Indian Institute of Technology Guwahati

ME 101: Engineering Mechanics (2016-2017, Sem II)

Quiz-1 (30.01.2017)

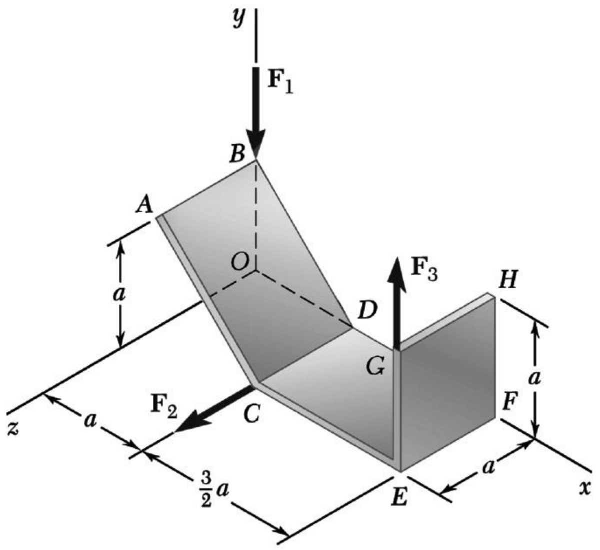
Time: 8:00 AM – 8:55 AM Full Marks: 60

Q1. A piece of sheet metal is bent into the shape shown (Fig.1) and is acted upon by **three forces**. If the forces have the same magnitude **P**, replace them with an equivalent wrench. Determine the following:

(a) Magnitude and the direction of the resultant force [4 marks]

(b) Magnitude and the direction of moment of the wrench [8 marks]

(c) Locate the point where the axis of the wrench intersects the *xy* plane [8 marks]



Model Answer for Q1:

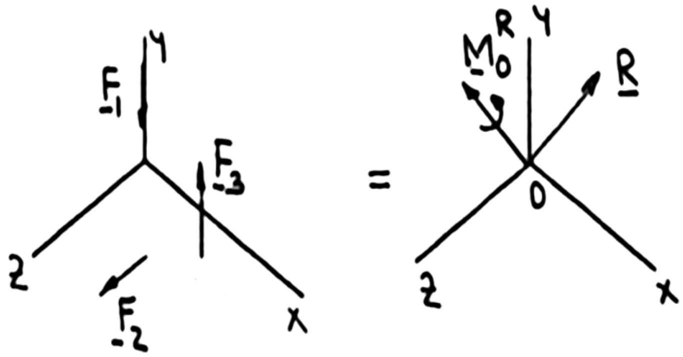
First reduce the given forces to an equivalent force-couple system ($\underline{R}$, $\underline{M}_O^R$) at the origin.

(a) We have

$$\underline{R} = \sum \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= -P\hat{j} + P\hat{k} + P\hat{j} = P\hat{k}$$

R = *P*
Direction of resultant force is along +ve z axis



(b)

$$\underline{M}_O^R = \sum \underline{M}_o = \sum \underline{r} \times \underline{F} \quad \text{(using Varignon's Theorem)}$$

$$= \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2 + \underline{r}_3 \times \underline{F}_3$$

$$= a\hat{j} \times (-P\hat{j}) + (a\hat{i} + a\hat{k}) \times (P\hat{k}) + (\tfrac{5}{2}a\hat{i} + a\hat{k}) \times (P\hat{j})$$

$$= -(aP)\hat{i} - (aP)\hat{j} + \left(\frac{5}{2}aP\right)\hat{k}$$

$$\underline{M}_O^R = aP(-\hat{i} - \hat{j} + \tfrac{5}{2}\hat{k})$$

Then for the wrench,

and $\hat{\lambda}_{axis} = \left(\underline{R}/R\right) = \hat{k}$

$$\cos\theta_x = 0 \quad \cos\theta_y = 0 \quad \cos\theta_z = 1$$

or $$\theta_x = 90^\circ \quad \theta_y = 90^\circ \quad \theta_z = 0^\circ$$

Now,

$$M_1 = \hat{\lambda}_{axis} \cdot \underline{M}_O^R$$

$$= \hat{k} \cdot aP\left(-\hat{\imath} - \hat{\jmath} + \tfrac{5}{2}\hat{k}\right) = \tfrac{5}{2}aP$$

