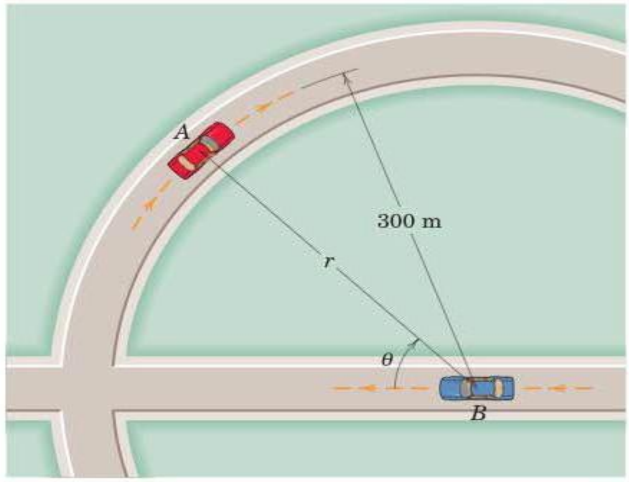
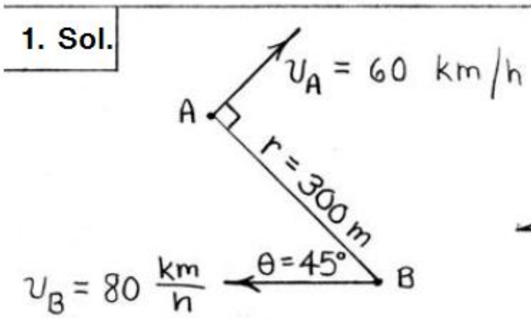
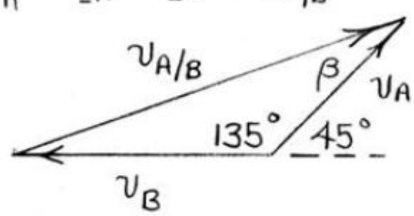
Ques.1 – Car A is traveling at the constant speed of 60 km/h as it rounds the circular curve of 300-m radius and at instant represented is at position θ = 45°. Car B is traveling at the constant speed of 80 km/h and passes the center of circle at this same instant. Car A is located with respect to car B by polar coordinates r and θ with the pole moving with B. For this instant determine $v_{A/B}$ and the value of $\dot{r}$ and $\dot{\theta}$ as measured by an observer in car B.



1. Sol.



$\underline{v}_A = \underline{v}_B + \underline{v}_{A/B}$



$$v_{A/B}^2 = 60^2 + 80^2 - 2(60)(80)\cos 135°$$

$$v_{A/B} = 129.6 \text{ km/h} \quad \text{or} \quad \underline{36.0 \text{ m/s}}$$

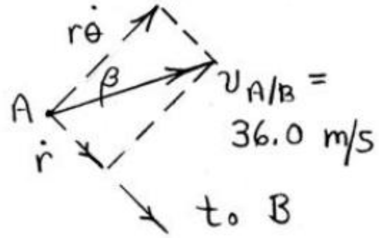
$$\frac{36.0}{\sin 135°} = \frac{80/3.6}{\sin\beta}, \quad \beta = 25.9°$$

$$r\dot{\theta} = v_{A/B}\cos\beta : \quad 300\,\dot{\theta} = 36.0\cos 25.9°$$

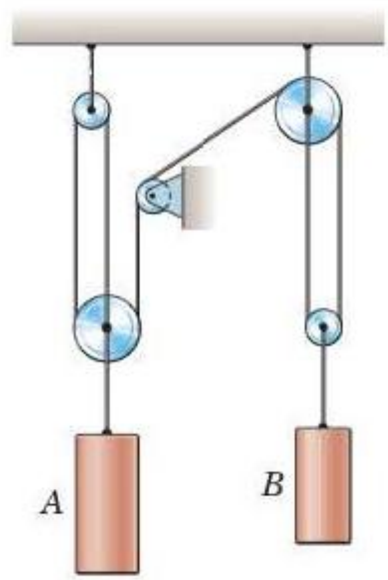
$$\underline{\dot{\theta} = 0.1079 \text{ rad/s}}$$

