Magnetic Materials (E)

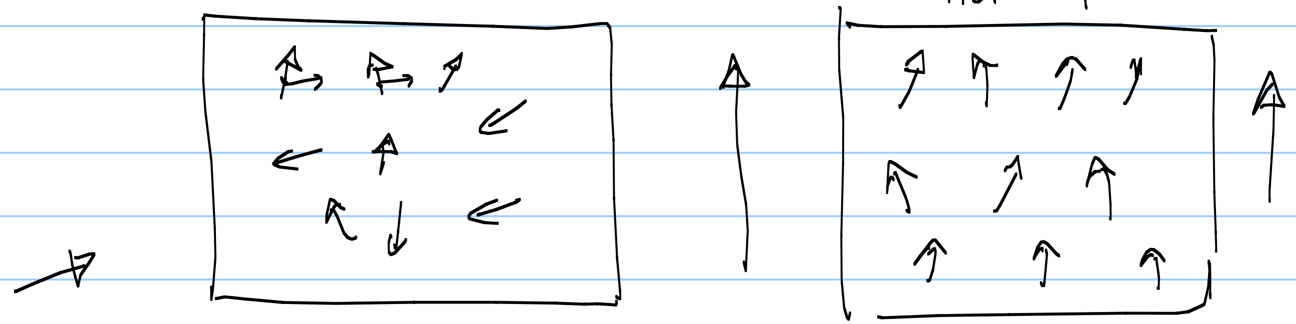
Note Title 4/8/2009

Electrons: spin $\longrightarrow$ magnetic dipole moment

Paramagnetic
Diamagnetic
Ferromagnets

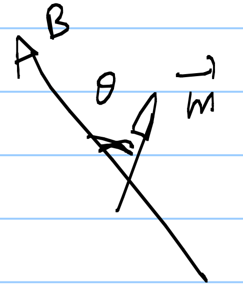
Paramagnetic

net $\bar{m} = 0$ net $\bar{m} \neq 0$



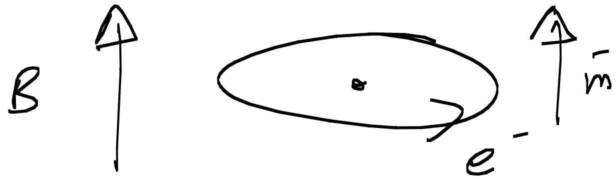
electrons are ind.

$B = 0$ $B \neq 0$

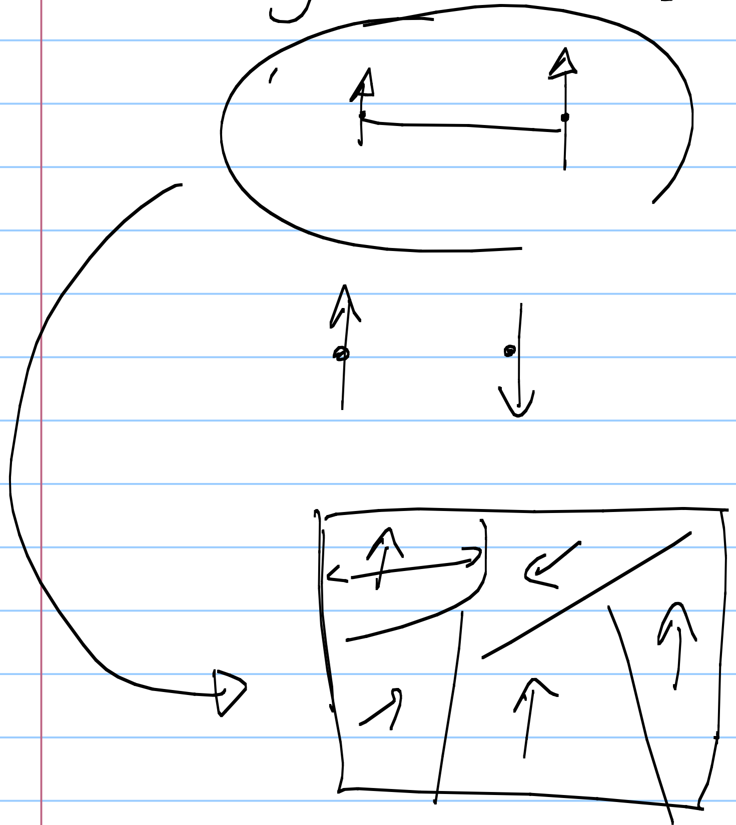


$$\vec{T} = \vec{m} \times \vec{B}$$

Diamagnets (Quantum effect)

