Indian Institute of Technology Guwahati

ME 101: Engineering Mechanics (2016-2017, Sem II)

Tutorial 4 (13.02.2017) (Div 1 & 4)

Time: 8:00 AM – 8:55 AM **Full Marks**: 40

Q1. A cylinder of 20 cm diameter and having weight 10 KN as shown in Fig.1 is held at rest on the 30° incline by a weight P suspended from a cord wrapped around the cylinder. If slipping impends, determine P and the coefficient of friction.

Q.2. A 6 m long ladder has a mass of 18 kg and its center of gravity is 2.4 m from the bottom. The ladder is placed against a vertical wall so that it makes an angle of 60° with the ground as shown in Fig. 2. How far up the ladder can a 72-kg man climb before the ladder is on the verge of slipping? The angle of friction at all contact surfaces is 15°.

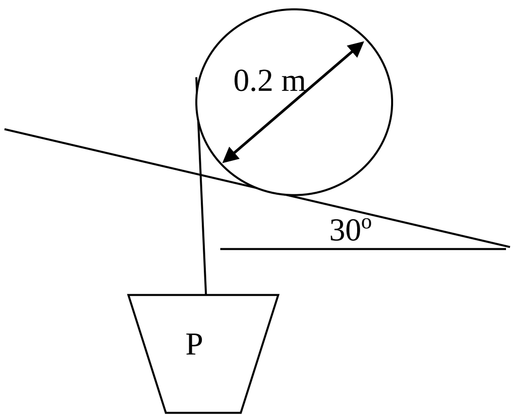


Fig. 1

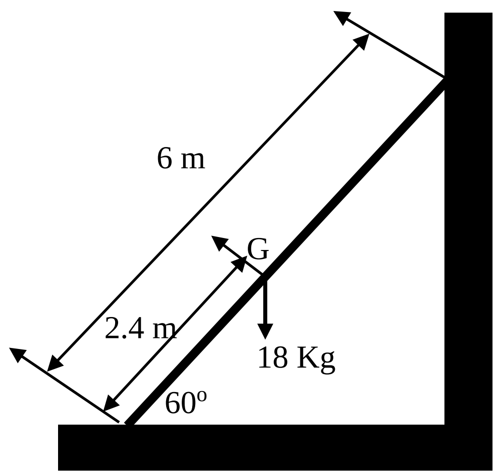


Fig. 2.

Q.3. Determine the value of P just sufficient to start the 10° wedge under the 40-kN block as shown in Fig 3. The angle of friction is 20° for all contact surfaces.

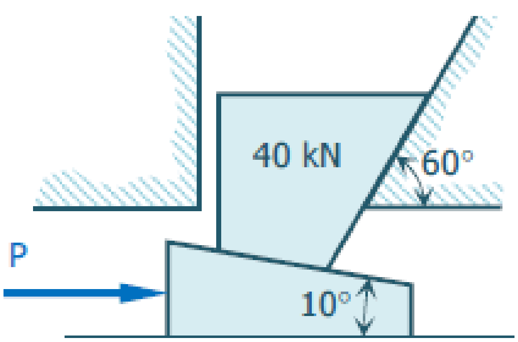


Fig. 3

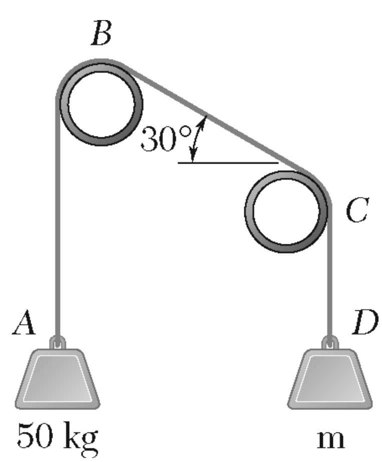


Fig. 4.

Q.4 A rope *ABCD* is looped over two pipes as shown in Fig. 4. The coefficient of static friction is 0.25 for all contacts, determine (*a*) the smallest value of the mass *m* for which equilibrium is possible, (*b*) the corresponding tension in portion *BC* of the rope.

Q.5. The speed of the brake drum as shown in Fig. 5 is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 100 N is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that *a* = 5 cm., and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise

Q 6. In the machinist's vise shown, the movable jaw D is rigidly attached to the tongue AB that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 2 cm and a pitch of 0.6 cm. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 4 KN.

