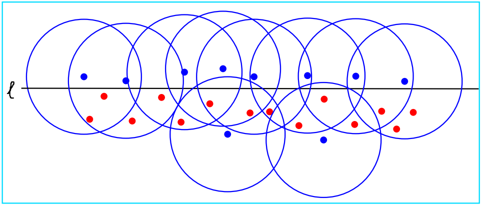

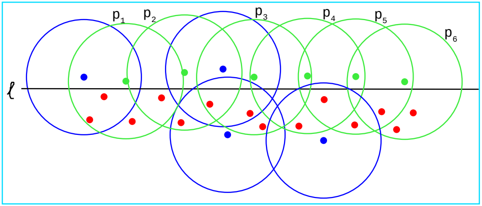

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- Let the set of red points are $P_1, P_2, \ldots P_s$ from left to right

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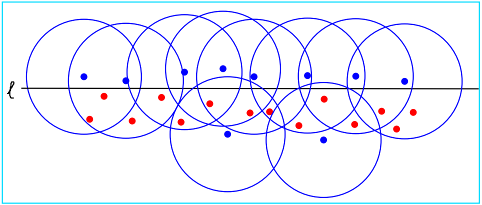

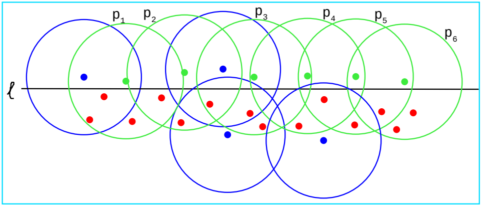

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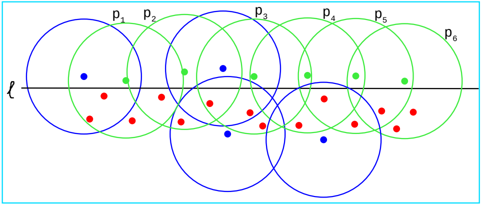
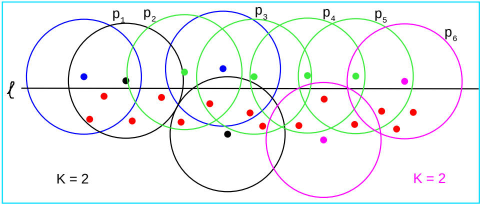
Repeat the following step:

Choose k disks to cover all points in $\cup_{i=1,2,\ldots,t} P_i$, where t is the maximum possible value

PTAS for Line-Separable DUDC(LSDUDC)





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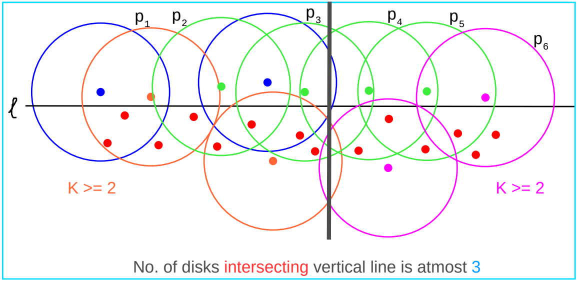
PTAS for Line-Separable DUDC(LSDUDC)

Result:



$(1 + \frac{3}{k-3})$-approximation results in $O(m^k n \log n)$ time, where m is number of disks and n is number of red points.

Solution of Discrete Unit Disk Cover (DUDC) Problem

Das et al., 2012:

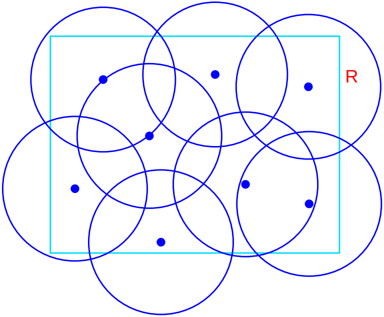
Approximation factor of DUDC problem is
$6\times$ (approximation factor of LSDUDC)
$+$
approximation factor of WSDUDC problem.

