[Griffiths] 7.8, 7.11, 7.16, 7.22, 7.26, 7.38, 7.41,7.52

[Griffiths] 8.2, 8.4, 9.9, 9.10, 9.12, 9.13, 9.15,

Answers

G7.8 (a) Flux through the square loop $= \frac{\mu_0 I a}{2\pi} \ln\left(\frac{s+a}{s}\right)$.
(b) emf $= \frac{\mu_0 I a^2 v}{2\pi s(s+a)}$. Current flows counterclockwise in the figure.
(c) emf $= 0$.

G7.11 Emf in loop $\mathcal{E} = Blv$. Current $I = Blv/R$. Upward force on loop $= IlB = B^2l^2v/R$. Then the equation of motion is

$$m\frac{dv}{dt} = mg - \frac{B^2l^2v}{R}.$$

Upon solving,

$$v = \frac{mgR}{B^2l^2}\left(1 - e^{-\alpha t}\right)$$

where $\alpha = \frac{B^2l^2}{mR}$.

G7.16 (a) Suppose current in the inside wire is in $\hat{k}$ direction. The magnetic field is circumferential. Thus electric field is in $\hat{k}$ direction.
(b) $\mathbf{E}(s,\phi,z) = \frac{\mu_0 I_0 \omega}{2\pi}\sin(\omega t)\ln(a/s)\hat{k}$.

G7.22 Flux through single turn $= \mu_0 n I \pi R^2$. In one unit length n such turns. The total flux $= \mu_0 n^2 I \pi R^2$. Self Inductance $= \mu_0 n^2 \pi R^2$

G7.26 (a) Inductance per unit length $= \mu_0 n^2 \pi R^2$. Energy per unit length $W = \frac{1}{2}LI^2 = \mu_0 n^2 \pi R^2 I^2/2$.
(b) $W = \frac{1}{2}\oint(\mathbf{A}\cdot\mathbf{I})dl$. Now, $A = (\mu_0 n I R/2)\hat{\phi}$ on the surface. Then $W = \frac{1}{2}((\mu_0 n I R/2)I(2\pi R)$
(c) $W = \frac{1}{2\mu_0}\int B^2 d\tau$. Now, $B = \mu_0 n I$ inside and zero outside. Take a volume with one unit length along the axis. Then $W = \frac{1}{2\mu_0}(\mu_0 n I)^2 \pi R^2$.

G7.38 The figure shows a hemispherical surface. The current through the surface is $I = \int J.da = \sigma \int E.da$. Here σ is the conductivity of the matierial. The surface can be converted a closed Gaussian surface by closing it in the conductor, where E is zero. Thus $I = \sigma Q_{enc}/\epsilon_0$. Where $Q_{enc} = \int \sigma_e da$. However, (From eq 3.77) $\sigma_e = 3\epsilon_0 E_0 \cos\theta$. Here $E_0 = V_0/d$. Thus $I = 3\sigma\pi V_0 a^2/d$.

G7.41 The potential:

$$\begin{aligned} V_i(s,\phi) &= \textstyle\sum_{k=1}^{\infty} c_k s^k \sin(k\phi) \qquad && s < a \\ V_o(s,\phi) &= \textstyle\sum_{k=1}^{\infty} b_k s^{-k} \sin(k\phi) \qquad && s > a \end{aligned}$$

Cosines will not contribute since the potential function is odd on the surface.
At $s = a$, $V_i = V_o = V_0\phi/2\pi$. Thus

$$\begin{aligned} c_k &= \frac{V_0}{2\pi^2 a^k}\int_{-\pi}^{\pi}\phi\sin(k\phi)\,d\phi = \frac{V_0}{2\pi^2 a^k}\left[-\frac{2\pi}{k}(-1)^k\right] \\ b_k &= \frac{V_0 a^k}{2\pi^2}\int_{-\pi}^{\pi}\phi\sin(k\phi)\,d\phi = \frac{V_0 a^k}{2\pi^2}\left[-\frac{2\pi}{k}(-1)^k\right] \end{aligned}$$

then

$$\begin{aligned} V_i(s,\phi) &= -\tfrac{V_0}{\pi}\textstyle\sum_{k=1}^{\infty}\frac{1}{k}\left(-\frac{s}{a}\right)^k\sin(k\phi) &= \frac{V_0}{\pi}\tan^{-1}\left(\frac{s\sin\phi}{a+s\cos\phi}\right) \\ V_o(s,\phi) &= -\tfrac{V_0}{\pi}\textstyle\sum_{k=1}^{\infty}\frac{1}{k}\left(-\frac{a}{s}\right)^k\sin(k\phi) &= \frac{V_0}{\pi}\tan^{-1}\left(\frac{a\sin\phi}{s+a\cos\phi}\right) \end{aligned}$$

G7.52 A point on the lower loop: $\mathbf{r}_1 = (b\cos\phi_1, b\sin\phi_1, 0)$and a point on upper loop: $\mathbf{r}_2 = (a\cos\phi_2, a\sin\phi_2, z)$. Then

$$\begin{aligned} r^2 &= (\mathbf{r}_1 - \mathbf{r}_2)^2 \\ &= a^2 + b^2 + z^2 - 2ab\cos(\phi_1 - \phi_2) \\ &= \frac{ab}{\beta}[1 - 2\beta\cos(\phi_1 - \phi_2)] \end{aligned}$$

