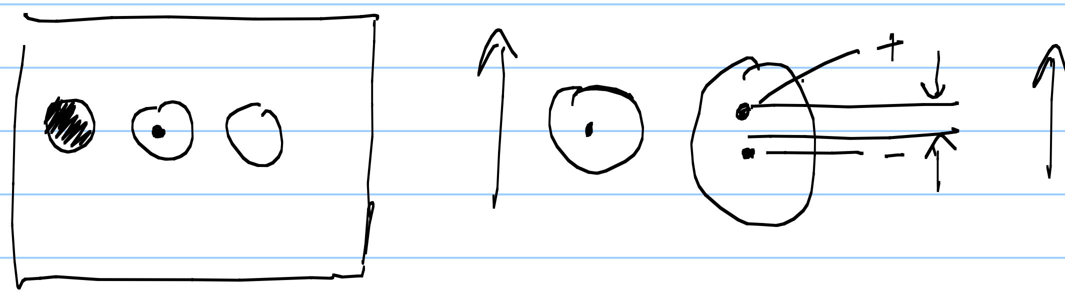
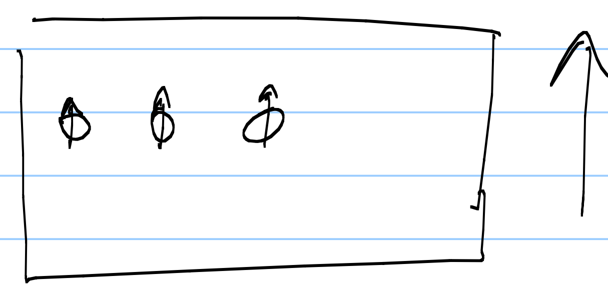
Dielectric Materials

Note Title 3/18/2009

- Conductors → Contain unlimited no of charges (both kinds)
- Insulators → Electrons bound ~~to~~ atoms
  - ↳ Atomic/Molecular properties

(a)

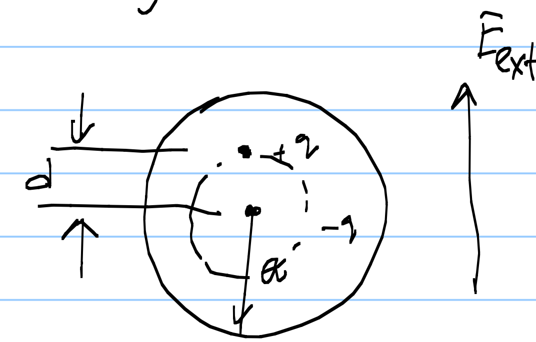




Crude Model

Nucleus a point charge : q

Electron cloud is rigid sphere with uniform charge density



Net force on nucleus

$$F_N = q\left(E_{ext} + E_e\right)$$

E_e : field of electron cloud

$$E_e = \frac{-1}{4\pi\epsilon_0 d^2} \cdot q\left(\frac{d^3}{a^3}\right)$$

In equilibrium $\quad F_N = 0$

$$\Rightarrow \qquad q\,E_{ext} = \frac{q^2 d}{4\pi\epsilon_0 a^3}$$

$$\Rightarrow \quad \vec{p} = q\vec{d} = \underbrace{(4\pi\epsilon_0 a^3)}\, \vec{E}_{ext}$$

In general, if E_{ext} is small

$$\vec{p} = \underbrace{\alpha}\, \vec{E}_{ext} \qquad \text{(empirical)}$$

Atomic Polarizability

In our model

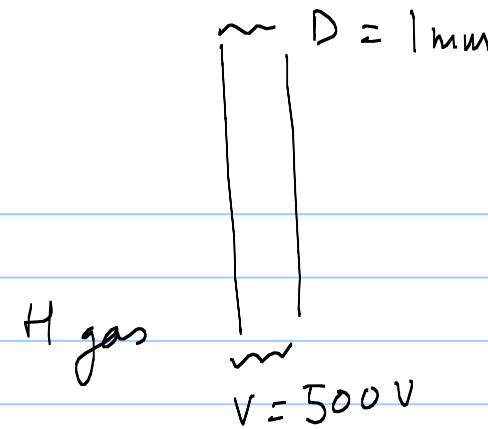
$$\alpha/(4\pi\epsilon_0) = a^3 \quad (m^3)$$