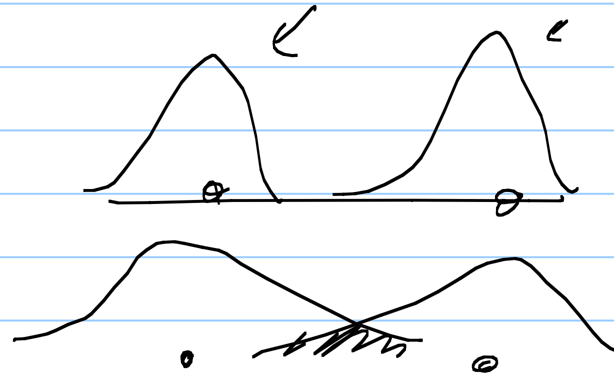


Ferromagnet (Insulating materials) & (Typical)



$\left\{ \begin{array}{l} \text{Coulomb interaction} \\ + \text{ Pauli principle} \end{array} \right\}$

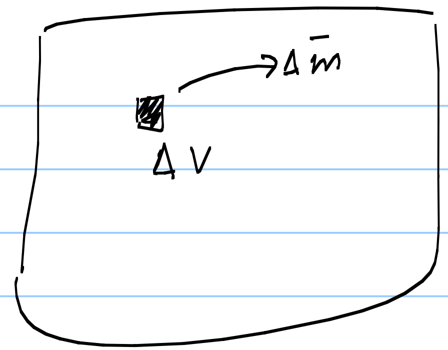
Magnetic interaction $\sim 10^{-3}$



Magnetization

$$\vec{M}(\vec{r}) = \lim_{\Delta V \to 0} \frac{\Delta \bar{m}}{\Delta V}$$

$$= \text{density of dipole moments}$$



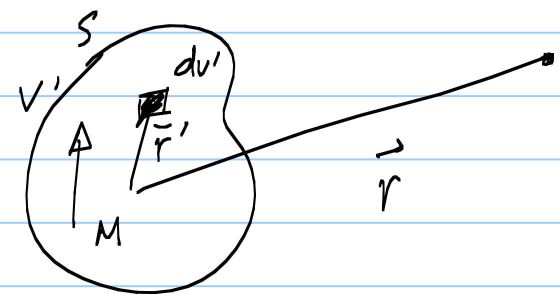
Field of Magnetic Material

Vector Potential

$$A(\vec{r}) = \frac{\mu_0}{4\pi} \int_V \frac{d\vec{m} \times (r - r')}{|r - r'|^3}$$

$$= \frac{\mu_0}{4\pi} \int_V \frac{\bar{M}(r') \times (r - r')}{|r - r'|^3} \, dv'$$

Magnetized Material



in dv', $d\vec{m}$

$$\nabla \times (\vec{F}\,\lambda) = \lambda\, \nabla \times F - \vec{F} \times \bar{\nabla}\lambda \qquad A_d = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \hat{r}}{r^2}$$

$$\nabla' \times \left(\vec{M}(\vec{r}\,') \frac{1}{|r - r'|} \right) = \frac{1}{|r - r'|} \nabla' \times M(r') \underbrace{- \vec{M}(\vec{r}\,') \times \bar{\nabla}\left(\frac{1}{|r - r'|} \right)}$$

$$- M(r') \times \frac{(\bar{r} - \bar{r}\,')}{|r - r'|^3}$$

$$A(\vec{r}) = \underbrace{\frac{\mu_0}{4\pi} \int_v \frac{\nabla' \times M}{|r - r'|} dv'}_{} - \frac{\mu_0}{4\pi} \underbrace{\int_v \nabla \times \left(\frac{M(r')}{|r - r'|} \right) dv'}_{}$$

Compare with $\quad A = \frac{\mu_0}{4\pi} \int \frac{J\, dv'}{|r - r'|} \quad \leftarrow$

define bound current density $\quad J_b = \nabla \times M$

$$\int_V \nabla \times \left(\frac{M(r')}{|r-r'|} \right) dv' = -\oint_S \frac{(M \times \hat{n})}{|r-r'|} ds \leftarrow$$