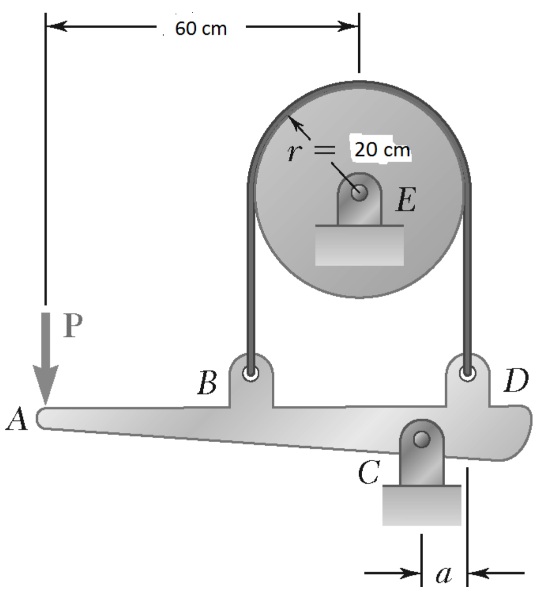


Fig. 5

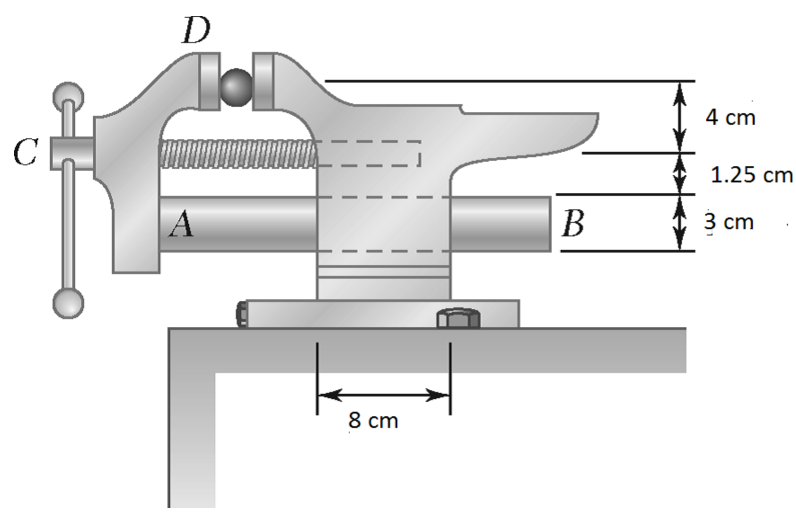


Fig. 6.

Q1. A cylinder of 20 cm diameter and having weight 10 KN as shown in Fig.1 is held at rest on the 30° incline by a weight P suspended from a cord wrapped around the cylinder. If slipping impends, determine P and the coefficient of friction.

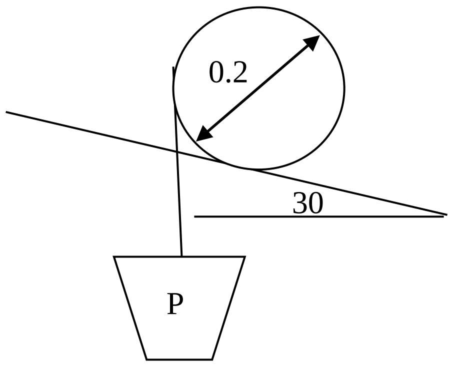


Fig.1

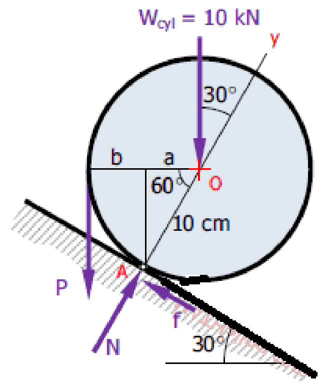


Fig. 2

Solution:

Draw the FBD as shown in fig. 2

Find out

$$a = 10\cos 60^\circ = 5 \text{ cm}$$

$$b = 10 - a = 5 \text{ cm}$$

Taking moment about A

$$\Sigma M_A = 0\;; \qquad Pb = Wa \;\Rightarrow\; P(5) = 10(5) \;\Rightarrow\; P = 10 \text{ kN}$$