accurate upto 25% for most atoms

$$\sim 10^{-30} \; m^3$$

Example (4.1 G)

$$\alpha = (4\pi\epsilon_0)\left(0.667 \times 10^{-30}\ m^3\right)$$



$$E_{ext} = \frac{V}{D} = 5 \times 10^{5}\ \frac{V}{m}$$

$$p = qd = \alpha E_{ext}$$

$$d = \frac{\alpha E_{ext}}{e} = 2.3 \times 10^{-16}\ m$$

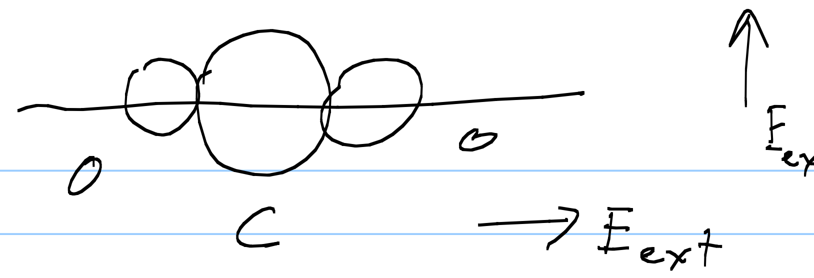
$$a = 0.79\ \mathring{A} \qquad \frac{d}{a} \sim 10^{-6}$$

$$\vec{p} = qd = 4 \times 10^{-35}\ C\text{-}m$$

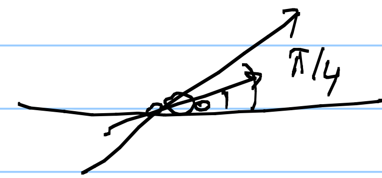
Molecules : CO_2

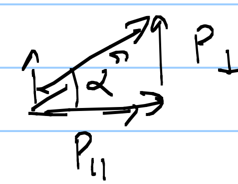
$$\alpha_{\parallel} = 4.05 \times 10^{-30}\, m^3 \,(4\pi\epsilon_0)$$

$$\alpha_{\perp} = 1.75 \times 10^{-30}\, m^3 \,(4\pi\epsilon_0)$$



$$\frac{P_{\parallel}}{P_{\perp}} = \frac{\alpha_{\parallel}}{\alpha_{\perp}} \neq \cot \alpha$$



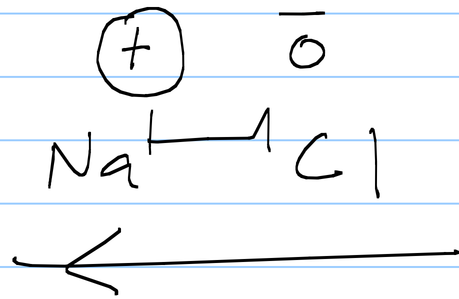


Very general case

$$\begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix} = \underbrace{\begin{bmatrix} \alpha_{xx} & \alpha_{xy} & \alpha_{xz} \\ \alpha_{yx} & \alpha_{yy} & \\ & & \end{bmatrix}}_{\text{Polarizability tensor.}} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

Material is linear
Anisotropic

(b) Polar Molecules NaCl



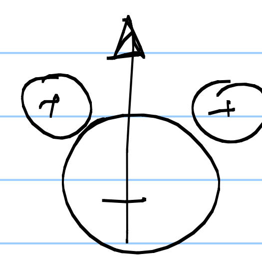
polar molecules

Bond length = 2.36 Å

$\vec{p} = q \times \text{Bond length} = 3.8 \times 10^{-29}$ C-m (gas form)

$\vec{p}_{ext} = 3 \times 10^{-29}$ C-m 80% charge transfer

Another Example H_2O



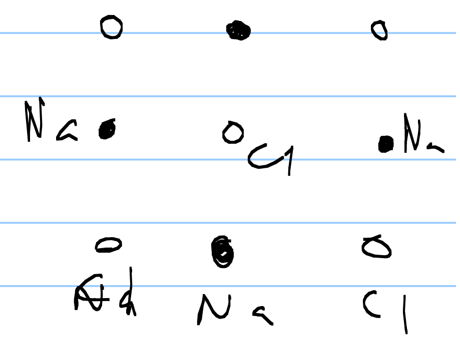
$\vec{p}_{H_2O} = 6.1 \times 10^{-30}$ C-m (room)

