Indian Institute of Technology Guwahati

ME 101: Engineering Mechanics (2016-2017, Sem II)

Tutorial 4 (20.02.2017) (Div 1 & 4)

Time: 8:00 AM – 8:55 AM **Full Marks**: 40

Q1. A couple **M** of magnitude 100 N m is applied as shown in Fig 1 to the crank of the engine system. Knowing that *AB* = 50 mm and *BC* = 200 mm, determine the force **P** required to maintain the equilibrium of the system when (*a*) θ= 60°, (*b*) θ =120°.

Q.2. A vertical force **P** of magnitude 150 N is applied to end *E* of cable *CDE*, which passes over a small pulley *D* and is attached to the mechanism at *C*. The constant of the spring is *k* = 4 kN/m, and the spring is unstretched when θ = 0. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium

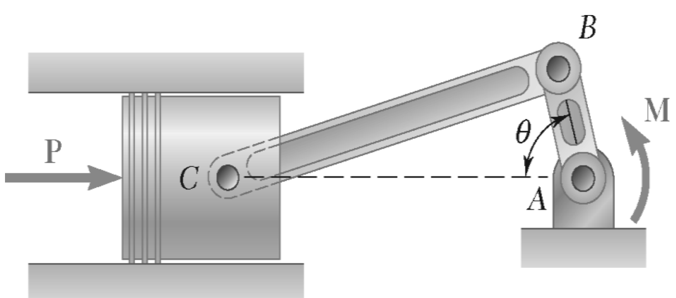


Fig. 1

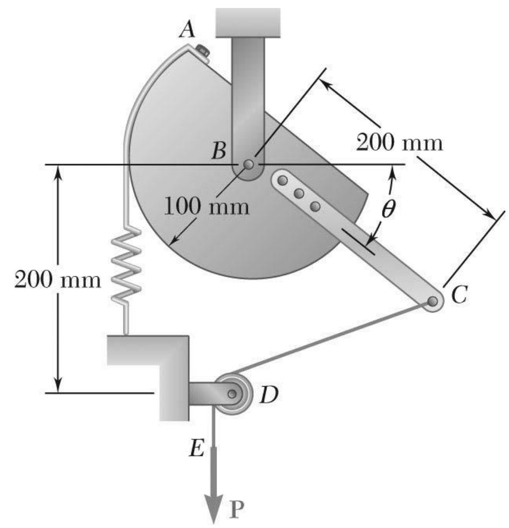


Fig. 2.

Q.3. Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal (Fig 3), determine the value of θ corresponding to equilibrium for the data indicated. *P* = 300 N, *l* = 400 mm, *k* = 5 kN/m..