$$\dot{r} = -v_{A/B}\sin\beta : \quad \dot{r} = -36.0 \sin 25.9°$$

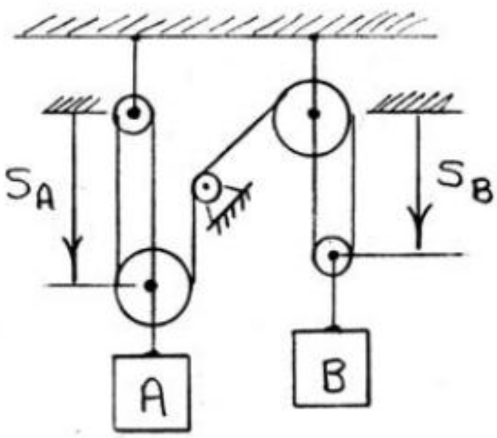
$$\underline{\dot{r} = -15.71 \text{ m/s}}$$



Ques.2 At a certain instant, cylinder *A* has a downward velocity of 0.8 *m/s* and an upward acceleration of 2 m/s^2. Determine the corresponding velocity and acceleration of cylinder B.



2 Sol.



The length of the main cable is

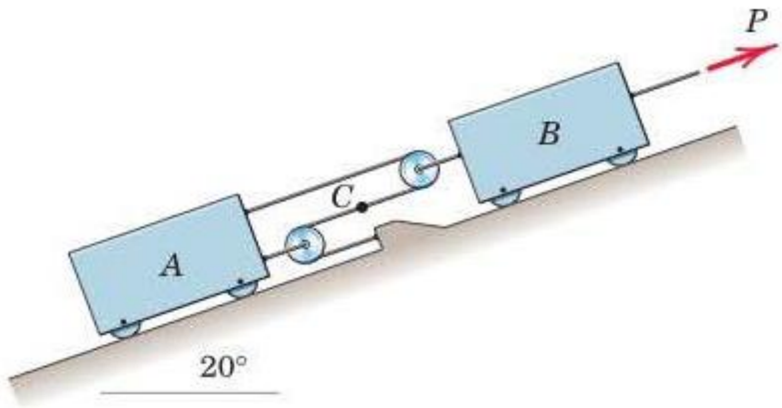
$L = 3s_A + 2s_B + \text{constants}$

$\Rightarrow 0 = 3v_A + 2v_B \; ; \quad 0 = 3a_A + 2a_B$

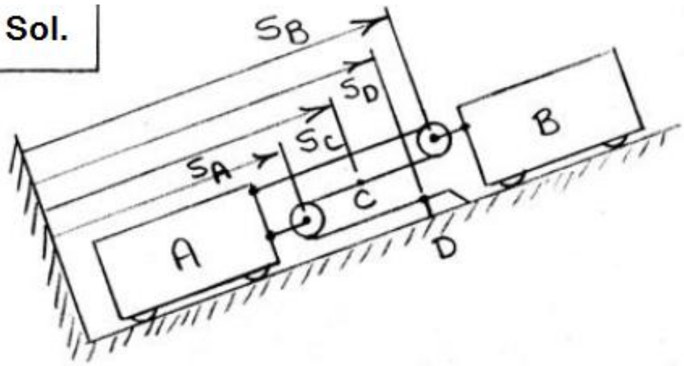
So $v_B = -\frac{3}{2} v_A = -\frac{3}{2}(0.8) = \underline{-1.2 \text{ m/s}}$ (up)

and $a_B = -\frac{3}{2} a_A = -\frac{3}{2}(-2) = \underline{3 \text{ m/s}^2}$ (down)

Ques.3- Under the action of force *P*, the constant acceleration of block *B* is 6 ft/sec^2 up the incline. For the instant when the velocity of *B* is 3 ft/sec up the incline, determine the velocity of *B* relative to *A*, and the acceleration of B relative to A, and the absolute velocity of point *C* of the cable.