$(NaCl)_{solid} = 2.8Å$

At room temp

$K_{H_2O} = 88\ (0°C), 80\ (25°C), 66\ (50°C)$

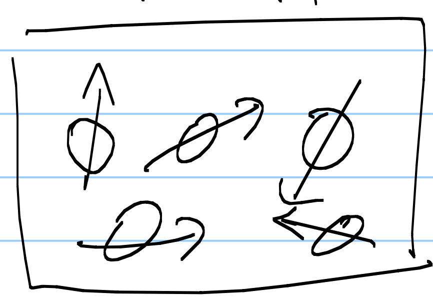
$K_{NaCl} = 6$



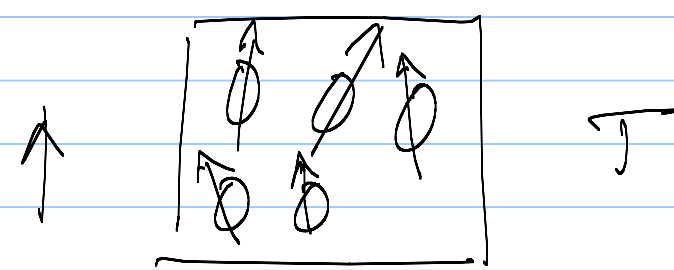
Room

When in Ext Field

No field

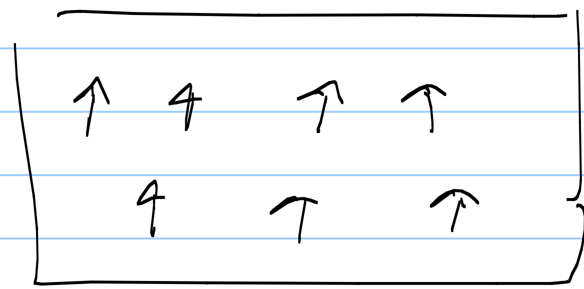


Net dipole moment $= 0$



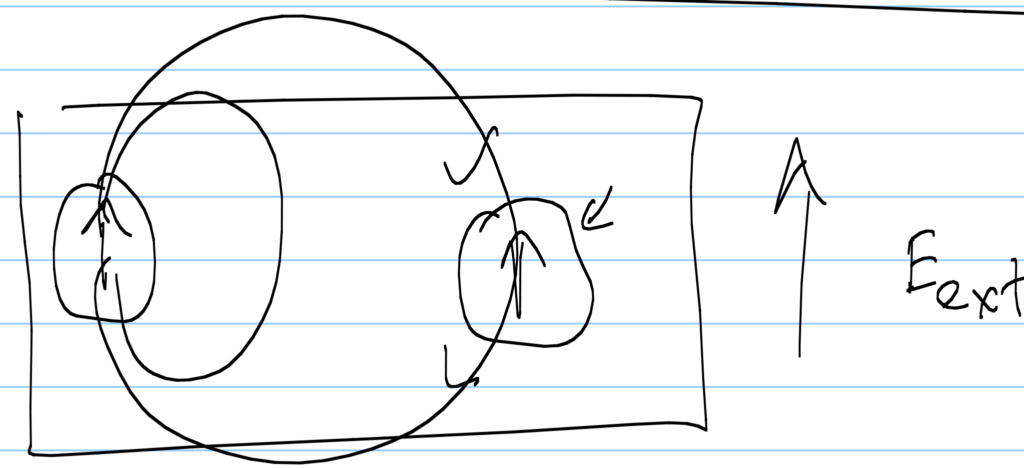
Net dipole moment $\neq 0$

Whatever be the mechanism Ext field



in ext field.

