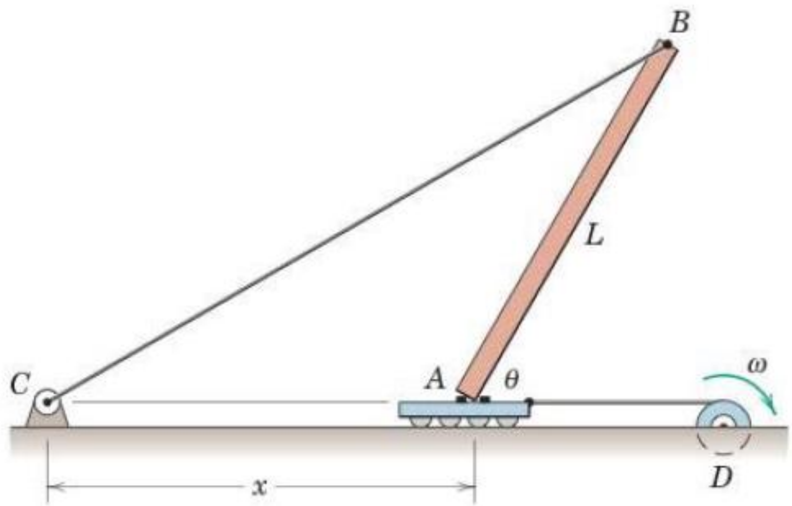
Indian Institute of Technology Guwahati

ME 101: Engineering Mechanics (2016-2017, Sem II)

Tutorial 9 Solution (Div 1 & 4)

Time: 8:00 AM – 8:55 AM **Full Marks**: 40

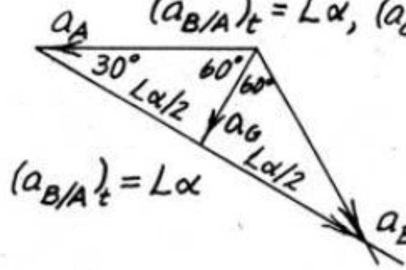
Ques.1 – The figure shows the edge view of a uniform concrete slab with a mass of 12 Mg. The slab is being hoisted slowly by the winch *D* with a cable attached to the dolly. At the position θ = 60^o, the distance *x* from the fixed ground position to the dolly is equal to the length L = 4 m of the slab. If the hoisting cable should break at this position, determine the initial acceleration a_A of the small dolly, whose mass is negligible, and the initial tension *T* in the fixed cable. End *A* of the slab will not slip on the dolly.

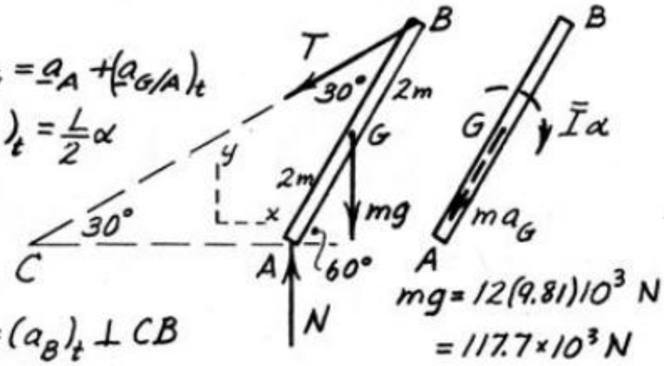


Sol. 1

$\underline{a}_B = \underline{a}_A + (\underline{a}_{B/A})_t, \quad \underline{a}_G = \underline{a}_A + (\underline{a}_{G/A})_t$

$(a_{B/A})_t = L\alpha, \quad (a_{G/A})_t = \frac{L}{2}\alpha$





$mg = 12(9.81)10^3$ N $= 117.7\times10^3$ N

From accel. diag. $a_B = \frac{L}{2}\alpha \sec 30° = \frac{4}{2}\sec 30°\alpha = 2.30\,\alpha$ m/s²

Since $\underline{a}_G$ passes through A,

$\Sigma M_A = \bar{I}\alpha: \; 117.7(10^3)2\cos 60° - T\times 4\sin 30° = \frac{1}{12}12(10^3)4^2\alpha$