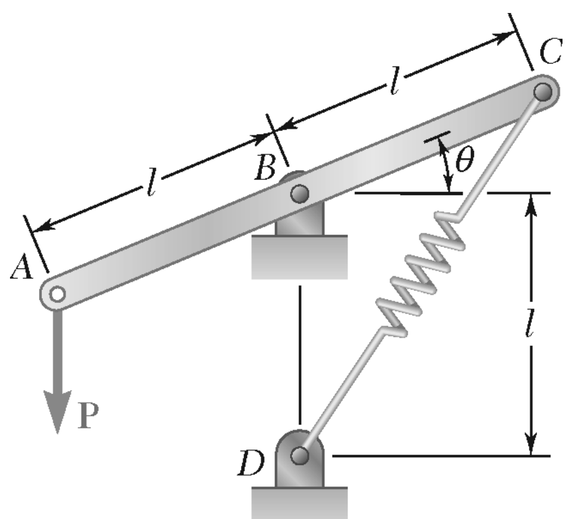


Fig. 3

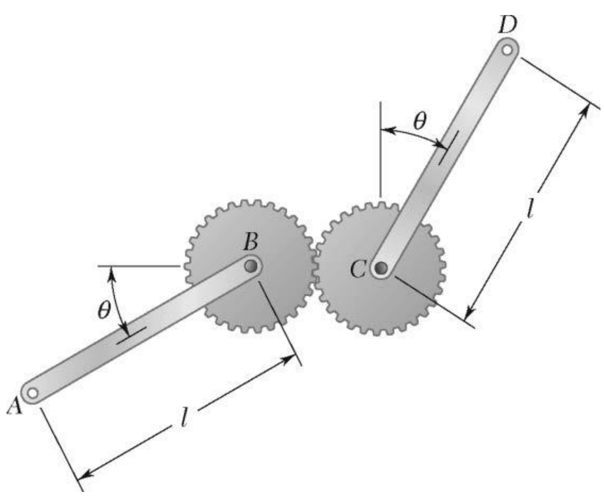


Fig. 5

Q.4 Solve the question 2 and question 3 using the method of potential energy.

Q.5. Two uniform rods, each of mass *m*, are attached to gears of equal radii as shown in Fig 5. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.

Q.6. A vertical bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown in Fig 6. Determine the range of values of the magnitude *P* of two equal and opposite vertical forces **P** and −**P** for which the equilibrium position is stable if (*a*) $AB = CD$, (*b*) $AB = 2CD$.

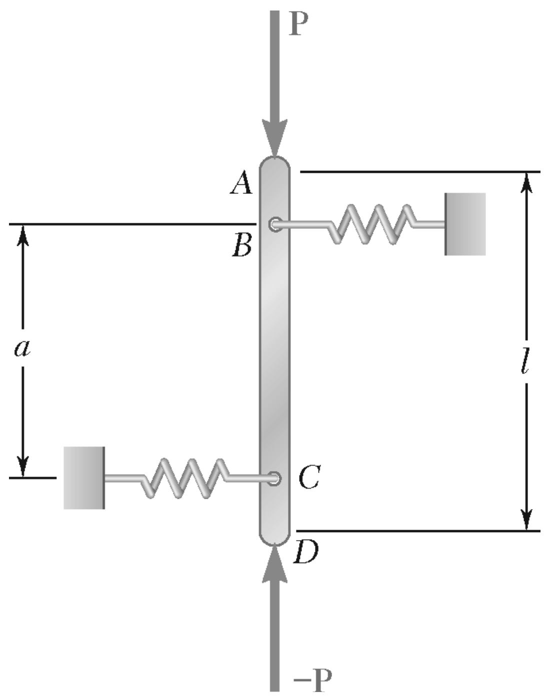
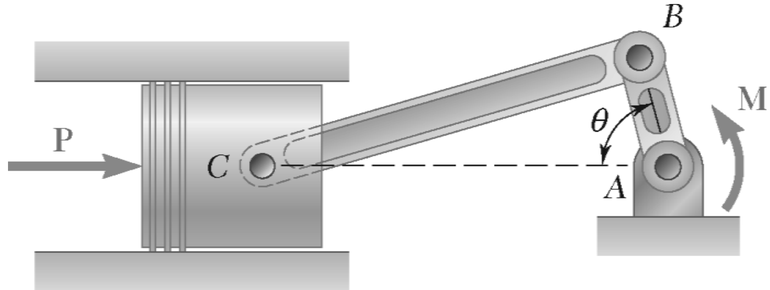


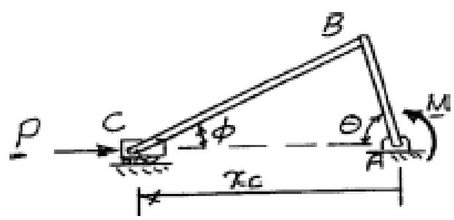
Fig. 6

Q1. A couple **M** of magnitude 100 N m is applied as shown in Fig 1 to the crank of the engine system. Knowing that *AB* = 50 mm and *BC* = 200 mm, determine the force **P** required to maintain the equilibrium of the system when (*a*) θ= 60°, (*b*) θ =120°.