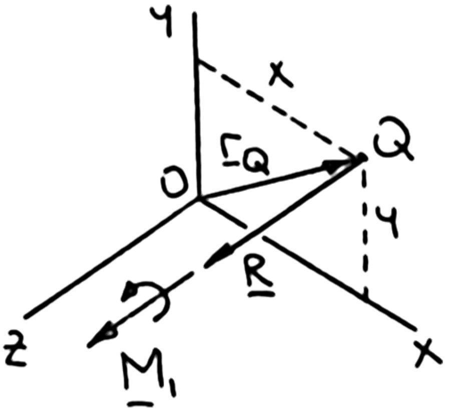
$M_1 = \frac{5}{2}aP$

Direction of moment of the wrench is along +ve z axis

(c) The components of the wrench are $(\underline{R}, \underline{M}_1)$, where $\underline{M}_1 = M_1\hat{\lambda}_{axis}$, and the axis of the wrench is assumed to intersect the xy-plane at point Q, whose coordinates are $(x, y, 0)$. Thus, we require

$$\underline{M}_2 = \underline{r}_Q \times \underline{R}$$

Where $$\underline{M}_2 = \underline{M}_O^R - \underline{M}_1$$



Then

$$\underline{r}_Q \times \underline{R} = \underline{M}_O^R - \underline{M}_1$$

$$(x\hat{\imath} + y\hat{\jmath}) \times P\hat{k} = aP\left(-\hat{\imath} - \hat{\jmath} + \tfrac{5}{2}\hat{k}\right) - \tfrac{5}{2}aP\hat{k}$$

$$-xP\,\hat{\jmath} + yP\,\hat{\imath} = aP\left(-\hat{\imath} - \hat{\jmath}\right)$$

Equating coefficients:

$\hat{\imath}: yP = -aP$ or $y = -a$

$\hat{\jmath}: -xP = -aP$ or $x = a$

The axis of the wrench is parallel to the z-axis and intersects the xy-plane at $x = a, \quad y = -a$

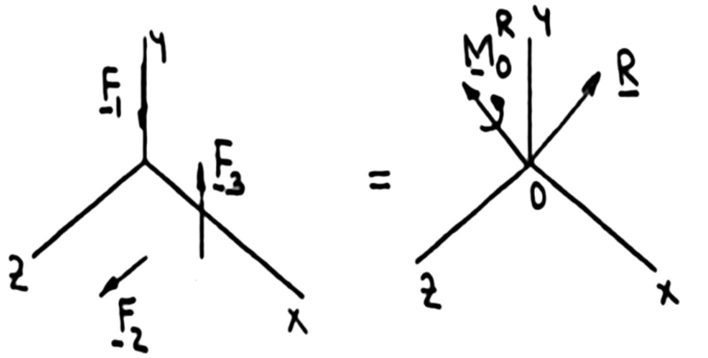
Alternate Model Answer for Q1:

First reduce the given forces to an equivalent force-couple system $(\underline{R}, \underline{M}_O^R)$ at the origin.

(a) We have

$$\underline{R} = \sum \underline{F} = \underline{F}_1 + \underline{F}_2 + \underline{F}_3$$

$$= -P\hat{\jmath} + P\hat{k} + P\hat{\jmath} = P\hat{k}$$



or ,

R = P
Direction of resultant force is along +ve z axis

(b), (c)

Let Q $(x, y, 0)$ be the point of intersection of the line of action of the moment of the wrench and xy-plane. The components of the wrench are $(\underline{R}, \underline{M}_1)$ and $\underline{r}_Q$ is the position vector of point Q.

Now, $\underline{M}_O^R = \underline{M}_1 + \underline{r}_Q \times \underline{R}$, for moment about O

$\underline{M}_Q^R = \underline{M}_O^R - \underline{r}_Q \times \underline{R}$, for moment about Q

So, $\underline{M}_1 = \underline{M}_Q^R$

The direction of the moment of the wrench i.e. $\underline{M}_1$ and resultant force $\underline{R}$ should be same.