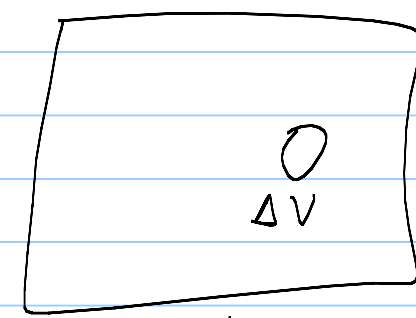
Model of insulator



$$
\begin{tikzcd}
E_{ext} \arrow[r] & \text{dipoles} \arrow[r] & \text{internal field} \arrow[ll, bend left]
\end{tikzcd}
$$

Define Polarization:

$$P = \lim_{\Delta v \to 0} \frac{\Delta p}{\Delta v}$$



Δp = dipole mom't in ΔV

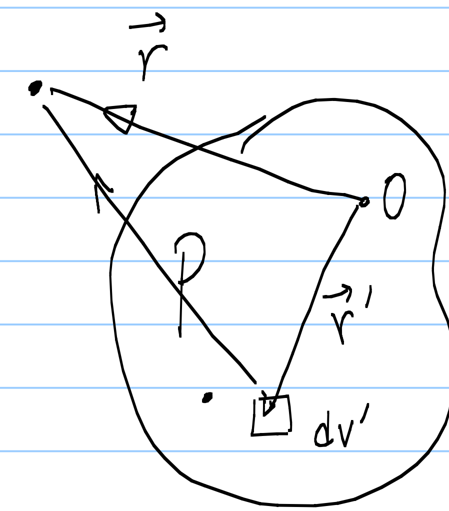
$$V = \frac{1}{4\pi\epsilon_0} \frac{p \cdot \hat{r}}{r^2}$$

Potential due to Polarized object

dipole moment in $dp' = \underline{P(\vec{r}')}\, dv'$

Potential at $\vec{r}$ due to dv'

$$dV = \frac{dv'}{4\pi\epsilon_0} \frac{\vec{P}(r')\cdot(\vec{r}-\vec{r}')}{|\vec{r}-\vec{r}'|^3}$$ ← pure dipole



Volume V

Surface S

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{P(\vec{r}')\cdot(\vec{r}-\vec{r}')}{|r-r'|^3}\, dv'$$

$$\nabla'\cdot\left(\frac{\vec{P}(\vec{r}')}{|r-r'|}\right) = \frac{1}{|\vec{r}-\vec{r}'|}\nabla'\cdot\vec{P} + \underbrace{\vec{P}\cdot\nabla'\left(\frac{1}{|\vec{r}-\vec{r}'|}\right)}$$

$$\nabla \cdot (f\vec{A}) = f\,\nabla \cdot A + \vec{A} \cdot \nabla f$$

$$\vec{p} \cdot \frac{(\bar{r} - \bar{r}')}{|\bar{r} - \bar{r}'|^3}$$

$$V(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \nabla' \cdot \left(\frac{\bar{P}(r')}{|r - r'|} \right) dv' + \underbrace{\frac{1}{4\pi\epsilon_0} \int \frac{(-\nabla' \cdot \vec{P})}{|r - \bar{r}'|} dv'}$$

σ_b

$$= \frac{1}{4\pi\epsilon_0} \int \frac{\vec{P} \cdot \hat{n}\, ds}{|r - r'|} + \frac{1}{4\pi\epsilon_0} \int \frac{\rho_b(r')}{|r - r'|} dv'$$

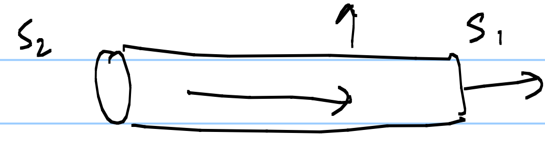
σ $\quad V = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma\, ds'}{|r - r'|}$ $\qquad \rho$ $\quad V = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(r')\, dv'}{|r - r'|}$

