Chapter 16

Kinetics of Plane Motion of Rigid bodies

(A) Moment of Momentum Equation

$$M_A = \left(\frac{dH_A}{dt}\right)_{xyz} + \omega \times H_A \qquad (1)$$

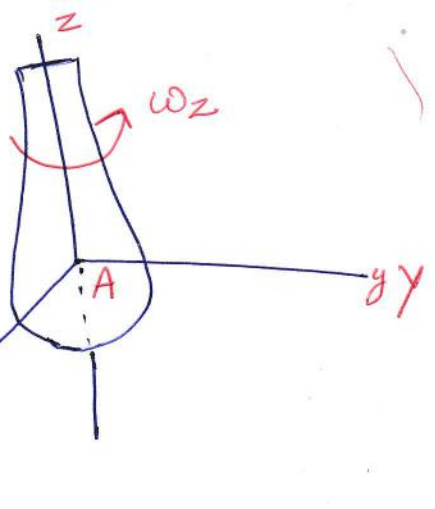
ω = is the angular velocity of xyz and thus of the body relative to ~~XYZ~~.

The scalar form of the equation

$$\left.\begin{aligned} (M_A)_x &= -I_{xz}\dot{\omega} + I_{yz}\omega^2 \\ (M_A)_y &= -I_{yz}\dot{\omega} - I_{xz}\omega^2 \\ (M_A)_z &= I_{zz}\dot{\omega} \end{aligned}\right] \qquad (2)$$

(A) Pure Rotation of a body of Revolution about its axis of revolution

A uniform body of revolution is shown. If the body undergoes pure rotation about the axis of revolution fixed in inertial space reference XYZ.



⇒ Plane motion parallel to any plane for which the axis of revolution is a normal

A reference xyz is fixed to body

z is collinear with axis of revolution.

All the points along the axis of revolution are fixed in inertial space XYZ.

If body is symmetric around z axis and z-axis is axis of revolution

Then $I_{xy} = I_{xz} = 0$, $I_{yx} = I_{yz} = 0$

$$M_z = I_{zz}\dot{\omega}_z$$

$$M_x = 0$$

$$M_y = 0$$

z- principal axis of Inertia

Newton's Law = $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$

if a body Translates along x- axis then

$$M'_z = I_{zz}\dot{\omega}_z$$

$$F_x = ma_x \qquad (3)$$

Valid :-

(1) Body having two planes of mass symmetry on z through A.

(2) Body of Revolution with z- axis of revolution,

(3) Slab like body cylindrical body with z- parallel to generator.

iF angular velocity is constant, Then

eq. (2) yields.

$$(M_A) = I_{yz}\omega^2 \hat{j} - I_{xz}\omega^2 \hat{i} \qquad (\dot{\omega} = 0)$$

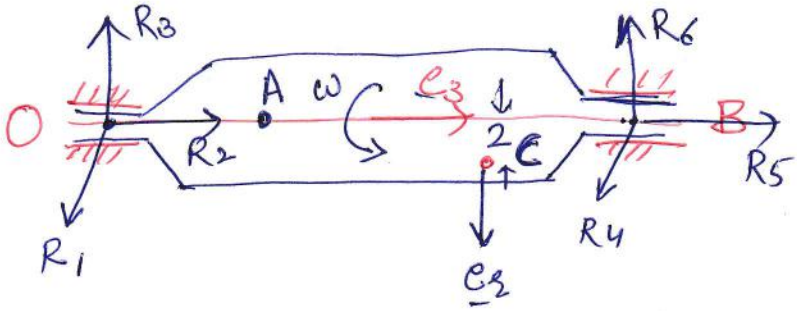
$$(M_A)_x = I_{yz}\omega^2 \; ; \quad (M_A)_y = -I_{xz}\omega^2$$

Hence, a vanable moment is needed for motion at constant ω since $\hat{i}\,(e_1)$ & $\hat{j}\,(e_2)$ rotate with the body.

However, if $e_3\,(k)$ is principal axis of inertia at A then $\boxed{M_A = 0}$ $I_{yz} = I_{xz} = 0$

i.e. No moment M_A is needed to maintain motion at constant speed.

A body rotating about a fixed axis is said to be BALANCED if the bearing reactions at O and B are zero when it rotates at constant angular velocity



$$m a_c = -m\omega^2 r_c e_2 = F = 0 \Rightarrow r_c = 0$$

~~$I_{13}^{2}\omega^2 e_1 + I_{23}^{2}\omega^2$~~ $\quad I_{yz}^{C}\omega^2 e_1 - I_{xz}^{C}\omega^2 e_2 = M_c = 0$

$$\Rightarrow I_{yz}^{C} = I_{xz}^{C} = 0$$

i.e C is on the axis of rotation and the axis of rotation is a principal axis at C

$$I_{xz}^{A} = \cancel{I_{xz}^{C}} + m x_{cx} x_{cz} = 0 \quad , \quad I_{yz}^{A} = I_{yz}^{C} + m x_{cy} x_{cz} = 0$$

$\Rightarrow$ Axis of rotation is principal axis at every point on it. Hence the necessary and sufficient conditions of balancing are

(1) Center of mass C lies on axis of rotation

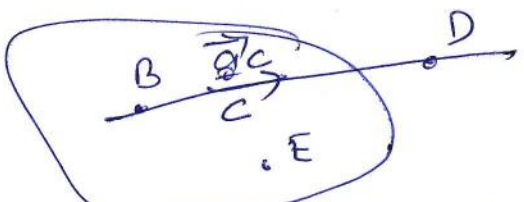
(2) Axis of rotation is a principal axis at one A of itself.

