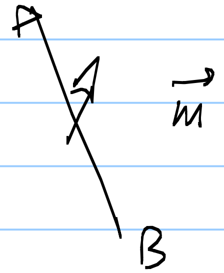
Magnetic Materials

Note Title 4/8/2009

Electrons: Spin $\longrightarrow$ permanent dipole moment

$$\vec{\tau} = \vec{m} \times \vec{B}$$



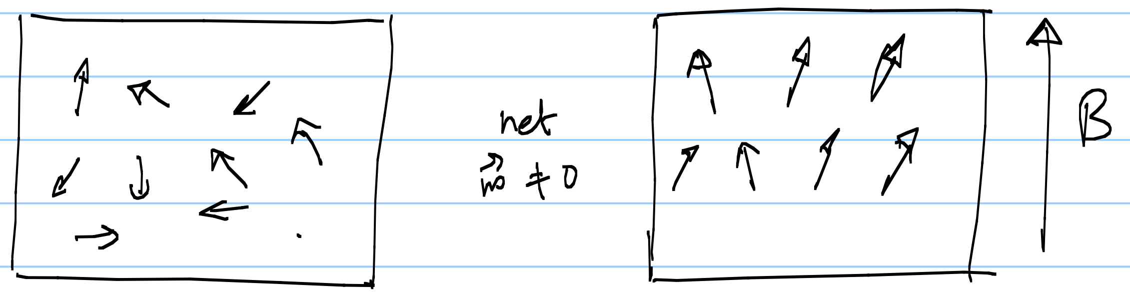
Paramagnetic

Diamagnetic

Ferromagnetic

Paramagnets

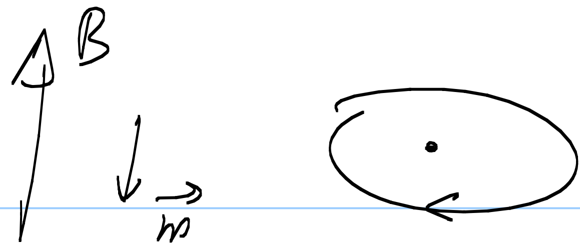
net $\vec{m} = 0$



net $\vec{m} \neq 0$

$B = 0$ $\qquad$ $B \neq 0$

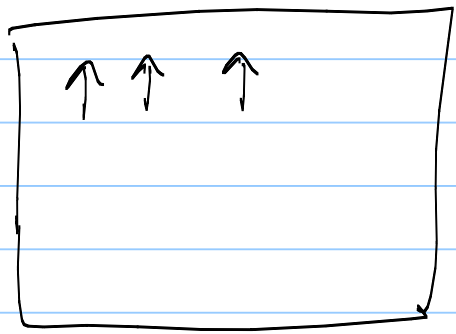
Diamagnet (Quantum effect)



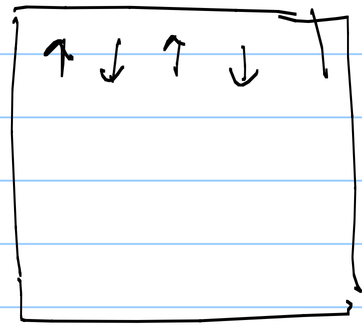
Ferromagnets (Quantum)

$\uparrow \uparrow$ 10^3 small Coulomb + Pauli principle

$B = 0$



Ferromagnet



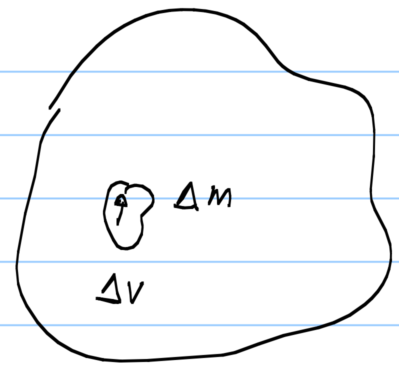
Antiferromagnet

net $\vec{m} = 0$

Magnetization

Magnetized Material

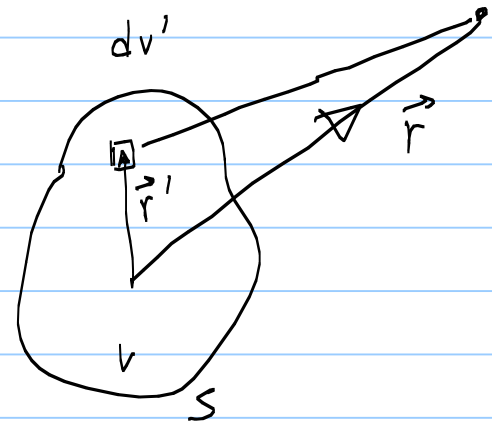
$$\vec{M} = \lim_{\Delta V \to 0} \frac{\Delta \vec{m}}{\Delta V}$$