$117.7(10^3) - 2T = 16(10^3)\alpha \qquad \text{-- (a)}$

$\Sigma F_x = m\bar{a}_x: \; T\cos 30° = 12(10^3)(a_G)_x$ where $a_G = \frac{L}{2}\alpha \tan 30°$,

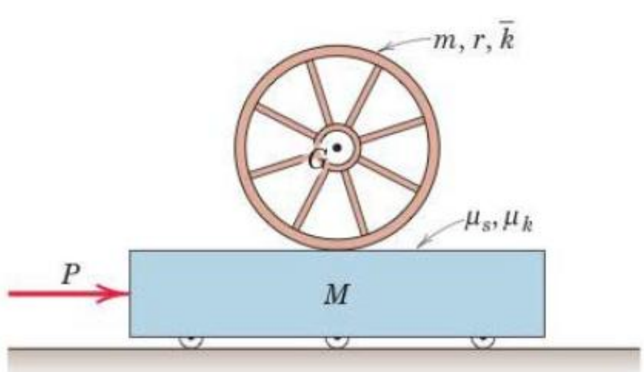
$(a_G)_x = \frac{L}{2}\tan 30°\alpha\cos 60° = 0.577\alpha \frac{m}{s^2}$

so $T = 8(10^3)\alpha \qquad \text{--- (b)}$

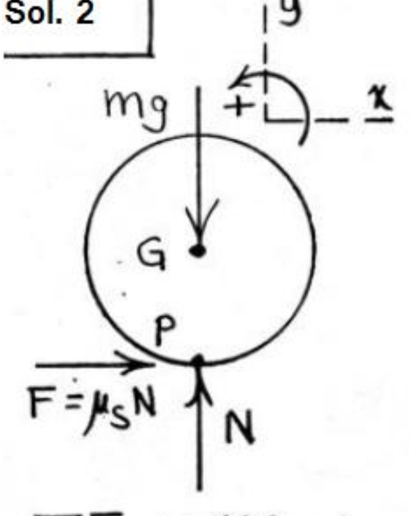
Solve (a) & (b) & get $\alpha = 3.68$ rad/s², $\underline{T = 29.4 \text{ kN}}$

$a_A = \frac{L}{2}\alpha/\cos 30° = \frac{4}{2}(3.68)/\cos 30°, \quad \underline{a_A = 8.50 \text{ m/s}^2}$

Ques.2 – Determine the maximum horizontal force *P* which may be applied to the cart of mass *M* for which the wheel will not slip as it begins to roll on the cart. The wheel has mass *m*, rolling radius *r*, and radius of gyration $\bar{k}$. The coefficients of the static and kinetic friction between the wheel and the cart are μ_s and μ_k, respectively.



Sol. 2



The no-slip constraint is found by equating the horizontal acceleration of point P to the acceleration a_c of the cart.

$$(a_P)_{hor} = a_G + r\alpha = a_C \qquad (1)$$

$$\Sigma F_y = 0 \Rightarrow N = mg \qquad (2)$$

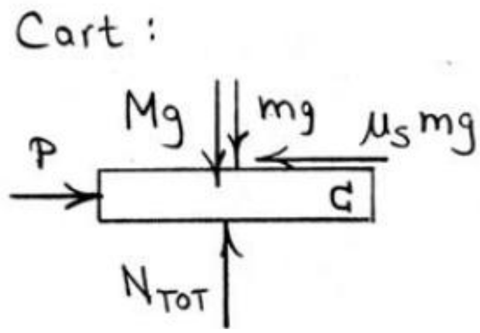
$$\Sigma F_x = m a_G : \quad \mu_s \cancel{m} g = \cancel{m} a_G \qquad (3)$$

$$\Sigma M_G = \bar{I}\alpha : \quad +\mu_s \cancel{m} g r = \cancel{m} \bar{k}^2 \alpha \qquad (4)$$

Solution of (1), (3), & (4) :

$$\begin{cases} a_G = \mu_s g \\ a_C = \mu_s g \left[1 + \frac{r^2}{\bar{k}^2}\right] \\ \alpha = +\mu_s g r / \bar{k}^2 \end{cases}$$

