[Griffiths] 5.8, 5.13, 5.15, 5.18, 5.24, 5.25, 5.27, 5.29, 5.36, 5.37

[Jackson 5.1] Starting with the differential expression

$$d\mathbf{B} = \frac{\mu_0 I}{4\pi} d\mathbf{l}' \times \frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}$$

for the magentic induction at the point P with coordinates $\mathbf{x}$ produced by an increment of current $I\,d\mathbf{l}'$ at $\mathbf{x}'$, show explicitly that for a closed loop carrying current I the magnetic induction at P is

$$\mathbf{B} = \frac{\mu_0 I}{4\pi}\nabla\Omega$$

where Ω is the solid angle subtended by the loop at the point P. This corresponds to a magentic scalar potential, $\Phi_M = -\mu_0 I\Omega/4\pi$. The sign convention for the solid angle is that Ω is positive if the point P views the inner side of the surface spanning the loop, that is if a unit normal$\mathbf{n}$ to the surface is defined by the direction of the current flow via the right hand rule, Ω is positive if $\mathbf{n}$ points away from the point P and negative otherwise.

[Jackson 5.3] A right circular solenoid of finite length L and radius a has N turns per unit length and carries a current I. Show that the mangetic induction on the cylinder axis in the limit $NL \to \infty$ is

$$B_z = \frac{\mu_0 NI}{2}(\cos\theta_1 + \cos\theta_2)$$

where the angles are defined in the figure.

[Jackson 5.6] A cylindrical conductor of radius a has a hole of radius b bored parallel to, and centered a distance d from, the cylinder axis $(d + b < a)$. The current density is uniform throughout the remaining metal of the cylinder and is parallel to the axis. Use Ampere's law and principle of superposition to find the magnitude and the direction of the magnetic flux density in the hole.

[Jackson 5.8] A localized cylindrically symmetric current distribution is such that the current flows only in the azimuthal direction; the current density is function of r and θ: $\mathbf{J} = \hat{\mathbf{\Phi}}J(r,\theta)$. The distribution is hollow in the sense that there is a current free region near the origin as well as outside.

1. Show that the magnetic field can be derived from the azimuthal component of the vector potential, with a multipole expansion

$$A_\phi(r,\theta) = \frac{\mu_0}{4\pi}\sum_L m_L r^L P_L^1(\cos\theta)$$

in the interior and

$$A_\phi(r,\theta) = -\frac{\mu_0}{4\pi}\sum_L \mu_L r^{-L-1} P_L^1(\cos\theta)$$

outside the current distribution.

2. Show that the internal and external multipole moments are

$$m_L = -\frac{1}{L(L+1)} \int d^3x \, r^{-L-1} P_L^1(\cos\theta) \, J(r,\theta)$$

and

$$\mu_L = -\frac{1}{L(L+1)} \int d^3x \, r^L P_L^1(\cos\theta) \, J(r,\theta)$$

[Jackson 5.9] The two circular coils of radius a and separation b of problem 5.7 can be described in cylindrical coordinates by the current density

$$\mathbf{J} = \hat{\Phi} I \delta(\rho - a)[\delta(z - b/2) + \delta(z + b/2)]$$

1. Using the formalism of Problem 5.8, calculate the internal and external multipole moments for $L = 1, \ldots, 5$.
2. Using the internal multipole expansion of the problem 5.8, write down explicitly an expression for B_z on the z axis and relate it to the answer of problem 5.7b