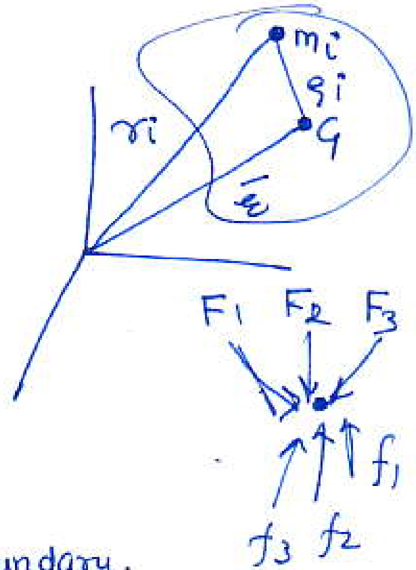
Kinetics of System of Particles.

⇒ Generalised Newton's law.



$$\Sigma F + \Sigma f = \Sigma m_i \ddot{r}_i$$

where

ΣF = external forces applied on the system

Σf = forces acting on m_i from sources internal to the system boundary.

External forces :→ external forces are due to contact with external bodies or to external gravitational, electric & magnetic effects

Internal forces : forces of reaction with other mass particles within the boundary

$$m \bar{r} = \Sigma m_i \bar{r}_i$$

$$m \ddot{\bar{r}} = \Sigma m_i \ddot{r}_i$$

$$\Sigma F = m \ddot{\bar{r}} \Rightarrow \Sigma F = m \bar{a}_c$$

↳ acceleration of the center of mass

$$\Sigma F_x = m\vec{a}_{cx}, \quad \Sigma F_y = m\vec{a}_{cy}, \quad \Sigma F_z = m\vec{a}_{cz}$$

(2) Work Energy :

$$\boxed{\Delta T = U_{1-2}}$$

ΔT → Change in kinetic Energy

U_{1-2} → Work done by all forces external, internal on all particles.

$$U_{1-2} = \underbrace{U'_{1-2}}_{\text{non conservative fore}} - \Delta V$$

ΔV → conservative force, Potential Energy

$$\boxed{\Delta T + \Delta V = U'_{1-2}}$$

OR

$$\boxed{T_1 + V_1 + U'_{1-2} = T_2 + V_2}$$

(3) Kinetic Energy

$$T = \Sigma \tfrac{1}{2} m_i v_i^2$$

$$v_i = \bar{v}_c + \dot{\rho}_i$$

$$T = \tfrac{1}{2}\Sigma m_i v_i \cdot v_i = \Sigma \tfrac{1}{2} m_i v_c^2 + \tfrac{1}{2} m_i |\dot{\rho}_i|^2 + \underbrace{\Sigma m_i \bar{v}_c \cdot \dot{\rho}_i}_{0}$$

$$\bar{v}_c \frac{d}{dt}(\Sigma m_i \rho_i) = 0$$

$$T = \tfrac{1}{2} m \bar{v}_c^2 + \tfrac{1}{2}\Sigma m_i |\dot{\rho}_i|^2$$

Impulse - Momentum

Linear Momentum.

$$G_i = m_i v_i$$

$$v_i = v_c + \dot{\rho}_i$$

Vector sum of liner momenta of all of its particles.

$$G = \Sigma m_i (v_c + \dot{\rho}_i)$$

$$= \Sigma m_i v_c + \frac{d}{dt}\underbrace{(\Sigma m_i \rho_i)}_{0}$$

$$\boxed{G = m \bar{v}_c}$$

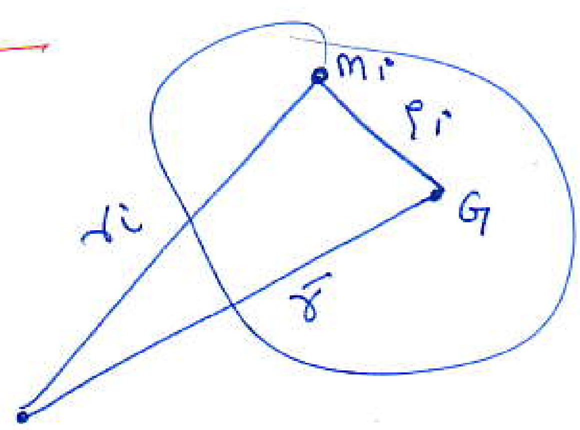
Linear momentum of any system of constant mass is the product of the mass and velocity of its center of mass.

