ED-ME111

L-2

Dr. Poonam Kumari

PLANE CURVES

Designing certain objects require shapes made out of curves which satisfy specific mathematical equations to meet the functional, aesthetic and ergonomic requirement of the object.

Basic Construction skills

- How to divide a line into given number of equal parts
- How to divide a circle into given number of equal parts
- How to find the center of an arc or a circle
- How to draw a normal and a tangent to the arc/ circle
- How to construct regular polygons
- How to construct arc/ circle tangent to line(s)/ arc(s)

ENGINEERING CURVES

Part- I {Conic Sections}

ELLIPSE

1.Concentric Circle Method

2.Rectangle Method

3.Oblong Method

4.Arcs of Circle Method

5.Rhombus Metho

6.Basic Locus Method
(Directrix – focus)

PARABOLA

1.Rectangle Method

2 Method of Tangents
(Triangle Method)

3.Basic Locus Method
(Directrix – focus)

HYPERBOLA

1.Rectangular Hyperbola
(coordinates given)

2 Rectangular Hyperbola
(P-V diagram - Equation given)

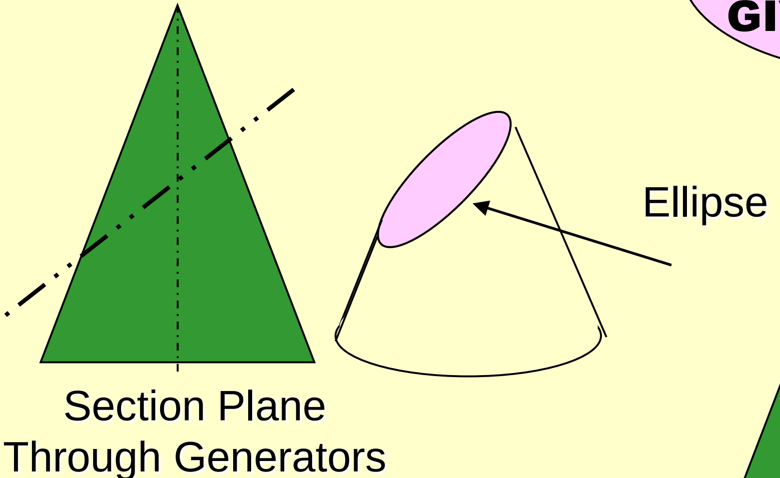
3.Basic Locus Method
(Directrix – focus)

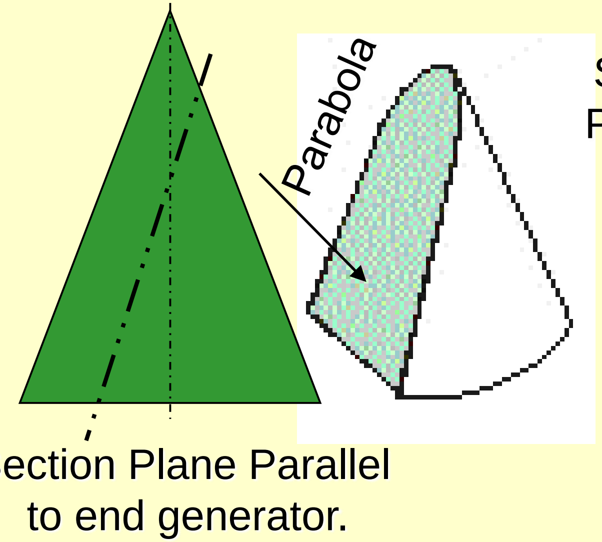
Methods of Drawing Tangents & Normals To These Curves.

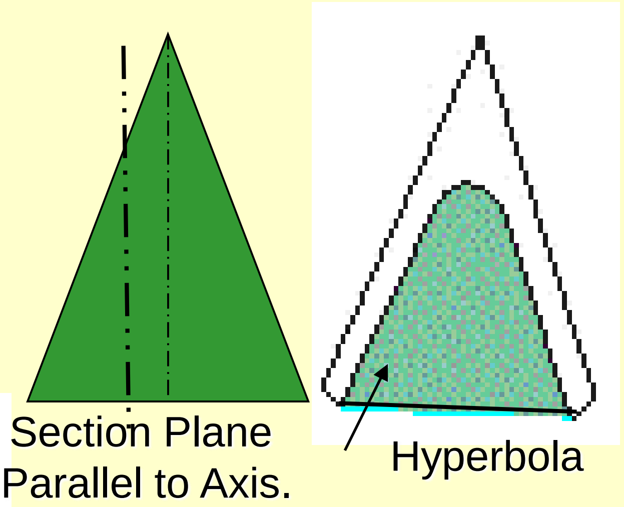
CONIC SECTIONS

ELLIPSE, PARABOLA AND HYPERBOLA ARE CALLED CONIC SECTIONS BECAUSE
THESE CURVES APPEAR ON THE SURFACE OF A CONE WHEN IT IS CUT BY SOME TYPICAL CUTTING PLANES.

OBSERVE ILLUSTRATIONS GIVEN BELOW..







COMMON DEFINATION OF ELLIPSE, PARABOLA & HYPERBOLA:

These are the loci of points moving in a plane such that the ratio of it's distances from a ***fixed point*** And a ***fixed line*** always remains constant.

The Ratio is called **ECCENTRICITY. (E)**

A) For Ellipse E<1

B) For Parabola E=1

C) For Hyperbola E>1

Refer Problem nos. 6. 9 & 12

SECOND DEFINATION OF AN ELLIPSE:-

It is a locus of a point moving in a plane such that the SUM of it's distances from TWO fixed points always remains constant.

{And this ***sum equals*** to the length of ***major axis***.}

These TWO fixed points are FOCUS 1 & FOCUS 2

Refer Problem no.4

Ellipse by Arcs of Circles Method.

Loci of points

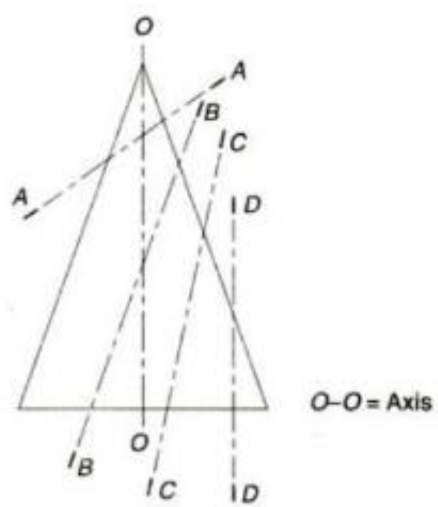
Loci of the point is the path taken by the point as it moves in space.

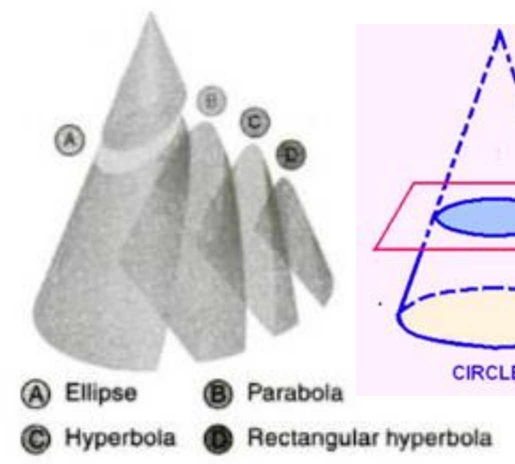
E.g.

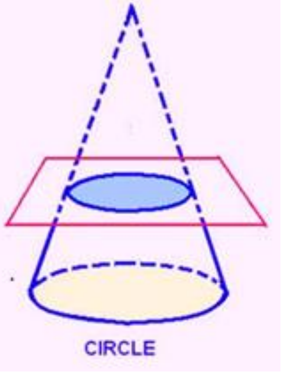
1. If a point moves on a plane in such a way that it keeps its distance constant from a fixed straight line, the locus of the point is a straight line parallel to and at a distance equal to the given distance from the given fixed line.
2. If a point moves on a plane in such a way that it keeps its distance constant from a given fixed point, the locus is a circle with the given fixed point as the center and the radius equal to the distance from the fixed point.
3. If a point moves on a plane in such a way that it keeps its distance constant from a fixed circular arc, the locus of the point is a circular arc with the same center and a radius equal to the radius of the given arc plus or minus the fixed distance.

Conic sections (conics)

- Curves formed by the intersection of a plane with a right circular cone. Right circular cone is a cone that has a circular base and the axis is inclined at 90^o to the base and passes through the center of the base.
- Conic sections are always "smooth". More precisely, they never contain any inflection points.
- Important for many applications, such as aerodynamics, civil engineering, mechanical engineering, etc.