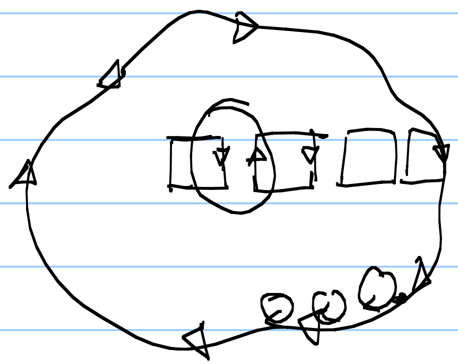
$$\text{Second term} = \frac{\mu_0}{4\pi} \oint_S \frac{(M \times \hat{n})}{|r-r'|} ds$$

Define: bound surface current density $= M \times \hat{n} = K_b$

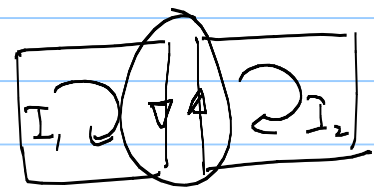
Interpretation

uniformly magnetized

$\vec{M} \perp$ to plane



Non uniform $\vec{M}$



J_b

$M_1 < M_2$

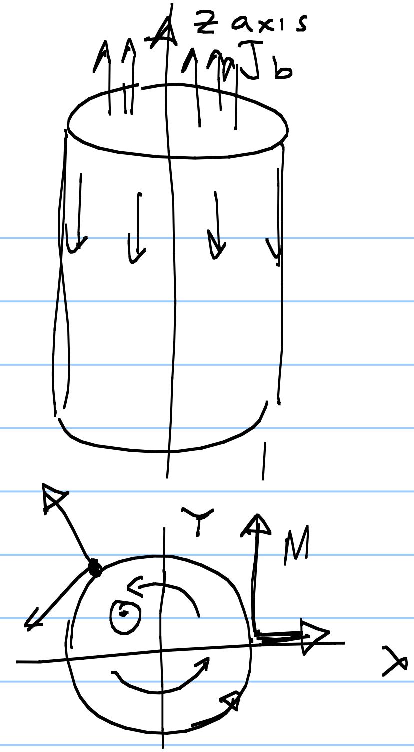
$I_1 < I_2$

Ex $\vec{M} = ks^2\hat{\phi}$

K_b : Surface current density

$= \vec{M} \times \hat{s} = kR^2\hat{\phi} \times \hat{s}$

$= kR^2(-\hat{z})$



J_b = Volume current density

$= \nabla \times M$

$= \frac{\hat{z}}{s}\frac{\partial}{\partial s}(sM_\phi) = 3ks\hat{z}$

S Net current $= K_b \cdot 2\pi R = -(2\pi)kR^3\hat{z}$

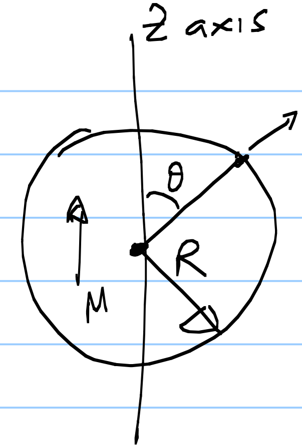
V Net current $= \int J_b \cdot d\bar{s} = \int_0^R\int_0^{2\pi}(3ks)\hat{z}\cdot\hat{z}\, s\,ds\,d\phi = (2\pi)kR^3\hat{z}$

Ex Uniformly Magnetized Sphere

$$\vec{M} = M\,\hat{z}$$

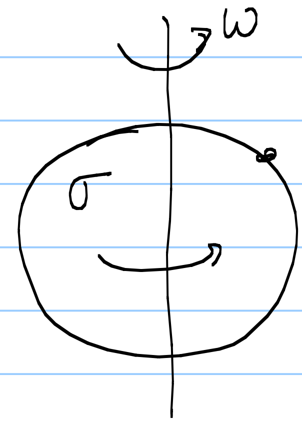
$$\vec{J}_b = \bar{\nabla} \times \bar{M} = 0$$

$$\vec{K}_b = M\,\hat{z} \times \hat{r} = M \sin\theta\,\hat{\phi}$$



$$\vec{B}_{inside} = \frac{2}{3}\mu_0 \vec{M}$$

$$\vec{B}_{outside} = \frac{\mu_0}{4\pi}\left(\frac{4}{3}\pi R^3 \vec{M}\right)\left[2\cos\theta\,\hat{r} + \sin\theta\,\hat{\theta}\right]$$