SOLUTION

Analysis of the geometry:



Law of sines

$$\frac{\sin\phi}{AB} = \frac{\sin\theta}{BC}$$

$$\sin\phi = \frac{AB}{BC}\sin\theta \qquad (1)$$

Now

$$x_C = AB\cos\theta + BC\cos\phi$$

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\delta\phi \qquad (2)$$

Now, from Equation (1)

$$\cos\phi\delta\phi = \frac{AB}{BC}\cos\theta\delta\theta$$

or

$$\delta\phi = \frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta \qquad (3)$$

From Equation (2)

$$\delta x_C = -AB\sin\theta\delta\theta - BC\sin\phi\left(\frac{AB}{BC}\frac{\cos\theta}{\cos\phi}\delta\theta\right)$$

or

$$\delta x_C = -\frac{AB}{\cos\phi}(\sin\theta\cos\phi + \sin\phi\cos\theta)\delta\theta$$

Then

$$\delta x_C = -\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta$$

Virtual Work: $\delta U = 0$: $\quad -P\delta x_C - M\,\delta\theta = 0$

$$-P\left[-\frac{AB\sin(\theta+\phi)}{\cos\phi}\delta\theta\right] - M\,\delta\theta = 0$$

Thus, $$M = AB\frac{\sin(\theta+\phi)}{\cos\phi}P \qquad (4)$$

(*a*) $$M = 100 \text{ N}\cdot\text{m}, \quad \theta = 60^\circ$$

Eq. (1): $$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 60^\circ \qquad \phi = 12.504^\circ$$

Eq. (4): $$100 \text{ N}\cdot\text{m} = (0.05 \text{ m})\frac{\sin(60^\circ + 12.504^\circ)}{\cos 12.504^\circ}P$$

$$P = 2047 \text{ N}$$

P = 2.05 kN ⟶ ◀

(*b*) $$M = 100 \text{ N}\cdot\text{m}, \quad \theta = 120^\circ$$

Eq. (1): $$\sin\phi = \frac{50 \text{ mm}}{200 \text{ mm}}\sin 120^\circ \quad \phi = 12.504^\circ$$

Eq. (4): $$100 \text{ N}\cdot\text{m} = (0.05 \text{ m})\frac{\sin(120^\circ + 12.504^\circ)}{\cos 12.504^\circ}P$$

$$P = 2649 \text{ N}$$

P = 2.65 kN ⟶ ◀

Q.2. A vertical force **P** of magnitude 150 N is applied to end *E* of cable *CDE*, which passes over a small pulley *D* and is attached to the mechanism at *C*. The constant of the spring is *k* = 4 kN/m, and the spring is unstretched when θ = 0. Neglecting the weight of the mechanism and the radius of the pulley, determine the value of θ corresponding to equilibrium

$$l = BC = 0.2 \text{ m}$$

$$r = 0.1 \text{ m}$$

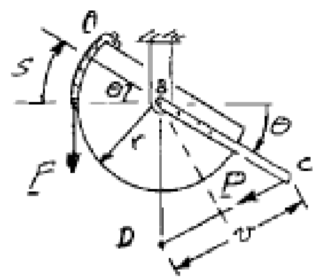
$< CBD = 90^\circ - \theta$

$$v = 2l \sin\left(45^\circ - \frac{\theta}{2}\right)$$

$$\delta v = -l \cos\left(45^\circ - \frac{\theta}{2}\right)\delta\theta$$

$$s = r\theta \quad \delta s = r\delta\theta$$

$$F = ks = kr\theta$$



Virtual Work:

$$\delta U = 0: \quad -P\delta v - F\delta s = 0$$

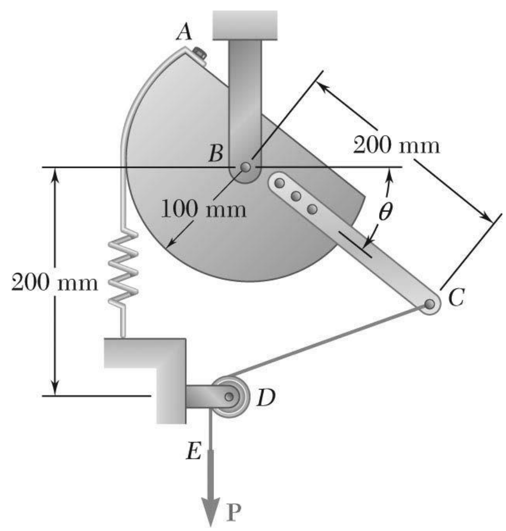
$$-P\left[-l\cos\left(45^\circ - \frac{\theta}{2}\right)\right]\delta\theta - kr\theta(r\delta\theta) = 0$$

$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$

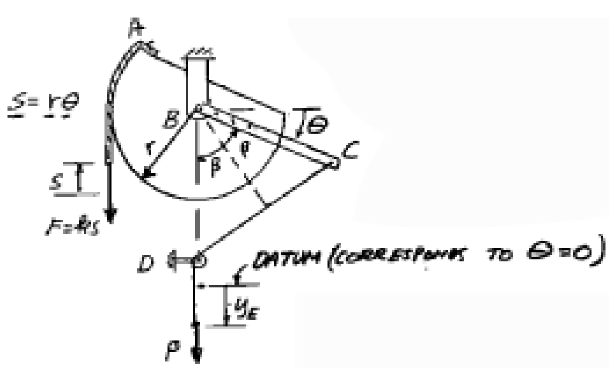
$$0.75 = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

$$\theta = 0.67623 \text{ rad} = 38.745^\circ$$



Q4. Answer using method of Potential Energy

SOLUTION



$$\beta = \frac{1}{2}(90^\circ - \theta) = 45^\circ - \frac{\theta}{2}$$

$$BC = BD = l$$

$$CD = 2l \sin\beta = 2l \sin\left(45 - \frac{\theta}{2}\right)$$

For $\theta = 0$: $\quad (CD)_0 = 2l\sin 45 = \sqrt{2}\, l$

$$y_E = (CD)_0 - CD$$

$$= \sqrt{2}\, l - 2l\sin\left(45^\circ - \frac{\theta}{2}\right)$$

Potential energy:

$$V = \frac{1}{2}ks^2 - Py_E$$

$$V = \frac{1}{2}k(r\theta)^2 - P\left[\sqrt{2}l - 2l\sin\left(45^\circ - \frac{\theta}{2}\right)\right]$$

$$\frac{dV}{d\theta} = kr^2\theta + 2Pl\cos\left(45^\circ - \frac{\theta}{2}\right)\left(-\frac{1}{2}\right) = 0$$

$$\frac{Pl}{kr^2} = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

$$\frac{Pl}{kr^2} = \frac{(150 \text{ N})(0.2 \text{ m})}{(4000 \text{ N/m})(0.1 \text{ m})^2} = 0.75$$

$$0.75 = \frac{\theta}{\cos\left(45^\circ - \frac{\theta}{2}\right)}$$

Solve by trial and error: $\theta = 0.67623$ rad

$$\theta = 38.745^\circ$$

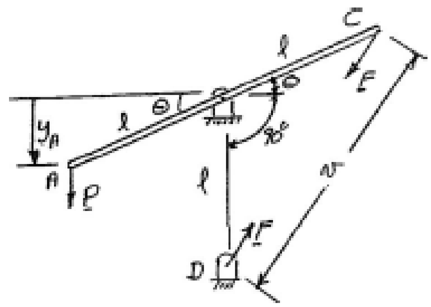
$\theta = 38.7^\circ$

Q3. Knowing that the constant of spring *CD* is *k* and that the spring is unstretched when rod *ABC* is horizontal (Fig 3), determine the value of θ corresponding to equilibrium for the data indicated. *P* = 300 N, *l* = 400 mm, *k* = 5 kN/m..