Angular impluse about axis e_3 through A

$$I_{ang\,A_3}(t_1, t_2) = I_{33}^{A}\left(\omega(t_2) - \omega(t_1)\right)$$

Moment Equation for Translating ~~body~~ System

A translating system behaves as a rigid body.



$\omega = 0, \; M_c = 0$

$$M_A = M_c + r_{CA} \times F$$

$\Rightarrow M_c = 0, \; M_A = 0$ if A coincides with C

But $M_E \neq 0$ (For translating system)

Euler's Equation for 3D motion of a rigid body

Let A be a point of the rigid body and $A \equiv C$ or $\bar{a}_A|_I = 0$ or $\bar{a}_A/_I$ is along AC, then w.r.t. principal axes of inertia e_i at A

$$[\bar{H}_A] = \begin{bmatrix} I_{11}^A & 0 & 0 \\ 0 & I_{22}^A & 0 \\ 0 & 0 & I_{33}^A \end{bmatrix} \begin{matrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{matrix} = \begin{bmatrix} I_{11}^A \omega_1 \\ I_{22}^A \omega_2 \\ I_{33}^A \omega_3 \end{bmatrix}$$

$$\bar{H}_A = I_{11}^A \omega_1 \bar{e}_1 + I_{22} \omega_2 \bar{e}_3 + I_{33} \omega_3 \bar{e}_3$$

Choose $\bar{e}_i$ fixed to the rigid body so that

$$\dot{I}_{ij} = 0$$

$$\bar{M}_A = \dot{\bar{H}}_A/_I = \dot{H}_A/_{e_i = \text{constant}} + \bar{\omega} \times \bar{H}_A$$

$$(M_A)_1 = I_{11}^A \dot{\omega}_1 - (I_{22}^A - I_{33}^A)\, \omega_2 \omega_3$$

$$(M_A)_2 = I_{22}^A \dot{\omega}_2 - (I_{33}^A - I_{11}^A)\, \omega_3 \omega_1$$

$$(M_A)_3 = I_{33} \dot{\omega}_3 - (I_{11}^A - I_{22}^A)\, \omega_1 \omega_2$$

~~Conservative~~

Example

Find the elements of the inertia matrix for point A w.r.t. x, y, z axes for the system shown The density of body i is ρ_i. Neglect the mass of the shaft.

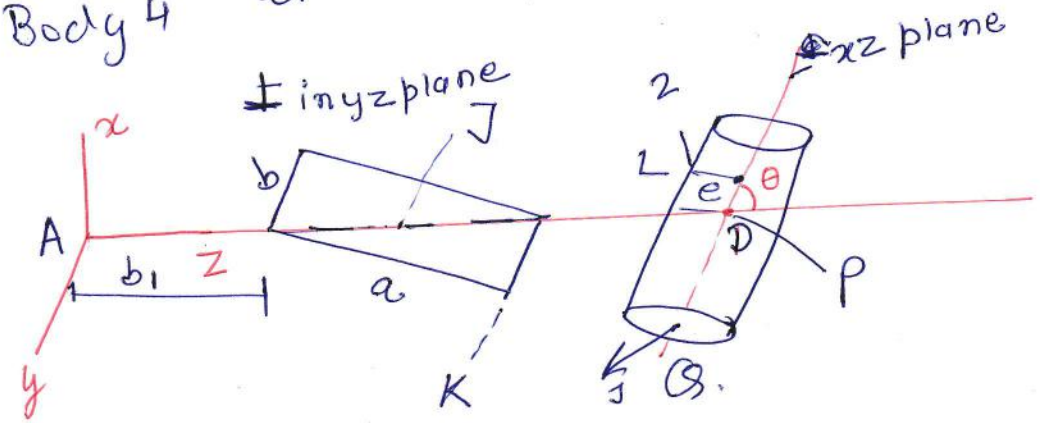
List the principal axes of inertia which can be determined by inspection for various bodies at the following points

Body 1 at A, J, K

Body 2 at A, D, P, Q

Body 3 at A, B, F, G, H

Body 4 at E, S

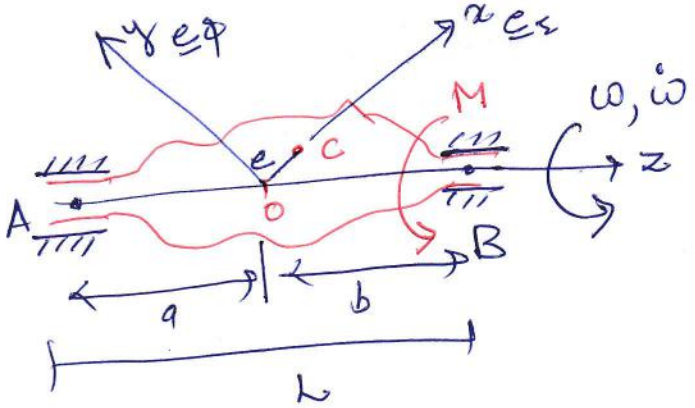


Example 3.2

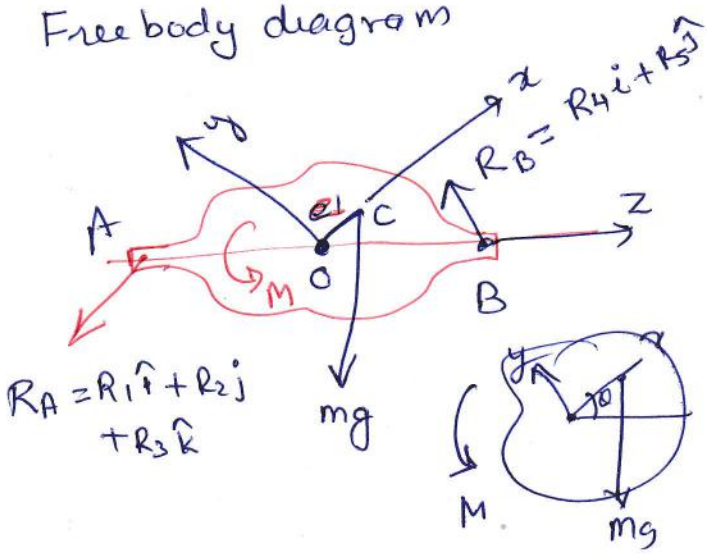
Prof. Dumir

A rigid body, of mass m with center of mass C is rotating about a non principal axis of inertia. The inertia materix at point O for axes x, y, z attached to the body has elements I^O_{xx}, I^O_{xy} ...