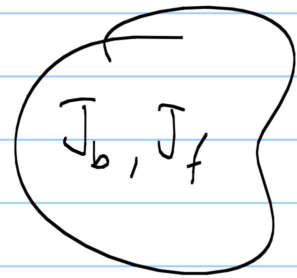


$$K = \sigma \bar{V} = \sigma \omega R \sin\theta\,\hat{\phi}$$

Compare $M \sin\theta\,\hat{\phi}$

Ampere's Law for Magnetic Materials

$$\bar{J} = \bar{J}_f + \bar{J}_b$$



$$\frac{1}{\mu_0}\bar{\nabla}\times\bar{B} = J_f + \bar{\nabla}\times\bar{M}$$

$$\vec{\nabla}\times\underbrace{\left(\frac{1}{\mu_0}B - M\right)}_{H:\ \text{field.}} = \bar{J}_f$$

$$\boxed{\nabla\times\vec{H} = \bar{J}_f}$$

$$\boxed{\oint_C \vec{H}\cdot d\bar{l} = \bar{I}_{f,enc}}$$

$\nabla \cdot B = 0$ $\quad\quad$ $\nabla \cdot \vec{H}$ may not be zero

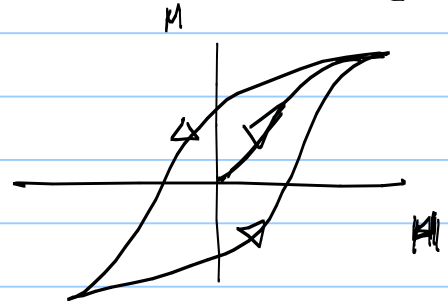
$$\nabla \cdot B = 0$$

$$\nabla \times H = J_f \qquad \vec{H} = \frac{1}{\mu_0}\bar{B} - \bar{M}$$

$\vec{M}(\vec{H})$ or $\vec{M}(\vec{B})$ $\quad$ Constitutive relations

Linear Materials

Ferromagnet



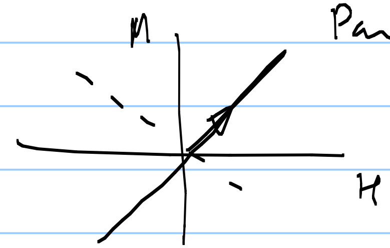
$$\vec{M} = \chi_m \vec{H}$$

χ_m: Magnetic susceptibility

$$\vec{B} = \mu_0(\vec{H} + \vec{M})$$

$$= \mu_0(1 + \chi_m)\vec{H} = \mu\vec{H}$$

Para

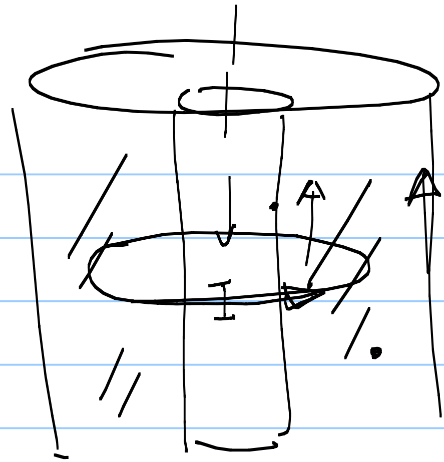


μ: Permeability of material.

Ex Inner shell, radius a, $\vec{I} = -I\hat{z}$

Outer shell, " b, $\vec{I} = I\hat{z}$

Magnetic material, χ_m



$$2\pi s H = I_{f,encl} = 0 \qquad s<a$$

$$H = 0$$

$$\vec{H} = \frac{-I}{2\pi s}\hat{\phi} \qquad a<s<b$$

$$\vec{H} = 0 \qquad b<s$$

Magnetisation

$$\vec{M} = 0 \qquad r<a$$

$$= 0 \qquad b<r$$

$$= \chi_m \vec{H} = -\frac{\chi_m I}{2\pi s}\hat{\phi} \qquad a<s<b$$

Bound currents

$$J_b = \vec{\nabla}\times\vec{M} = \frac{1}{s}\frac{\partial}{\partial s}\left(s M_\phi\right)\hat{z} = 0$$