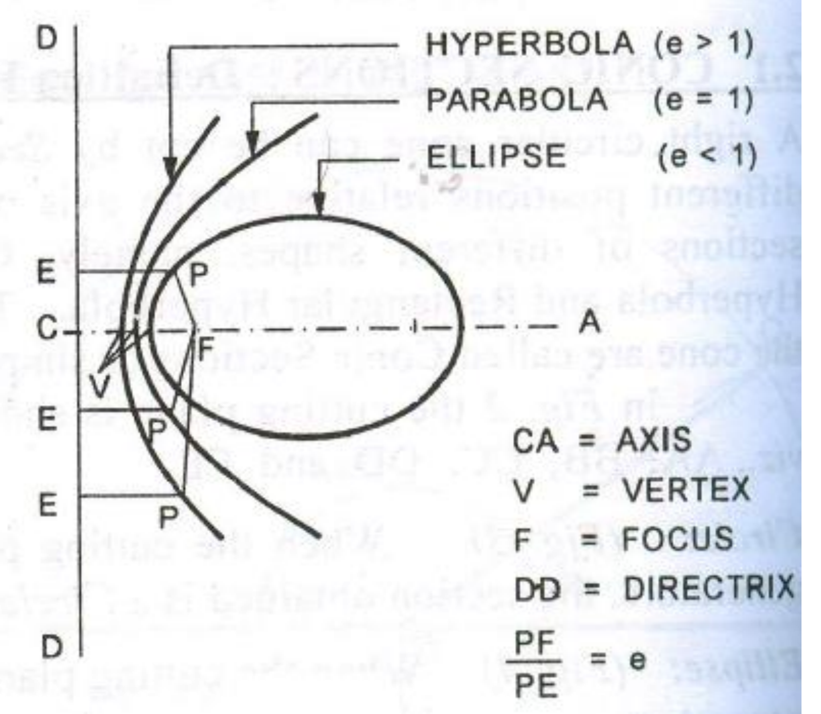


Conic

- A conic is defined as the curve traced by the point moving in a plane such that the ratio of its distance from a fixed point and a fixed line is always constant. The fixed point is called the focus and the fixed line the directrix. The ratio is called the eccentricity (e)
- Eccentricity (e) = $\frac{\text{Distance of a point from focus}}{\text{Distance of a point from directrix}}$

For Ellipse, e<1
For Parabola, e=1
For Hyperbola, e>1

Latus rectum: the chord of a conic perpendicular to the axis & passing through the focus is latus rectum.



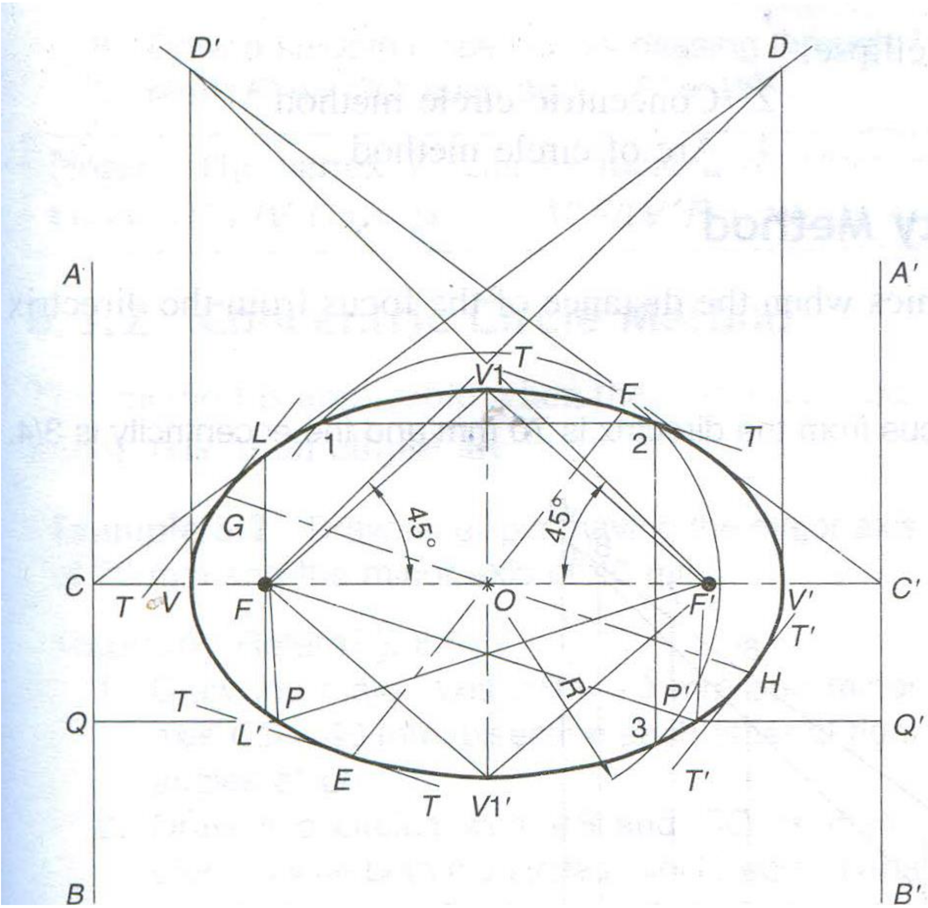
Ellipse

An ellipse is also defined as a curve traced by a point, moving in a plane such that the sum of its distances from two fixed points is always the same.

Terminologies

Elliptical curves application

Architectural, engineering design, arches, bridges, elliptical gears, bullet nose etc.



$e = PF/PQ = PF/P'Q' < 1$

F & E' = FOCI

AB & AB' = Directrices

V–$V1'$ = Major Axis

$V1$–$V1'$ = Minor Axis

O = Centre

V, V', $V1$ & $V1'$ = Vertices

EF & GH = Conjugate Axis

T–T = Tangent to Ellipse at E

T'–T' = Tangent to Ellipse at G

T–T || GH & T'–T' || EF

$PF + PF' = V1F + V1F' = P'F + P'F' = VF + VF'$ = Major Axis

$V1F = V1F' = V1'F = V1'F'$ = 1/2 (Major Axis)

Slope of CD = Slope of $C'D'$ = e

LL' = Latus Rectum

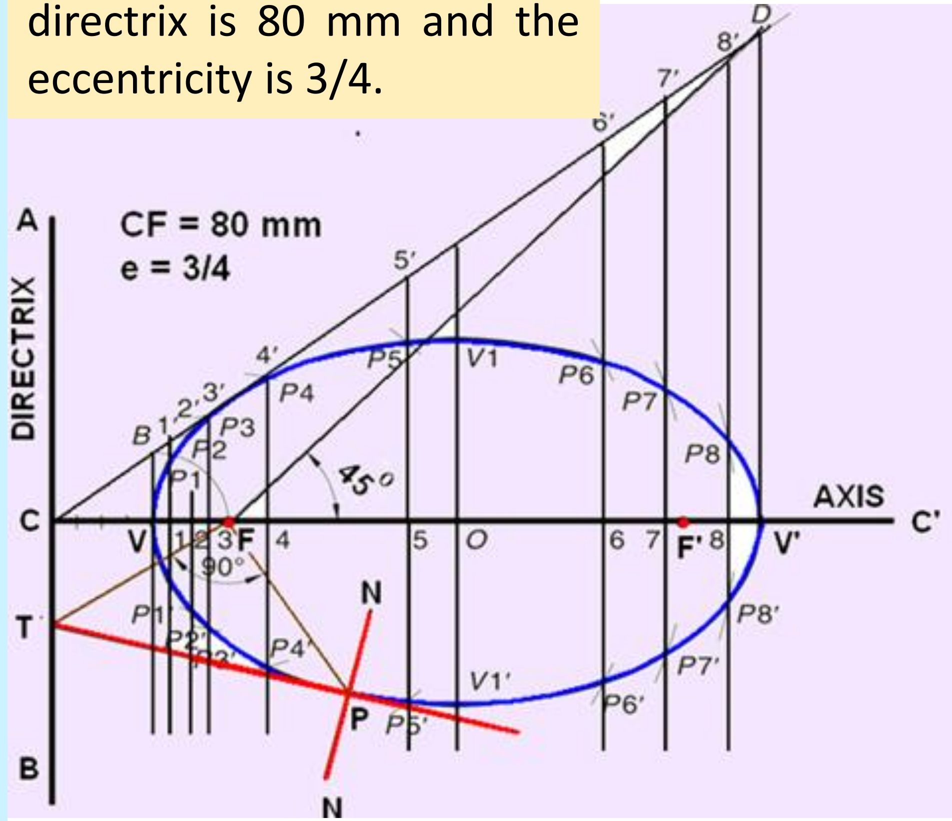
Generating an ellipse

1. **Focus-Directrix Or Eccentricity Method**

   General method of constructing any conics when the distance of the focus from the directrix and its eccentricity are given.

Drawing an ellipse if the distance of focus from the directrix is 80 mm and the eccentricity is 3/4.

1. Draw the directrix AB & axis CC’
2. Mark F on CC’ such that CF = 80 mm.
3. Divide CF into 7 equal parts and mark V at the fourth division from C.
4. At V, erect a perpendicular VB = VF. Join CB. Through F, draw a line at 45° to meet CB produced at D. Through D, drop a perpendicular DV’ on CC’. Mark O at the midpoint of V– V’.

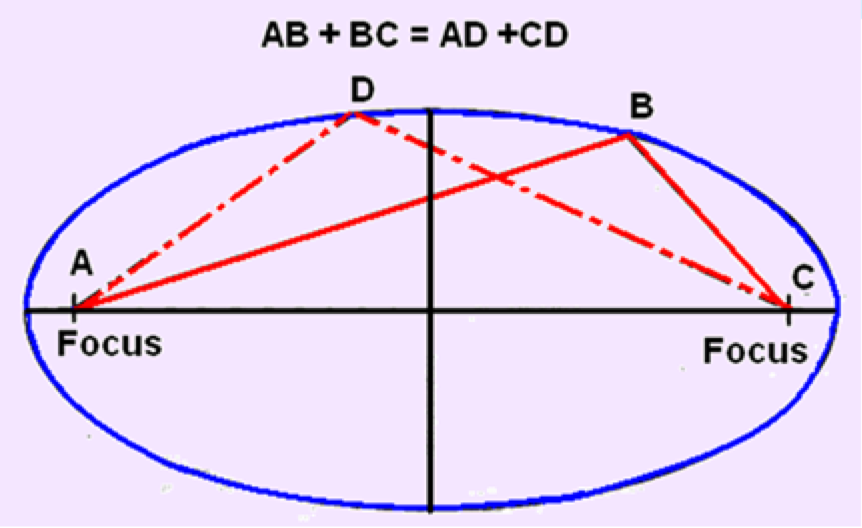


Generating an ellipse

5. Mark a few points, 1, 2, 3, … on V– V' and erect perpendiculars though them meeting CD at 1', 2', 3'…. Also erect a perpendicular through O.