Find the bearing reactions for the given position. The bearing at B does not provide any axial constraint.



Free body diagram



Solution.

Fig. $\bar{\omega} = \omega \hat{k}$, $\overline{OC} = e\hat{i}$

Let $\bar{\dot{\omega}} = \dot{\omega}\hat{k}$ — angular acceleration

$\bar{R}_A = R_1\hat{i} + R_2\hat{j} + R_3\hat{k}$ — Bearing reaction at A

$\bar{R}_B = R_4\hat{i} + R_5\hat{j}$ — Bearing reaction at B (∵ at B, does not have axial component.

Example 3.2 Continue

The acceleration of C is obtained from $\bar{a}_O = 0$ — Body is not translating

$$\bar{a}_c = \bar{a}_O + \dot{\bar{\omega}} \times \bar{OC} + \bar{\omega} \times (\bar{\omega} \times \bar{OC})$$

$$\bar{a}_c = \dot{\omega}\hat{k} \times e\hat{i} + \omega^2 e\hat{i} \Rightarrow \dot{\omega} e\hat{j} + \omega^2 e\hat{i} \quad (1)$$

Now We have 6 unknowns, $\dot{\omega}, R_1, R_2, R_3, R_4, R_5$ are determined using moment about fixed point O and $\bar{F} = m\bar{a}_c$

$$\bar{M}_O = -a\hat{k} \times (R_1\hat{i} + R_2\hat{j} + R_3\hat{k}) + b\hat{k} \times (R_4\hat{i} + R_5\hat{j}) + m\hat{k} + e\hat{i} \times mg(-\sin\theta\hat{i} - \cos\theta\hat{j})$$

$$= (I^O_{xx}\dot{\omega} - I_{yz}\omega^2)\hat{i} + (I^O_{yz}\dot{\omega} + I^O_{xz}\omega^2) + I^O_{zz}\dot{\omega}\,k$$

After simplification and equating i, j, k

$$-aR_1 + bR_4 = I^O_{yz}\dot{\omega} + I_{xz}\omega^2 \quad (2)$$

$$+aR_2 - bR_5 = I^O_{xx}\dot{\omega} - I_{yz}\omega^2 \quad (3)$$

$$M - mge\cos\theta = I^O_{zz}\dot{\omega} \quad (4)$$

$$\bar{F} = m\bar{a}_c \Rightarrow mg(-\sin\theta\hat{i} - \cos\theta\hat{j}) + (R_1\hat{i} + R_2\hat{j} + R_3\hat{k}) + R_4\hat{i} + R_5\hat{j} = m(-\omega^2 e\hat{i} + \dot{\omega} e\hat{j})$$

$$\Rightarrow R_1 + R_4 - mg\sin\theta = -m\omega^2 e \quad (5)$$

$$\Rightarrow R_2 + R_5 - mg\cos\theta = m\dot{\omega}e \quad (6)$$

$$R_3 = 0 \quad (7)$$

Using Eq 3, 5

$$R_1 = \frac{-(I^0_{yz}\dot{\omega} + I^0_{xz}\omega^2 + m\omega^2 eb - mgb\sin\theta)}{L}$$

$$R_4 = (I^0_{yz}\dot{\omega} - I^0_{xz}\omega^2 - m\omega^2 ea + mga\sin\theta)/L$$

Using eq. ② & 6

$$R_2 = (I^0_{xz}\omega - I^0_{yz}\omega^2 + m\dot{\omega}eb + mgb\cos\theta)/L$$

$$R_5 = (-I^0_{xz}\dot{\omega} + I_{yz}\omega^2 + m\dot{\omega}ea + mga\cos\theta)/L$$

Comments

These reactions are very large for high speed rotors due to presence of ω^2 term and change their directors as the axes rotates. Consequently, their components with respect to the fixed direction in ground reference vary with time and cause heavy forced vibration and wear So Rotors must be balanced to avoid this

Special cases

(1) $e = 0$ the center of mass C is on the axis of rotation.

$$R_1 = -(I^0_{yz}\dot{\omega} - I^0_{xz}\omega^2 + mgb\sin\theta)/L$$

$$R_2 = (I^0_{xz}\dot{\omega} - I^0_{yz}\omega^2 + mgb\cos\theta)/L$$

$$R_4 = (I^0_{yz}\dot{\omega} + I^0_{xz}\omega^2 + mga\sin\theta)/L$$

$$R_5 = (-I^0_{xz}\dot{\omega} + I^0_{yz}\omega^2 + mga\cos\theta)/L$$

(2) $I_{xz}^{O} = I_{yz}^{O}$ - z-axis is a principal axis of inertia at O

$$R_1 = -(m\omega^2 eb - mgb\sin\theta)/L$$

$$R_2 = (m\dot{\omega} eb + mgb\cos\theta)/L$$

$$R_4 = (-m\omega^2 ea + mga\sin\theta)/L$$

$$R_5 = (m\dot{\omega} ea + mga\cos\theta)/L$$

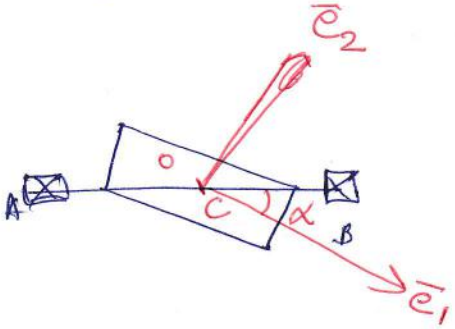
(3) $e = 0$, $I_{xz}^{O} = I_{yz}^{O} = 0$, the axis of rotation is a principal axis of inertia at G which lies on it.

