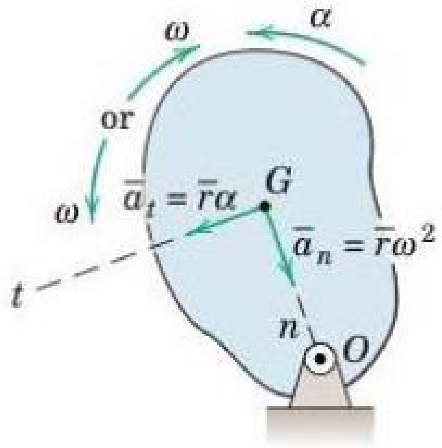
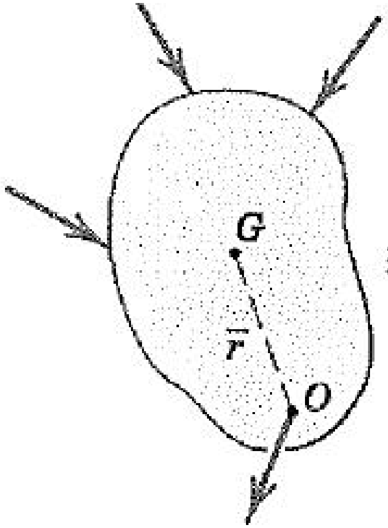
Fixed axis Rotation

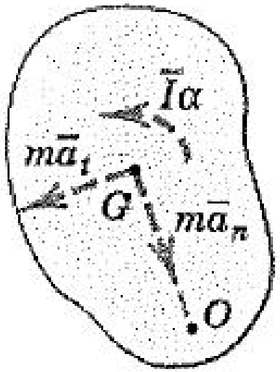


Fixed-Axis Rotation
(a)



Free-Body Diagram
(b)

≡



Kinetic Diagram
(c)

$$\Sigma \mathbf{F} = m\bar{\mathbf{a}}$$

$$\Sigma M_G = \bar{I}\alpha$$

$$\Sigma M_O = I_O\alpha$$

$$\Sigma M_O = \bar{I}\alpha + m\bar{a}_t\bar{r}.$$

Application of the parallel-axis theorem for mass moments

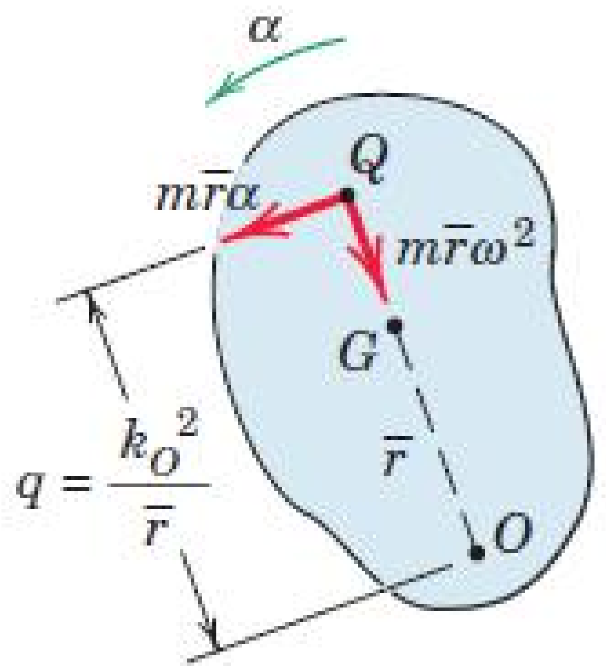
$$I_O = \bar{I} + m\bar{r}^2,$$

$$\Sigma M_O = (I_O - m\bar{r}^2)\alpha + m\bar{r}^2\alpha = I_O\alpha$$

We may combine the resultant-force component and resultant couple by moving to a parallel position through point *Q* on line *OG*,

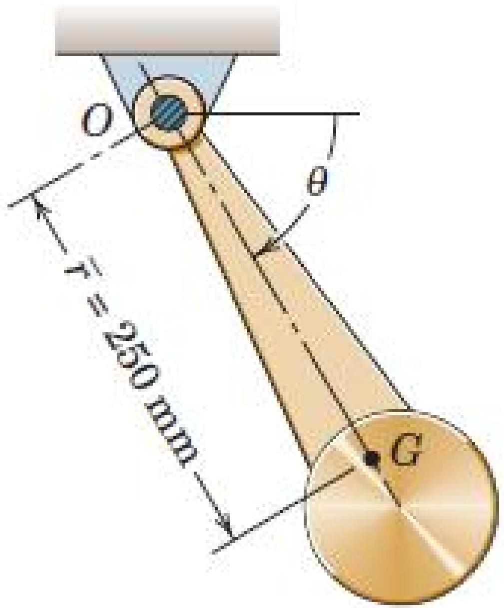
located by $m\bar{r}\alpha q = \bar{I}\alpha + m\bar{r}\alpha(\bar{r})$.

$I_O = k_O{}^2 m$ gives $q = k_O{}^2/\bar{r}$.



Point Q is called the *center of percussion* and has the unique property that the resultant of all forces applied to the body must pass through it. It follows that the sum of the moments of all forces about the center of percussion is always zero, $\Sigma M_Q = 0$.

The pendulum has a mass of 7.5 kg with center of mass at G and has a radius of gyration about the pivot O of 295 mm. If the pendulum is released from rest at $\theta = 0$, determine the total force supported by the bearing at the instant when $\theta = 60°$. Friction in the bearing is negligible.