Cart :

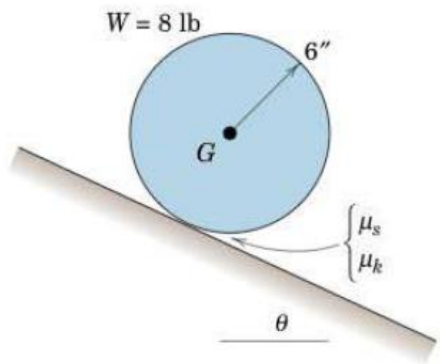


$$\Sigma F_x = M a_C :$$

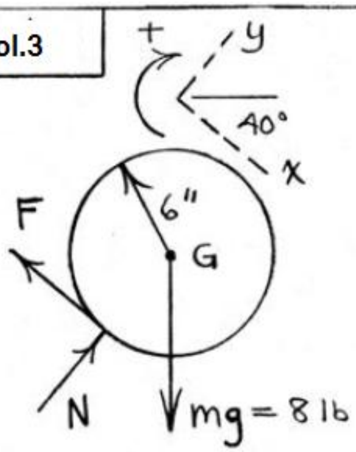
$$P - \mu_s mg = M \mu_s g \left[1 + \frac{r^2}{\bar{k}^2}\right]$$

$$\underline{P = \mu_s g \left[m + M\left(1 + \frac{r^2}{\bar{k}^2}\right)\right]}$$

Ques.3- The solid homogenous cylinder is released from rest on the ramp. If $\theta = 40^o$, $\mu_s = 0.30$, $\mu_k = 0.20$, determine the acceleration of the mass center G and the friction force exerted by the ramp on the cylinder.



Sol.3



$$\begin{cases} mg = 8 \text{ lb}, \quad \bar{I} = \frac{1}{2} m r^2 \\ \mu_s = 0.3, \quad \mu_k = 0.20 \\ \theta = 40^\circ \end{cases}$$

$$\Sigma F_x = m\bar{a}_x : \quad -F + 8 \sin 40^\circ = \frac{8}{32.2} a \qquad (1)$$

$$\Sigma F_y = 0 : \quad N - 8 \cos 40^\circ = 0 \qquad (2)$$

$$\Sigma M_G = \bar{I}\alpha : \quad F\left(\frac{6}{12}\right) = \frac{1}{2} \frac{8}{32.2} \left(\frac{6}{12}\right)^2 \alpha \qquad (3)$$

Assume rolling with no slip : $a = \frac{6}{12}\alpha$ (4)

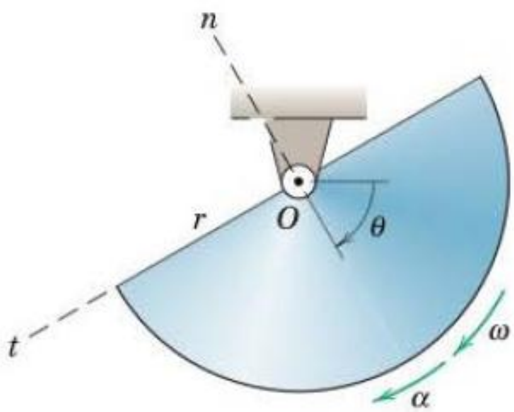
Solution of (1) - (4) : $\underline{F = 1.714 \text{ lb}}$ $\quad \underline{a = 13.80 \frac{ft}{sec^2}}$

$N = 6.13$ lb $\quad \alpha = 27.6 \frac{rad}{sec^2}$

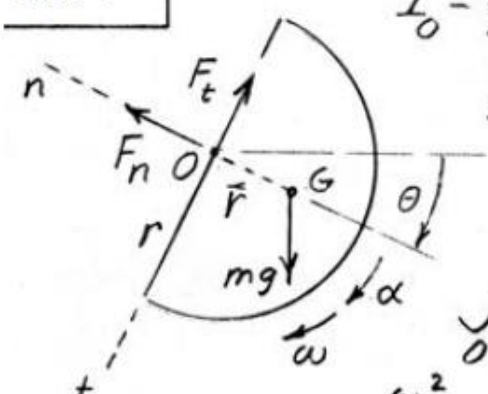
$F_{max} = \mu_s N = 0.3 \, (6.13) = 1.839 \text{ lb} > F$

Assumption valid.