SOLUTION

$$y_A = l\sin\theta$$

$$\delta y_A = l\cos\theta\,\delta\theta$$



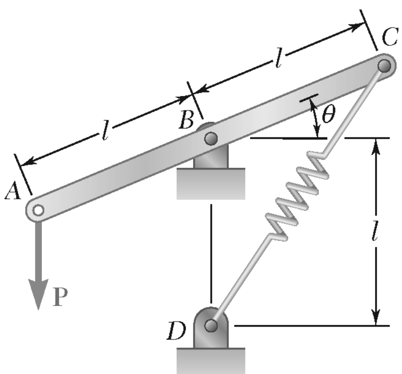
Spring: $v = CD$

Unstretched when $\theta = 0$

so that $v_0 = \sqrt{2}l$

For θ: $v = 2l\sin\left(\dfrac{90^\circ+\theta}{2}\right)$

$$\delta v = l\cos\left(45^\circ+\frac{\theta}{2}\right)\delta\theta$$



Stretched length: $s = v - v_0 = 2l\sin\left(45^\circ+\dfrac{\theta}{2}\right) - \sqrt{2}l$

Then $F = ks = kl\left[2\sin\left(45^\circ+\dfrac{\theta}{2}\right) - \sqrt{2}\right]$

<u>Virtual Work</u>:

$$\delta U = 0: \quad P\delta y_A - F\delta v = 0$$

$$Pl\cos\theta\delta\theta - kl\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)-\sqrt{2}\right]l\cos\left(45^\circ+\frac{\theta}{2}\right)\delta\theta = 0$$

or

$$\frac{P}{kl} = \frac{1}{\cos\theta}\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)\cos\left(45^\circ+\frac{\theta}{2}\right)-\sqrt{2}\cos\left(45^\circ+\frac{\theta}{2}\right)\right]$$

$$= \frac{1}{\cos\theta}\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)\cos\left(45^\circ+\frac{\theta}{2}\right)\cos\theta-\sqrt{2}\cos\left(45^\circ+\frac{\theta}{2}\right)\right]$$

$$= 1-\sqrt{2}\frac{\cos\left(45^\circ\frac{\theta}{2}\right)}{\cos\theta}$$

Now, with $P = 300$ N, $l = 400$ mm, and $k = 5$ kN/m

$$\frac{(300\text{ N})}{(5000\text{ N/m})(0.4\text{ m})} = 1-\sqrt{2}\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta}$$

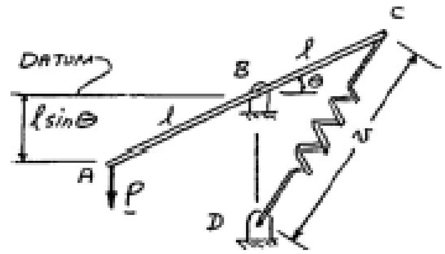
or

$$\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $\theta = 22.6^\circ$ ◀

Solution Q 4. Using method of potential energy

SOLUTION



Spring $$v = 2l\sin\left(\frac{90^\circ+\theta}{2}\right)$$

$$v = 2l\sin\left(45^\circ+\frac{\theta}{2}\right)$$

Unstretched $(\theta = 0)$ $$v_0 = 2l\sin 45^\circ = \sqrt{2}l$$

Deflection of spring $$s = v - v_0 = 2l\sin\left(45^\circ+\frac{\theta}{2}\right) - \sqrt{2}l$$

$$V = \frac{1}{2}ks^2 + Py_A = \frac{1}{2}kl^2\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)-\sqrt{2}\right]^2 + P(-l\sin\theta)$$

$$\frac{dV}{d\theta} = kl^2\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)-\sqrt{2}\right]\cos\left(45^\circ+\frac{\theta}{2}\right) - Pl\cos\theta = 0$$

$$\left[2\sin\left(45^\circ+\frac{\theta}{2}\right)\cos\left(45^\circ+\frac{\theta}{2}\right)-\sqrt{2}\cos\left(45^\circ+\frac{\theta}{2}\right)\right] = \frac{P}{kl}\cos\theta$$

$$\cos\theta - \sqrt{2}\cos\left(45^\circ+\frac{\theta}{2}\right) = \frac{P}{kl}\cos\theta$$