$[\Sigma M_O = I_O\alpha]$ $\qquad 7.5(9.81)(0.25)\cos\theta = (0.295)^2(7.5)\alpha$

$$\alpha = 28.2 \cos\theta \text{ rad/s}^2$$

and for $\theta = 60°$

$[\omega\, d\omega = \alpha\, d\theta]$ $\qquad \int_0^{\omega} \omega\, d\omega = \int_0^{\pi/3} 28.2 \cos\theta\, d\theta$

$$\omega^2 = 48.8 \text{ (rad/s)}^2$$

The remaining two equations of motion applied to the 60° position yield

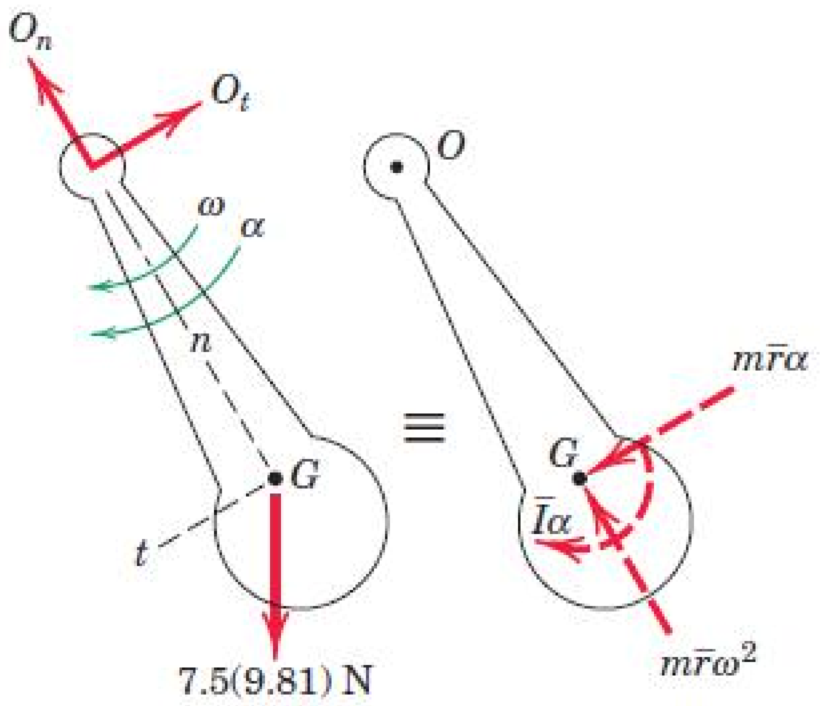
$[\Sigma F_n = m\bar{r}\omega^2]$ $\qquad O_n - 7.5(9.81)\sin 60° = 7.5(0.25)(48.8)$

$$O_n = 155.2 \text{ N}$$

$[\Sigma F_t = m\bar{r}\alpha]$ $\qquad -O_t + 7.5(9.81)\cos 60° = 7.5(0.25)(28.2)\cos 60°$

$$O_t = 10.37 \text{ N}$$

$$O = \sqrt{(155.2)^2 + (10.37)^2} = 155.6 \text{ N}$$ A



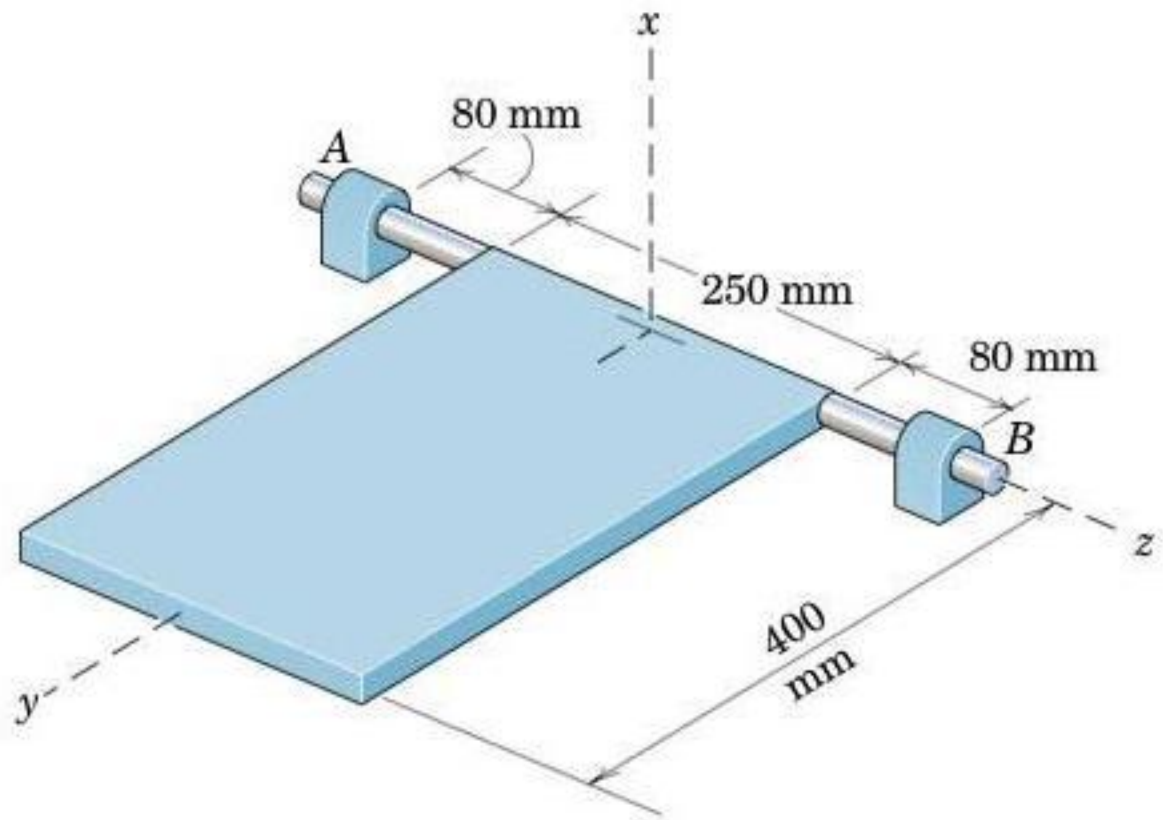
The proper sense for O_t may be observed at the outset by applying the moment equation $\Sigma M_G = \bar{I}\alpha$, where the moment about G due to O_t must be clockwise to agree with α. The force O_t may also be obtained initially by a moment equation about the center of percussion Q, shown in the lower figure, which avoids the necessity of computing α. First, we must obtain the distance q, which is