So, $\underline{M}_1 = M_1\,\hat{k}$

$\underline{M}_Q^R = \sum \underline{M}_Q = \sum \underline{r} \times \underline{F}$ (using Varignon's Theorem)

$= \underline{r}_1 \times \underline{F}_1 + \underline{r}_2 \times \underline{F}_2 + \underline{r}_3 \times \underline{F}_3$

Here, $\underline{r}_1 = [-x\hat{\imath} + (a-y)\hat{\jmath}]$, $\underline{r}_2 = [(a-x)\hat{\imath} - y\hat{\jmath} + a\hat{k}]$, $\underline{r}_3 = \left[\left(\frac{5}{2}a - x\right)\hat{\imath} - y\hat{\jmath} + a\hat{k}\right]$

Hence,

$$\underline{M}_Q^R = [-x\hat{\imath} + (a-y)\hat{\jmath}] \times (-P\hat{\jmath}) + [(a-x)\hat{\imath} - y\hat{\jmath} + a\hat{k}] \times (P\hat{k}) + \left[\left(\frac{5}{2}a - x\right)\hat{\imath} - y\hat{\jmath} + a\hat{k}\right] \times (P\hat{\jmath})$$

$$= -P\,(y+a)\hat{\imath} - P(a-x)\hat{\jmath} + \left[Px + P\left(\frac{5}{2}a - x\right)\right]\hat{k}$$

$$M_1\hat{k} = -P(y+a)\hat{\imath} - P(a-x)\hat{\jmath} + \left[Px + P\left(\frac{5}{2}a - x\right)\right]\hat{k}$$

Equating coefficients:

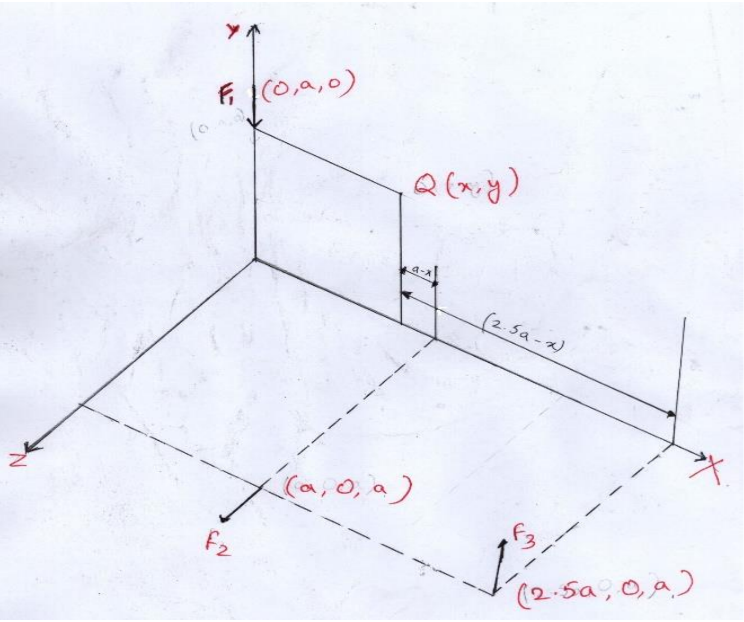
Hence, $\hat{\imath}: -yP - aP = 0 \Rightarrow y = -a$

$\hat{\jmath}: -P \times (a-x) = 0 \Rightarrow x = a$

$\hat{k}: M_1 = P\,x + P\left(\frac{5}{2}a - x\right) = Pa + \frac{3}{2}\,Pa = \frac{5}{2}\,aP$

Answer (b): $M_1 = \frac{5}{2}\,aP$, Direction of moment of the wrench is along +ve z axis

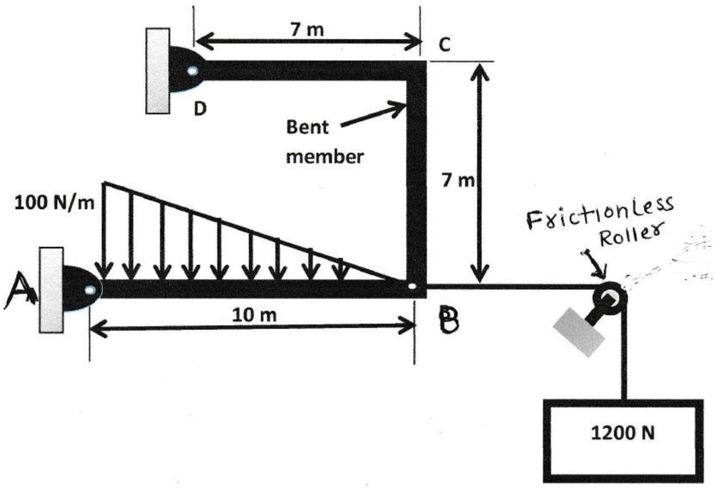
Answer (c): $x = a, \quad y = -a$



Q2. A weight of **1200 N** is supported by a rope through a frictionless pulley in conjunction with a system of beam and bent member as shown in Fig. 2. Please note that joints **A, B and D** are pinned joints.