$$K_b\Big|_{s=a} = \vec{M}\times\hat{n} = -\frac{\chi_m I}{2\pi a}\,\hat{\phi}\times(-\hat{s}) = \frac{\chi_m I}{2\pi a}(-\hat{z})$$

$$K_b\Big|_{s=b} = \frac{\chi_m I}{2\pi b}\,\hat{z}$$

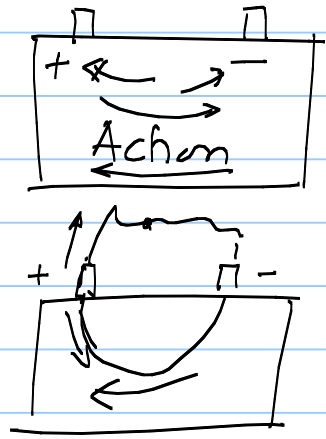
$$B = \mu_0\left(\bar{H}+\bar{M}\right) = -\frac{\mu_0(1+\chi_m)I}{2\pi s}\,\hat{\phi}$$

($\chi_m > 0$, paramagnetic)

($\chi_m < 0$, diamagnetic)

Electromotive Force

$$\int \vec{E}\cdot\vec{dl} = 0$$



$\mathcal{E} = \oint \bar{f}_s \cdot \bar{dl}$ = emf in a circuit

f_s: force per unit charge (battery)

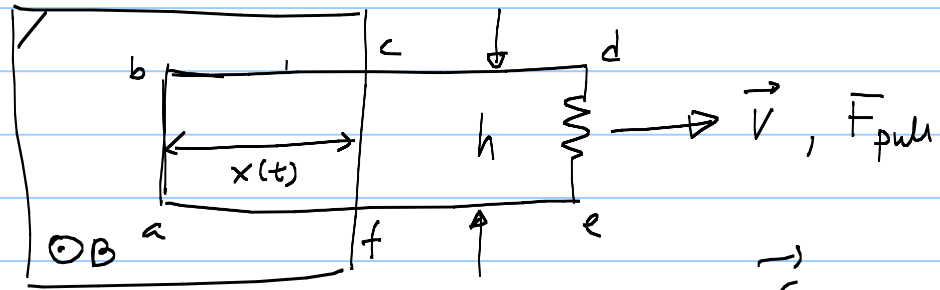
When no current flows (in battery)

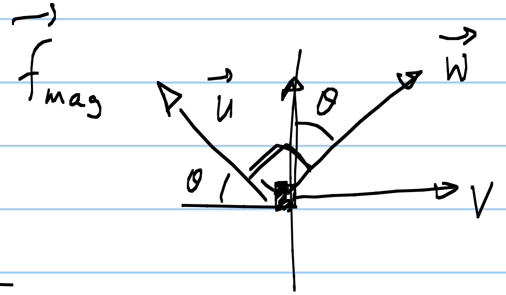
$$\vec{f}_s = -\vec{E}$$

$\mathcal{E}$ = Pot. diff bet terminals

Motional Emf : Emf generated in a wire moving in Magnetic field.

Ex

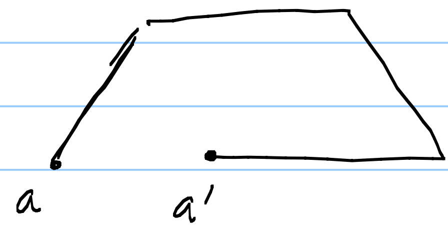




$$\boxed{\mathcal{E} = \oint \bar{f}_{mag} \cdot d\bar{l}}$$

$$= \left[\int_a^b + \int_b^c + \int_f^a \right] \bar{f}_{mag} \cdot d\bar{l}$$

$$= \int_a^b (vB)\, dl = vBh$$



$$F_{pull} = \vec{f}_{mag} \cos\theta$$

$\underline{\text{Work done}}$ F_{pull}
per unit charge

$$\int \vec{F}_{pull} \cdot dl = vBh \qquad \text{(Verify)}$$

Calculate Emf by flux rule.