$[q = k_O{}^2/\bar{r}]$ $$q = \frac{(0.295)^2}{0.250} = 0.348 \text{ m}$$

$[\Sigma M_Q = 0]$ $$O_t(0.348) - 7.5(9.81)(\cos 60°)(0.348 - 0.250) = 0$$

$$O_t = 10.37 \text{ N}$$ *Ans.*

The 20-kg uniform steel plate is freely hinged about the z-axis as shown. Calculate the force supported by each of the bearings at A and B an instant after the plate is released from rest in the horizontal y-z plane.



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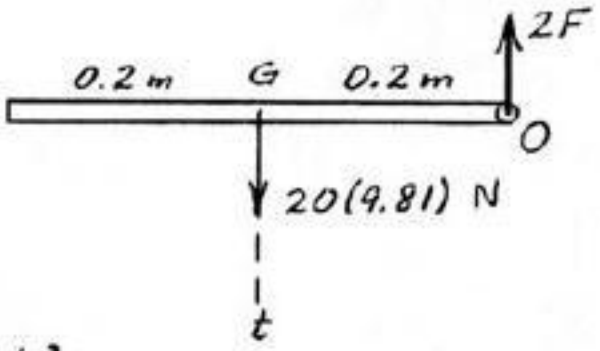


$\Sigma M_O = I_O \alpha$, $20(9.81)(0.2) = \frac{1}{3} 20(0.4)^2 \alpha$

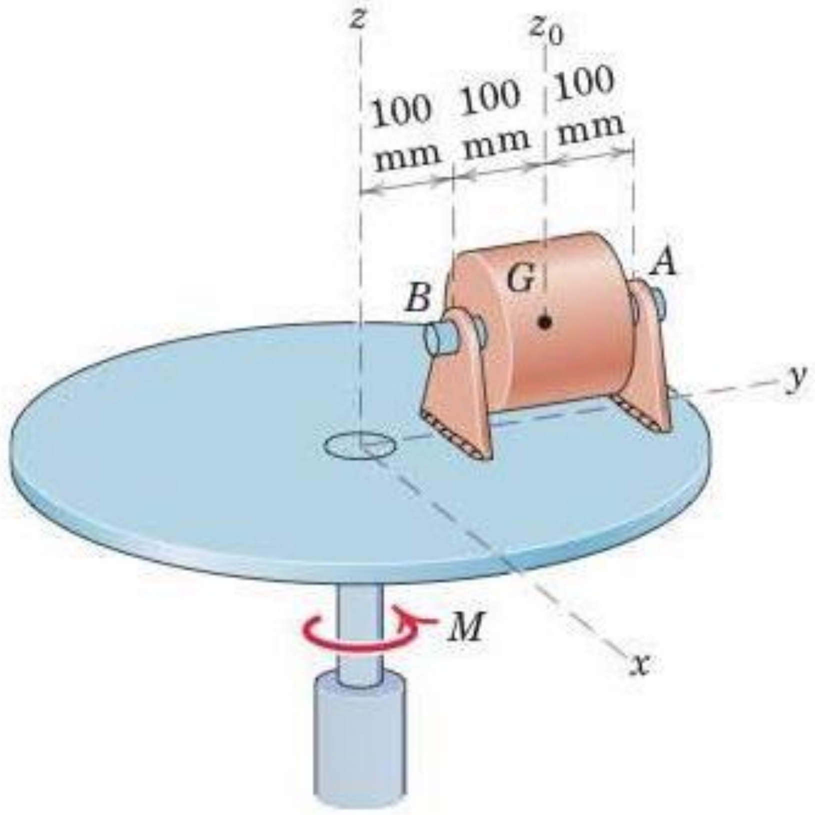
$\alpha = 36.8 \text{ rad/s}^2$

$\bar{a} = \bar{r}\alpha$, $\bar{a} = 0.2 \times 36.8 = 7.36 \text{ m/s}^2$

$\Sigma F_t = m\bar{a}_t$, $20(9.81) - 2F = 20 \times 7.36$

$2F = 49.0$, $\underline{F_A = F_B = F = 24.5 \text{ N}}$

6/55 The 12-kg cylinder supported by the bearing brackets at A and B has a moment of inertia about the vertical z_0-axis through its mass center G equal to $0.080\ \text{kg}\cdot\text{m}^2$. The disk and brackets have a moment of inertia about the vertical z-axis of rotation equal to $0.60\ \text{kg}\cdot\text{m}^2$. If a torque $M = 16\ \text{N}\cdot\text{m}$ is applied to the disk through its shaft with the disk initially at rest, calculate the horizontal x-components of force supported by the bearings at A and B.