3. Sol.



The cable length is $L = 2(s_B - s_A) + s_D - s_A$ + Constants

Differentiating:

$0 = 2v_B - 3v_A \;;\quad 0 = 2a_B - 3a_A$

So $v_A = \frac{2}{3} v_B = \frac{2}{3}(3) = 2$ ft/sec

$a_A = \frac{2}{3} a_B = \frac{2}{3}(6) = 4$ ft/sec^2

$v_{B/A} = v_B - v_A = 3 - 2 = \underline{1 \text{ ft/sec}}$

$a_{B/A} = a_B - a_A = 6 - 4 = \underline{2 \text{ ft/sec}^2}$

The length of cable between A and C is

$L' = (s_B - s_A) + (s_B - s_C) = 2s_B - s_A - s_C$ + constants

$\Rightarrow 0 = 2v_B - v_A - v_C \;;\; v_C = 2v_B - v_A = 2(3) - 2 = \underline{4 \text{ ft/sec}}$

(All answers are quantities directed up incline.)

Ques.4– A particle has the following position, velocity and acceleration components: x = 50 ft, y = 25 ft, $\dot{x}$ = -10 ft/sec, $\dot{y}$ = 10 ft/sec, $\ddot{x}$ = -10 ft/sec^2, and $\ddot{y}$ = 5 ft/sec^2. Determine the following quantities: v, a, e_t, e_n, a_t, a_n, ρ, e_r, e_θ, v_r, V_r, v_θ, V_θ, a_r, a_θ, r, $\dot{r}$, $\ddot{r}$, θ, $\dot{\theta}$, $\ddot{\theta}$. Express all vectors in terms of **i** and **j**, and graph all vectors on one set of *x-y* axes as you proceed.

4. Sol.

$$\begin{cases} x = 50 \text{ ft}, & \dot{x} = -10 \text{ ft/sec}, & \ddot{x} = -10 \text{ ft/sec}^2 \\ y = 25 \text{ ft}, & \dot{y} = 10 \text{ ft/sec}, & \ddot{y} = 5 \text{ ft/sec}^2 \end{cases}$$

$$v = \sqrt{\dot{x}^2 + \dot{y}^2} = \sqrt{(-10)^2 + 10^2} = \underline{10\sqrt{2} \text{ ft/sec}}$$

$$a = \sqrt{\ddot{x}^2 + \ddot{y}^2} = \sqrt{(-10)^2 + 5^2} = \underline{11.18 \text{ ft/sec}^2}$$

$$\underline{e}_t = \underline{v}/v = (-10\underline{i} + 10\underline{j})/10\sqrt{2} = \underline{\tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j})}$$

$$a_t = \underline{a} \cdot \underline{e}_t = (-10\underline{i} + 5\underline{j}) \cdot \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{10.61 \text{ ft/sec}^2}$$

$$\underline{a}_t = a_t \underline{e}_t = 10.61 \tfrac{\sqrt{2}}{2}(-\underline{i} + \underline{j}) = \underline{-7.5\underline{i} + 7.5\underline{j} \text{ ft/sec}^2}$$

$$\underline{a}_n = \underline{a} - \underline{a}_t = (-10\underline{i} + 5\underline{j}) - (-7.5\underline{i} + 7.5\underline{j}) = \underline{-2.5(\underline{i} + \underline{j}) \text{ ft/sec}^2}$$

$$a_n = \sqrt{2.5^2 + 2.5^2} = \underline{3.54 \text{ ft/sec}^2}$$

$$\rho = v^2/a_n = (10\sqrt{2})^2/3.54 = \underline{56.6 \text{ ft}}$$

$$\underline{e}_n = \underline{a}_n/a_n = -2.5(\underline{i} + \underline{j})/3.54 = \underline{-\tfrac{\sqrt{2}}{2}(\underline{i} + \underline{j})}$$