$$\phi(t) = Bh\,x(t)$$

$$\frac{d\phi}{dt} = -Bhv = -\mathcal{E}$$

Emf in a circuit is given

$$\boxed{\mathcal{E} = -\frac{d\phi}{dt}}$$

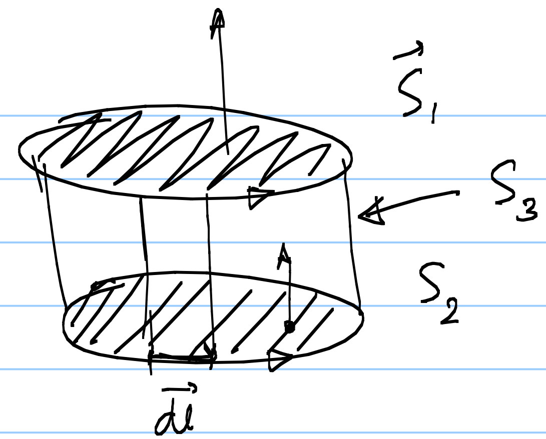
ϕ is flux thro' the circuit

Proof of flux Rule

$B(\vec{r})$ is ind. of time

At $t_0 + dt$

At t_0



$S_1 + S_3 + (-S_2)$ is closed surface

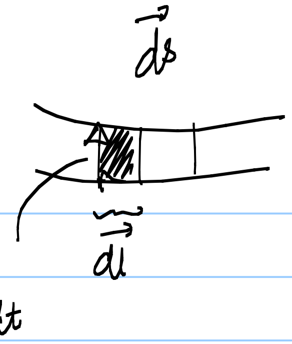
$$\oint_{S_1 - S_2 + S_3} \vec{B} \cdot \vec{ds} = 0$$

$$\int_{S_1} \vec{B} \cdot \bar{ds} - \int_{S_2} \vec{B} \cdot \bar{ds} = -\int_{S_3} \bar{B} \cdot \bar{ds}$$

$$\underbrace{\phi(t_0 + dt) - \phi(t_0)}_{d\phi} = -\int_{S_3} \bar{B} \cdot \bar{ds}$$

On S_3, $\vec{ds} = \vec{dl} \times (\vec{v}\,dt)$

$= -(\vec{v} \times \vec{dl})\,dt$

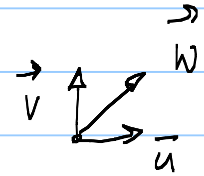


$$d\phi = \int B \cdot (\vec{v} \times \vec{dl})\, dt$$

$$\frac{d\phi}{dt} = \int B \cdot (w \times \vec{dl})$$

$$= -\int (\bar{w} \times \vec{B}) \cdot \vec{dl}$$

$$= -\int \bar{f}_{mag} \cdot \bar{dl} = -\mathcal{E}$$

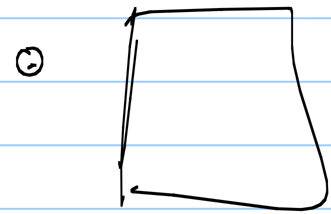


$\vec{v} \times \bar{dl} = \vec{w} \times dl$

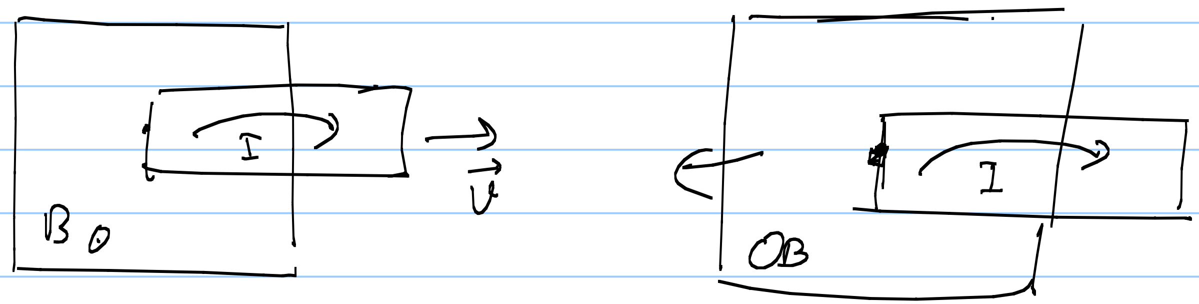
$= (\vec{v} + \vec{u}) \times \vec{dl}$

$=$

Eddy currents



Faraday's Law



$qV \times B$

$\downarrow$

0