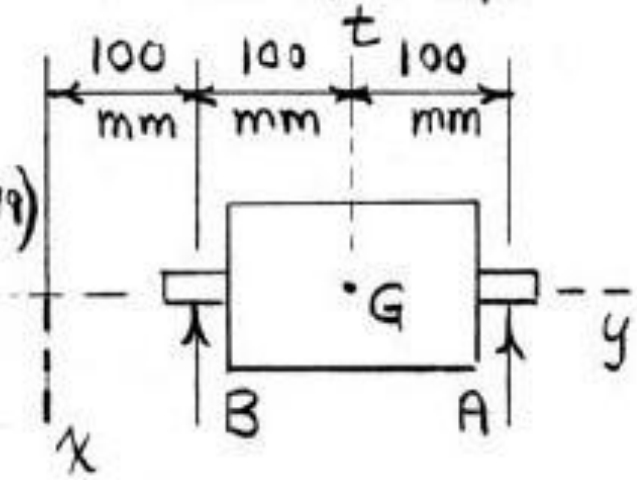


6/55 For entire assembly,

$$I_{zz} = 0.60 + \left(0.080 + 12(0.2)^2\right) = 1.160 \text{ kg}\cdot\text{m}^2$$

$$\Sigma M_z = I_{zz}\alpha : \quad 16 = 1.160\,\alpha, \quad \alpha = 13.79 \text{ rad/s}^2$$

For cylinder:



$$\Sigma F_t = ma_t : A + B = 12(0.2)(13.79)$$

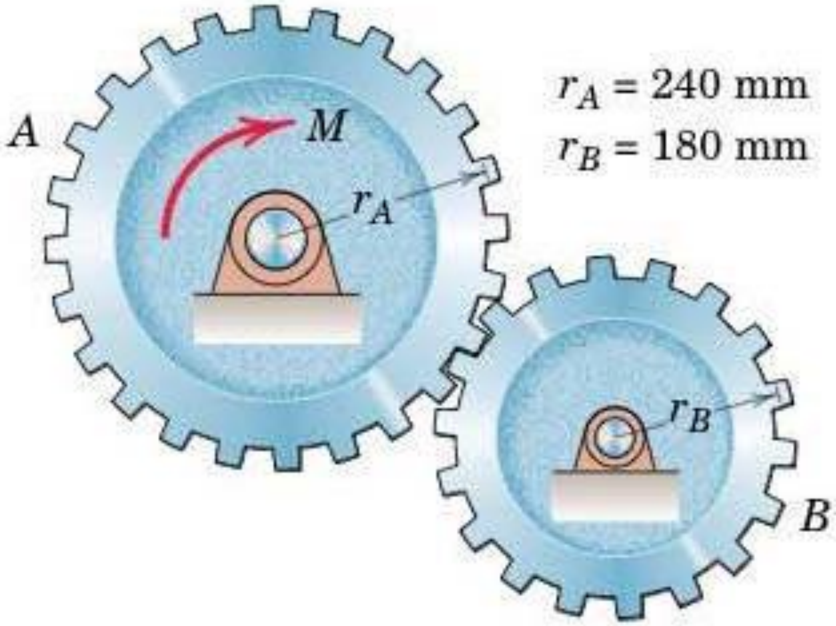
$$= 33.1 \text{ N}$$

$$\Sigma M_o = I_{zz}\alpha :$$

$$0.3A + 0.1B = \left[0.080 + 12(0.2)^2\right] 13.79$$

Simultaneous solution: $\underline{A = 22.1 \text{ N}}$, $\underline{B = 11.03 \text{ N}}$

6/56 The mass of gear A is 20 kg and its centroidal radius of gyration is 150 mm. The mass of gear B is 10 kg and its centroidal radius of gyration is 100 mm. Calculate the angular acceleration of gear B when a torque of 12 N·m is applied to the shaft of gear A. Neglect friction.



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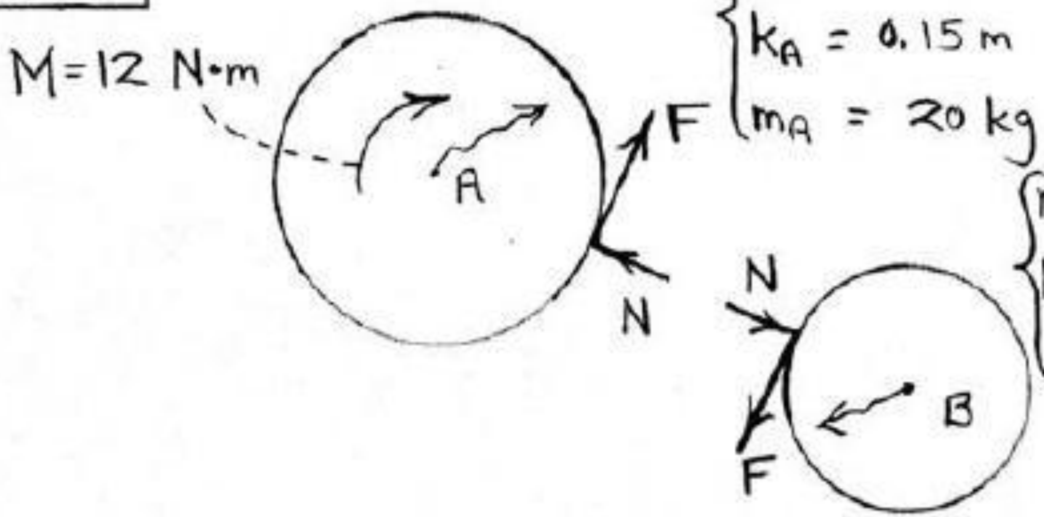


$$\begin{cases} r_A = 0.24 \text{ m} \\ k_A = 0.15 \text{ m} \\ m_A = 20 \text{ kg} \end{cases} \qquad \begin{cases} r_B = 0.18 \text{ m} \\ k_B = 0.1 \text{ m} \\ m_B = 10 \text{ kg} \end{cases}$$

$$\overset{\curvearrowright}{+} \Sigma M_A = I_A \alpha_A : \quad 12 - F(0.24) = 20(0.15)^2 \alpha_A \quad (1)$$