$$\underline{e}_r = \underline{r}/r = 50\underline{i} + 25\underline{j}/\sqrt{50^2 + 25^2} = \underline{0.894\underline{i} + 0.447\underline{j}}$$

$$\underline{e}_\theta = \underline{e}_r \text{ rotated CCW } 90° \doteq \underline{-0.447\underline{i} + 0.894\underline{j}}$$

$$v_r = \underline{v} \cdot \underline{e}_r = (-10\underline{i} + 10\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-4.47 \text{ ft/sec}}$$

$$\underline{v}_r = v_r \underline{e}_r = -4.47(0.894\underline{i} + 0.447\underline{j}) = \underline{-4\underline{i} - 2\underline{j} \text{ ft/sec}}$$

$$v_\theta = \underline{v} \cdot \underline{e}_\theta = (-10\underline{i} + 10\underline{j}) \cdot (-0.447\underline{i} + 0.894\underline{j}) = \underline{13.42 \text{ ft/sec}}$$

$$\underline{v}_\theta = v_\theta \underline{e}_\theta = 13.42(-0.447\underline{i} + 0.894\underline{j}) = \underline{-6\underline{i} + 12\underline{j} \text{ ft/sec}}$$

$$a_r = \underline{a} \cdot \underline{e}_r = (-10\underline{i} + 5\underline{j}) \cdot (0.894\underline{i} + 0.447\underline{j}) = \underline{-6.71 \text{ ft/sec}^2}$$

$$\underline{a}_r = a_r \underline{e}_r = -6.71(0.894\underline{i} + 0.447\underline{j}) = \underline{-6\underline{i} - 3\underline{j} \text{ ft/sec}^2}$$

$$a_\theta = \underline{a}\cdot\underline{e}_\theta = (-10\underline{i}+5\underline{j})\cdot(-0.447\underline{i}+0.894\underline{j}) = \underline{8.94 \text{ ft/sec}^2}$$

$$\underline{a}_\theta = a_\theta \underline{e}_\theta = 8.94(-0.447\underline{i}+0.894\underline{j}) = \underline{-4\underline{i}+8\underline{j} \text{ ft/sec}^2}$$