6. With F as a centre and radius = 1–1', cut two arcs on the perpendicular through 1 to locate P1 and P1¢. Similarly, with F as a centre and radii = 2–2', 3–3', etc., cut arcs on the corresponding perpendiculars to locate P/2 and P/2', P/3 and P/3', etc. Also, cut similar arcs on the perpendicular through O to locate V1 and V1'.

An ellipse is also the set of all points in a plane for which the sum of the distances from the two fixed points (the foci) in the plane is constant.



Generating an ellipse (Concentric circle method)

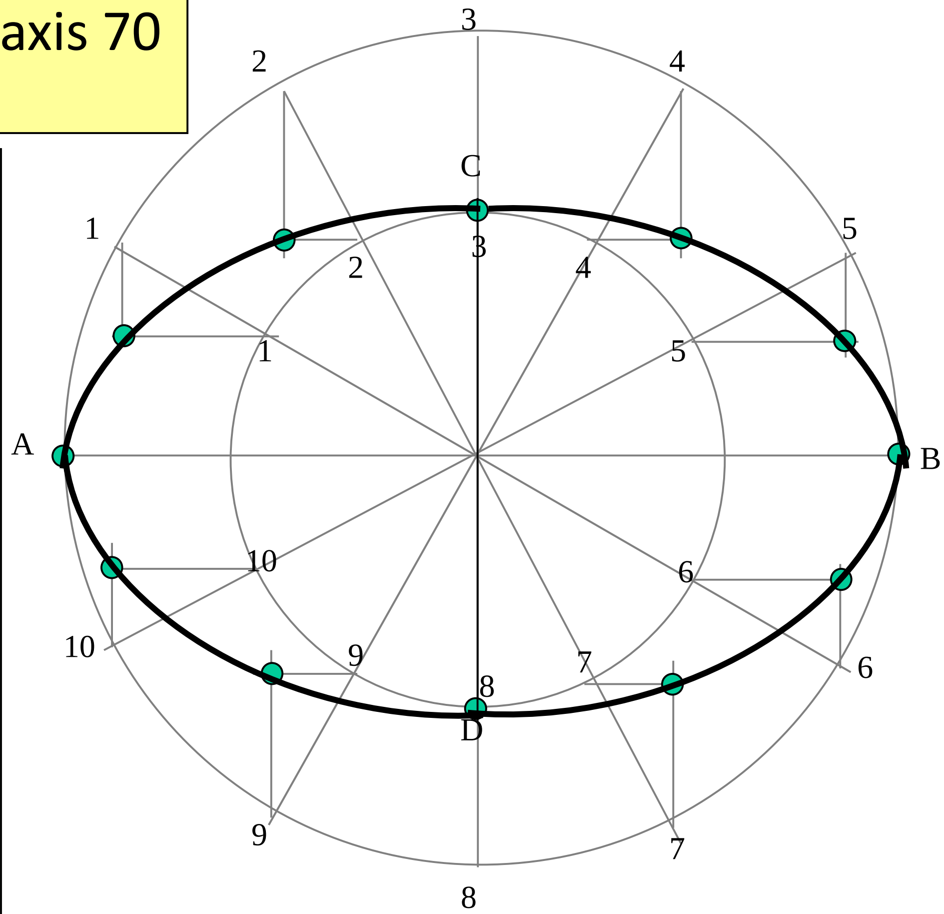
Draw ellipse by **concentric circle method**. Take major axis 100 mm and minor axis 70 mm long.

Steps:

1. Draw both axes as perpendicular bisectors of each other & name their ends as shown.
2. Taking their intersecting point as a center, draw two concentric circles considering both as respective diameters.
3. Divide both circles in 12 equal parts & name as shown.
4. From all points of outer circle draw vertical lines downwards and upwards respectively.

5.From all points of inner circle draw horizontal lines to intersect those vertical lines.

6. Mark all intersecting points properly as those are the points on ellipse.
7. Join all these points along with the ends of both axes in smooth possible curve.

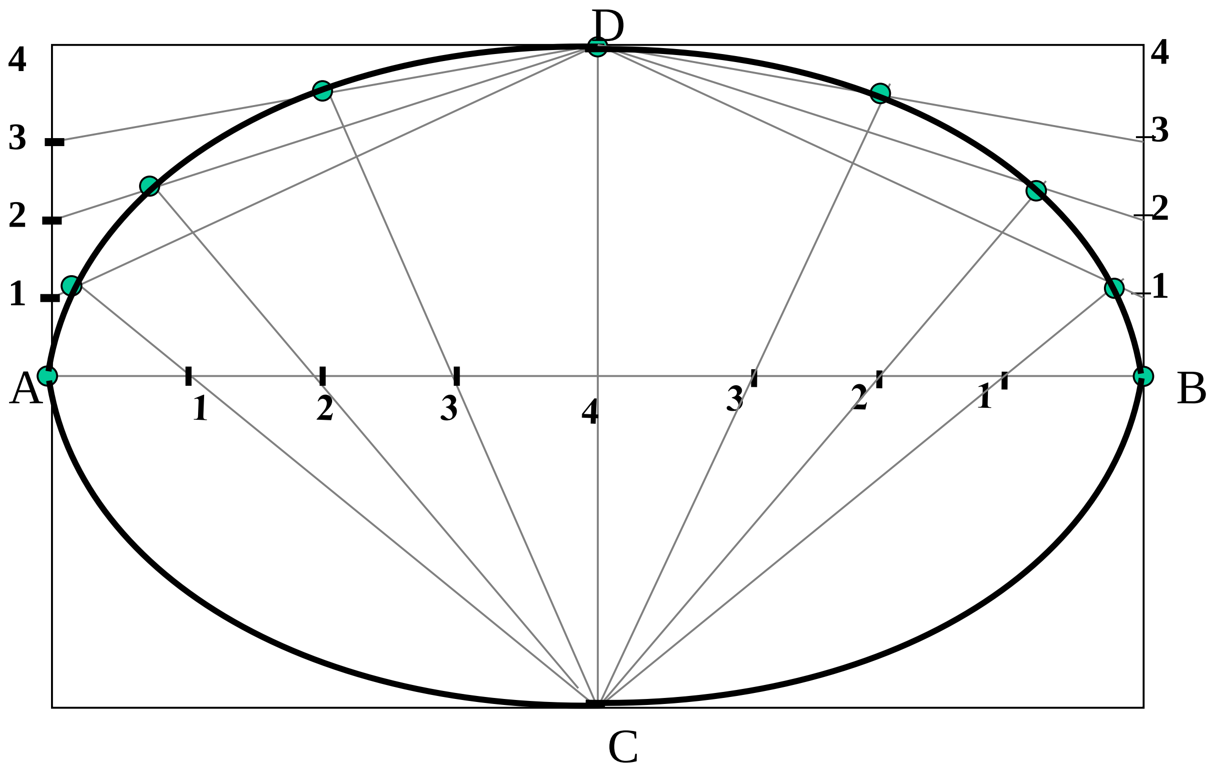


Used when major axis and minor axis of an ellipse are given.

Generating an ellipse (Oblong - rectangular Method)

1 Draw a rectangle taking major and minor axes as sides.
2. In this rectangle draw both axes as perpendicular bisectors of each other..
3. For construction, select upper left part of rectangle. Divide vertical small side and horizontal long side into same number of equal parts.(here divided in four parts)
4. Now join all vertical points 1,2,3,4, to D. And all horizontal points i.e.1,2,3,4 to C.
5. Then extend C-1 line upto D-1 and mark that point. Similarly extend C-2, C-3, C-4 lines up to D-2, D-3, & D-4 lines.
7. Mark all these points properly and join all along with ends A and D in smooth possible curve. Do similar construction in right side along with lower half of the rectangle. Join all points in smooth curve for required ellipse.

Draw ellipse by **Rectangle** *method.*
Take major axis 100 mm and minor axis 70 mm long.



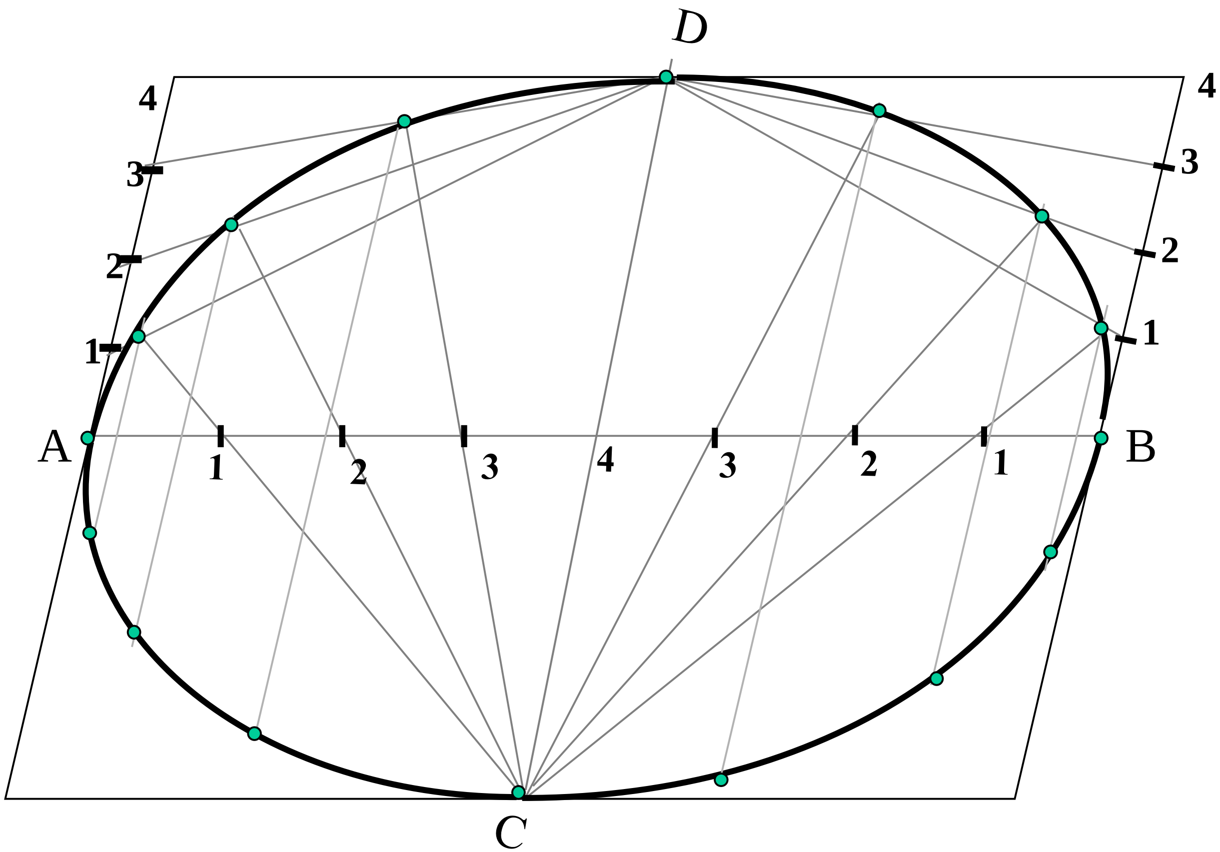
Used when major axis and minor axis of an ellipse are given.