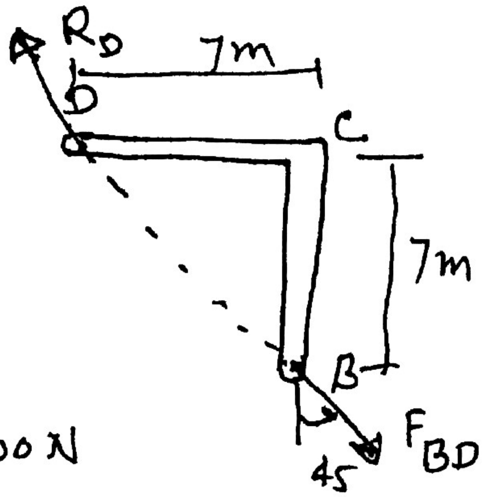
(a) Draw the free body diagrams for members AB and BCD. [8 marks]

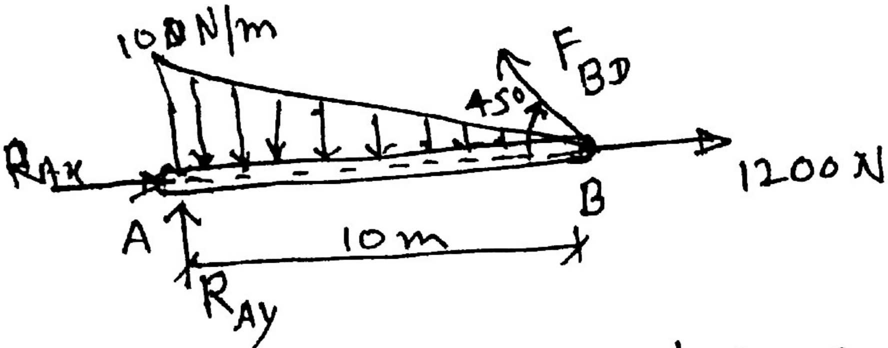
(b) Find the support reaction at A [12 marks]



Q-2

(a) Free body diagrams





Note :- BD makes 45° angle to AB.

(b) Equivalent force due to to triangular force

$$= \frac{100 \times 10}{2} = 500\,N$$

which acts at distance $10/3$ from A.

<u>From Equilibrium of AB</u>

Taking moment about A of all the forces on AB

$(M_A)_z = 0$

$$-500 \times \frac{10}{3} + F_{BD} \sin 45 \times 10 = 0 \Rightarrow F_{BD} = \frac{500\sqrt{2}}{3} N$$

$F_y = 0$ gives $R_{Ay} + \frac{500\sqrt{2}}{3} \times \frac{1}{\sqrt{2}} - 500 = 0$

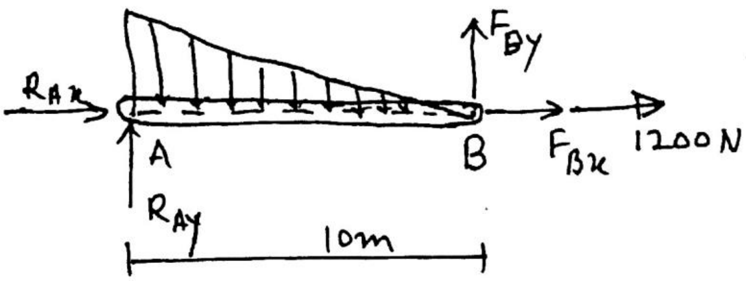
$$\boxed{R_{Ay} = \frac{1000}{3} N}$$

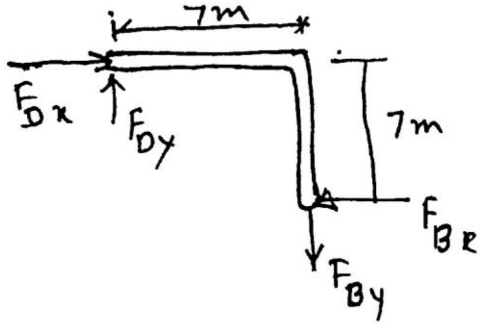
$F_x = 0$ gives $R_{Ax} + 1200 - \frac{500\sqrt{2}}{3} \frac{1}{\sqrt{2}} = 0$

$$\boxed{R_{Ax} = -\frac{3100}{3} N}$$

Alternate Model Answer for Q2:

(a) Free body diagrams





(b) Equivalent force due to triangular ~~load~~ force

$$= \frac{100 \times 10}{2} = 500\,N$$

which acts at distance $10/3$ from A

<u>Equilibrium of AB</u>

Taking moment about A ie $(M_A)_2 = 0$

$$-500 \times \frac{10}{3} + F_{By} \times 10 = 0 \Rightarrow F_{By} = \frac{500}{3}\,N$$

$F_y = 0$ gives

$$R_{Ay} + \frac{500}{3} - 500 = 0$$

$$\Rightarrow \boxed{R_{Ay} = \frac{1000}{3}\,N}$$

$F_x = 0$ gives

$$R_{Ax} + F_{Bx} + 1200 = 0 \qquad \text{—— (1)}$$

Equilibrium of BCD

Taking moment about D ie $(M_D)_z = 0$

$$-F_{By} \times 7 - F_{Bx} \times 7 = 0$$

$$F_{Bx} = -F_{By} = -\frac{500}{3} N \quad \text{--- (2)}$$

Substituting F_{Bx} from eq(2) in eq(1) we get

$$R_{Ax} - \frac{500}{3} + 1200 = 0$$

$$\boxed{R_{Ax} = -\frac{3100}{3} N}$$

Q3. Analyze the truss as shown in Fig. 3 using the method of joints. Determine the following:

(a) Find out the support reactions [3 marks]
(b) Identify the zero force members [4 marks]
(c) Calculate force in the each member with clearly identifying whether the member is in tension (T) or compression (C). [13 marks]

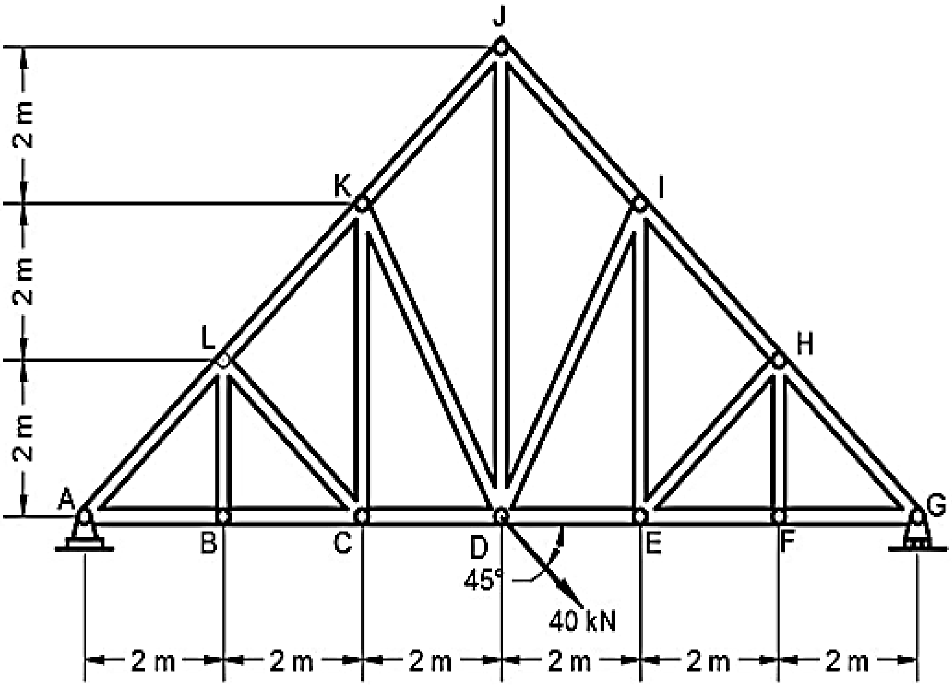
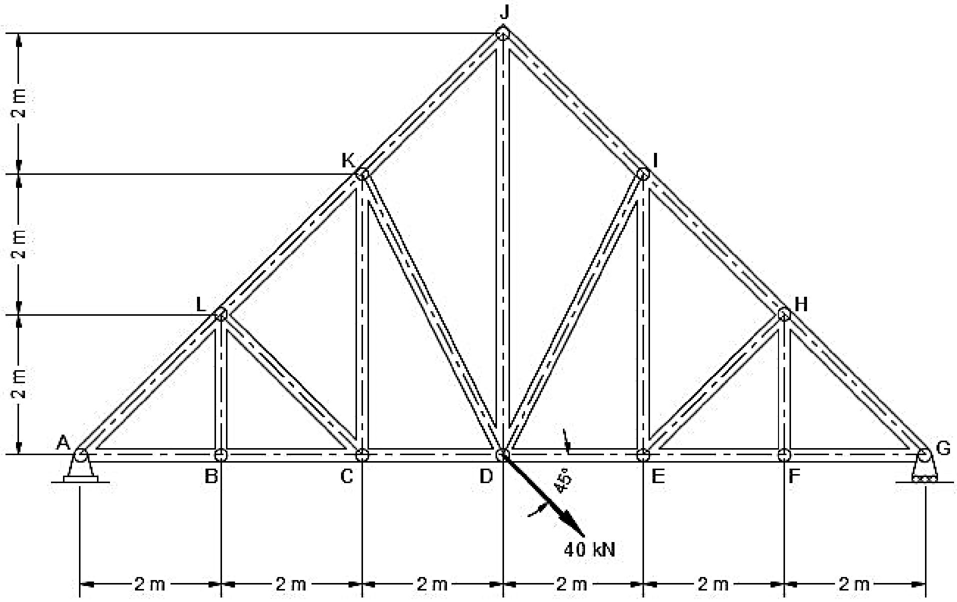