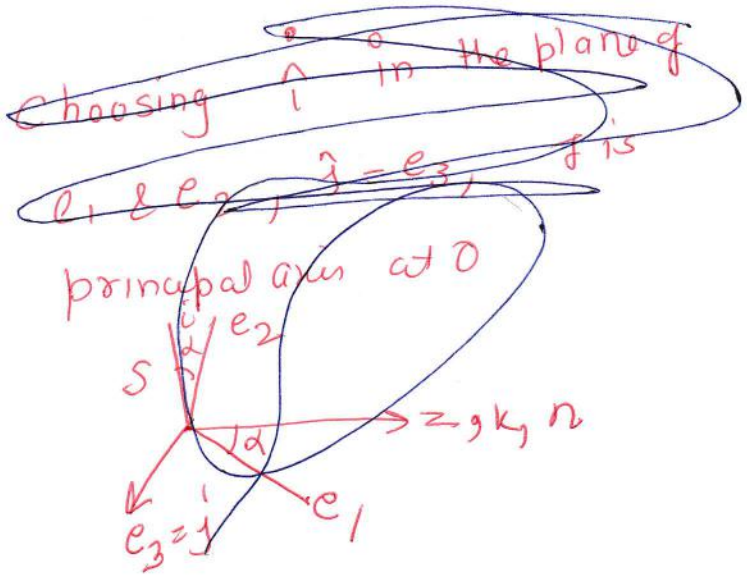
$$R_1 = \frac{mgb\sin\theta}{L}, \quad R_2 = \frac{mgb\cos\theta}{L}, \quad R_4 = \frac{mga\sin\theta}{L}$$

$$R_5 = \frac{mga\cos\theta}{L}$$

$\therefore$ only static reactions due to forces mg act since rotor is balanced.

4). $e = 0$ and the axis of rotation $\hat{k}$ is coplanar with principal axes of inertia $\bar{e}_1, \bar{e}_2$ at O with $\bar{e}_1$ at angle α to $\hat{k}$



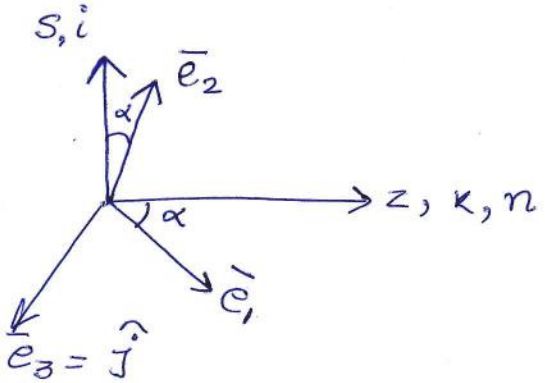


(5) Ex

(5)

Choosing $\hat{i}$ is in the plane of $\bar{e}_1$ & $\bar{e}_2$, and $e_3 = \hat{j}$. Hence $\hat{j}$ is principal axis at O. $I_{yz} = 0 = I_{yx}$. We compute I^o_{zz} & I^o_{xz} using the transformation rule.

$\hat{k} = \bar{n}$



$\hat{k} = \bar{n} = \cos\alpha\, \bar{e}_1 + \sin\alpha\, \bar{e}_2$

$i = s = -\sin\alpha\, \bar{e}_1 + \cos\alpha\, \bar{e}_2$

$n_1 = \cos\alpha, \quad n_2 = \sin\alpha, \quad n_3 = 0,$

$s_1 = -\sin\alpha, \quad s_2 = \cos\alpha, \quad s_3 = 0$

$\Rightarrow \quad I^o_{zz} = I^o_{nn} = I^o_{ij}\, n_i n_j$

$= I^o_{11} n_1^2 + I^o_{22} n_2^2 + I^o_{33} n_3^2 = I^o_{11}\cos^2\alpha + I^o_{22}\sin^2\alpha$

$I^o_{xz} = I^o_{sn} = I^o_{ij}\, s_i n_j = I^o_{11} s_1 n_1 + I^o_{22} s_2 n_2 + I^o_{33} s_3 n_3$ (10)

$= (I_{22} - I_{11})\sin\alpha\cos\alpha$

Now $R_1, R_2, R_4, R_5 \longrightarrow$ will yield.

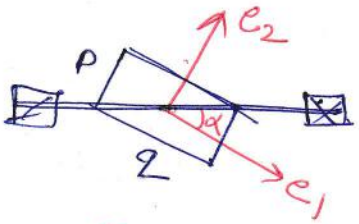
$R_1 = -(I^o_{xz}\omega^2 - mgb\sin\theta)/L, \quad R_2 = (I^o_{xz}\omega + mgb\cos\theta)/L$

$R_4 = (I^o_{xz}\omega^2 + mga\sin\theta)/L \quad R_5 = (-I^o_{xz}\omega + mgb\cos\theta)/L$

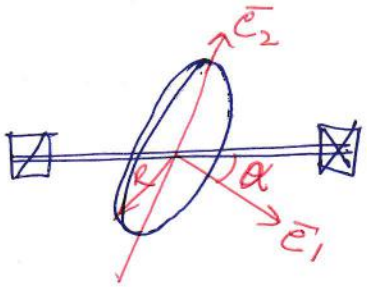
where I^{o}_{xz} is given by eq (10) For bodies in