Bound volume charge density $\rho_b = -\bar{\nabla}\cdot\bar{P}$

Bound surface " $\sigma_b = \vec{p}\cdot\hat{n}$

$\Big\}$ Bound Charges

Example Interpretation

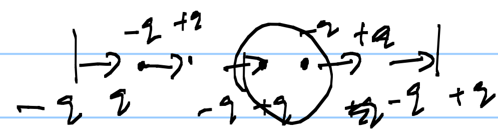


$\boxed{\rho_b = 0}$

$\sigma_b = 0$ on curved

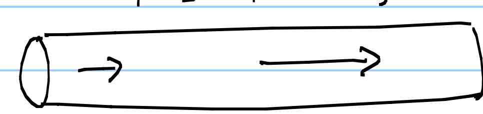
$= P$ on S_1

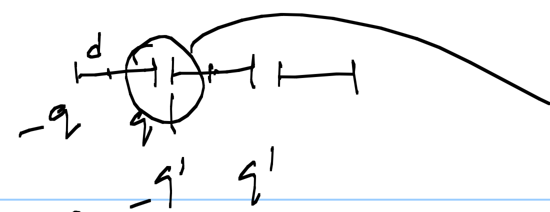
$= -P$ on S_2



$\frac{q}{A}$ $\qquad \frac{q}{(A/\cos\theta)} = \frac{q}{A}\cos\theta$

P is increasing





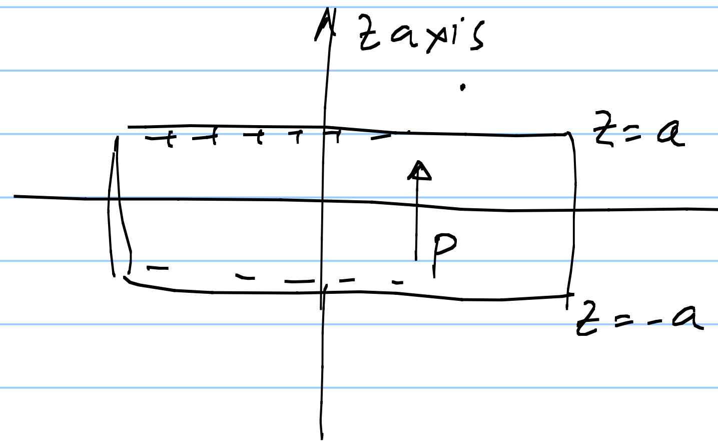
net -ve charg

$-\nabla \cdot P$

$qd = p \qquad p' = q'd$

$$p' > p \Rightarrow q' > q$$

Example:



P is uniform

$$\rho_b = -\nabla \cdot P = 0$$

$$\sigma_b \Big|_{z=a} = P$$

$$\sigma_b \Big|_{z=-a} = -P$$

$$\sigma_b \Big|_{\text{edges}} = 0$$

$$E(z) = \frac{\sigma_b}{\epsilon_0} = \frac{P}{\epsilon_0} \qquad -a < z < a$$

$$= 0 \qquad \text{otherwise}$$

Example: $\quad P(\vec{r}) = k\,\vec{r} \qquad |\vec{r}| < R$

$\quad = 0 \qquad |\vec{r}| > R$

Spherical

$$\rho_b = -\nabla\cdot P = -3k$$

$$\sigma_b\Big|_{r=R} = kR$$

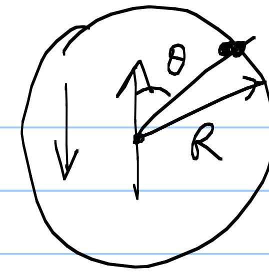
$$\text{Total Bound charge} = (-3k)\,\frac{4}{3}\pi R^3 + (kR)\cdot\underbrace{4\pi R^2}_{\text{surface area}}$$

$$= 0$$

$$E_{\text{outside}} = 0$$

$$\vec{E}_{\text{inside}} = \frac{\hat{r}}{4\pi\epsilon_0 r^2}\,(-3k)\,\frac{4}{3}\pi r^3 = -\frac{kr}{\epsilon_0}\,\hat{r}$$

Example: Uniformly Polarized.