Result:

Approximation factor of DUDC problem is $6\times\ (1+\epsilon)+3=9+\mu$.

Rectangular Region Cover(RRC)

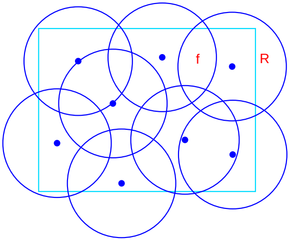
- Given a set D of m unit disks and a rectangular region R such that $R \subseteq \cup_{d \in D} d$, the objective of RRC problem is to choose a minimum cardinality set $D^{**}(\subseteq D)$ such that $R \subseteq \cup_{d \in D^{**}} d$.

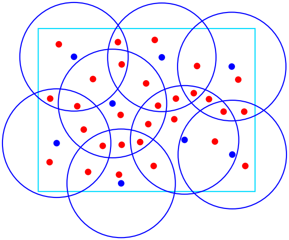


Approximation Algorithm for RRC Problem

Definitions and Notations

- A sector f inside R is a maximal region formed by the intersection of a set of disks.
- F - set of all sectors (inside R) formed by D and $|F|=O(m^2)$.
- Construct a set of points T such that there is a point $p \in T$ corresponding to each sector $f \in F$ and $p \in f \in F$, and $|T|=|F|=O(m^2)$.

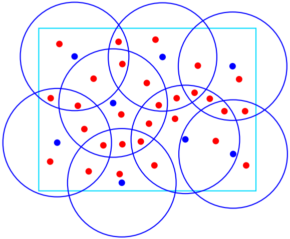




Approximation Algorithm for RRC Problem

Result:

RRC(R,D) $\Longrightarrow$ DUDC(T,D) & $n = O(m^2)$.



The RRC problem has $(9+\epsilon)$-factor approximation algorithm with running time $O(\max(m^{6(1+\frac{3}{\epsilon})} \log m, m^8))$ for $0 < \epsilon \leq 6$.

RRC Problem in Reduce Radius Setup

Definitions and Notations

- Given a set D of unit disks and a rectangular region R such that R is covered by the disks in D after reducing their radius to $(1-\delta)$, the objective is to choose a minimum cardinality set $D^{**}(\subseteq D)$ whose union covers R.
- Place a grid with cells of size $\nu \times \nu$ over the region R, where $\nu = \sqrt{2}\delta$ for $0 < \delta < 1$, and snap the center of each disk $d \in D$ to the closest vertex of the grid and set its radius to $(1-\delta)$.
- Let D' be the set of disks with radius $(1-\delta)$ after snapping their centers.
- Let R' be a square of size 4×4 on the plane contained in R.
- A disk $d \in D'$ dominates another disk $d' \in D'$ with respect to the region R' if $d \cap R' \supseteq d' \cap R'$.

RRC Problem in Reduce Radius Setup

Definitions and Notations

• Construct a set $D_{RS}(\subseteq D')$ such that any disk $d \in D'$ and $d \notin D_{RS}$ can not participate in the optimal solution for covering the region R' by D'.

RRC Problem in Reduce Radius Setup

Result:

- If $d \in D'$ and $d \notin D_{RS}$, then d can not participate in the optimal solution for covering R' by minimum number of disks in D'.
- $|D_{RS}| \leq \frac{16}{\nu^2} + \frac{20}{\nu}$
- In the reduce radius setup, the RRC problem has an 2.25-factor approximation algorithm with running time $O(q2^{\frac{16}{\nu^2}+\frac{20}{\nu}})$, where q is the minimum number of squares of size 4×4 covering R.

Conclusion

We have proposed (i) a PTAS for Line Separable DUDC problem and (ii) $(9+\epsilon)$-factor approximation algorithm for DUDC problem.
(iii) a $(9+\epsilon)$-factor approximation algorithm for RRC problem and (iv) 2.25-factor approximation algorithm for RRC problem in reduce radius setup.

THANK YOU