Generating an ellipse (Oblong – parallelogram Method)

Draw ellipse by **Oblong method.**
Draw a parallelogram of 100 mm and 70 mm long sides with included angle of 75^0. Inscribe Ellipse in it.

STEPS ARE SIMILAR TO THE PREVIOUS CASE (RECTANGLE METHOD) ONLY IN PLACE OF RECTANGLE, HERE IS A PARALLELOGRAM.

Used when the conjugate axes with the angle between them is given.



Generating an ellipse (Arc of circle method)

Major axis AB & minor axis CD are 100 & 70mm long respectively. Draw ellipse by arcs of circles method.

STEPS:

1.Draw both axes as usual.Name the ends & intersecting point

2.Taking AO distance I.e.half major axis, from C, mark F_1 & F_2 On AB . (focus 1 and 2.)

3.On line F_1- O taking any distance, mark points 1,2,3, & 4

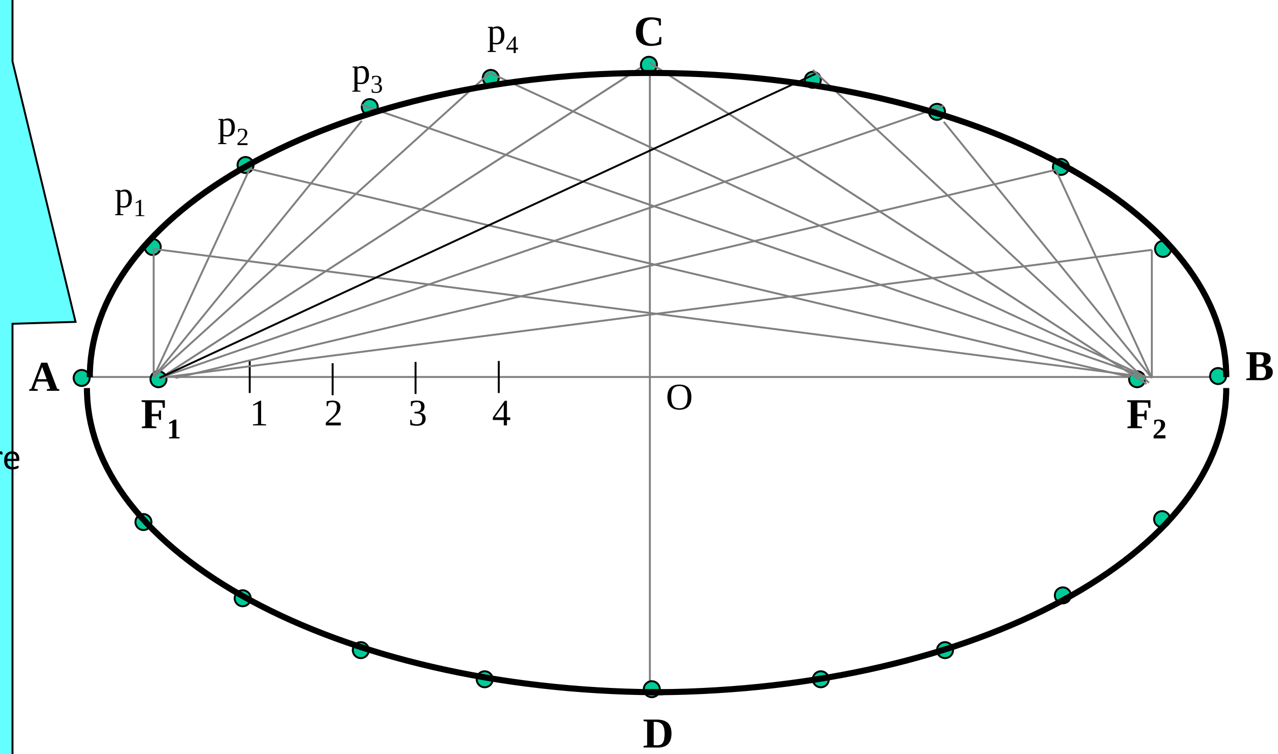
4.Taking F_1 center, with distance A-1 draw an arc above AB and taking F_2 center, with B-1 distance cut this arc. Name the point p_1

5. Intersection points of the two arcs are points on the ellipse.

6.Repeat this step with same centers but taking now A-2 & B-2 distances for drawing arcs. Name the point p_2

7.Similarly get all other P points. With same steps positions of P can be located below AB.

8.Join all points by smooth curve to get an ellipse. (use a french curve).

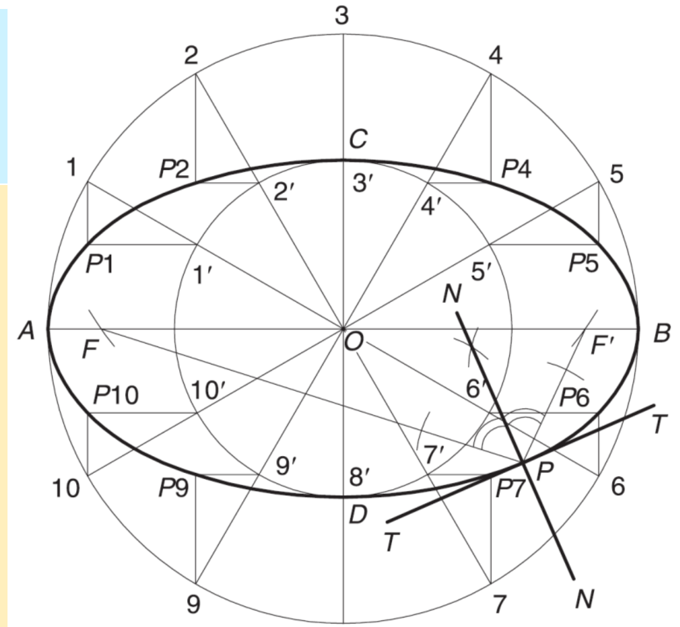


As per the definition Ellipse is locus of point P moving in a plane such that the SUM of it's distances from two fixed points (F_1 & F_2) remains constant and equals to the length of major axis AB.(Note A .1+ B .1=A . 2 + B. 2 = AB)

Draw Tangent & Normal to Ellipse at any point on it

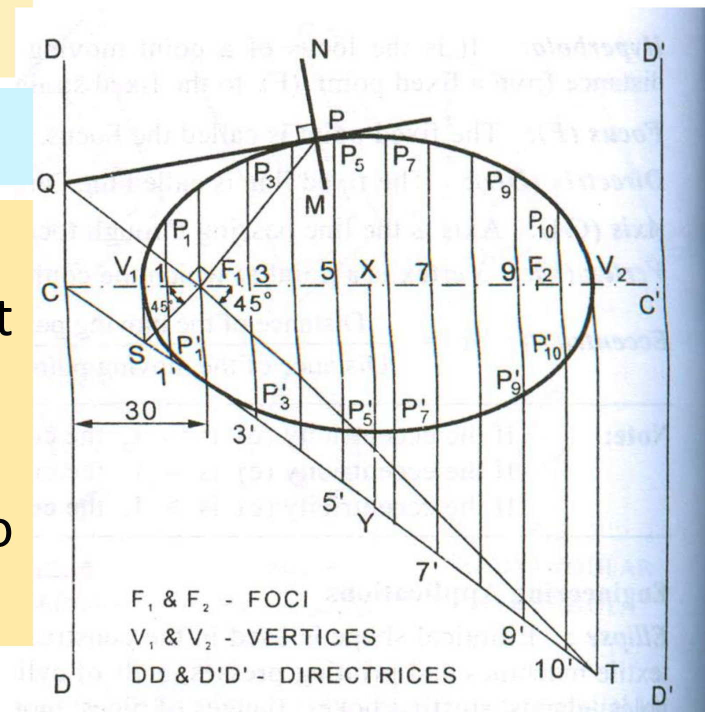
Bisector Method when Focus and Directrix are not known

1. First obtain the foci F and F′ by cutting the arcs on major axis with C as a centre and radius =OA
2. Obtain NN, the bisector of ∠FPF′. N-N is the required normal
3. Draw TT perpendicular to N-N at P. T-T is the required tangent