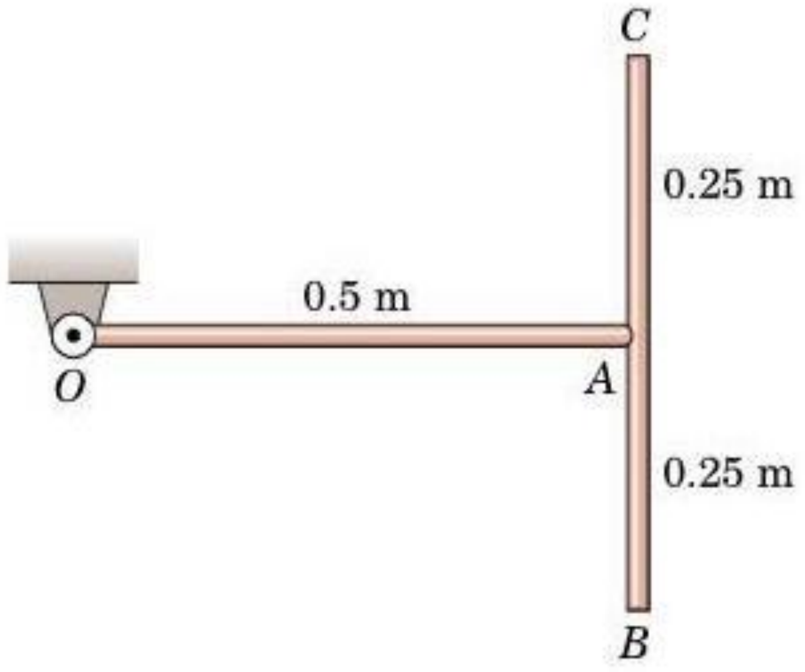
$$\overset{\curvearrowleft}{+} \Sigma M_B = I_B \alpha_B : \quad F(0.18) = 10(0.1)^2 \alpha_B \quad (2)$$

Tangential accelerations match: $r_A \alpha_A = r_B \alpha_B$

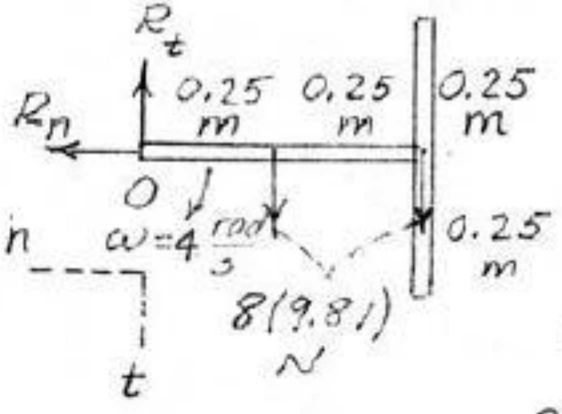
$$0.24\, \alpha_A = 0.18\, \alpha_B \quad (3)$$

Solution of Eqs. (1)–(3): $\begin{cases} F = 14.16 \text{ N} \\ \alpha_A = 19.12 \text{ rad/s}^2 \text{ (CW)} \\ \alpha_B = 25.5 \text{ rad/s}^2 \text{ (CCW)} \end{cases}$

6/68 Each of the two uniform slender bars OA and BC has a mass of 8 kg. The bars are welded at A to form a T-shaped member and are rotating freely about a horizontal axis through O. If the bars have an angular velocity ω of 4 rad/s as OA passes the horizontal position shown, calculate the total force R supported by the bearing at O.



6/68 $I_O = \frac{1}{3}ml^2 + \left(\frac{1}{12}ml^2 + ml^2\right) = \frac{17}{12}(8)(0.5)^2$ kg·m²