Ques.4– The semicircular disk of mass *m* and radius *r* is released from rest at $\theta = 0^0$, and rotates freely in the vertical plane about its fixed bearing at *O*. Derive expression for the *n*- and *t*-components of the force *F* of the bearing as function of θ.



Sol. 4



$$I_O = \frac{1}{2}mr^2\ ; \quad \bar{r} = \frac{4r}{3\pi}$$

$$\Sigma M_O = I_O\alpha\ ; \quad mg\bar{r}\cos\theta = I_O\alpha$$

$$\alpha = mg\cos\theta\,\frac{\bar{r}}{I_O} = \frac{8}{3\pi}\,\frac{g}{r}\cos\theta$$

$$\int_0^\omega \omega\, d\omega = \int_0^\theta \alpha\, d\theta$$

$$\frac{\omega^2}{2} = \frac{8}{3\pi}\,\frac{g}{r}\sin\theta\Big]_0^\theta, \quad \omega^2 = \frac{16}{3\pi}\,\frac{g}{r}\sin\theta$$

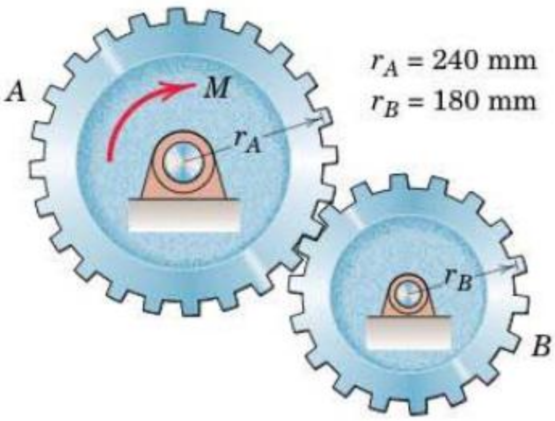
$$\Sigma F_n = m\bar{r}\omega^2\ ; \quad F_n - mg\sin\theta = m\,\frac{4r}{3\pi}\,\frac{16}{3\pi}\,\frac{g}{r}\sin\theta$$

$$F_n = \left(\frac{64}{9\pi^2} + 1\right)mg\sin\theta = \underline{1.721\, mg\sin\theta}$$

$$\Sigma F_t = m\bar{r}\alpha\ ; \quad mg\cos\theta - F_t = m\,\frac{4r}{3\pi}\,\frac{8}{3\pi}\,\frac{g}{r}\cos\theta$$

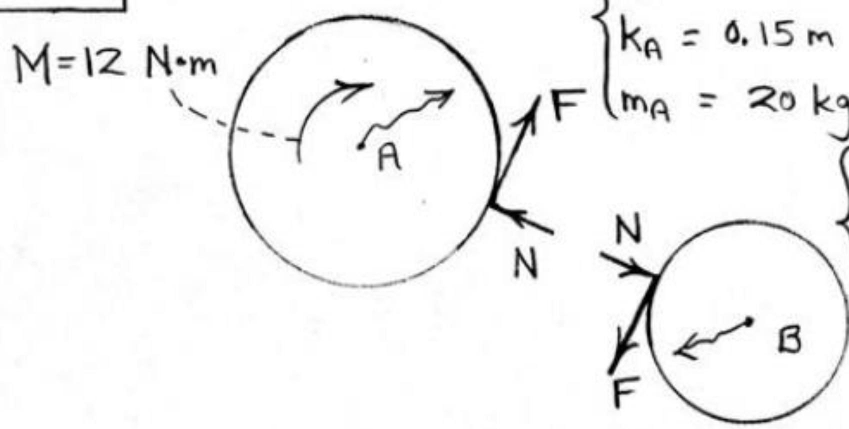
$$F_t = \left(1 - \frac{32}{9\pi^2}\right)mg\cos\theta = \underline{0.640\, mg\cos\theta}$$

Ques.5 – The mass of gear *A* is 20 kg and its centroidal radius of gyration is 150 mm. The mass of gear *B* is 10 kg and its centroidal radius of gyration is 100 mm. Calculate the angular acceleration of gear *B* when a torque of 12 N.m is applied to shaft of gear *A*. Neglect friction.