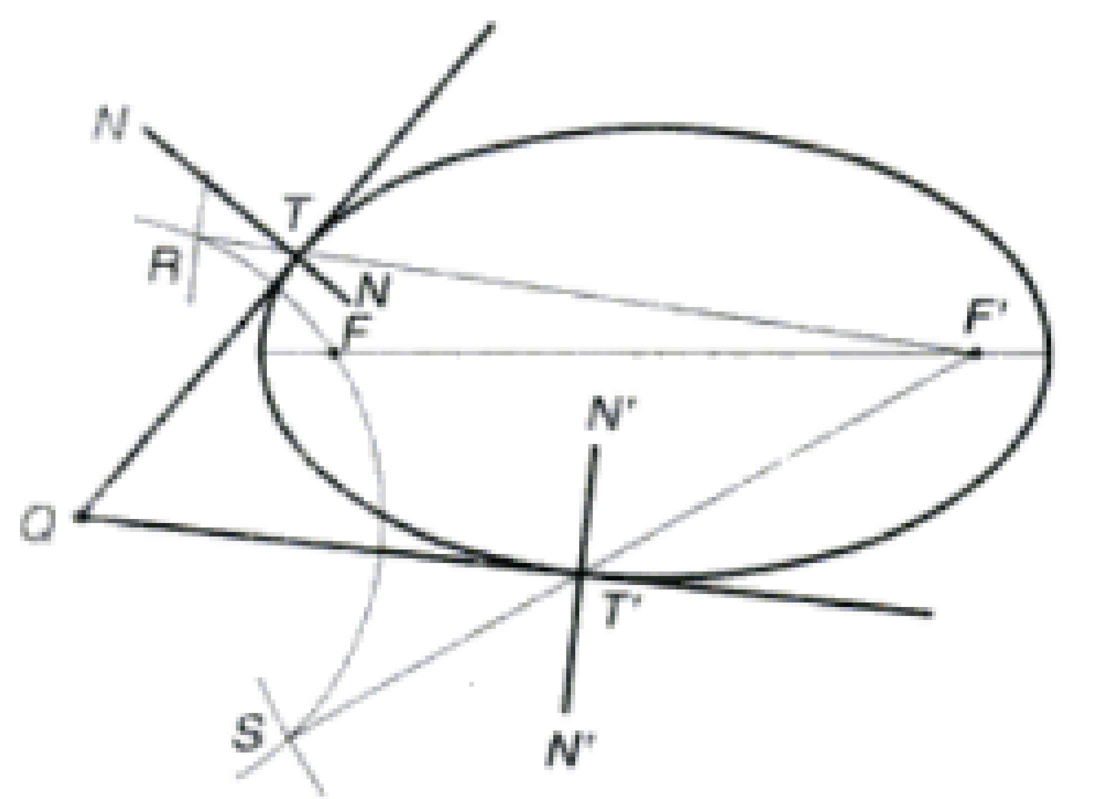


When Focus and Directrix are known

1. Mark the given point P and join PF_1 .
2. At F_1 draw a line perpendicular to PF_1 to cut DD at Q.
3. Join QP and extend it. QP is the tangent at P
4. Through P, draw a line NM perpendicular to QP. NM is the normal at P



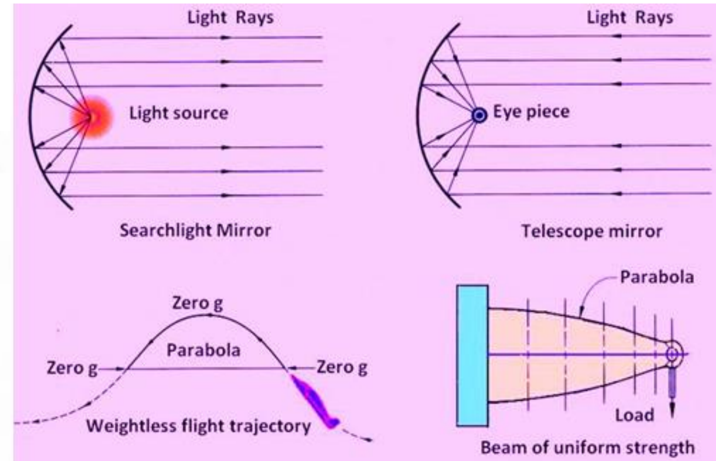
From a point outside ellipse

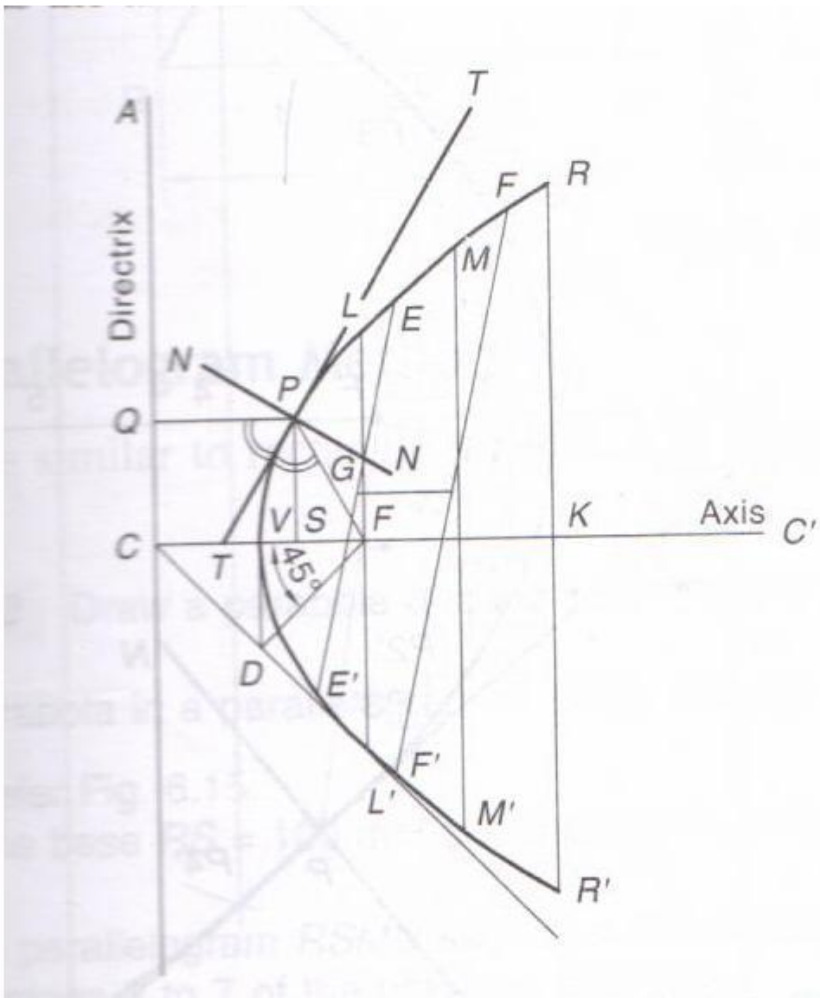


Parabola

A parabola is a conic whose eccentricity is equal to 1. It is an open-end curve with a focus, a directrix and an axis.

Terminologies





$e = PF/PQ = 1$

F = Focus

AB = Directrix

V = Vertex

M–M' = Double ordinate

R/R' = Base

L–L' = Latus rectum

RK = Ordinate

KV = Abscissa

Slope of $CD = e = 1$

$EE \| FF'$

$GH \| C$–C'

$RS | C'$–C'

$TV = VS$

TT = Tangent to Parabola at P

$\angle QPT = \angle TPF$

PARABOLA
DIRECTRIX-FOCUS METHOD

PROBLEM 9: Point F is 50 mm from a vertical straight line AB. Draw locus of point P, moving in a plane such that it always remains equidistant from point F and line AB.

SOLUTION STEPS:

1.Locate center of line, perpendicular to AB from point F. This will be initial point P and also the vertex.

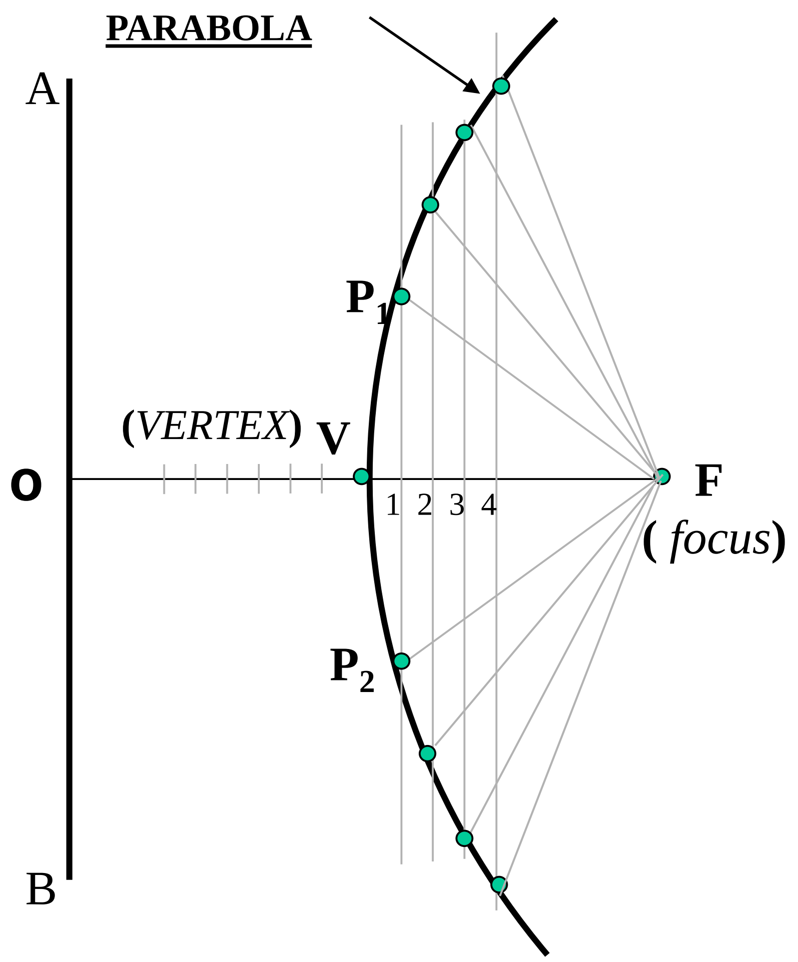
2.Mark 5 mm distance to its right side, name those points 1,2,3,4 and from those draw lines parallel to AB.