$$r = \sqrt{x^2+y^2} = \sqrt{50^2+25^2} = \underline{55.9 \text{ ft}}$$

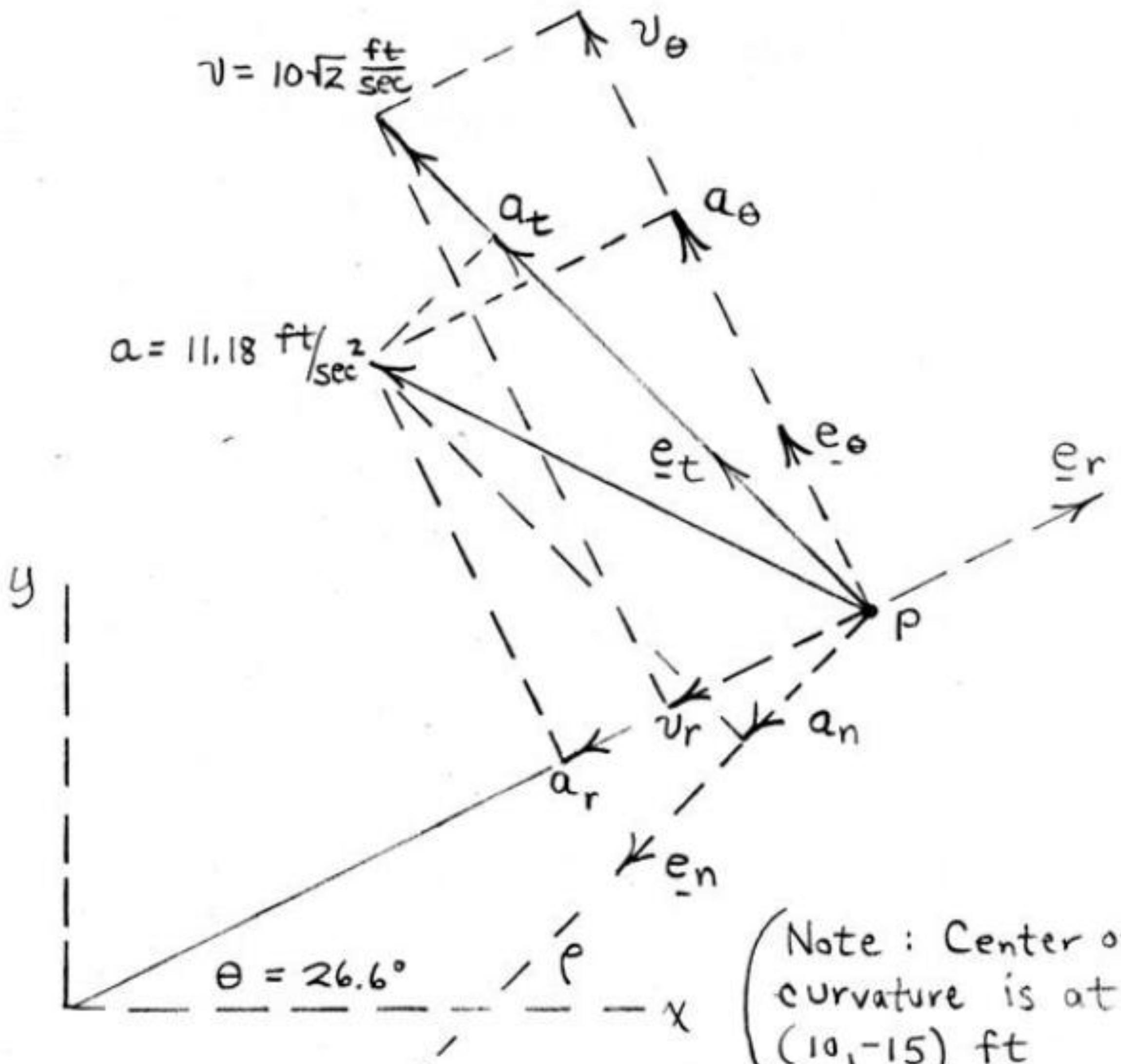
$$\dot{r} = v_r = \underline{-4.47 \text{ ft/sec}}$$

$$v_\theta = r\dot{\theta}, \quad \dot{\theta} = v_\theta/r = 13.42/55.9 = \underline{0.240 \text{ rad/sec}}$$

$$a_r = \ddot{r} - r\dot{\theta}^2, \quad \ddot{r} = a_r + r\dot{\theta}^2 = -6.71 + 55.9\,(0.240)^2$$
$$= \underline{-3.49 \text{ ft/sec}^2}$$

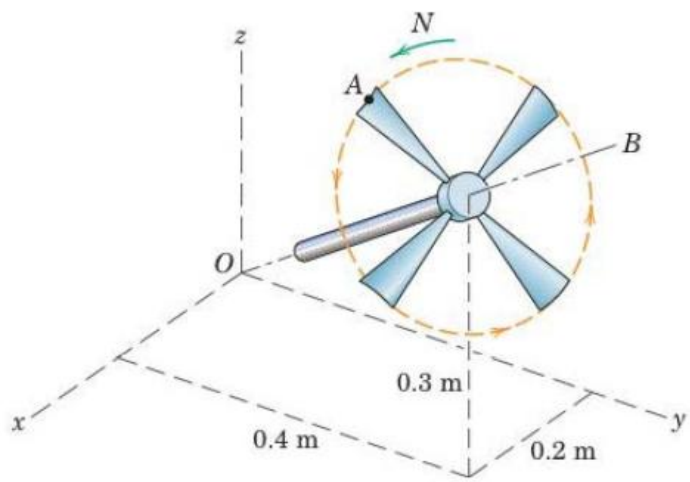
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}, \quad \ddot{\theta} = \frac{1}{r}(a_\theta - 2\dot{r}\dot{\theta})$$
$$= \frac{1}{55.9}\left[8.94 - 2(-4.47)(0.240)\right] = \underline{0.1984 \text{ rad/sec}^2}$$