$$P(\vec{r}) = p_0 \hat{k} \qquad |\bar{r}| < R$$

$$= 0 \qquad |\bar{r}| > R$$

Bound volume charge density $= \rho_b = 0$

" Surface " $= \sigma_b = P_0 \cos\theta$

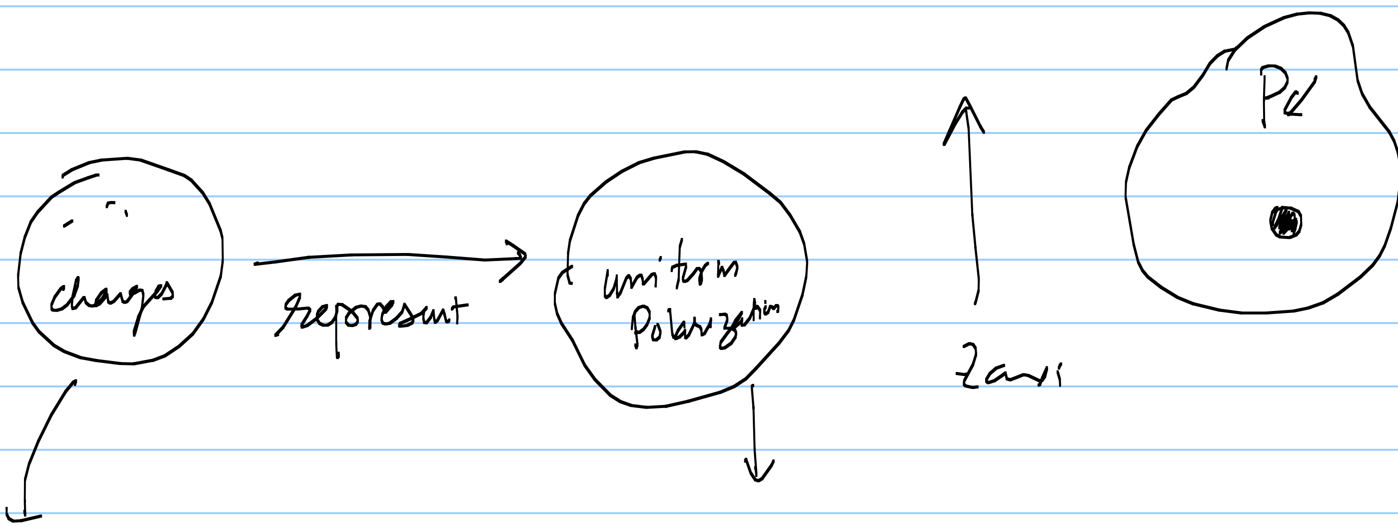
$$V(r,\theta) = \frac{P_0}{3\epsilon_0} r\cos\theta \qquad |r| < R \Rightarrow \bar{E} = -\frac{P}{3\epsilon_0}\hat{k}$$

$$= \frac{P_0 R^3}{3\epsilon_0}\frac{\cos\theta}{r^2} \qquad |r| > R$$

Electric field

(a) In Vacuum → actual electric field

(b) In material → Averaged electric field

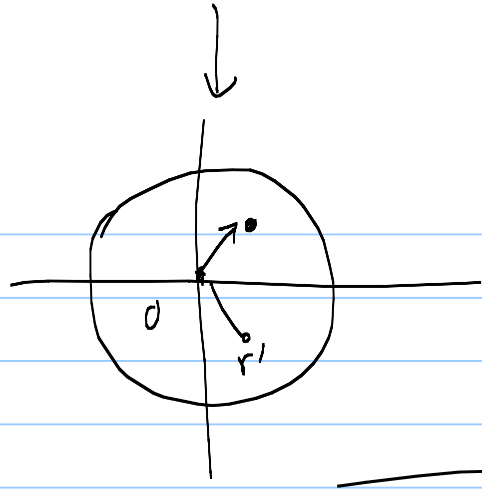


E_{ave} due to all charge inside sphere

$$E = -\frac{P}{3\epsilon_0}\hat{k}$$

q at $\vec{r}$

$$\frac{q}{V} = -\rho = \text{cons}$$



$$E_{ave} = \frac{1}{\frac{4}{3}\pi R^3}\int E(\vec{r}')\cdot dv'$$

$$= \frac{q}{V 4\pi\epsilon_0}\int \frac{(r'-r)}{|r'-r|^3}\, dv'$$

$$E(r') = \frac{q}{4\pi\epsilon_0}\frac{(r'-r)}{|r'-r|^3}$$

$$= \underbrace{\frac{1}{4\pi\epsilon_0}\int \frac{\rho(r')(r-r')}{|r-r'|^3}\, dv'}$$