For rectangular plate

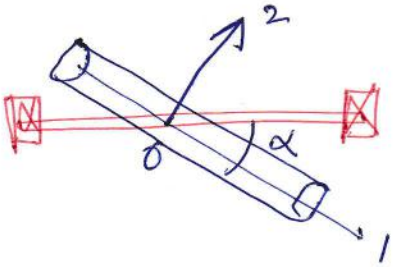
(i) $I^{o}_{xz} = \left(\frac{mq^2}{12} - \frac{mp^2}{12}\right) \sin\alpha\cos\alpha$



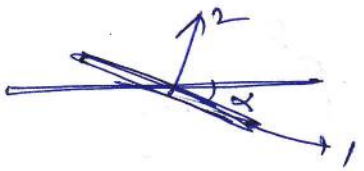
Rectangular plate



$I^{o}_{xz} = \left(\frac{mR^2}{4} - \frac{mR^2}{2}\right) \sin\alpha\cos\alpha$



$I^{o}_{xz} = \left[m(R^2/4 + h^2/12) - \frac{mR^2}{2}\right] \cos\alpha\sin\alpha$



$I^{o}_{xz} = (0 - mh^2/12) \sin\alpha\cos\alpha$

$= -(mh^2/12) \sin\alpha\cos\alpha$

16.10

A shaft has protruding arms each of which weighs 40 N/m. A Torque T gives shaft an angular acceleration $\ddot{\omega}$ of 2 rad/sec^2. At the instant shown in the diagram, ω is 5 rad/sec. If the Shaft without arms weights 180 N, Compute Vertical & horizontal forces at bearings A & B.

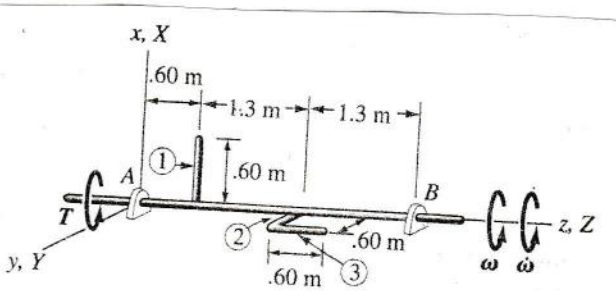


Figure 16.30. Rotating shaft with arms.

Free body diagram

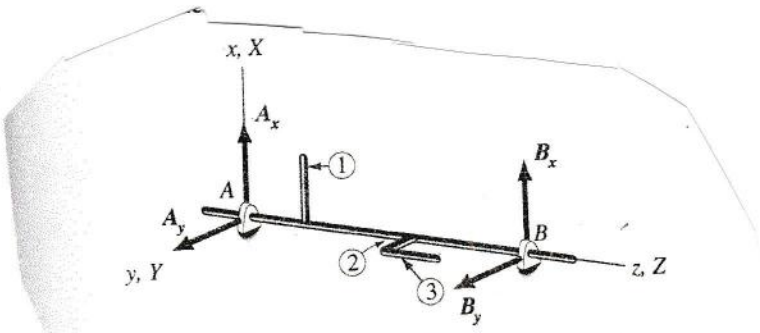


Figure 16.31. Supporting forces.

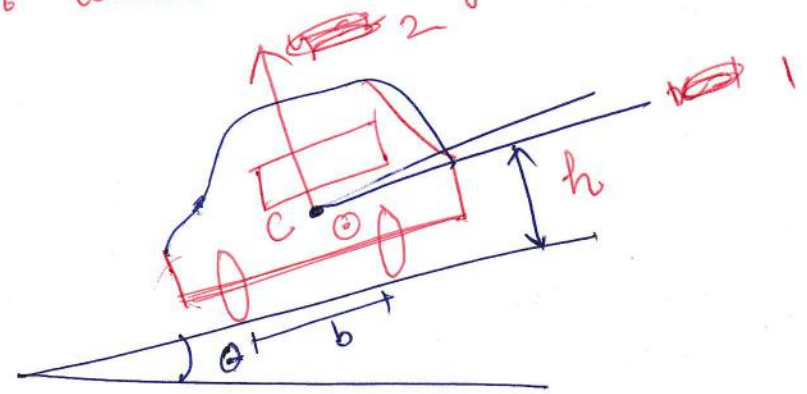
Example 3.4

Banking or superelevation of highway and railroad ~~curve~~ curves.

An automobile of mass m is travelling over a circular road 'banked' at angle θ. Find the proper value of θ if its centre of mass C travels at constant speed v in a horizontal circle of radius R. The height of C above the base is h. Neglect the weight of the wheels. The principal axes of inertia at C are the axes 1, 2, 3 with the principal moment of inertia I_{11}^C, I_{22}^C, I_{33}^C. Find also the condition of tipping or sliding sideways