$\Sigma M_O = I_O \alpha$

$8(9.81)(0.50 + 0.25) = \frac{17}{12}(8)(0.5)^2 \alpha$

$\alpha = 20.8 \text{ rad/s}^2$

$\Sigma F_t = \Sigma m\bar{a}_t$

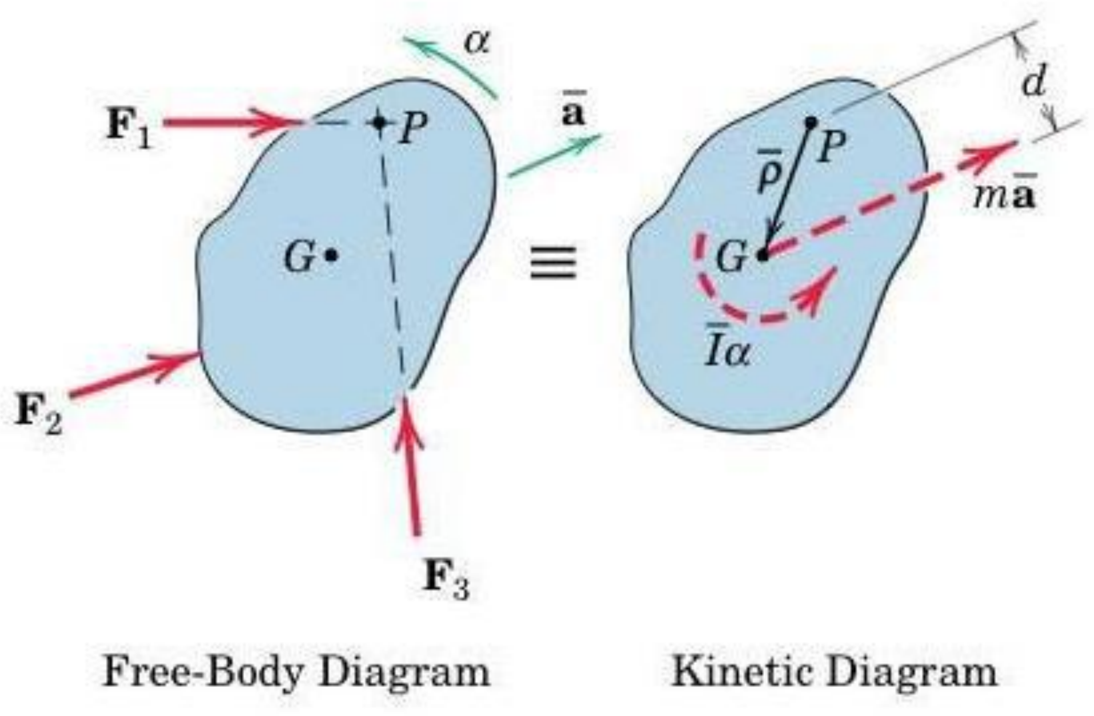
$2(8)(9.81) - R_t = 8(0.25)(20.8) + 8(0.50)(20.8)$

$R_t = 32.3 \text{ N}$

$\Sigma F_n = \Sigma m\bar{a}_n$; $R_n = 8(0.25)4^2 + 8(0.50)4^2 = 96.0 \text{ N}$

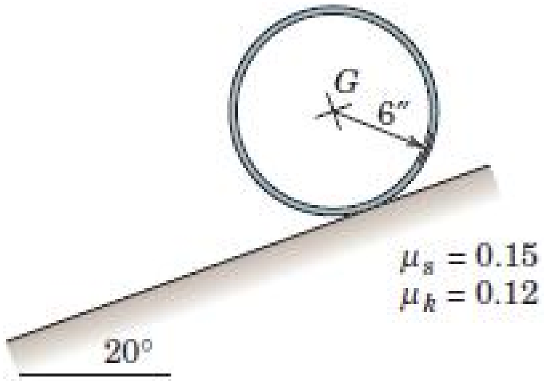
$R = \sqrt{32.3^2 + 96.0^2} = \underline{101.3 \text{ N}}$

General Plane Motion



$$\Sigma M_P = \bar{I}\alpha + m\bar{a}d$$

A metal hoop with a radius $r = 6$ in. is released from rest on the 20° incline. If the coefficients of static and kinetic friction are $\mu_s = 0.15$ and $\mu_k = 0.12$, determine the angular acceleration α of the hoop and the time t for the hoop to move a distance of 10 ft down the incline.



$[\Sigma F_x = m\bar{a}_x]$ $\qquad mg \sin 20° - F = m\bar{a}$

$[\Sigma F_y = m\bar{a}_y = 0]$ $\qquad N - mg \cos 20° = 0$

1 $[\Sigma M_G = \bar{I}\alpha]$ $\qquad Fr = mr^2\alpha$

Elimination of F between the first and third equations and substitution of the kinematic assumption $\bar{a} = r\alpha$ give

2 $$\bar{a} = \frac{g}{2} \sin 20° = \frac{32.2}{2}(0.342) = 5.51 \text{ ft/sec}^2$$

Alternatively, with our assumption of $\bar{a} = r\alpha$ for pure rolling, a moment sum about C by Eq. 6/2 gives $\bar{a}$ directly. Thus,

$[\Sigma M_C = \bar{I}\alpha + m\bar{a}d]$ $\qquad mgr \sin 20° = mr^2\frac{\bar{a}}{r} + m\bar{a}r \qquad \bar{a} = \frac{g}{2}\sin 20°$

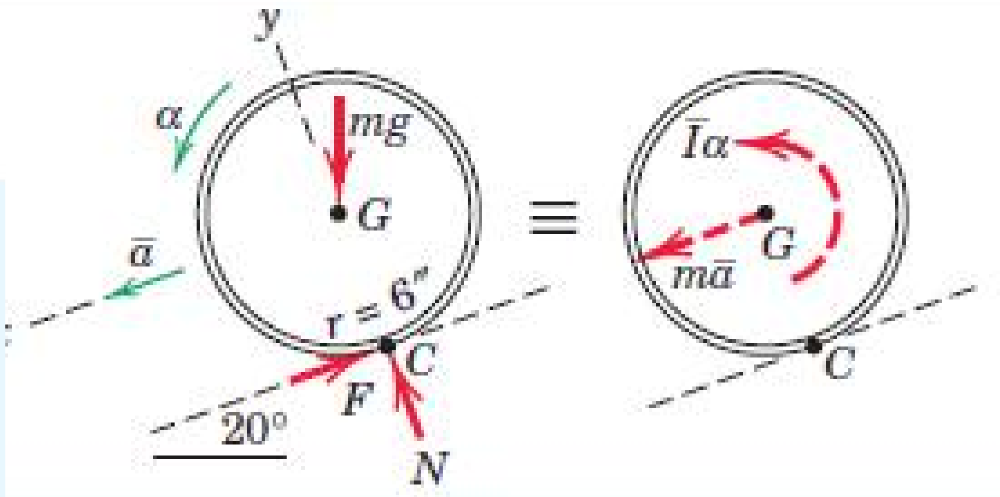
To check our assumption of no slipping, we calculate F and N and compare F with its limiting value. From the above equations,