= Electric field of uniform charge sphere with density $\left(-\frac{q}{V}\right)$

by Gauss Law

$$= \frac{\rho \vec{r}}{3\epsilon_0} = -\frac{q\,\vec{r}}{4\pi\epsilon_0 R^3}$$

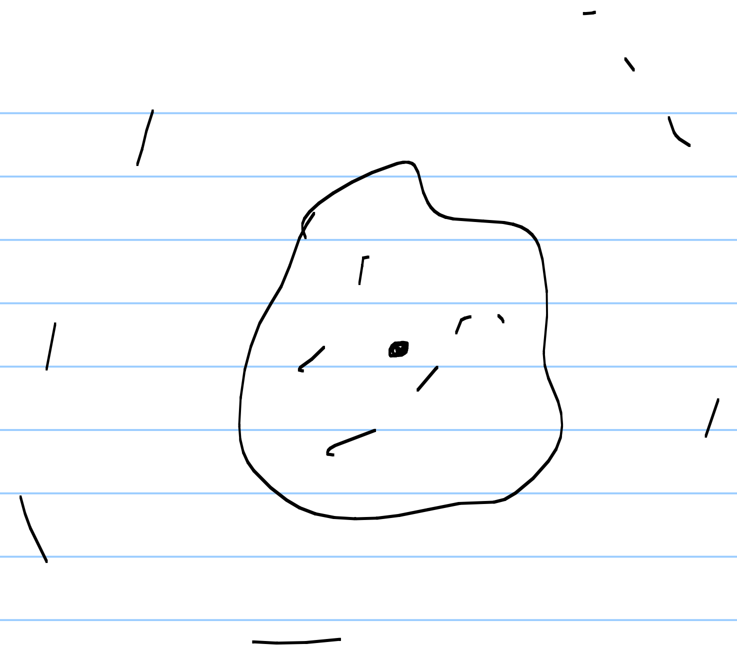
$$= -\frac{\text{net dipole moment in sphere}}{4\pi\epsilon_0 R^3}$$

net dipole mom

$$E_{ave} = -\frac{\left(p / \frac{4\pi}{3} R^3\right)}{3\epsilon_0}$$

$$= -\frac{P}{3\epsilon_0}$$

Gauss Law for Dielectrics



made up of bound charge

+ free charges

Net charge density

$$\rho = \rho_f + \rho_b$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho_f + (-\vec{\nabla} \cdot P)$$

$$\vec{\nabla} \cdot \underbrace{(\epsilon_0 E + P)} = \rho_f \Rightarrow \boxed{\nabla \cdot D = \rho_f}$$

displacement field

electric displacemen

$$D = \epsilon_0 E + P$$

Integral from $\oint D \cdot \hat{n}\, ds = Q_{f,\text{enclosed}}$.

Example: 4.15 "Frozen Polarization" No free changes

$$\vec{P} = \frac{k}{r}\hat{r} \qquad r < R$$

$$= 0 \qquad r > R$$

$$\sigma_b \Big|_{r=R} = \frac{k}{R}$$

$$\rho_b = -\frac{k}{r^2} = -\nabla \cdot P = -\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{k}{r}\right)$$

$$E = -\frac{k}{\epsilon_0 r}\hat{r} \qquad \text{inside}$$

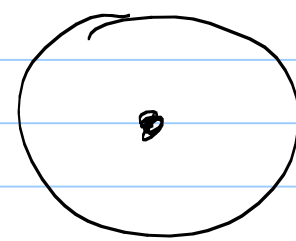
$$= 0 \qquad \text{outside}$$

$$(4\pi r^2) E = \frac{1}{\epsilon_0}\int \rho_b \, dv = \frac{1}{\epsilon_0}\int -\frac{k}{r'^2}\, 4\pi r'^2 \, dr'$$

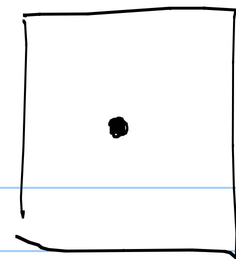
$$4\pi r^2 D = 0 \qquad D = 0$$

$$\epsilon_0 E = D - P = -P$$

ρ_f = point



$$D = \frac{1}{4\pi}\frac{q}{r^2}$$



$$D = \frac{1}{4\pi}\frac{q}{r^2} \; ?$$