Taking moment about O

$$\Sigma M_O = 0;\; 10f = 10P \;\Rightarrow\; f = P \;\Rightarrow\; f = 10 \text{ kN}$$

$$\Sigma F_y = 0; \qquad N = 10\cos 30^\circ + P\cos 30^\circ$$

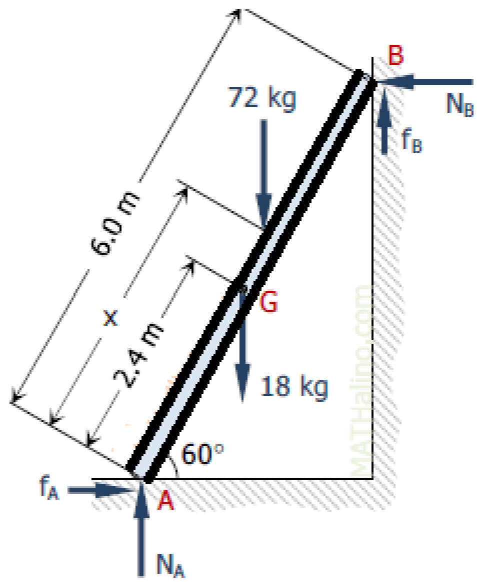
$$N = 10\cos 30^\circ + 10\cos 30^\circ$$

$$N = 17.32 \text{ kN}$$

Now since motions is impending: $f = \mu N$

$$10 = \mu(17.32) \;\Rightarrow\; \boldsymbol{\mu = 0.577}$$

Q.2. A 6 m long ladder has a mass of 18 kg and its center of gravity is 2.4 m from the bottom. The ladder is placed against a vertical wall so that it makes an angle of 60° with the ground as shown in Fig. 2. How far up the ladder can a 72-kg man climb before the ladder is on the verge of slipping? The angle of friction at all contact surfaces is 15°.



Solution:

Step 1: Draw the free body diagram

Find out Coefficient of friction

$\mu = \tan\phi \implies \mu = \tan 15^o$

Amount of friction at contact surfaces

$f_A = \mu N_A = N_A \tan 15^o$

$f_B = \mu N_B = N_B \tan 15^o$

$\Sigma F_V = 0;\ N_A + f_B = 18 + 72$

$N_A = 90 - f_B$

$N_A = 90 - N_B \tan 15^o$

$\Sigma F_H = 0;\ f_A = N_B \implies N_A \tan 15^o = N_B$

$(90 - N_B \tan 15^o) \tan 15^o = N_B$

$90 \tan 15^o - N_B \tan^2 15^o = N_B$

$90 \tan 15^o = N_B + N_B \tan^2 15^o$

$N_B (1 + \tan^2 15^o) = 90 \tan 15^o$

$N_B = 90 \tan 15^o / (1 + \tan^2 15^o)$

$N_B = 22.5$ kg

$f_B = 22.5 \tan 15^o$ $f_B = 6.03$ kg

$\Sigma M_A = 0$

Now taking moment about A

$\Sigma M_A = 0$

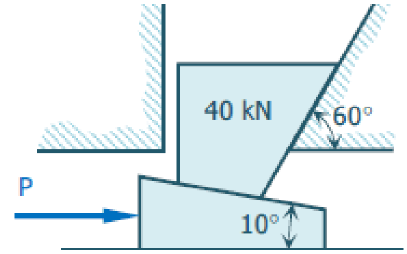
$N_B(6\sin 60^o) + f_B(6\cos 60^o) = 18(2.4\cos 60^o) + 72(x\cos 60^o)$

$N_B(6\tan 60^o) + 6f_B = 18(2.4) + 72x$

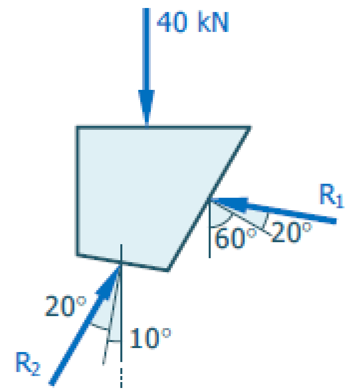
$6(22.5)\tan 60^o + 6(6.03) = 43.2 + 72x$

$72x = 226.81$ $\quad$ **$x = 3.15$ m**

Q.3. Determine the value of P just sufficient to start the 10° wedge under the 40-kN block as shown in Fig 3. The angle of friction is 20° for all contact surfaces.