Divide each member by cos θ $$1-\sqrt{2}\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = \frac{P}{kl}$$

Then with $P = 300$ N, $l = 0.4$ m and $k = 5000$ N/m

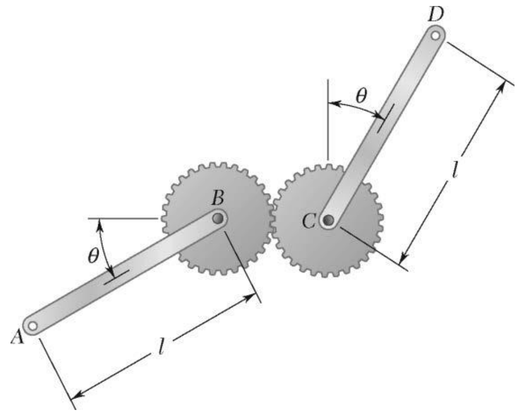
$$1-\sqrt{2}\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = \frac{300 \text{ N}}{(5000 \text{ N/m})(0.4 \text{ m})}$$

$$= 0.15$$

or $$\frac{\cos\left(45^\circ+\frac{\theta}{2}\right)}{\cos\theta} = 0.60104$$

Solving numerically $$\theta = 22.6^\circ \blacktriangleleft$$

Q.5. Two uniform rods, each of mass *m*, are attached to gears of equal radii as shown. Determine the positions of equilibrium of the system and state in each case whether the equilibrium is stable, unstable, or neutral.



SOLUTION

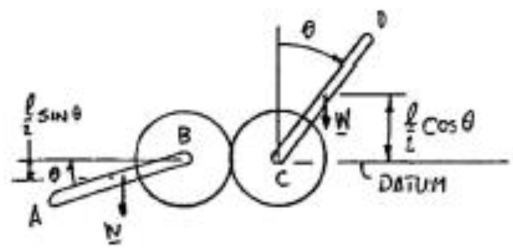
Potential energy

$$V = W\left(-\frac{l}{2}\sin\theta\right) + W\left(\frac{l}{2}\cos\theta\right) \quad W = mg$$

$$= W\frac{l}{2}(\cos\theta - \sin\theta)$$

$$\frac{dV}{d\theta} = \frac{Wl}{2}(-\sin\theta - \cos\theta)$$

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin\theta - \cos\theta)$$



For equilibrium: $\frac{dV}{d\theta} = 0: \quad \sin\theta = -\cos\theta$

or $\tan\theta = -1$

Thus $\theta = -45.0^\circ$ and $\theta = 135.0^\circ$

Stability:

At $\theta = -45.0^\circ$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}[\sin(-45^\circ) - \cos 45^\circ]$$

$$= \frac{Wl}{2}\left(-\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\right) < 0$$