$$F = mg \sin 20° - m\frac{g}{2}\sin 20° = 0.1710mg$$

$$N = mg \cos 20° = 0.940mg$$

But the maximum possible friction force is

$[F_{max} = \mu_s N]$ $\qquad F_{max} = 0.15(0.940mg) = 0.1410mg$



Because our calculated value of $0.1710mg$ exceeds the limiting value of $0.1410mg$, we conclude that our assumption of pure rolling was wrong. Therefore, the hoop slips as it rolls and $\bar{a} \neq r\alpha$. The friction force then becomes the kinetic value

$$[F = \mu_k N] \qquad F = 0.12(0.940mg) = 0.1128mg$$

The motion equations now give

$$[\Sigma F_x = m\bar{a}_x] \qquad mg \sin 20° - 0.1128mg = m\bar{a}$$

$$\bar{a} = 0.229(32.2) = 7.38 \text{ ft/sec}^2$$

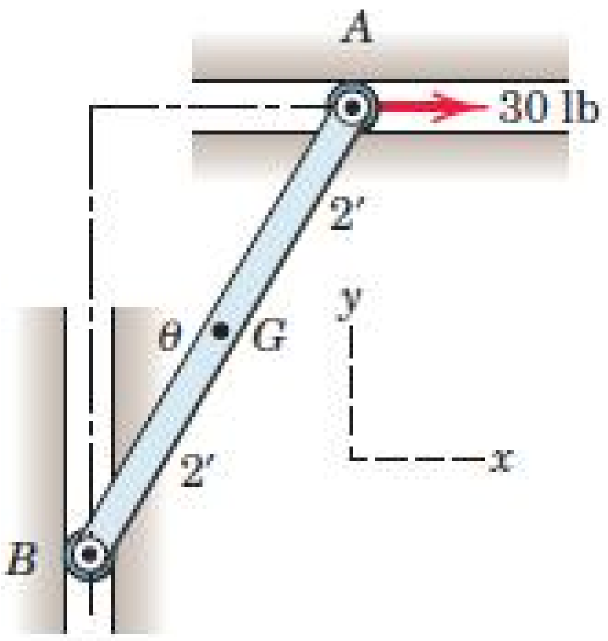
3 $$[\Sigma M_G = \bar{I}\alpha] \qquad 0.1128mg(r) = mr^2\alpha$$

$$\alpha = \frac{0.1128(32.2)}{6/12} = 7.26 \text{ rad/sec}^2 \qquad \textit{Ans.}$$

The time required for the center G of the hoop to move 10 ft from rest with constant acceleration is

$$[x = \tfrac{1}{2}at^2] \qquad t = \sqrt{\frac{2x}{\bar{a}}} = \sqrt{\frac{2(10)}{7.38}} = 1.646 \text{ sec} \qquad \textit{Ans.}$$

The slender bar AB weighs 60 lb and moves in the vertical plane, with its ends constrained to follow the smooth horizontal and vertical guides. If the 30-lb force is applied at A with the bar initially at rest in the position for which $\theta = 30°$, calculate the resulting angular acceleration of the bar and the forces on the small end rollers at A and B.



③ $[\Sigma M_C = \bar{I}\alpha + \Sigma m\bar{a}d]$

$$30(4\cos 30°) - 60(2\sin 30°) = \frac{1}{12}\frac{60}{32.2}(4^2)\alpha$$

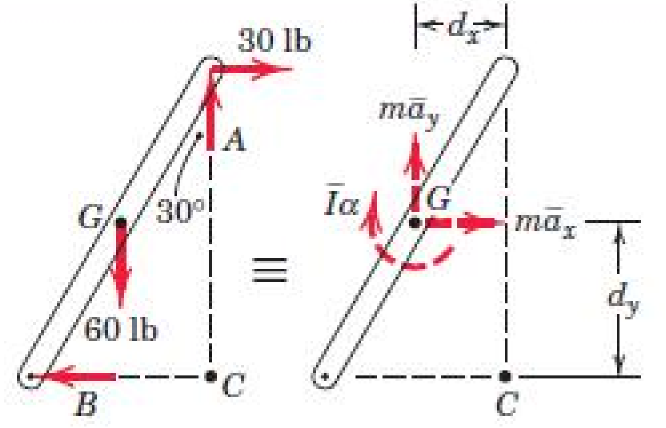
$$+\frac{60}{32.2}(1.732\alpha)(2\cos 30°) + \frac{60}{32.2}(1.0\alpha)(2\sin 30°)$$

$$43.9 = 9.94\alpha \qquad \alpha = 4.42 \text{ rad/sec}^2$$ *Ans.*

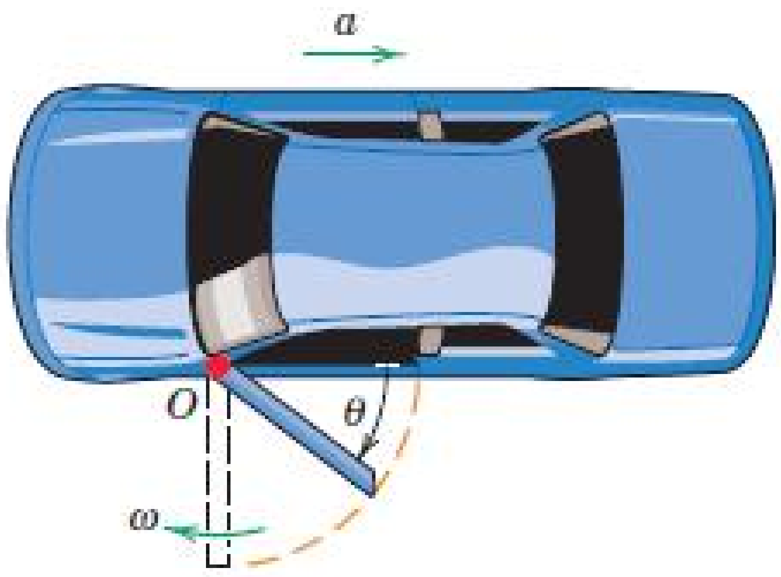
With α determined, we can now apply the force equations independently and get