Solution:



From the FBD of 40 kN block

$\Sigma F_H = 0$; $R_1 \sin 80^o = R_2 \sin 30^o$

$R_1 = (R_2 \sin 30^o) / \sin 80^o$

$R_1 = 0.5077 R_2$

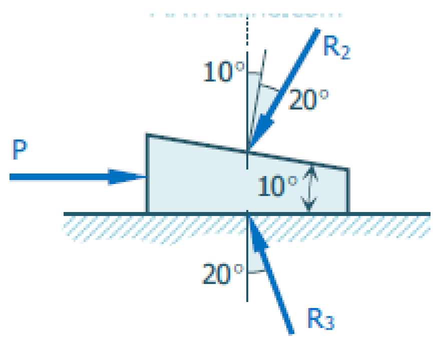
$\Sigma F_V = 0$

$R_2 \cos 30^o + R_1 \cos 80^o = 40$

$R_2 \cos 30^o + (0.5077 R_2) \cos 80^o = 40$

$0.9542 R_2 = 40$

$R_2 = 41.92$ kN



From the FBD of lower block

$\Sigma F_V = 0$

$R_3 \cos 20^o = R_2 \cos 30^o$

$R_3 \cos 20^o = 41.92 \cos 30^o$

$R_3 = 38.634$ kN

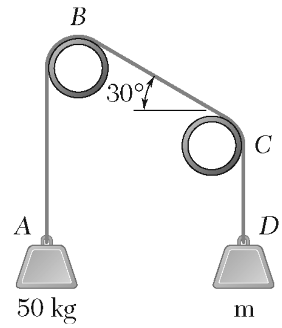
$\Sigma F_H = 0$

$P = R_2 \sin 30^o + R_3 \sin 20^o$

$P = 41.92 \sin 30^o + 38.634 \sin 20^o$

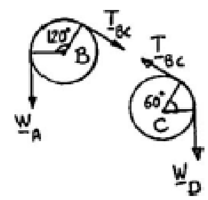
$P = 34.174$ kN

Q.4 A rope *ABCD* is looped over two pipes as shown in Fig. 4. The coefficient of static friction is 0.25 for all contacts, determine
(*a*) the smallest value of the mass *m* for which equilibrium is possible,
(*b*) the corresponding tension in portion *BC* of the rope.



We apply Eq. (8.14) to pipe *B* and pipe *C*.

$$\frac{T_2}{T_1} = e^{\mu_s \beta} \qquad (8.14)$$



<u>Pipe *B*</u>:

$$T_2 = W_A, \quad T_1 = T_{BC}$$

$$\mu_s = 0.25, \quad \beta = \frac{2\pi}{3}$$

$$\frac{W_A}{T_{BC}} = e^{0.25(2\pi/3)} = e^{\pi/6} \qquad (1)$$

<u>Pipe *C*</u>:

$$T_2 = T_{BC}, \quad T_1 = W_D, \quad \mu_s = 0.25, \quad \beta = \frac{\pi}{3}$$

$$\frac{T_{BC}}{W_D} = e^{0.025(\pi/3)} = e^{\pi/12} \qquad (2)$$

(*a*) Multiplying Eq. (1) by Eq. (2):

$$\frac{W_A}{W_D} = e^{\pi/6} \cdot e^{\pi/12} = e^{\pi/6+\pi/12} = e^{\pi/4} = 2.193$$

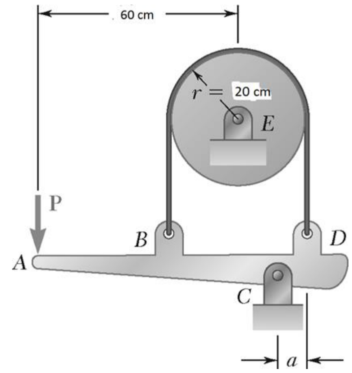
$$W_D = \frac{W_A}{2.193} \qquad m = \frac{W_D}{g} = \frac{\frac{W_A}{g}}{2.193} = \frac{m_A}{2.193} = \frac{50\text{ kg}}{2.193}$$

$$m = 22.8\text{ kg} \ ◀$$

(*b*) From Eq. (1):

$$T_{BC} = \frac{W_A}{e^{\pi/6}} = \frac{(50\text{ kg})(9.81\text{ m/s}^2)}{1.688} = 291\text{ N} \ ◀$$