$d\mathbf{l}_1 = bd\phi_1\hat{\phi}_1$; $d\mathbf{l}_2 = ad\phi_2\hat{\phi}_2$. Then $d\mathbf{l}_1\cdot d\mathbf{l}_2 = ab\,d\phi_1 d\phi_2\,\hat{\phi}_1\cdot\hat{\phi}_2 = ab\,d\phi_1 d\phi_2\,\cos(\phi_1-\phi_2)$.

$$M = \frac{\mu_0}{4\pi}\oint\oint\frac{d\mathbf{l}_1\cdot d\mathbf{l}_2}{r} = \frac{\mu_0}{4\pi}\frac{ab}{\sqrt{ab/\beta}}\int\int\frac{\cos(\phi_1-\phi_2)}{\sqrt{1-2\beta\cos(\phi_1-\phi_2)}}d\phi_1 d\phi_2$$

With some simplification

$$M = \frac{\mu_0}{4\pi}\sqrt{ab\beta}\int_0^{2\pi}\frac{\cos u}{\sqrt{1-2\beta\cos u}}du$$

If a is small compared to b and z, then $\beta << 1$. Use Binomial theorem to obtain results given in the book.

G8.2 Discussed in class.

G8.4 Let the positive charge be at $(0,0,a)$ and the negative charge at $(0,0,-a)$. The two regions are $z > 0$ and $z < 0$. To find the force on the positive charge we will consider the region $z > 0$. The Maxwells stress tensor is given by $T_{ij} = \epsilon_0(E_iE_j - \delta_{ij}E^2/2)$, since this is electrostatic case with no magnetic field. Thus on $z = 0$ surface, $E(r,\pi/2,\phi) = (0,0,E_z)$ Where $E_z = \frac{2qa}{4\pi\epsilon_0(r^2+a^2)^{3/2}}$

$$T = \begin{pmatrix} -\epsilon_0 E_z^2/2 & 0 & 0 \\ 0 & -\epsilon_0 E_z^2/2 & 0 \\ 0 & 0 & \epsilon_0 E_z^2/2 \end{pmatrix}$$

Now the force on the upper region is given by $F = \oint_s T.da$. Since $da = -rdrd\phi\hat{k}$, the force is

$$\begin{aligned} F &= -\int T_{zz} rdrd\phi \\ &= -\frac{1}{2}\left(\frac{2qa}{4\pi\epsilon_0}\right)^2 \epsilon_0 \int \frac{rdrd\phi}{(r^2+a^2)^3} \\ &= -\frac{q}{4\pi\epsilon_0}\frac{1}{4a^2} \end{aligned}$$

G9.9 (a)

$$\begin{aligned} \mathbf{E}(x,y,z,t) &= \hat{\mathbf{z}} E_0 \cos(\frac{\omega}{c}x + \omega t) \\ \mathbf{B}(x,y,z,t) &= \hat{\mathbf{y}} \frac{E_0}{c} \cos(\frac{\omega}{c}x + \omega t) \end{aligned}$$

(b) $\mathbf{k} = \frac{\omega}{\sqrt{3}c}(1,1,1)$ and $\mathbf{n} = \frac{1}{\sqrt{2}}(\hat{\mathbf{x}} - \hat{\mathbf{z}})$, Thus $\mathbf{k} \times \mathbf{n} = \frac{1}{\sqrt{6}}(-\hat{\mathbf{x}} + 2\hat{\mathbf{y}} - \hat{\mathbf{z}})$.

$$\begin{aligned} \mathbf{E}(x,y,z,t) &= E_0 \cos\left(\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right)\left(\frac{\hat{\mathbf{x}} - \hat{\mathbf{z}}}{\sqrt{2}}\right) \\ \mathbf{B}(x,y,z,t) &= \frac{E_0}{\sqrt{6}c} \cos\left(\frac{\omega}{\sqrt{3}c}(x+y+z) - \omega t\right)(-\hat{\mathbf{x}} + 2\hat{\mathbf{y}} - \hat{\mathbf{z}}) \end{aligned}$$

G9.10 Pressure $P = I/c = 4.3 \times 10^{-6}\mathrm{N/m}^2$.

G9.12 $\mathbf{E}(x,y,z,t) = \hat{\mathbf{x}} E_0 \cos(\frac{\omega}{c}z - \omega t)$ and $\mathbf{B}(x,y,z,t) = \hat{\mathbf{y}}\frac{E_0}{c}\cos(\frac{\omega}{c}z - \omega t)$. Thus,

$$T_{xx} = \epsilon_0\left(E_xE_x - \frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\frac{1}{2}B^2\right) = \frac{1}{2}\left(\epsilon_0 E^2 - \frac{1}{\mu_0}B^2\right) = 0$$

Simillarly for $T_{yy} = 0$. And

$$T_{xx} = \epsilon_0\left(-\frac{1}{2}E^2\right) + \frac{1}{\mu_0}\left(-\frac{1}{2}B^2\right) = -\frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right) = -u$$

G9.13

G9.15 If the given equation is true for all x, the derivative of the equation is also true at all x. Then

$$\begin{aligned} A + B &= C && \text{put x=0} \\ aA + bB &= cC && \text{differentiate and put x=0} \\ a^2A + b^2B &= c^2C && \text{differentiate twice and put x=0} \end{aligned}$$

For non-trivial solution for A, B and C, the determinant of coefficients must be zero. This quickly gives $a = b$. In addition all A, B and C must be nonzero gives $b = c$.