$$\boxed{\Sigma F = \dot{G}}$$

Angular Momentum

About a fixed point O

Angular momentum of the mass system about the point O, fixed in inertial reference ~~its~~ is defined as the vector sum of linear momenta about O of



$$H_o = \Sigma(r_i \times m_i v_i)$$

$$\dot{H}_o = \underline{\Sigma \dot{r}_i \times m_i v_i} + \Sigma r_i \times m_i \dot{v}_i$$

$\dot{r}_i$, $v_i \rightarrow$ same direction

$$\dot{H}_o = \Sigma r_i + m_i \dot{v}_i \Rightarrow \Sigma M_o = \dot{H}_o$$

About the mass center,

$$H_G = \Sigma \rho_i \times m_i \dot{r}_1$$

$$H_G = \Sigma \rho_i \times m_i (v_c + \dot{\rho}_1)$$

$$= \Sigma \rho_i \times m_i v_c + \Sigma \rho_i \times m_i \rho_i$$

$$= -v_c \times \underbrace{\Sigma m_i \rho_i}_{0} + \Sigma \rho_i \times m_i \dot{\rho}_1$$

$$\boxed{H_G = \Sigma \rho_i \times m_i \dot{\rho}_i}$$

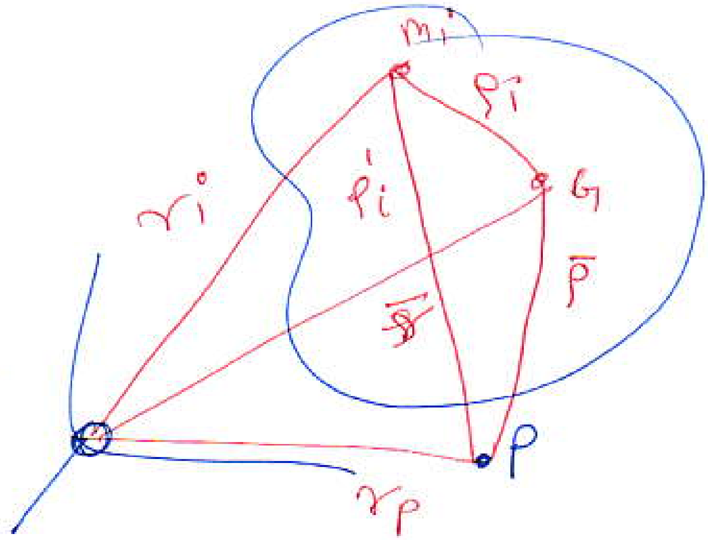
$\Sigma M_G = \dot{H}_G$,

About an arbitrary Point P

$$H_P = \Sigma \rho_i' \times m_i \dot{r}_i$$

$$= \Sigma (\bar{\rho} + \rho_i) \times m_i \dot{r}_i$$

$$= \Sigma \bar{\rho} \times m_i \dot{r}_i + \Sigma \rho_i \times m_i \dot{r}_i$$



$$\boxed{H_P = H_G + \bar{\rho} \times m \bar{v}_c}$$

$$\boxed{\Sigma M_P = \dot{H}_G + \bar{\rho} \times m \bar{a}_c}$$

Conservation of Energy & Momentum.

1. if $U_{1-2}' =$ Work done by non conservative forces $= 0$

$$\Delta T + \Delta V = 0$$

$$T_1 + V_1 = T_2 + V_2$$

2. Conservation of Momentum

$$\boxed{G_1 = G_2}$$ Linear momentum

$$(H_O)_1 = (H_O)_2, \quad (H_G)_1 = (H_G)_2$$

→ Conservation of Angular momentum.

Steady Mass Flow

Steady Mass flow conditions = rate at which mass enters a given volume equals the rate at which mass leaves the same volume

'Volume' — May be a rigid container
- → fixed or moving blades
- → (nozzles of jet aircraft
- → space between blades in a gas turbine
- → the volume within the casing of a centrifugal pump.
- → Volume within the bend of a pipe through which a fluid is flowing at a steady rate.