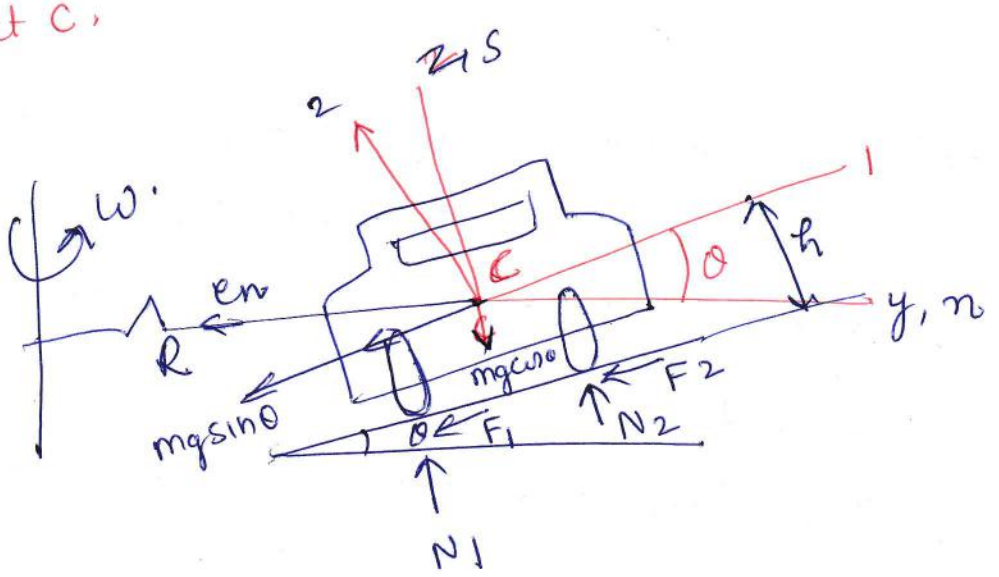
Solution:

Superelevation or banking is the difference in the elevation of the outer and inneer edges of a roadways or railways track to reduce the tendency to slide away from the centre of curvature or to overturn while travelling on a curve



Main body rotates about a fixed vertical axis.

The vertical axis z is not a principal axis of inertia at C.



The angular velocity $\bar{\omega} = \omega \hat{k}$ of the main body is obtained by $v_c = \omega R$, $\omega = v/R$, $\dot{\omega} = \dot{v}/R = 0$ ($\because$ v = constant speed)

Acceleration of G moving in a circle of radius R.

$$\bar{a}_c = \left(\frac{v^2}{R}\right)\bar{e}_n = \frac{v^2}{R}\left(-\cos\theta \bar{e}_1 + \sin\theta \bar{e}_2\right)$$

N_1, F_1 = Normal & frictional components of the sum of the forces exerted on the front and rear inner wheel

N_2, F_2 = ~~are th~~ Normal & frictional components of the sum of the forces exerted on the front and ~~outer~~ rear outer ~~wh~~ wheel.

Note: This implies that the front and rear wheels are pointed in the same direction which is a gross simplication of the actual situation. No force is shown in $\bar{e}_3$ direction.

⇒ Forward frictional force exerted by the road is balanced by air resistances.

Thus the formation is reduced to a coplanar force system. We use moment equation for G w.r.t. non principal axes. $I^G_{xz} = 0$ as $i = \bar{e}_3$ is a principal axis of inertia at G

$$\hat{j} = \bar{n} = \cos\theta e_1 - \sin\theta e_1$$

$$k = s = \sin\theta e_1 + \cos\theta e_2$$

$$n_1 = \cos\theta, \quad n_2 = -\sin\theta$$

$$n_3 = 0, \quad s_1 = \sin\theta, \quad s_2 = \cos\theta, \quad s_3 = 0$$

$$I^G_{yz} = I^G_{ns} = I^G_{ij} n_j s_j = I^G_{11} n_1 s_1 + I^G_{22} n_2 s_2 + I^G_{33} n_3 s_3$$

$$= (I^G_{11} - I^G_{22}) \sin\theta\cos\theta$$

Using Newton's law:

$$F = m\bar{a}_G$$

$\bar{e}_1$: $-F_1 - F_2 - mg\sin\theta = m\left(\frac{-v^2\cos\theta}{R}\right)$

$$\Rightarrow F_1 + F_2 = m\left(\frac{v^2\cos\theta}{R} - g\sin\theta\right) \quad \text{—— (2)}$$

$\bar{e}_2$: $N_1 + N_2 - mg\cos\theta = m\frac{v^2\sin\theta}{R}$ ——

$$\Rightarrow N_1 + N_2 = m(v^2\sin\theta/R + g\cos\theta) \quad \text{—— (3)}$$

Using Euler equation

$$M_G = (I^G_{xz}\dot{\omega} - I^G_{yz}\omega^2)\,i + (I^G_{yz}\dot{\omega} + I^G_{xz}\omega^2)\,j + I^G_{zz}\dot{\omega}\hat{k} \Rightarrow -I^G_{yz}\omega^2 i$$

$$(N_2 - N_1)b - h(F_1 + F_2) = -I^G_{yz}\omega^2 = (I^G_{22} - I^G_{11})\sin\theta\cos\theta\,(v/R)^2$$

Using eq (2) $N_2 - N_1 = m(v^2\cos\theta/R - g\sin\theta)h/b + (I^G_{22} - I^G_{11})\sin\theta\cos\theta\,(v/R)^2$

Solving.

$$N_1 = \frac{1}{2} m \left(v^2 \sin\theta/R + g\cos\theta - (v^2 \cos\theta/R - g\sin\theta) h/b \right) - (I_{22}^c - I_{11}^c) \sin\theta\cos\theta \left(v/R\right)^2 / 2b$$

$$N_2 = \frac{1}{2} m \left(v^2 \sin\theta/R + g\cos\theta + (v^2 \cos\theta/R - g\sin\theta) h/b \right] + (I_{22}^c - I_{11}^c) \sin\theta\cos\theta \left(v/R\right)^2 / 2b$$

special case

(1) For proper Banking, Total frictional forces should be zero. therefore

$$F_1 + F_2 = m(v^2\cos\theta/R - g\sin\theta) = 0$$

$$\boxed{\tan\theta = v^2/Rg}$$

(2) For no outward slip $(F_1 + F_2) \leq \mu (N_1 + N_2)$

$\mu = \tan\phi$ is the static coefficient of friction.