Q.5. The speed of the brake drum as shown in Fig. 5 is controlled by a belt attached to the control bar *AD*. A force **P** of magnitude 100 N is applied to the control bar at *A*. Determine the magnitude of the couple being applied to the drum, knowing that the coefficient of kinetic friction between the belt and the drum is 0.25, that *a* = 5 cm., and that the drum is rotating at a constant speed (*a*) counterclockwise, (*b*) clockwise

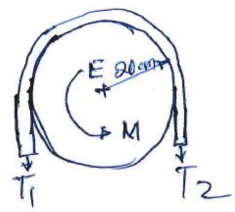


(a) Counter clockwise rotation

Free body diagram: Drum

R=20cm=0.2m; $\beta = 180^0 = \pi\, radians$

$$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$$

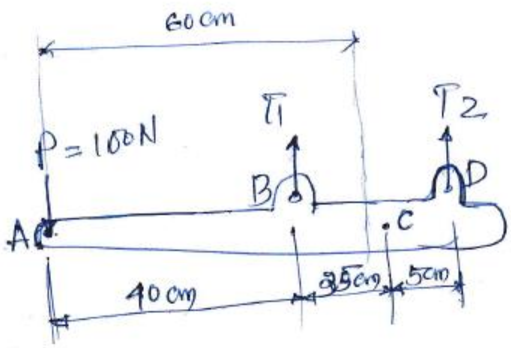


$T_2 = 2.1933 T_1$

Free body diagram: Control bar

$+\circlearrowleft \sum M_C = 0$

$T_1 \times (0.35\,m) - T_2(0.05\,m) - (100\,N)(0.75\,m) = 0$



$\Rightarrow 0.35\, T_1 - 2.1933 \times 0.05\, T_1 - 75 = 0$

$\Rightarrow 0.240335\, T_1 = 75$

$\Rightarrow T_1 = 312.0644$ N.

and $T_2 = 684.45$ N,

Now considering moment equilibrium at drum centre, E

$+\circlearrowleft \sum M_E = 0$: $M + T_1(0.2m) - T_2(0.2m) = 0.$

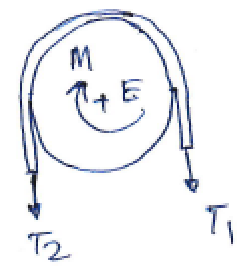
$\Rightarrow M + 62.41288 - 136.89 = 0$

$\Rightarrow M = 74.47712$ N.m.

(a) Clockwise rotation:

Free body diagram: Drum

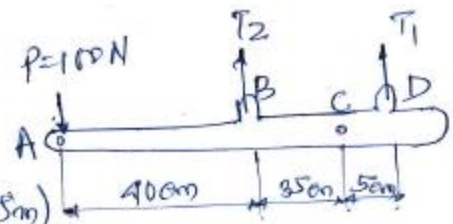
R=20cm=0.2m; $\beta = 180^0 = \pi\, radians$



$\frac{T_2}{T_1} = e^{\mu_k \beta} = e^{0.25\pi} = 2.1933$

$T_2 = 2.1933 T_1$

Free body of control rod:



$+\circlearrowleft \sum M_C = 0$:

$T_2(0.35m) - T_1(0.05m) - (100N)(0.75m) = 0$

$\Rightarrow 0.35(2.1933 T_1) - 0.05 T_1 - 75 = 0$

$\Rightarrow 0.717655 T_1 - 75 = 0$

$\Rightarrow$ $T_1 = 104.507$ N. and $T_2 = 229.2152$ N.

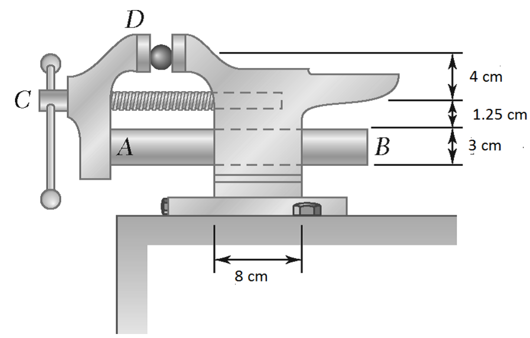
Considering moment equilibrium at drum centre, E

$+\circlearrowleft \sum M_E = 0$: $M + T_1(0.2m) - T_2(0.2m) = 0$

$\Rightarrow M + 20.9014 - 45.84304 = 0$

$\Rightarrow$ $M = 24.94164$ N.m.