Fig. 3

Solution: **(METHOD-1)**



Part-a)

Total vertical force acting on Joint D = $40 \sin 45 = 28.28\ kN$

Total horizontal force acting on Joint D = $40 \cos 45 = 28.28\ kN$

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad A_x + 28.28\ kN = 0$

$$\therefore \boldsymbol{A_x = 28.28\ kN} \ (\leftarrow)$$

Due to symmetry w.r.t the vertical load,

$$\boldsymbol{A_y = G_y = \frac{28.28}{2} = 14.14\ kN}\ (\uparrow)$$

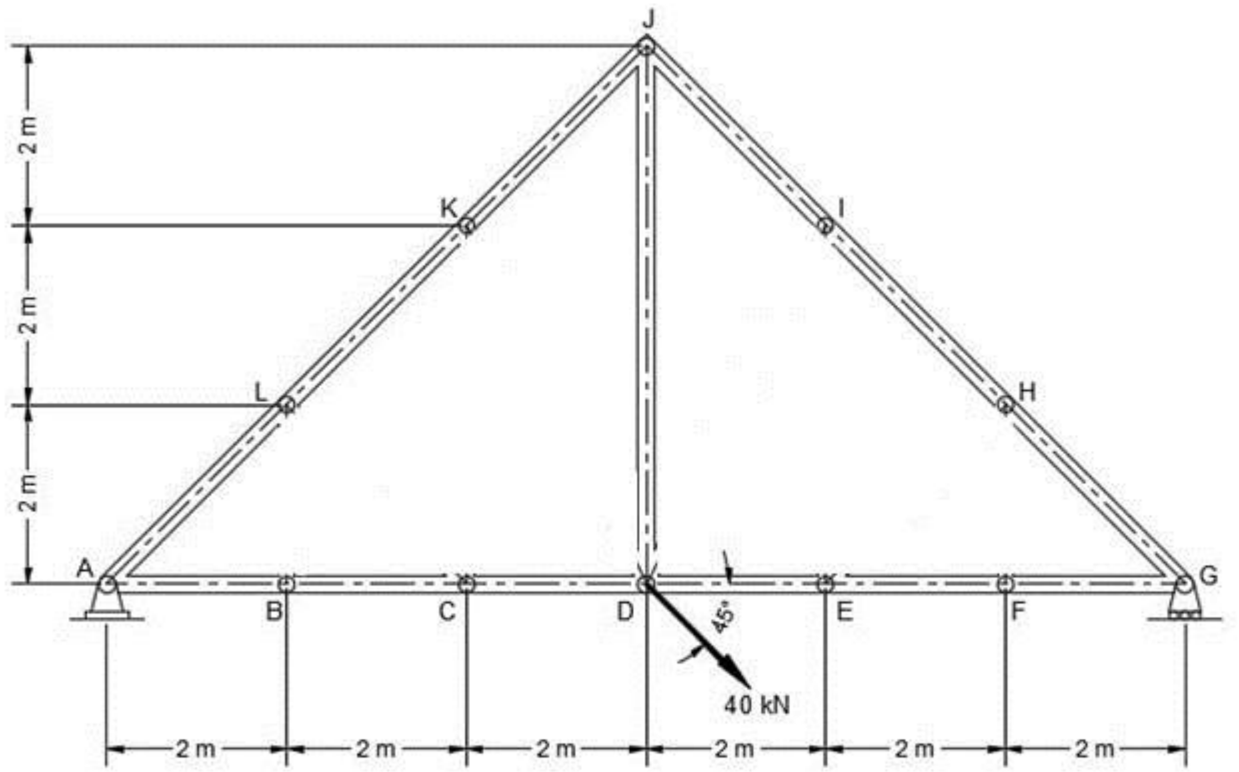
Part-b) Using the principles of zero force member identification, referring to lecture-5 slides 8, 9 and 10, we have:

Stepwise with correct sequence:

- LB and HF are zero force members
- LC and HE are zero force members
- KC and IE are zero force members
- KD and ID are zero force members

Part-c)

Now that the zero force members have been identified, the truss to be solved, effectively, is (The zero force members have been grayed out) –



Applying force equilibrium at joints, we get,

Joint A: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad F_{AB} + F_{AL}\cos 45 - 28.28 = 0$

$$\therefore \boldsymbol{F_{AB} = 42.42\ kN\ (T)}$$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\qquad F_{AL}\sin 45 + 14.14 = 0$

$$\therefore \boldsymbol{F_{AL} = -20\ kN\ (C)}$$

Joint L:

Only members AL and LK are joined here and they are connected in a straight line (LB and LC are zero force members). Thus,

$\overset{+}{\rightarrow} \boldsymbol{\Sigma \mathrm{F} = 0}$ $\qquad \boldsymbol{F_{LK} = F_{AL} = -20\ kN\ (C)}$

Joint K: Similar to Joint L,

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F = 0}$ $\qquad \boldsymbol{F_{KJ} = F_{LK} = -20\ kN\ (C)}$

Joint B: Similar to Joint L,

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad \boldsymbol{F_{BC} = F_{BA} = 42.42\ kN\ (T)}$

Joint C: Similar to Joint L,

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad\quad$ $\boldsymbol{F_{CD} = F_{BC} = 42.42\ kN\ (T)}$

Joint J:

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad\quad$ $F_{JI} cos\ 45 = F_{JK} \cos 45$

$$\therefore \boldsymbol{F_{JI} = -20\ kN\ (C)}$$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad\quad$ $-F_{JI} sin\ 45 - F_{JK} \sin 45 - F_{JD} = 0$

$$\therefore \boldsymbol{F_{JD} = 28.28\ kN\ (T)}$$

Joint D:

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad\quad$ $F_{DE} + 28.28 - F_{DC} = 0$

$$\therefore \boldsymbol{F_{DE} = 14.14\ kN (T)}$$

Due to geometrical symmetry and vertical load symmetry, the forces in the following members are equal, given as:

$$\boldsymbol{F_{HG} = F_{AL} = F_{LK} = F_{HI} = F_{KJ} = F_{IJ} = -20\ kN\ (C)}$$

Due to asymmetry w.r.t. horizontal load, the forces in the following members are obtained as given below:

Joint E: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad\quad$ $\boldsymbol{F_{EF} = F_{DE} = 14.14\ kN\ (T)}$

Joint F: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad\quad$ $\boldsymbol{F_{FG} = F_{EF} = 14.14\ kN\ (T)}$

The summary of all the member forces is provided in the next page.

$\boldsymbol{F_{HG} = F_{AL} = F_{LK} = F_{HI} = F_{KJ} = F_{IJ} = -20\ kN\ (C)}$

$\boldsymbol{F_{BL} = F_{FH} = F_{CL} = F_{EH} = F_{CK} = F_{EI} = F_{DK} = F_{DI} = 0}$ **(i.e., zero force members)**

$$\boldsymbol{F_{EF} = F_{DE} = F_{FG} = 14.14\ kN\ (T)}$$

$$\boldsymbol{F_{JD} = 28.28\ kN\ (T)}$$

$$\boldsymbol{F_{BC} = F_{BA} = F_{CD} = 42.42\ kN\ (T)}$$

Solution: (METHOD-2) (Not preferable)

Part-a)