Consider a rigid container, into which mass flows in a steady stream at the rate m' through the entrance section of area A_1

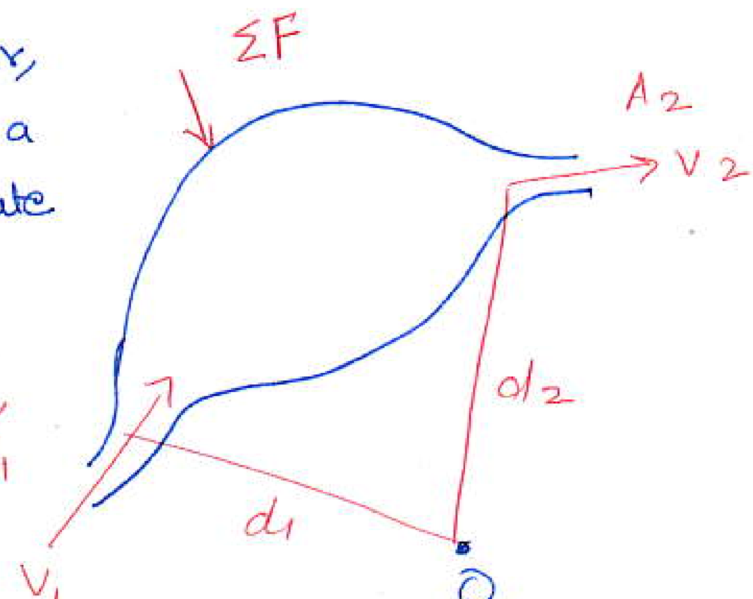
Mass leaves — exit section A_2

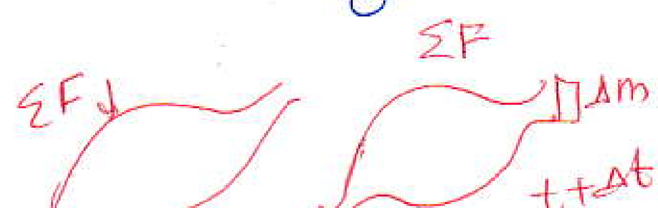
A_1

No accumulation or depletion of total mass.

V_1 = velocity of stream entering

V_2 = velocity of stream at exit.





$$\boxed{\rho_1 A_1 v_1 = \rho_2 A_2 v_2 = m'} \quad (1)$$

Now → External forces applied to the system:

1. The forces exerted on the container at a points of its attachment to the other structures, including attachement at A_1 & A_2

2. Forces acting on the fluid within the container at A_1 and A_2 due to any static pressure which may exist in the fluid at these positions

3. Weight of the fluid and structure if ~~applicable~~. appreciable.

A_1 & A_2 → remains unchanged during Δt

Δm → is also not changing.

So contribution in the change in linear momentum due to

$$\Delta G = \Delta m V_2 - \Delta m v_1 = \Delta m (V_2 - V_1) \quad (2)$$

Divide equation by time Δt

$$\frac{\Delta G}{\Delta t} = \frac{\Delta m}{\Delta t}(V_2 - V_1)$$

$$\Sigma F = m' \Delta V$$

Angular Momentum in Steady-flow system.

The resultant moment of all external forces about some fixed point O on or off the system, Equals the time rate of change of angular momentum of the system about O.

$$\boxed{\Sigma M_o = m'(v_2 d_2 - v_1 d_1)} \quad (3)$$

When the velocityies of the incoming and outgoing flows are not in the same plane

$$\boxed{\Sigma M_o = m'(d_2 \times V_2 - d_1 \times V_1)} \quad (4)$$

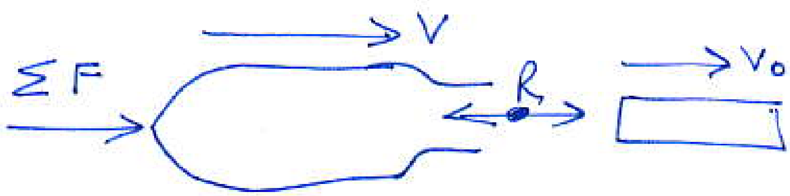
Variable Mass

$$\Sigma F = \dot{G}, \quad \Sigma M_o = \dot{H}_o, \quad \Sigma M_G = \dot{H}_G$$

$\Downarrow$

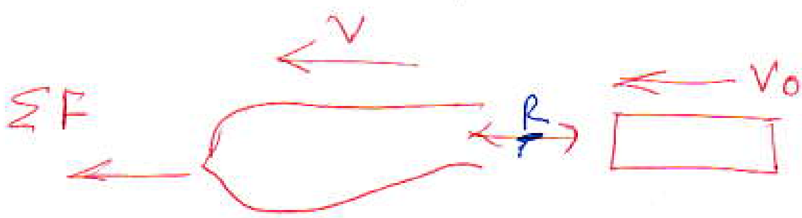
Valid for a fixed collection of ~~mass~~ particles so that the mass of the system to be analyzed was constant.