Magnetic Field due Magnetized materials

$\vec{M}(\vec{r}\,')$

In dv' $\qquad d\vec{m} = \vec{M}\, dv'$



$$\vec{A}_{dip} = \frac{\mu_0}{4\pi} \int_V \frac{d\vec{m} \times (\vec{r} - \vec{r}\,')}{|r - r'|^3}$$

$$A = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$= \frac{\mu_0}{4\pi} \int \frac{M(\vec{r}') \times (\bar{r}-\bar{r}')}{|r-r'|^3} dv'$$

$$\nabla' \times \left(M(\vec{r}') \cdot \frac{1}{|r-r'|} \right) = \frac{1}{|r-r'|} \nabla' \times M(\vec{r}') - M(\vec{r}') \times \left(\nabla' \frac{1}{|r-r'|} \right)$$

$$\nabla \times (\bar{a} \quad f) = f(\bar{\nabla} \times \vec{a}) - \vec{a} \times (\vec{\nabla} f)$$

$$= \frac{\nabla' \times M}{|r-r'|} - \frac{M(\vec{r}') \times (r-r')}{|r-r'|^3}$$

$$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \left[\int_v \frac{\nabla' \times M}{|r-r'|} dv' - \int_v \nabla' \times \left(\frac{M(r')}{|r-r'|} \right) dv' \right]$$

$$= \frac{\mu_0}{4\pi} \int_v \frac{J_b \, dv'}{|r-r'|} + \oint_s \frac{M \times \hat{n}}{|r-r'|} ds'$$

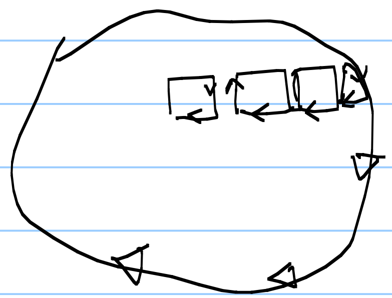
Define

Bound current density $= \nabla \times M$

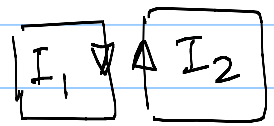
Bound surface current density $= \vec{M} \times \hat{n}$

$$\vec{A} = \frac{\mu_0}{4\pi} \int \frac{\bar{J}\, dv'}{|r - r'|}$$

$$= \frac{\mu_0}{4\pi} \int \frac{k\, ds'}{|r - r'|}$$

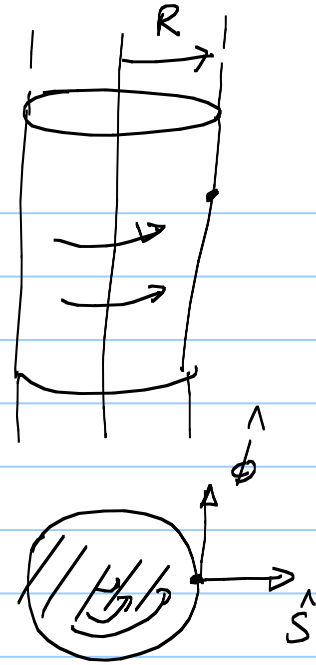


Uniformmagnetization



$I_2 > I_1$

$M(\vec{r}) < \vec{m}(r')$



<u>Ex</u> Infinite cylindrical sample

$$\vec{M} = ks^2\hat{\phi}$$

$$\vec{J}_b = \vec{\nabla}\times M$$

$$K_b = \vec{M}\times\hat{n}\Big|_{s=R} = kR^2\hat{\phi}\times\hat{s}$$

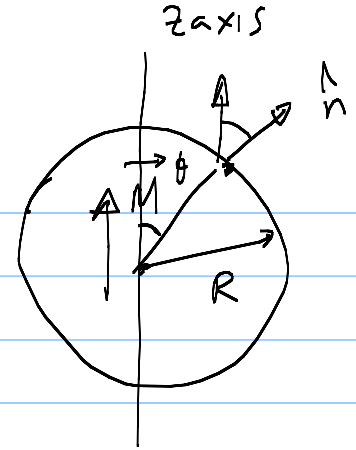
$$= kR^2(-\hat{z})$$

$$\vec{J}_b = \hat{z}\frac{1}{s}\frac{\partial}{\partial s}(sM_\phi) = 3ks\,\hat{z}$$