Sol. 5

$$\begin{cases} r_A = 0.24\text{ m} \\ k_A = 0.15\text{ m} \\ m_A = 20\text{ kg} \end{cases}$$



$$\begin{cases} r_B = 0.18\text{ m} \\ k_B = 0.1\text{ m} \\ m_B = 10\text{ kg} \end{cases}$$

$$\overset{\curvearrowright}{+} \Sigma M_A = I_A \alpha_A : \quad 12 - F(0.24) = 20(0.15)^2 \alpha_A \quad (1)$$

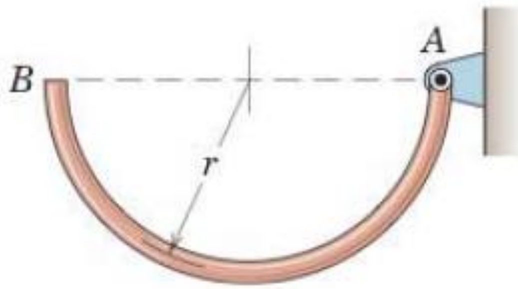
$$\overset{\curvearrowleft}{+} \Sigma M_B = I_B \alpha_B : \quad F(0.18) = 10(0.1)^2 \alpha_B \quad (2)$$

Tangential accelerations match: $r_A \alpha_A = r_B \alpha_B$

$$0.24\,\alpha_A = 0.18\,\alpha_B \quad (3)$$

Solution of Eqs. (1)–(3): $\begin{cases} F = 14.16\text{ N} \\ \alpha_A = 19.12\text{ rad/s}^2\text{ (CW)} \\ \alpha_B = 25.5\text{ rad/s}^2\text{ (CCW)} \end{cases}$

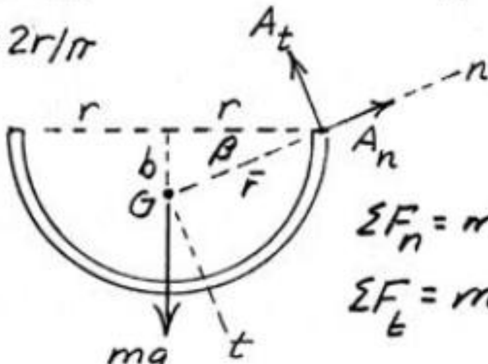
Ques.6 The uniform semicircular bar of mass *m* and radius *r* is hinged freely about a horizontal axis through *A*. If the bar is released from rest in position shown, where *AB* is horizontal, determine the initial angular acceleration α of the bar and the expression for force exerted on the bar by the pin at *A*. (Note carefully that the initial tangential acceleration of the mass center is not vertical.)



Sol. 6

$$I_A = \frac{1}{2}(2mr^2 + 2mr^2) = 2mr^2$$

$b = 2r/\pi$



$$\Sigma M_A = I_A \alpha; \quad mgr = 2mr^2\alpha$$

$$\underline{\alpha = g/2r}$$

$$\Sigma F_n = m\bar{a}_n = 0; \quad A_n = mg \sin\beta$$

$$\Sigma F_t = m\bar{a}_t; \quad mg\cos\beta - A_t = m\bar{r}\alpha$$

or $A_n = mg\frac{b}{\bar{r}}, \quad A_t = mg\left(\frac{r}{\bar{r}} - \frac{\bar{r}}{2r}\right)$

so $A = mg\sqrt{\frac{b^2}{\bar{r}^2} + \frac{r^2}{\bar{r}^2} - 1 + \frac{\bar{r}^2}{4r^2}} = mg\frac{\bar{r}}{2r} = \frac{mg}{2}\sqrt{1 + 4/\pi^2}$

or $\underline{A = 0.593\, mg}$