$$m(v^2\cos\theta/R - g\sin\theta) \leq \mu m(v^2 \sin\theta/R + g\cos\theta)$$

$$v^2 \leq gR(\tan\theta + \mu)/(1 - \mu\tan\theta) = gR\tan(\phi + \theta)$$

(3) For no inward slip

$$-(F_1 + F_2) \leq \mu (N_1 + N_2), \quad V_2 > gR\tan(\theta - \phi)$$

(4) For no tipping (overturning), the smaller normal reaction N_1 must remain positive.

$$N_1 \geq 0 \Rightarrow v^2 \leq gR(\tan\theta + b/h) / \left[(1 - b/h\tan\theta) + I_{22}^c - I_{11}^c \sin\theta / mRh \right] \quad \text{— (A)}$$

Equation (A) reveals that the car is more likely to overturn for smaller values of R, b, θ and for larger value of h, $(I_{22}^c - I_{11}^c)$.

$$v^2 \leq gR(\tan\theta + b/h) / (1 - (b/h)\tan\theta) \quad \text{— (11)}$$

(5)

if $(I_{22}^c - I_{11}^c)/mRh \leq 1$ then

$\cancel{v^2 \leq gR(}$

In this case tipping would occur $\mu > b/h$

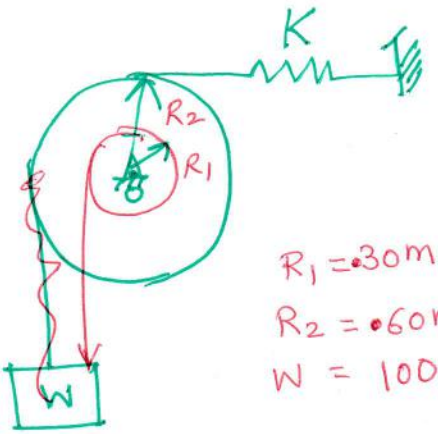
(5) For a straight banked road $R = \infty$

$$N_1, N_2 = \frac{1}{2} mg(\cos\theta \pm h/b \sin\theta)$$

In contrast to the circular case, the inner wheels are subjected to greater normal reaction than the outer wheels for the case of straight banked road.

Example No 16.1

A stepped cylinder having a radius of gyration $k = 0.4\,m$ and a mass of 200 kg is shown in Fig. The cylinder supports a weight W of mass 100 kg with an inextensible cord and is restrained by a linear spring whose constant K is 2 N/mm. What is angular acceleration of the stepped cylinder when it has rotated 10° after it is released from a state of rest? The spring is initially unstreched. What are the supporting forces at this time?



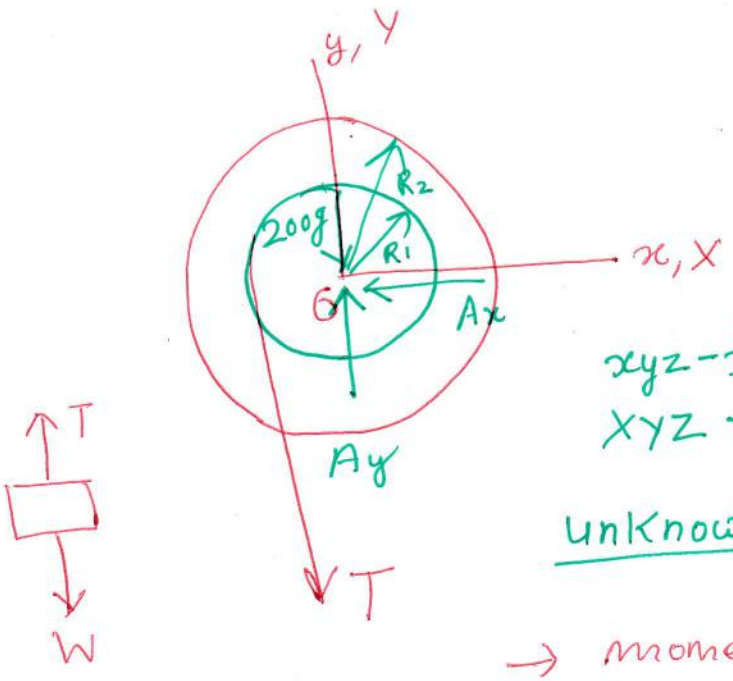
$R_1 = .30\,m$

$R_2 = .60\,m$

$W = 100g\ N$

Solution:

Free body diagram



xyz - fixed to body

XYZ - Inertial

Unknown: $\alpha\ (\ddot{\theta}),\ A_x,\ A_y,$

→ moment-of-momentum equation about center of the cylinder

$$\boxed{M_z = I_{zz}\dot{\omega}}$$

(Symmetric about axis of revolution
z - principal axis of inertia
$I_{zy} = I_{zx} = 0$

~~FBD~~

$$R_1(-\hat{i}) \times T(-\hat{j}) + R_2(\hat{j}) \times (KR_2\ddot{\theta})(\hat{i}) = mk^2\dot{\omega}\hat{k}$$

$$\boxed{R_1 T - KR_2^2\ddot{\theta} = (mk^2)\dot{\omega}} \quad -(1)$$

Now $\bar{a}_y = R_1 \dot{\omega} = R_1 \alpha$

Applying Newton's Law for cat weight

~~$T - W = m\bar{a}_y$~~

$T - W = m\bar{a}_y \Rightarrow \boxed{T - 100g = 100 R_1 \alpha}$ — (2)

$T = 100 R_1 \alpha + 100g$ — (3)

substituting into eq. (1)