Total vertical force acting on Joint D = $40 \sin 45 = 28.28\ kN$

Total horizontal force acting on Joint D = $40 \cos 45 = 28.28\ kN$

$\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad A_x + 28.28\ kN = 0$

$$\therefore \boldsymbol{A_x = 28.28\ kN\ (\leftarrow)}$$

Due to symmetry w.r.t the vertical load,

$$\boldsymbol{A_y = G_y = \frac{28.28}{2} = 14.14\ kN\ (\uparrow)}$$

Part-b) Using the principles of zero force member identification, referring to lecture-5 slides 8, 9 and 10, we have:

Stepwise with correct sequence:

- LB and HF are zero force members
- LC and HE are zero force members
- KC and IE are zero force members
- KD and ID are zero force members

Part c)

Joint A: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad F_{AB} + F_{AL} \cos 45 - 28.28 = 0$

$$\therefore \boldsymbol{F_{AB} = 42.42\ kN\ (T)}$$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\qquad F_{AL} \sin 45 + 14.14 = 0$

$$\therefore \boldsymbol{F_{AL} = -20\ kN\ (C)}$$

Joint L: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\qquad F_{LK} \cos 45 + F_{LC} \cos 45 - F_{AL} \cos 45 = 0$

⇨ $\quad F_{LK} + F_{LC} = -20$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad F_{LK} \sin 45 - F_{LC} \sin 45 - F_{AL} \sin 45 = 0$

$\Rightarrow \; F_{LK} - F_{LC} = -20$

$\therefore \boldsymbol{F_{LK} = -20\ kN\ (C)\ , F_{LC} = 0}$

Joint B: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad \boldsymbol{F_{BC} = F_{BA} = 42.42\ kN\ (T)}$

Joint C: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{CD} - F_{CB} - F_{CL} cos\ 45 = 0$

$\therefore \boldsymbol{F_{CD} = 42.42\ kN\ (T)}$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad F_{CK} + F_{CL} sin\ 45 = 0$

$\therefore \boldsymbol{F_{CK} = 0\ kN}$

Joint K: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{KJ} cos\ 45 + F_{KD} \cos 45 - F_{KL} \cos 45 = 0$

$\Rightarrow \; F_{KJ} + F_{KD} = -20$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad F_{KJ} sin\ 45 - F_{KD} \sin 45 - F_{KL} \sin 45 = 0$

$\Rightarrow \; F_{KJ} - F_{KD} = -20$

$\therefore \boldsymbol{F_{KJ} = -20\ kN\ (C)\ , F_{KD} = 0}$

Joint J: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{JI} cos\ 45 = F_{JK} \cos 45$

$\therefore \boldsymbol{F_{JI} = -20\ kN\ (C)}$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad -F_{JI} sin\ 45 - F_{JK} \sin 45 - F_{JD} = 0$

$\therefore \boldsymbol{F_{JD} = 28.28\ kN\ (T)}$

Joint D: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{DE} + 28.28 - F_{DC} + F_{DI} \cos 63.43 - F_{DK} \cos 63.43 = 0$

$\therefore F_{DE} + 0.447\ F_{DI} = 14.14$

$\uparrow + \boldsymbol{\Sigma F_y = 0}$ $\quad F_{DJ} - 28.28 + F_{DI} \sin 63.43 + F_{DK} \sin 63.43 = 0$

$\therefore \boldsymbol{F_{DI} = 0}$

On substituting, $\boldsymbol{F_{DE} = 14.14\ kN\ (T)}$

Due to geometrical symmetry and vertical load symmetry, the forces in the following members are equal, given as:

$$\boldsymbol{F_{HG} = F_{AL} = F_{LK} = F_{HI} = F_{KJ} = F_{IJ} = -20\ kN\ (C)}$$

$$\boldsymbol{F_{BL} = F_{FH} = F_{CL} = F_{EH} = F_{CK} = F_{EI} = F_{DK} = F_{DI} = 0}$$ **(i.e., zero force members)**

Due to asymmetry w.r.t. horizontal load, the forces in the following members are obtained as given below:

Joint E: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{EF} - F_{ED} + F_{EH} \cos 45 = 0$

$$\therefore \boldsymbol{F_{EF} = 14.14\ kN(T)}$$

Joint F: $\overset{+}{\rightarrow} \boldsymbol{\Sigma F_x = 0}$ $\quad F_{FG} - F_{FE} = 0$

$$\therefore \boldsymbol{F_{FG} = 14.14\ kN\ (T)}$$