3.Mark 5 mm distance to its left of P and name it 1.

4.Take O-1 distance as radius and F as center draw an arc cutting first parallel line to AB. Name upper point P_1 and lower point P_2. (FP_1=O1)

5.Similarly repeat this process by taking again 5mm to right and left and locate P_3P_4.

6.Join all these points in smooth curve.

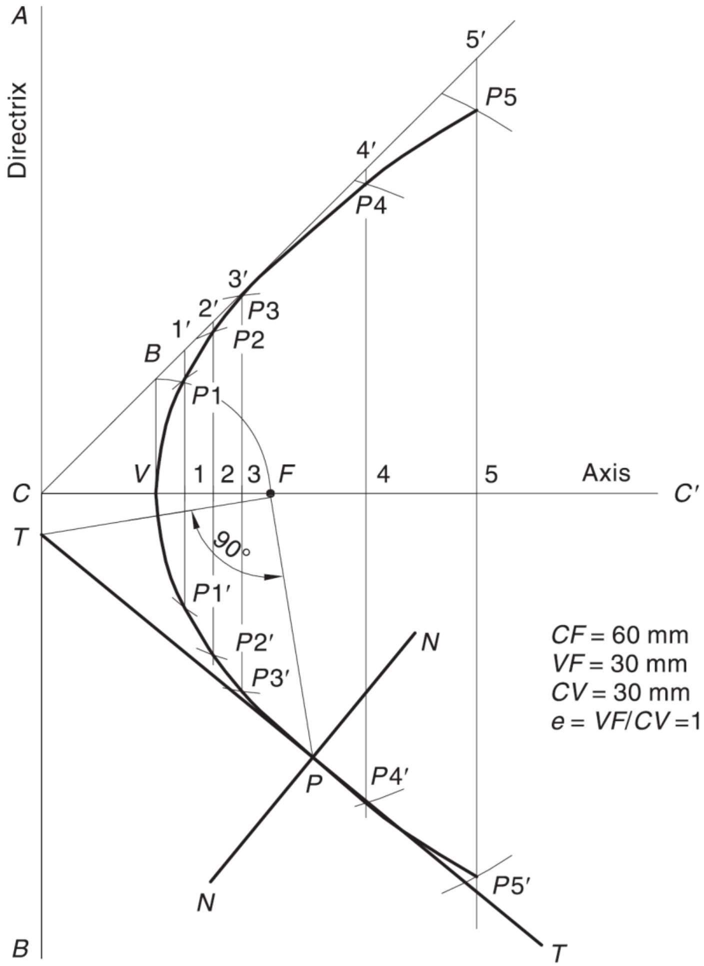


It will be the locus of P equidistance from line AB and fixed point F.

Generating a parabola

Focus-Directrix Or Eccentricity Method

Drawing a parabola if the distance of focus from the directrix is 60 mm

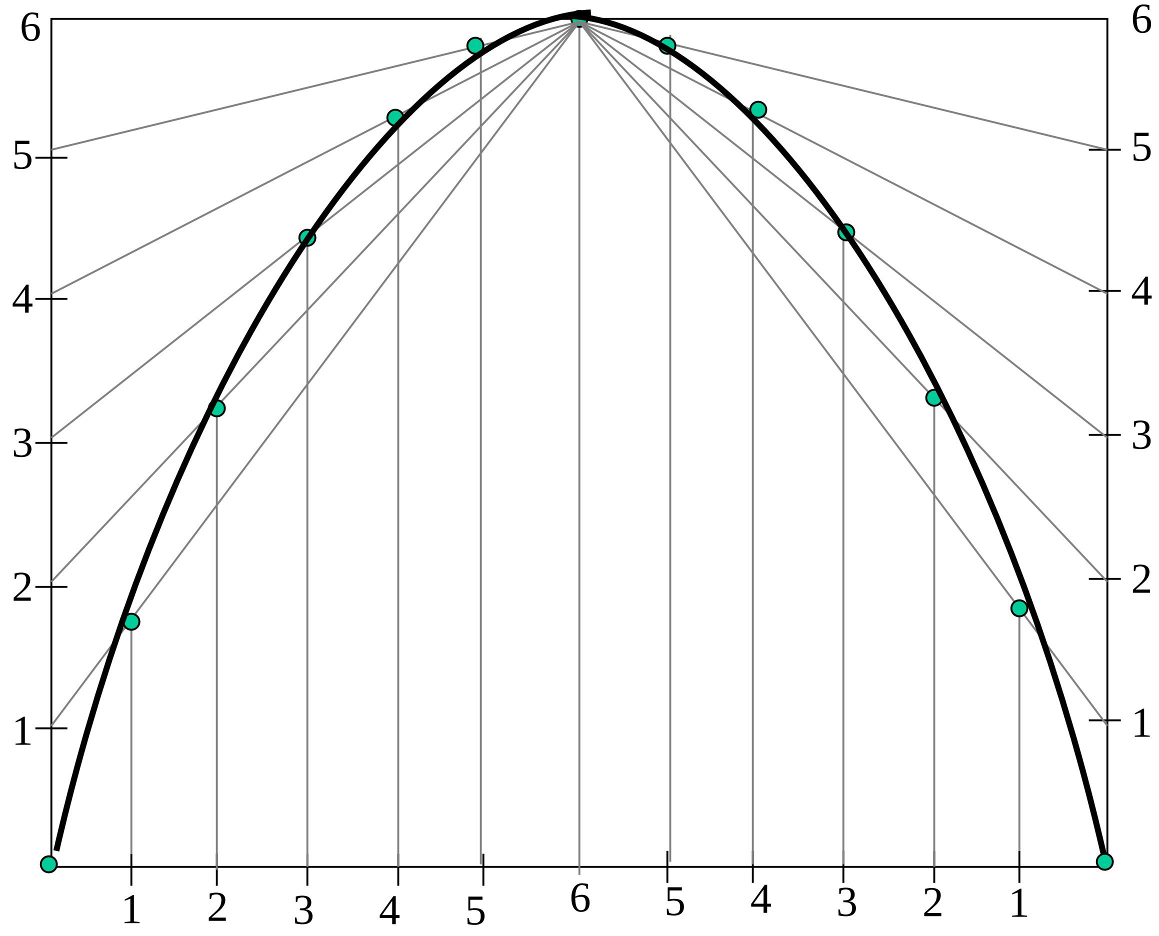


1. Draw directrix AB and axis CC
2. Mark F on CC’ such that CF = 60 mm.
3. Mark V at the midpoint of CF. Therefore, e = VF/ VC = 1.
4. At V, erect a perpendicular VE = VF. Join CE.
5. Mark a few points, say, 1, 2, 3, … on VC’ and erect perpendiculars through them meeting CE produced at 1’, 2’, 3’, …
6. With F as a centre and radius = 1–1’, cut two arcs on the perpendicular through 1 to locate P1 and P1’. Similarly, with F as a centre and radii = 2–2’, 3–3’, etc., cut arcs on the corresponding perpendiculars to locate P2 and P2’, P3 and P3’, etc.
7. Draw a smooth curve passing through V, P1, P2, P3 … P3

PROBLEM 7: A BALL THROWN IN AIR ATTAINS 100 M HIEGHT AND COVERS HORIZONTAL DISTANCE 150 M ON GROUND. Draw the path of the ball (projectile)-

PARABOLA
RECTANGLE METHOD

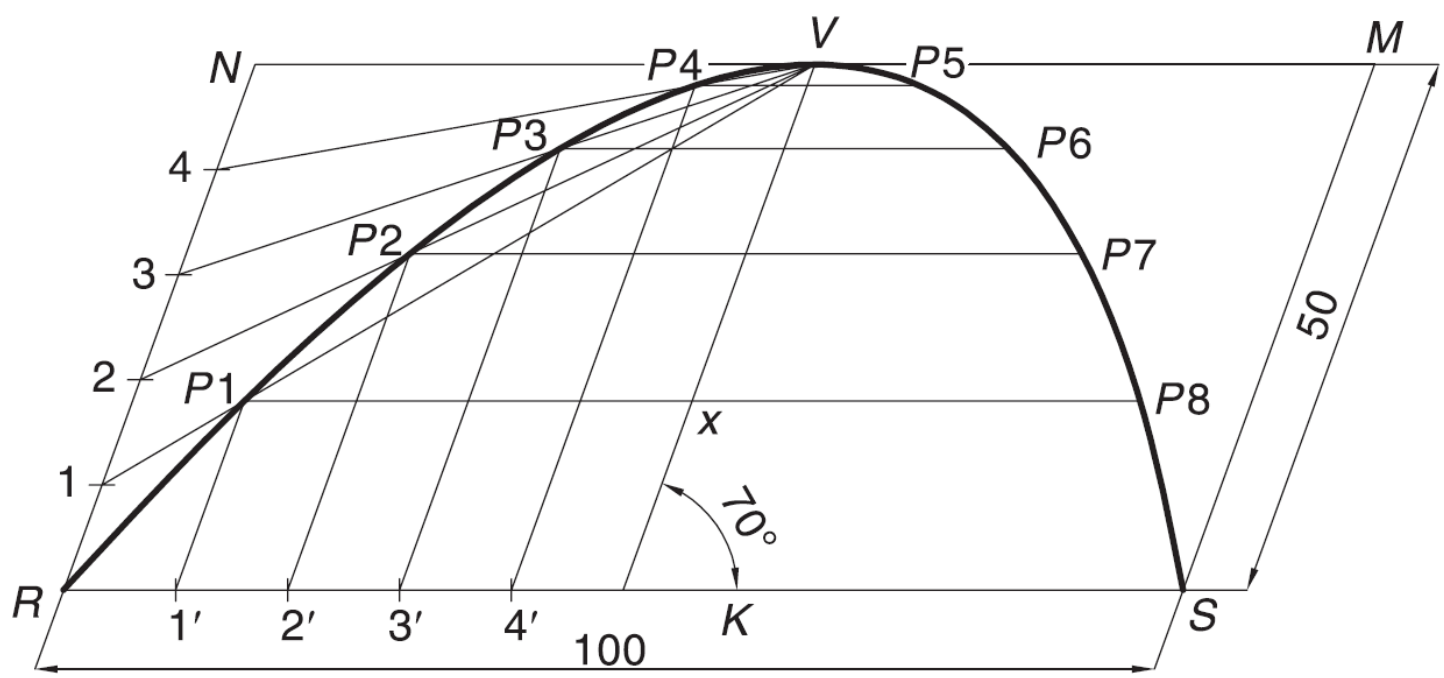
STEPS:
1.Draw rectangle of above size and divide it in two equal vertical parts
2.Consider left part for construction. Divide height and length in equal number of parts and name those 1,2,3,4,5& 6
3.Join vertical 1,2,3,4,5 & 6 to the top center of rectangle
4.Similarly draw upward vertical lines from horizontal1,2,3,4,5 And wherever these lines intersect previously drawn inclined lines in sequence Mark those points and further join in smooth possible curve.
5.Repeat the construction on right side rectangle also.Join all in sequence.
This locus is Parabola.