$$\theta = \tan^{-1}(y/x) = \tan^{-1}(25/50) = \underline{26.6^\circ}$$



(Note : Center of curvature is at (10, −15) ft

Ques.5 – The four-bladed fan rotates about the fixed axis *OB* with a constant angular speed N = 1200 rev/min. Write the vector expression for the velocity **v** and acceleration **a** of the tip A of the fan blade for the instant when its *x-y-z* coordinates are 0.260, 0.240, and 0.473m, respectively.



5. Sol. $\underline{r}_{OA} = \underline{r} = 0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k}$ m

Unit vector along OB is

$$\underline{n} = (0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k})/\sqrt{0.2^2 + 0.4^2 + 0.3^2}$$

$$\underline{\omega} = \omega\underline{n} = \frac{1200(2\pi)}{60}\ \frac{0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}}{0.539}$$

$$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \text{ rad/s}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (0.260\underline{i} + 0.240\underline{j} + 0.473\underline{k})$$

$$= 233(0.1172\underline{i} - 0.0166\underline{j} - 0.056\underline{k}) \text{ m/s}$$

$$= \underline{27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k} \text{ m/s}}$$

$$\underline{a} = \dot{\underline{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = 0 + \underline{\omega} \times \underline{v}$$

$$= 233(0.2\underline{i} + 0.4\underline{j} + 0.3\underline{k}) \times (27.3\underline{i} - 3.87\underline{j} - 13.07\underline{k})$$

$$= \underline{-949\underline{i} + 2520\underline{j} - 2730\underline{k} \text{ m/s}^2}$$

Ques.6 A circular disk rotate about a fixed axis with a constant angular velocity ω =10(i+2j+2k) rad/sec. At a certain instant, a point *P* on its rim has a velocity whose *x* and *y* component are 120 in./sec and -80 in./sec, respectively. Determine the magnitude *v* of the velocity of *P* and radial distance *R* from *P* to the rotation axis. Also find the magnitude *a* of the acceleration of *P*.

6. Sol.

$$\underline{\omega}\cdot\underline{v} = 0, \quad 10(\underline{i}+2\underline{j}+2\underline{k})\cdot(120\underline{i}-80\underline{j}+v_z\underline{k}) = 0$$

$$120 - 160 + 2v_z = 0, \quad v_z = 20 \text{ in./sec}$$

$$v = \sqrt{120^2+80^2+20^2} = \underline{145.6 \text{ in./sec}}$$

$$v = R\omega, \quad R = \frac{145.6}{30} = \underline{4.85 \text{ in.}}$$

$$\text{where } \omega = 10\sqrt{1^2+2^2+2^2} = 10(3) = 30 \text{ rad/sec}$$

$$\underline{a} = \dot{\underline{\omega}}\times\underline{r} + \underline{\omega}\times(\underline{\omega}\times\underline{r}) = \underline{0} + \underline{\omega}\times\underline{v}$$