$\theta = -45.0^\circ$, Unstable ◀

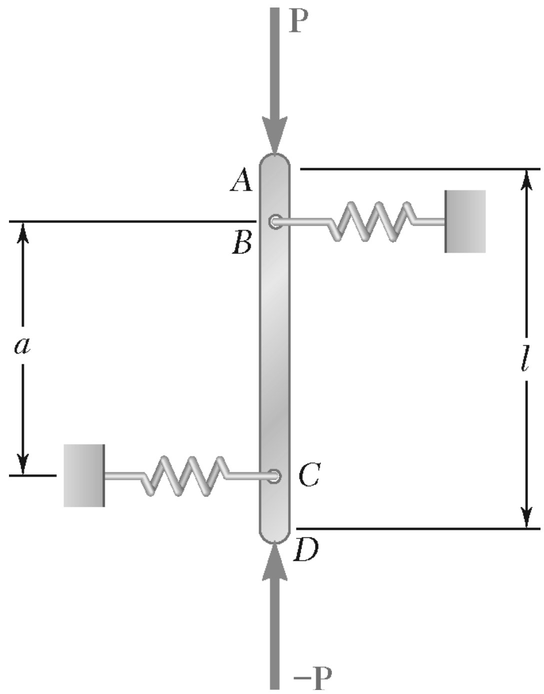
At $\theta = 135.0^\circ$:

$$\frac{d^2V}{d\theta^2} = \frac{Wl}{2}(\sin 135^\circ - \cos 135^\circ)$$

$$= \frac{Wl}{2}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) > 0$$

$\theta = 135.0^\circ$. Stable ◀

Q.6. A vertical bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. Determine the range of values of the magnitude *P* of two equal and opposite vertical forces **P** and −**P** for which the equilibrium position is stable if (*a*) *AB* = *CD*, (*b*) *AB* = 2*CD*.



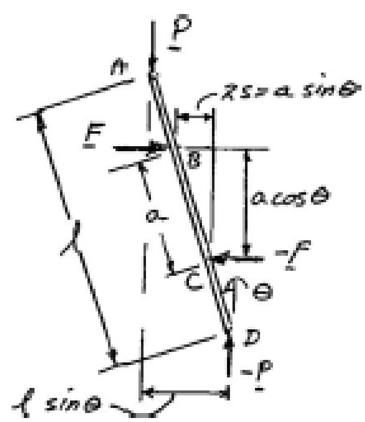
SOLUTION

For both (*a*) and (*b*): Since **P** and −**P** are vertical, they form a couple of moment

$$M_P = +Pl\sin\theta$$

The forces **F** and −**F** exerted by springs must, therefore, also form a couple, with moment

$$M_F = -Fa\cos\theta$$



We have $$dU = M_P d\theta + M_F d\theta$$

$$= (Pl\sin\theta - Fa\cos\theta)d\theta$$

but $$F = ks = k\left(\frac{1}{2}a\sin\theta\right)$$

Thus, $$dU = \left(Pl\sin\theta - \frac{1}{2}ka^2\sin\theta\cos\theta\right)d\theta$$

From Equation (10.19), page 580, we have

$$dV = -dU = -Pl\sin\theta d\theta + \frac{1}{4}ka^2\sin 2\theta d\theta$$

or $$\frac{dV}{d\theta} = -Pl\sin\theta + \frac{1}{4}ka^2\sin 2\theta$$

and $$\frac{d^2V}{d\theta^2} = -Pl\cos\theta + \frac{1}{2}ka^2\cos 2\theta \qquad (1)$$

For $\theta = 0$: $$\frac{d^2V}{d\theta^2} = -Pl + \frac{1}{2}ka^2$$

For Stability: $$\frac{d^2V}{d\theta^2} > 0, \quad -Pl + \frac{1}{2}ka^2 > 0$$

or (for Parts *a* and *b*) $$P < \frac{ka^2}{2l} \blacktriangleleft$$

Note: To check that equilibrium is unstable for $P = \frac{ka^2}{2l}$, we differentiate (1) twice:

$$\frac{d^3V}{d\theta^3} = +Pl\sin\theta - ka^2\sin 2\theta = 0, \quad \text{for} \quad \theta = 0,$$

$$\frac{d^4V}{d\theta^4} = Pl\cos\theta - 2ka^2\cos 2\theta$$

For $\theta = 0$ $$\frac{d^4V}{d\theta^4} = Pl - 2ka^2 = \frac{ka^2}{2} - 2ka^2 < 0$$

Thus, equilibrium is unstable when $$P = \frac{ka^2}{2l}$$