Generating parabola (Parallelogram method)

Draw a parabola of base 100 mm and axis 50 mm if the axis makes 70° to the base.

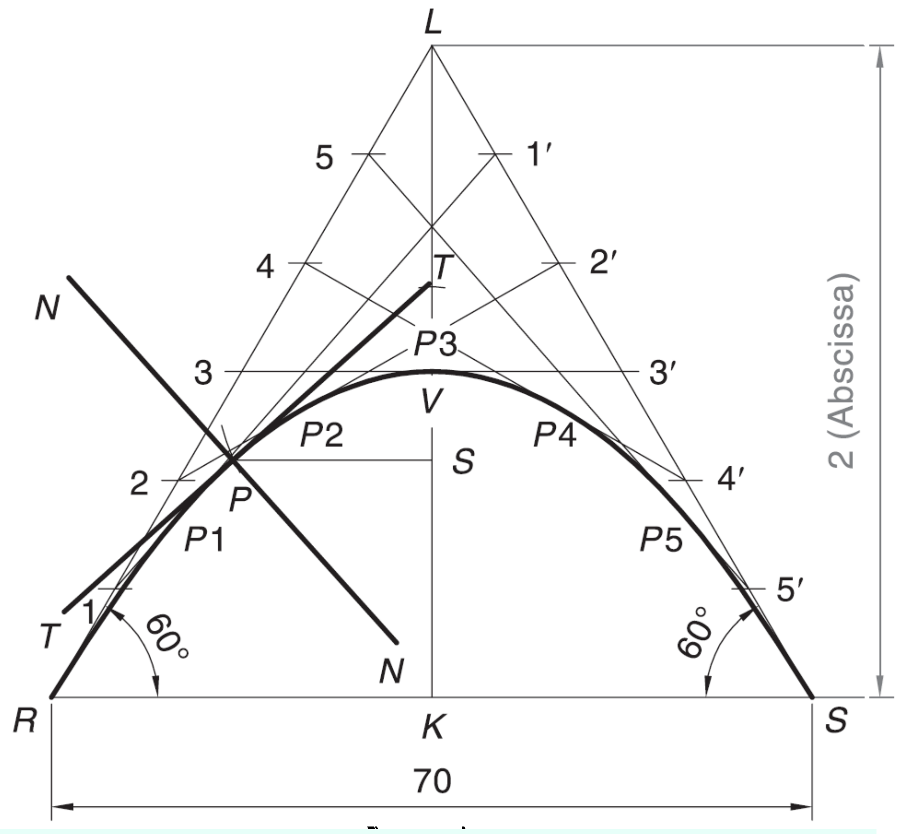
1. Draw the base RS = 100 mm and through its midpoint K, draw the axis KV = 50 mm, inclined at 70° to RS.
2. Draw a parallelogram RSMN such that SM is parallel and equal to KV.
3. Follow steps as in rectangle method



Generating parabola (Tangent method)

Draw a parabola if the base is 70 mm and the tangents at the base ends make 60° to the base.

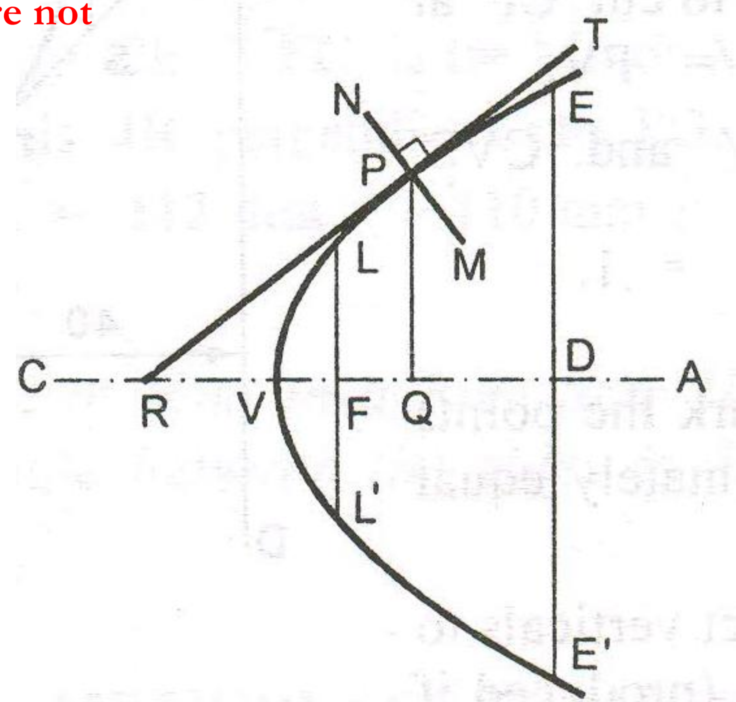
1. Draw the base RS = 70 mm. Through R and S, draw the lines at 60° to the base, meeting at L.
2. Divide RL and SL into the same number of equal parts, say 6. Number the divisions as 1, 2, 3 … and 1’, 2’, 3’, … as shown.
3. Join 1–1’, 2–2’, 3–3’, ….

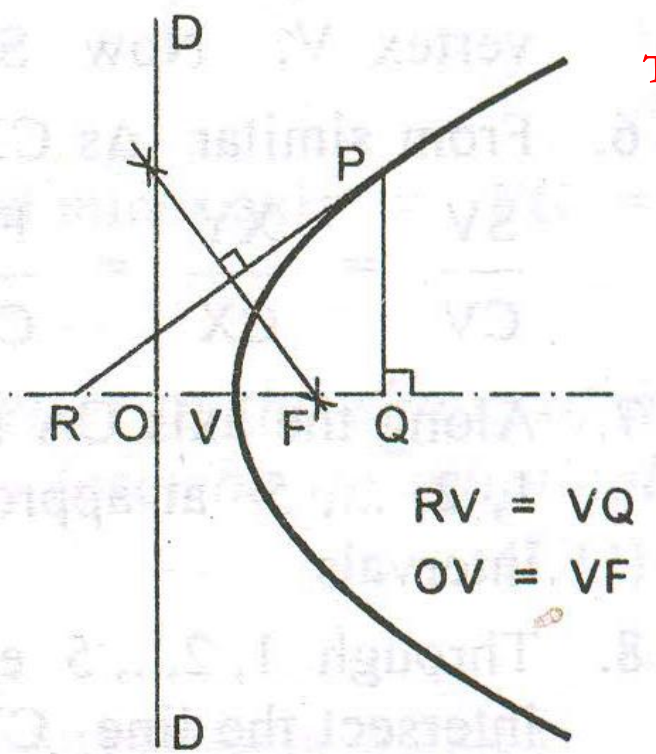


4. Draw a smooth curve, starting from R and ending at S and tangent to 1–1’, 2–2’, 3–3’, etc., at P1, P2, P3, etc., respectively

Tangent and Normal at any point P when Focus and Directrix are not known

1. Draw the ordinate PQ
2. Find the abscissa VQ
3. Mark R on CA such that RV=VQ
4. Draw the normal NM perpendicular to RP at P