$[\Sigma F_y = m\bar{a}_y]$ $\quad A - 60 = \frac{60}{32.2}(1.0)(4.42) \quad A = 68.2$ lb *Ans.*

$[\Sigma F_x = m\bar{a}_x]$ $\quad 30 - B = \frac{60}{32.2}(1.732)(4.42) \quad B = 15.74$ lb *Ans.*



A car door is inadvertently left slightly open when the brakes are applied to give the car a constant rearward acceleration a. Derive expressions for the angular velocity of the door as it swings past the 90° position and the components of the hinge reactions for any value of θ. The mass of the door is m, its mass center is a distance $\bar{r}$ from the hinge axis O, and the radius of gyration about O is k_O.



(1) spect to O. This equation becomes the kinematic equation of constraint and is

$$\overline{\mathbf{a}} = \mathbf{a}_G = \mathbf{a}_O + (\mathbf{a}_{G/O})_n + (\mathbf{a}_{G/O})_t$$

The magnitudes of the $m\overline{\mathbf{a}}$ components are then

(2) $$ma_O = ma \qquad m(a_{G/O})_n = m\bar{r}\omega^2 \qquad m(a_{G/O})_t = m\bar{r}\alpha$$

where $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$.

For a given angle θ, the three unknowns are α, O_x, and O_y. We can eliminate O_x and O_y by a moment equation about O, which gives

(3) $[\Sigma M_O = \bar{I}\alpha + \Sigma m\bar{a}d]$ $\qquad 0 = m(k_O{}^2 - \bar{r}^2)\alpha + m\bar{r}\alpha(\bar{r}) - ma(\bar{r}\sin\theta)$

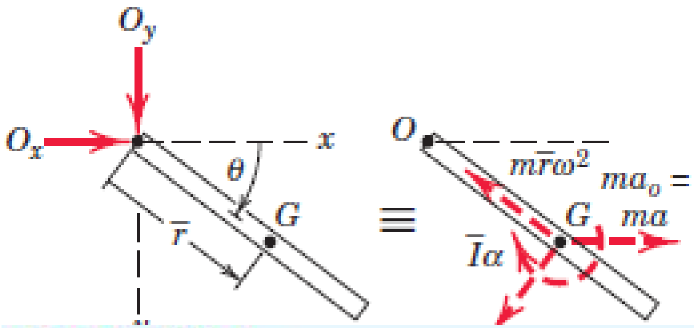
(4) Solving for α gives $\qquad \alpha = \dfrac{a\bar{r}}{k_O{}^2}\sin\,\theta$

Now we integrate α first to a general position and get

$[\omega\, d\omega = \alpha\, d\theta]$ $$\int_0^\omega \omega\, d\omega = \int_0^\theta \frac{a\bar{r}}{k_O{}^2}\sin\theta\, d\theta$$

$$\omega^2 = \frac{2a\bar{r}}{k_O{}^2}(1 - \cos\theta)$$

For $\theta = \pi/2$, $\qquad \omega = \dfrac{1}{k_O}\sqrt{2a\bar{r}}$ *Ans.*



Helpful Hints

(1) Point O is chosen because it is the only point on the door whose acceleration is known.

(2) Be careful to place $m\bar{r}\alpha$ in the sense of positive α with respect to rotation about O.

(3) The free-body diagram shows that there is zero moment about O. We use the transfer-of-axis theorem here and substitute $k_O{}^2 = \bar{k}^2 + \bar{r}^2$. If this relation is not totally familiar, review Art. B/1 in Appendix B.

(4) We may also use Eq. 6/3 with O as a moment center

$$\Sigma \mathbf{M}_O = I_O\boldsymbol{\alpha} + \bar{\boldsymbol{\rho}} \times m\mathbf{a}_O$$

where the scalar values of the terms are $I_O\alpha = mk_O{}^2\alpha$ and $\bar{\boldsymbol{\rho}} \times m\mathbf{a}_O$ becomes $-\bar{r}ma\sin\theta$.

5 To find O_x and O_y for any given value of θ, the force equations give

$$[\Sigma F_x = m\bar{a}_x] \qquad O_x = ma - m\bar{r}\omega^2 \cos\theta - m\bar{r}\alpha \sin\theta$$

$$= m\left[a - \frac{2a\bar{r}^2}{k_O^2}(1 - \cos\theta)\cos\theta - \frac{a\bar{r}^2}{k_O^2}\sin^2\theta\right]$$

$$= ma\left[1 - \frac{\bar{r}^2}{k_O^2}(1 + 2\cos\theta - 3\cos^2\theta)\right] \qquad \textit{Ans.}$$

$$[\Sigma F_y = m\bar{a}_y] \qquad O_y = m\bar{r}\alpha\cos\theta - m\bar{r}\omega^2\sin\theta$$

$$= m\bar{r}\frac{a\bar{r}}{k_O^2}\sin\theta\cos\theta - m\bar{r}\frac{2a\bar{r}}{k_O^2}(1 - \cos\theta)\sin\theta$$

$$= \frac{ma\bar{r}^2}{k_O^2}(3\cos\theta - 2)\sin\theta \qquad \textit{Ans.}$$