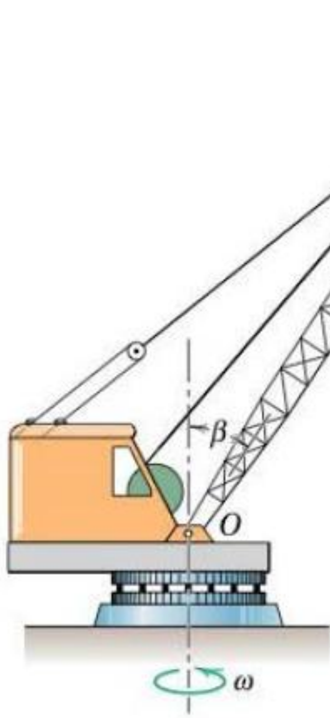
$$= 10(\underline{i}+2\underline{j}+2\underline{k})\times(120\underline{i}-80\underline{j}+20\underline{k})$$

$$= 10(200\underline{i}+220\underline{j}-320\underline{k})$$

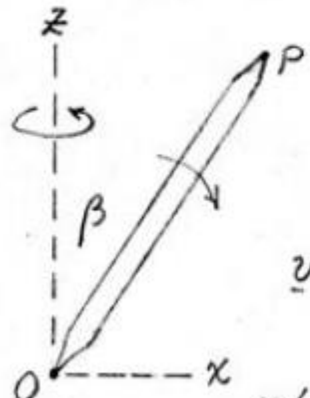
$$a = 10\sqrt{200^2+220^2+320^2} = 10\sqrt{190800} = \underline{4370 \text{ in./sec}^2}$$

$$\left(\text{or simply } a = a_n = r\omega^2 = 4.85(30^2) = 4370 \text{ in./sec}^2\right)$$

Ques.7 The crane has a boom of length *OP* = 24 m and is revolving about vertical axis at the constant rate of 2 rev/min in the direction shown. Simultaneously, the boom is lowered at the constant rate $\dot{\beta}$ = 0.10 rad/s. Calculate the magnitude of the velocity and acceleration of the end point P of the boom for the instant when it passes the position β = 30°.



7. Sol. $\overline{OP} = 24\ m,\quad \dot{\beta} = 0.10\ rad/s\ \text{const.},\quad \beta = 30^\circ$



$$\underline{r} = \overrightarrow{OP} = (24 \sin 30^\circ)\underline{i} + (24 \cos 30^\circ)\underline{k}$$

$$= 12\underline{i} + 20.78\underline{k}\ \ m$$

$$\underline{\omega} = \frac{2(2\pi)}{60}\underline{k} + 0.10\underline{j} = 0.209\underline{k} + 0.10\underline{j}\ \ \frac{rad}{s}$$

$$\underline{v} = \underline{\omega} \times \underline{r} = (0.209\underline{k} + 0.10\underline{j}) \times (12\underline{i} + 20.78\underline{k})$$

$$= 2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k}\ \ m/s$$

where $v = |\underline{v}| = \sqrt{(2.078)^2 + (2.513)^2 + (-1.2)^2} = \underline{3.48\ \frac{m}{s}}$

$$\underline{a} = \underline{\dot{\omega}} \times \underline{r} + \underline{\omega} \times (\underline{\omega} \times \underline{r}) = \underline{\alpha} \times \underline{r} + \underline{\omega} \times \underline{v}$$

$$\underline{\alpha} = \underline{\dot{\omega}} = \underline{\omega}_z \times \underline{\omega}_y = 0.209\underline{k} \times 0.10\underline{j} = -0.0209\underline{i}\ \ rad/s^2$$

$$\underline{\dot{\omega}} \times \underline{r} = \underline{\alpha} \times \underline{r} = -0.0209\underline{i} \times (12\underline{i} + 20.78\underline{k}) = 0.435\underline{j}\ \ m/s^2$$

$$\underline{\omega} \times \underline{v} = (0.209\underline{k} \times 0.10\underline{j}) \times (2.078\underline{i} + 2.513\underline{j} - 1.2\underline{k})$$

$$= -0.646\underline{i} + 0.435\underline{j} - 0.208\underline{k}\ \ m/s^2$$

$$\underline{a} = -0.646\underline{i} + 0.870\underline{j} - 0.208\underline{k}\ \ m/s^2$$

$$a = |\underline{a}| = \sqrt{(-0.646)^2 + (0.870)^2 + (-0.208)^2} = \underline{1.104\ m/s^2}$$