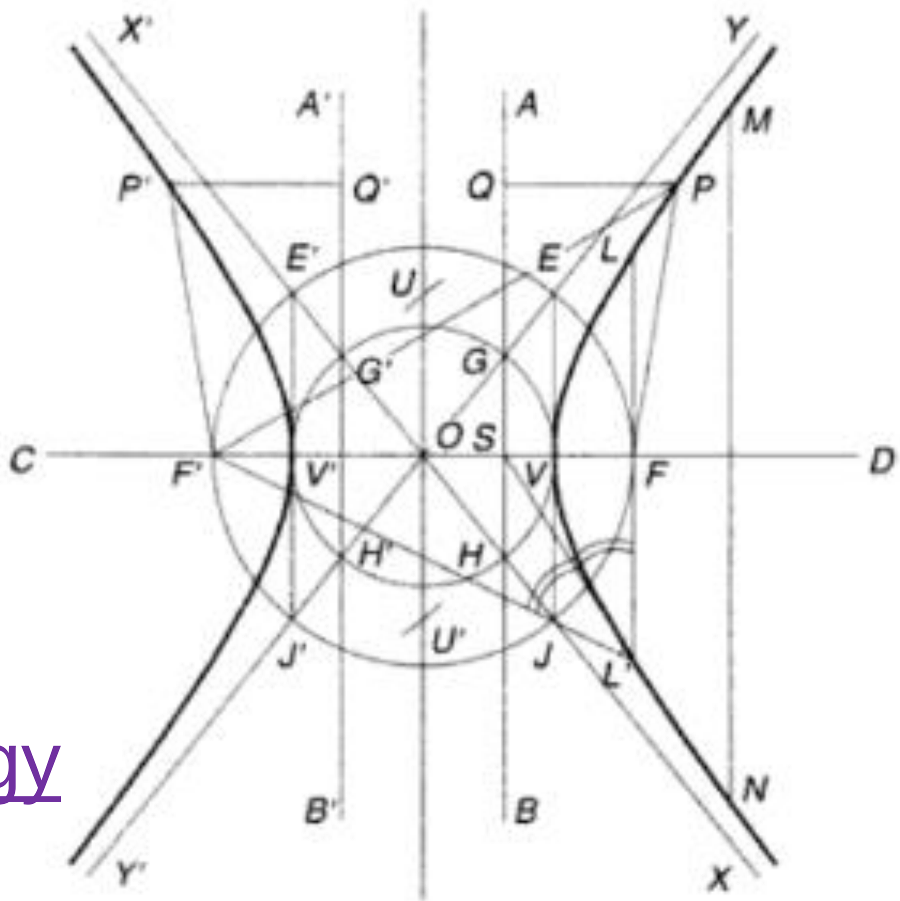


To find the focus and the directrix of a parabola given its axis

1. Mark any point P on the parabola
2. Draw a perpendicular PQ to the given axis
3. Mark a point R on the axis such that RV=VQ
4. Focus: Join RP. Draw a perpendicular bisector of RP cutting the axis at F, F is the focus
5. Directrix: Mark O on the axis such that OV= VF. Through O draw the directrix DD perpendicular to the axis

Hyperbola

- It is defined as a curve traced by the point moving in a plane such that the difference between its distance from 2 fixed points in the same plane is always the same. The fixed points represent the foci.
- The hyperbolas exist in a pair. It is a open-ended curve



Terminology

$e = PF/PQ = P'F'/P'Q' > 1$
F & F' = FOCI
AB & A'B' = Directrices
CD = Axis
O = Centre
V & V' = Vertices
V–V' = PF'–PF = Transverse Axis
U–U' = Conjugate Axes
X–X' & Y–Y' = Asymptotes
MN = Double Ordinate
L–L' = Latus Rectum
$\angle F'L'S = \angle SL'F$

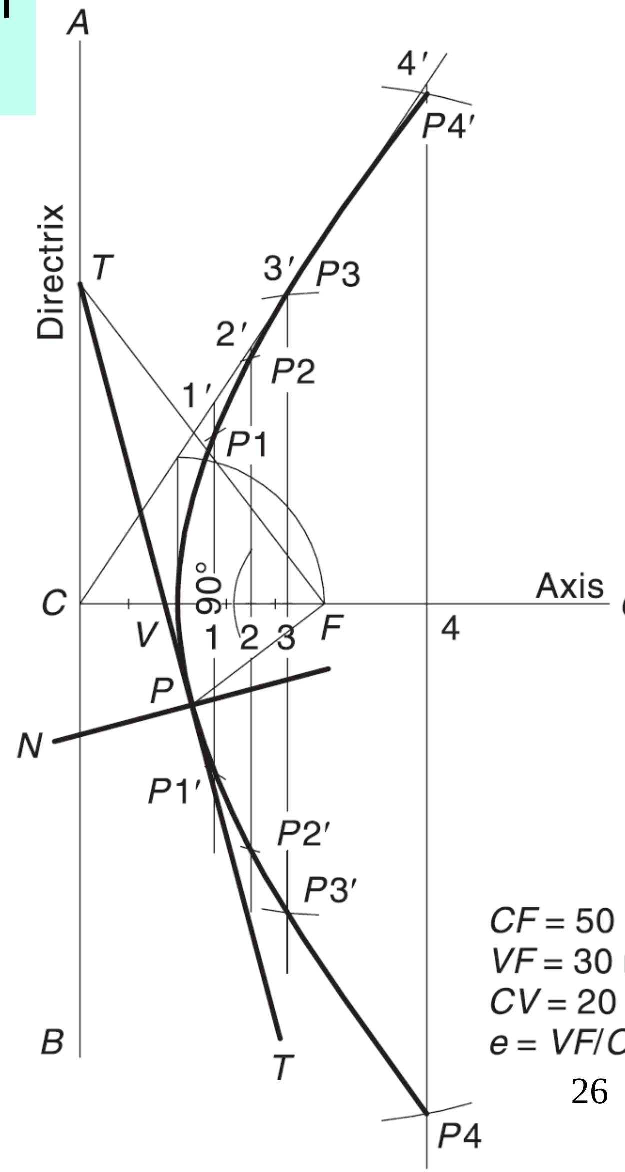
Draw a circle with O as centre and radius OF. Draw perpendicular to (V V′) at V and V′ cutting the circle at E′, J. The diagonals E, J′ are the asymptotes

- **Application**
- Cooling towers, Hyperbolic mirrors in telescopes

Generating hyperbola (Focus directrix or eccentricity method)

Draw a hyperbola of e = 3/2 if the distance of the focus from the directrix = 50 mm.

1. Draw directrix AB and axis CC' as shown.
2. Mark F on CC' such that CF = 50 mm.
3. Divide CF in to 3 + 2 = 5 equal parts and mark V at second division from C. Now, e = VF/ VC = 3/2.
4. Follow steps as in ellipse and parabola



CF = 50 mm
VF = 30 mm
CV = 20 mm
e = VF/CV = 3/2

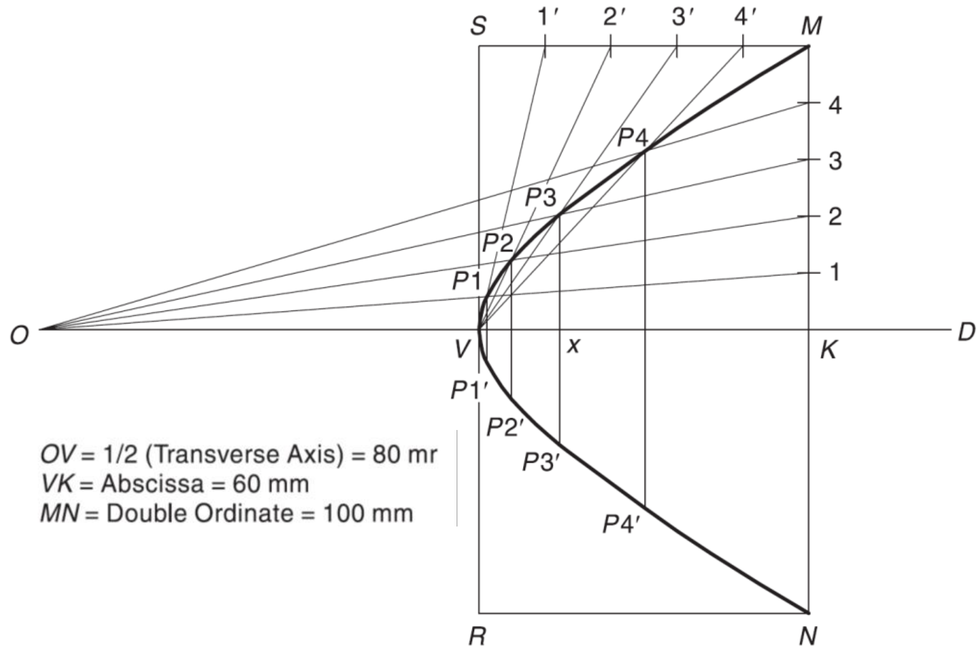
Generating hyperbola (rectangle or Abscissa- ordinate method)

Draw a hyperbola having the double ordinate of 100 mm, the abscissa of 60 mm and the transverse axis of 160 mm.

1. Draw axis OD and mark V and K on it such that OV = 1/2(Transverse Axis) = 80 mm and VK =Abscissa = 60 mm.
2. Through K, draw double ordinate MN = 100 mm.
3. Construct rectangle MNRS such that NR = VK.
4. Divide MK and MS into the same number of equal parts, say 5. Number the divisions as shown.

5. Join O–1, O–2, O–3, etc. Also join V–1’, V–2’, V–3’, etc. Mark P1, P2, P3, etc., at the intersections of O–1 and V–1’, O–2 and V–2’, O–3 and V–3’, etc., respectively.

6. Obtain P1’, P2’, P3’, etc., in other half in a similar way. Alternatively, draw P1/ P1’, P2/P2’, P3/P3’, etc., such that P3 –x = x–P3’ and likewise.



Generating hyperbola (Arc of circle method)

Draw a hyperbola if the transverse axis is 50mm, and the distance between the foci is 80 mm.

1. Draw axis CD and on it mark V-V’ = 50 mm & F-F’=80mm such that VF=V’F’.
2. On CD, mark a few points, 1,2,3,4 between F & D.
3. With F as a center and radius = V-1, draw 2 arcs on either side of CD. With F’ as a center and radius =V’-1, draw 2 arcs cutting the previous arcs at P1 & P1’. Note that (V’-1)-(V-1)= transverse axis.
4. Repeat step 3, for the pairs of radii = (V-2,V’-2), (V-3,V’-3) to obtain points (P2, P2’), (P3,P3’), etc. In each pair, difference of radii = Transverse axis. Therefore, (V’-P1)-(V-P1) = (V’-P2)-(V-P2) = (V’-P3)-(V-P3)=…= Transverse axis
5. Draw a smooth curve through V, P1, P2, P3.. P1’, P2’,…
6. To obtain the other half of hyperbola, mark a few points 1’,2’,3’,4’ between F’ & C’ and repeat the similar procedure.