Q 6. In the machinist's vise shown, the movable jaw D is rigidly attached to the tongue AB that fits loosely into the fixed body of the vise. The screw is single-threaded into the fixed base and has a mean diameter of 2 cm and a pitch of 0.6 cm. The coefficient of static friction is 0.25 between the threads and also between the tongue and the body. Neglecting bearing friction between the screw and the movable head, determine the couple that must be applied to the handle in order to produce a clamping force of 4 KN.

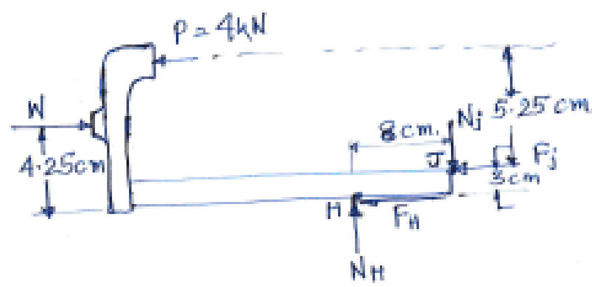


Solution:

<u>Free body: Jaw D and tongue AB</u>

P is due to elastic forces in clamped object

W is force exerted by screw



$+\uparrow \Sigma F_y = 0$

$N_H - N_j = 0 \; ; \; N_j = N_H = N.$

For final tightening

$F_H = F_j = \mu_s N = 0.25N.$

$\overset{+}{\rightarrow} \Sigma F_x = 0 : W - P - F_H - F_j = 0.$

$\Rightarrow W - (4\,kN) - \mu_s N - \mu_s N = 0$

$\Rightarrow W - 4 - 2 \times 0.25N = 0. \quad \Rightarrow N = 2(W-4)\,kN$ —— (1)

$\circlearrowleft + \Sigma M_H = 0 : P(0.0525 + 0.03) - W(0.0425) - N_j(0.08) + F_j(0.03) = 0$

$\Rightarrow 0.33 - 0.0425W - 0.08N + 0.0075N = 0$

$\Rightarrow W + 1.7058N - 7.7647 = 0$ —— (2)

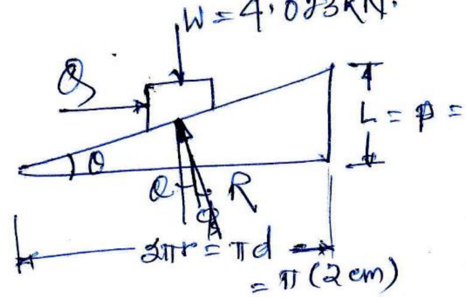
Substituting equation (1) into (2)

$W + 1.7058 \times 2(W-4) - 7.7647 = 0$

$\Rightarrow W + 3.4116W - 13.6464 - 7.7647 = 0$

$\Rightarrow \boxed{W = 4.854\,kN.}$

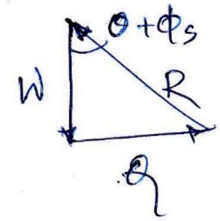
Block and incline analysis of screw:



$\tan \phi_s = \mu_s = 0.25$

$\phi_s = 14.0362^\circ$

$$\tan \theta = \frac{L \text{ or } p}{\pi d} = \frac{0.6 \text{ cm}}{\pi \times 2 \text{ cm}}$$



$\Rightarrow \tan \theta = 0.095$

$\Rightarrow \theta = \tan^{-1}(0.095) = 5.4548^\circ$

Now $\theta + \phi_s = 5.4548^\circ + 14.0362^\circ$

$= 19.491^\circ$

$Q = W \tan 19.491^\circ = 4.854 \times 0.35394$

$\Rightarrow Q = 1.718$ kN.

Torque, $T = Q \cdot r = 1.718 \times \left(\frac{0.02}{2}\right)$ kN.m

$\Rightarrow$ **$T = 0.01718$ kN.m $= 17.18$ N.m**