Net Current Surface $= K_b\times 2\pi R = -kR^3(2\pi)\,\hat{z}$

Volum $= \int_s J\cdot da = \hat{z}\int_0^R\int_0^{2\pi} 3ks\cdot s\,ds\,d\phi = kR^3(2\pi)\,\hat{z}$

Ex Uniformly Magnetized sphere



$$J_b = \nabla \times M = 0$$

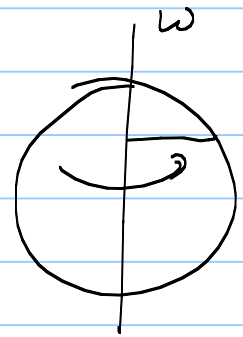
$$K_b = \vec{M} \times \hat{n} = M \sin\theta \, \hat{\phi}$$

$$\vec{M} = M \hat{z}$$

$$\vec{B}_{inside} = + \frac{2}{3} \mu_0 M$$

$$\vec{B}_{outside} = \frac{\mu_0}{4\pi} \left(\frac{4}{3} \pi R^3 M \right) \left[2\cos\theta \, \hat{r} + \sin\theta \, \hat{\theta} \right]$$

Charged sph. shell

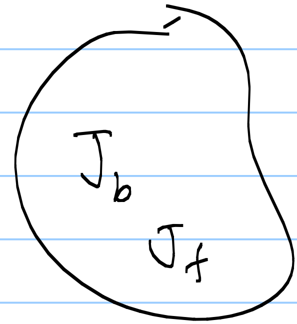


$$K = \sigma \vec{v}$$

$$= \frac{\sigma \omega R}{\sin\theta} \hat{\phi}$$

Ampere's Law in Magnetic Materials

$$J = J_b + J_f$$



$$\frac{1}{\mu_0}(\bar{\nabla} \times \bar{B}) = \nabla \times M + J_f$$

$$\nabla \times \underbrace{\left(\frac{1}{\mu_0} B - M \right)}_{H\text{-field}} = J_f$$

$$\boxed{\bar{\nabla} \times \bar{H} = J_f}$$

$$\boxed{\oint_C \vec{H} \cdot d\bar{l} = I_{f,\,\text{encl.}}}$$

$\vec{\nabla} \cdot B = 0$ $\nabla \cdot \vec{H}$ need not be zero.

need to know $\vec{M}(\bar{H})$ or $\vec{M}(\bar{B})$ } Constitutive Relations

$\chi \sim 10^{-6} \sim 10^{-7}$

Linear Materials (Paramagnetic, Diamagnetic)

$\chi \sim 10^{-4} \sim 10^{-5}$

$$\vec{M} = \underbrace{\chi_m}\, \vec{H}$$

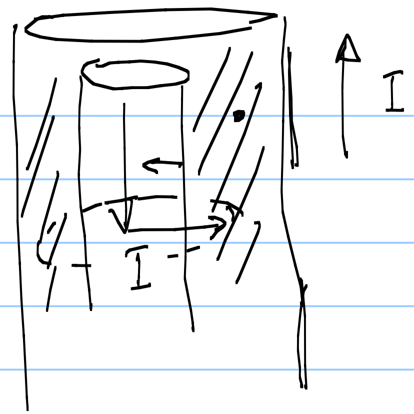
χ_m: Magnetic Susceptibility

$$\vec{B} = \mu_0 (H + M) = \mu_0 (1 + \chi_m) \vec{H}$$
$$= \underbrace{\mu}\, \vec{H}$$

