

1. Griffiths: 3.12, 3.14, 3.15
2. Jackson: 2.13, 2.14, 2.17, 2.20, 2.23

## Some Answers

### Griffiths (3.12)

$$\Phi(x, y) = \frac{8V_0}{\pi} \sum_{j=0}^{\infty} \frac{1}{4j+2} \exp\left(-\frac{(4j+2)\pi x}{a}\right) \sin\left(\frac{(4j+2)\pi y}{a}\right) \quad (1)$$

### Griffiths (3.14)

$$\Phi(x, y) = \frac{4V_0}{\pi} \sum_{\text{odd } n}^{\infty} \frac{\sinh(n\pi x/a) \sin(n\pi y/a)}{\sinh(n\pi b/a)} \quad (2)$$

### Griffiths (3.15)

$$\Phi(x, y) = \frac{16V_0}{\pi^2} \sum_{\text{odd } m} \sum_{\text{odd } n} \frac{\sinh(\sqrt{m^2 + n^2}\pi z/a) \sin(m\pi x/a) \sin(n\pi y/a)}{\sinh(\sqrt{m^2 + n^2}\pi)} \quad (3)$$

**Jackson (2.13)** (Unfortunately I chose wrong geometry. Change  $\phi \rightarrow \phi - \pi/2$  to get Jackson's answer). For  $\rho < b$ , the general solution is

$$\Phi(\rho, \phi) = a_0 + \sum_{n=1}^{\infty} \rho^n (a_n \sin(n\phi) + b_n \cos(n\phi)) \quad (4)$$

Boundary conditions:

$$\Phi(b, \phi) = V_1 \quad 0 \leq \phi \leq \pi/2 \quad (5)$$

$$= V_2 \quad \pi/2 \leq \phi \leq \pi \quad (6)$$

Then,  $a_0 = (V_1 + V_2)/2$ ,  $a_n = 0$  for all even  $n > 0$ ,  $a_n = 2(V_1 - V_2)/(\pi nb^n)$  for all odd  $n > 0$ .  $b_n = 0$  for all  $n$ . Thus,

$$\Phi(\rho, \phi) = (V_1 + V_2)/2 + \frac{2(V_1 - V_2)}{\pi} \sum_{n=0}^{\infty} \frac{1}{2n+1} \left(\frac{\rho}{b}\right)^{2n+1} \sin((2n+1)\phi) \quad (7)$$

$$= (V_1 + V_2)/2 + \frac{2(V_1 - V_2)}{\pi} \operatorname{Im} \sum_{n=0}^{\infty} \frac{1}{2n+1} \left(\frac{\rho}{b} e^{i\phi}\right)^{2n+1} \quad (8)$$

$$= (V_1 + V_2)/2 + \frac{(V_1 - V_2)}{\pi} \operatorname{Im} \ln \left( \frac{1 + \frac{\rho}{b} e^{i\phi}}{1 - \frac{\rho}{b} e^{i\phi}} \right) \quad (9)$$

$$= (V_1 + V_2)/2 + \frac{(V_1 - V_2)}{\pi} \tan^{-1} \left( \frac{2\rho b \sin \phi}{b^2 - \rho^2} \right) \quad (10)$$

And the charge density is

$$\sigma(\phi) = -\epsilon_0 \frac{(V_1 - V_2)}{\pi b \cos \phi} \quad (11)$$

**Jackson (2.14)**

**Jackson (2.17)** Use following identities to prove the result:

$$\delta(\mathbf{r} - \mathbf{r}') = \frac{1}{\rho} \delta(r - r') \delta(\phi - \phi') \quad (12)$$

$$\delta(\phi - \phi') = \frac{1}{2\pi} \sum_{-\infty}^{\infty} e^{im(\phi - \phi')} \quad (13)$$

To find  $g_m$  when  $m = 0$ , choose

$$g_0(\rho, \rho') = A \quad \rho < \rho' \quad (14)$$

$$= B \ln \rho \quad \rho' < \rho \quad (15)$$

But then  $g_0$  does not vanish as  $\rho \rightarrow \infty$ . When can you use this Green's Function?

**Jackson (2.20)** Use the Green's Function derived above.