Equation of motion for variable mass

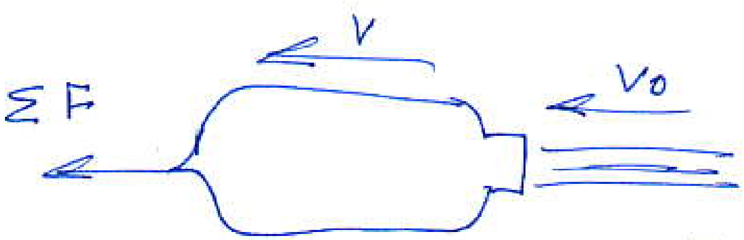


(a)

m m swallows mass ($V > V_0$)



m ~~m~~ m exples mass ($V > V_0$) (b)



m exples mass ($V > V_0$) (c)

For body (a)

$$R = m'(V - V_0) = \dot{m} u$$

u → magnitude of the relative velocity

$m' = \dot{m}$ → Time rate of increas of m

$$\Sigma F - R = m\dot{V}$$

$$\boxed{\Sigma F = m\dot{v} + \dot{m}u}$$

For body (b)

If the body loses mass by expelling it rearward so that its velocity v_0 is less than v. The force R required to decelerate the particles from a velocity V to a lesser velocity v_0 is

$$R = m'(-v_0 - [-v]) = m'(v - v_0)$$

But $m' = -\dot{m}$ since m is decreasing.

Thus $R = -\dot{m}u$

$$\boxed{\Sigma F = m\dot{v} + \dot{m}u}$$ — Same as (a)

Error Just applying the mathematic to the force momentum relation

$$\Sigma F = \frac{d}{dt}(mv) = \frac{dm}{dt}v + m\frac{dv}{dt}$$

$$\boxed{= m\dot{v} + \dot{m}\underset{v \neq u}{\underline{v}}}$$ (a)

· This will valid when body picks up mass initially at rest or when it exples mass which is left with zero ~~abosolute~~ absolute velocity.

$$\boxed{v_0 = 0, \quad v = u}$$

Another Approach, body (c)

System of mass = m

m_o = arbitrary portion of ejected mass.

So the mass of system = $m + m_o$ = Constant.

The ejected stream of mass is assumed to move undisturbed once separated from m_o and the only force external to the entire system is ΣF which is applied directly to m.

$\boxed{\cancel{R = -\mathring{m}u}}$ -1

The reaction $R = -\mathring{m}u$ is internal to the system and not disclosed as an external force on the system,

$$\Sigma F = \dot{G}$$

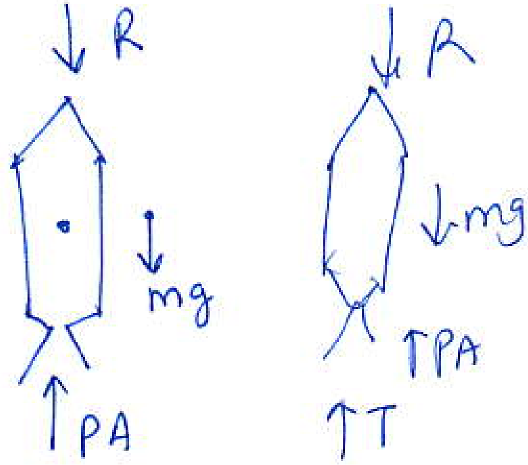
$$\Sigma F = \frac{d}{dt}(mv + m_o v_o)$$

$$\Sigma F = m\mathring{v} + \mathring{m}v + \mathring{m}_o v_o + m_o \mathring{v}_o$$

Now $\underline{\underline{\mathring{m}_o}} = -\mathring{m}$, $\dot{V}_o = 0$ since m_o moves undisturbured with no acceleration once freeom

$$\boxed{\Sigma F = m\mathring{v} + \mathring{m}(v - v_o) \Rightarrow m\dot{v} + \mathring{m}u}$$

Application to Rocket Propulsion



$$-R - mg + PA = m\dot{v} + \dot{m}u$$

$$m' = -\dot{m}$$

$$PA + \underbrace{m'u}_{\text{Thrust}} - R - mg = m\dot{v}$$