μ: Permeability of material

Ex Inner core (Radius a), $\vec{I} = I(-\hat{z})$

Outer Shell (Radius b), $\vec{I} = I\,\hat{z}$



Hollow space filled with linear magnetic material, susceptibility χ_m

Amperial loop is circular $\perp$ to z axis

$$\oint \bar{H} \cdot d\bar{l} = I_{f,encl} = 0 \qquad s < a$$

$$2\pi s H = 0 \Rightarrow H = 0$$

$$2\pi s H' = -I \qquad a < s < b$$

$$\vec{H} = -\frac{I}{2\pi s}\hat{\phi}$$

$$\vec{H} = 0 \qquad b < s$$

Magnetization $\vec{M} = 0$ $\quad r < a$

$= 0$ $\quad b < r$

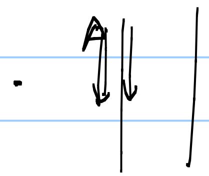
$$\vec{M} = \chi_m \vec{H} \qquad a < r < b$$

$$= -\frac{\chi_m I}{2\pi s}\hat{\phi}$$

Bound currents

$$J_b = \bar{\nabla} \times \bar{M} = \frac{1}{s}\frac{\partial}{\partial s}(s M_\phi)\hat{z} = 0$$

$$K_b\Big|_{s=a} = M \times \hat{n}\Big|_{s=a} = -\frac{\chi_m I}{2\pi a}\hat{\phi} \times (-\hat{s})$$



$$= \frac{\chi_m I}{2\pi a}(-\hat{z})$$

$$K_b\Big|_{s=b} = M \times \hat{n}\Big|_{s=b} = \frac{\chi_m I}{2\pi b}\hat{z}$$

$$\vec{B} = \mu_0(H + M)$$

$$= \mu_0(1 + \chi_m)\vec{H} = -\frac{(1+\chi_m) I}{2\pi s}\hat{\phi} \qquad a < r < b$$

Electromotive Force

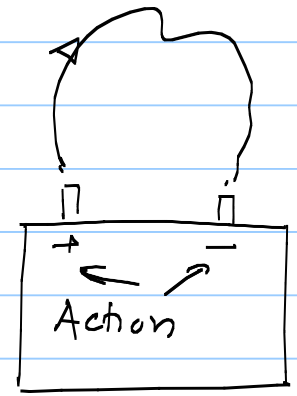
Define EMF

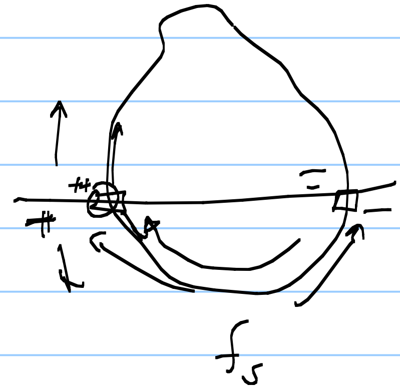
$\vec{f}_s$: force per unit charge

$$\boxed{\mathcal{E} = \oint \vec{f}_s \cdot \vec{dl}}$$

Some sense, work done / unit charge

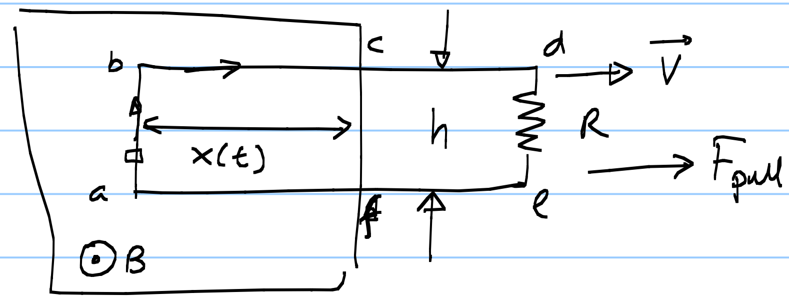
$$\oint \vec{E} \cdot \vec{dl} = 0$$



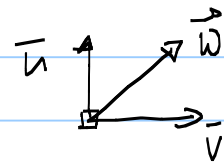


Motional EMF : Emf developed in a wire when moved thro' magnetic field.

Ex

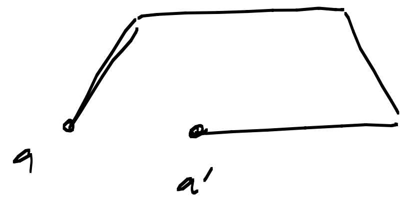


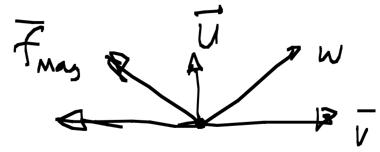
$f_s = f_{mag}$



$$\boxed{\mathcal{E} = \oint \bar{f}_{mag} \cdot d\bar{l}} = \left[\int_a^b + \int_b^c + \int_f^a \right] \bar{f}_{mag} \cdot d\bar{l}$$

$$= \int_a^b (vB)\, dl = vBh$$





$$F_{pull} = uB$$

$$\frac{\text{work done}}{\text{unit charge}} = \int \vec{F}_{pull} \cdot \bar{dl} = Bvh \qquad \text{(Verify)}$$

Flux of B thro' circuit

$$\phi(t) = Bh\,x(t)$$

$$\frac{d\phi}{dt} = -Bhv = -\mathcal{E}$$

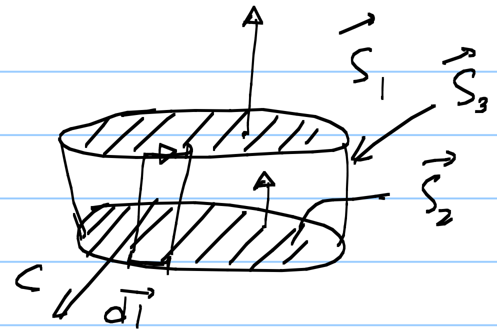
In general $\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$ flux rule

Proof $B(\vec{r})$ is ind of time

$S_1 - S_2 + S_3$ is closed surface

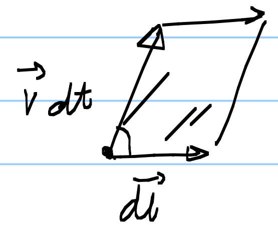
At $t = t_0 + dt$

At $t = t_0$



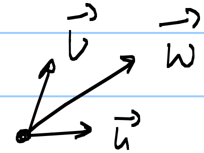
$$\int_{S_1 - S_2 + S_3} \bar{B} \cdot d\bar{s} = 0$$

$$\int_{S_1} B \cdot \vec{ds} - \int_{S_2} B \cdot \vec{ds} = - \int_{S_3} B \cdot d\bar{s}$$



$$\underline{\phi(t_0 + dt) - \phi(t_0)} = - \int_{S_3} \vec{B} \cdot \vec{ds}$$

on S_3 $\quad \vec{ds} = - \left(\vec{v} \times \vec{dl} \right) dt$



$$\Rightarrow \quad \frac{d\phi}{dt} = + \int_{S_3} \vec{B} \cdot (\vec{v} \times \vec{dl})$$

$$\vec{v} \times \vec{dl}$$
$$= \vec{w} \times \vec{dl}$$

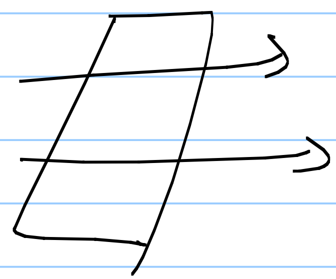
$$= \int_{S_3} (B \times \vec{w}) \cdot \vec{dl}$$

$$= - \int_{S_3} f_{mag} \cdot \vec{dl} = - \mathcal{E}$$

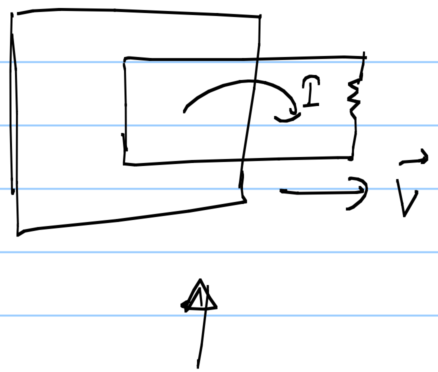
Loops

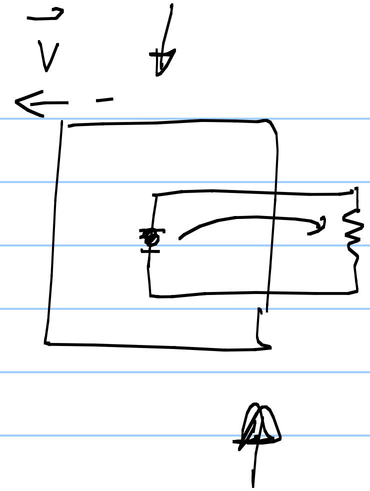
Eddy currents

difficult to calculate



Faraday's law





$$F = q\bar{v} \times \bar{B}$$

$$\bar{v} = 0$$

Changing magnetic field induces Electric field.