$R_1^2 100\alpha + R_1 100g - K R_2^2 \theta = mk^2 \alpha$

$\alpha (R_1^2 100 - mk^2) = -R_1 100g + K R_2^2 \theta$

$$\alpha = \frac{-R_1 100g + K R_2^2 \theta}{R_1^2 100 - mk^2}$$

$\theta = 10^\circ \Rightarrow \frac{10 \times 2\pi}{360} = .1745$ rad/sec

$\alpha = 4.11$ rad/sec^2

Applying Newtons Law at Bearing

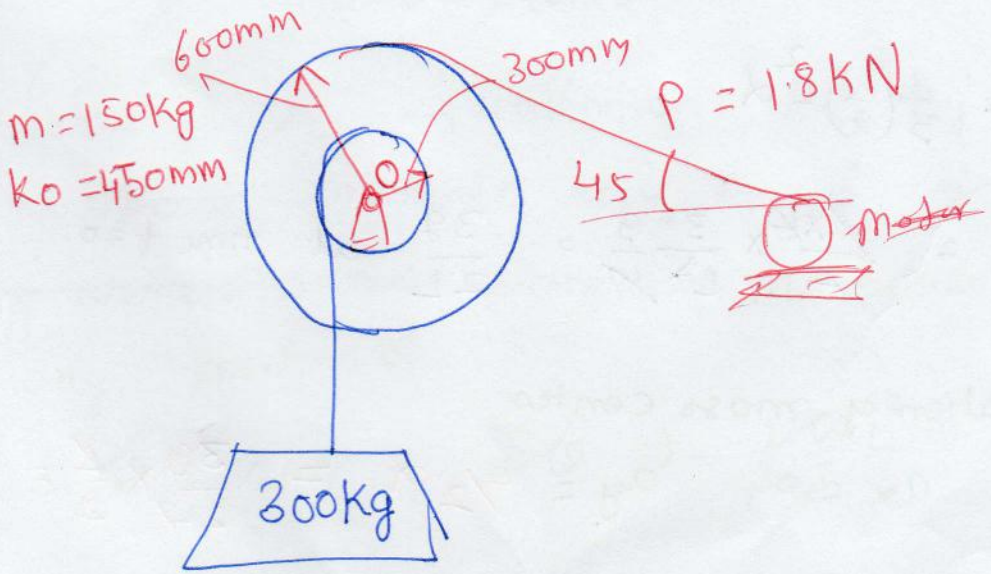
$A_y - 200g - T = 0$ — (4)

$A_x - K R_2 \theta = 0$ — (5)

~~Example 16.4~~

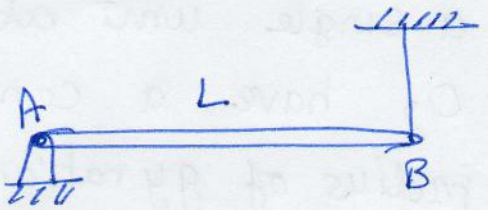
Class practice: 6/3

The 300 kg concrete block is elevated by the hoisting mechanism shown, where the cables are securely wrapped around the respective drums. The drums which are fastened together and turn as a single unit about their mass center at O, have a combined mass of 150kg and a radius of gyration about O of 450mm. if a constant tension P of 1.8 kN is maintained by the power unit at A, determine the vertical acceleration of the block and the resultant force on the bearing at O



$M_O = \bar{I}\alpha + mr$

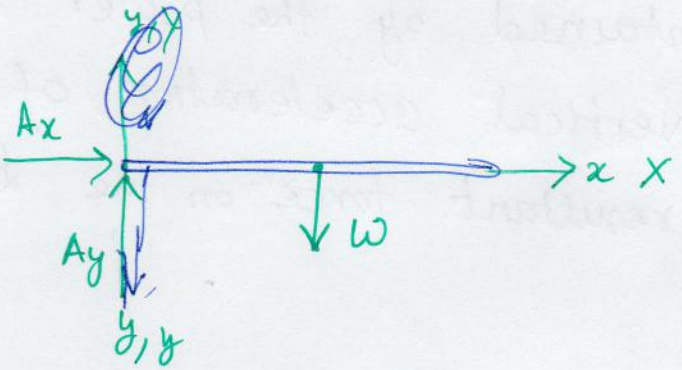
A uniform rod of weight W and length L supported by a pin connection at A and a wire at B is shown in Fig. What is the force on pin A at the instant that the wire is released? What is the force at A when the rod has rotated 45°?



Solution

At the Instant, wire is released at B.

The rod is rotating about A.



$$\Sigma M_A = I\dot{\omega} = I\alpha$$

$$W \times \frac{L}{2} = \frac{1}{3}\left(\frac{W}{g}\right)L^2\alpha$$

$$\alpha = \frac{W \times L}{2} \times \frac{3}{L^2}\frac{g}{W} = \frac{3g}{2L} \quad \text{at time } t = 0$$

Acceleration of mass center

$$a_x = 0, \quad a_y = \bar{r}\alpha = \frac{3g}{2L} \times \frac{L}{2} = \frac{3g}{4}$$

Applying Newton's Law

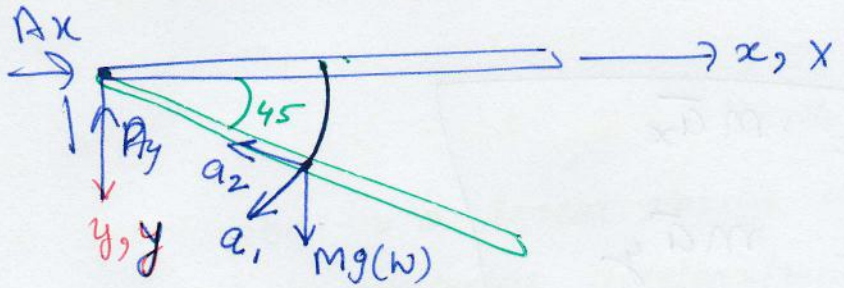
$$A_x = \frac{W}{g}a_x$$

~~$A_x = 0$~~

$A_y = W$

$$A_y = \frac{W}{g}a_y = \frac{W}{4}$$

(B) Rod at an position of 45°



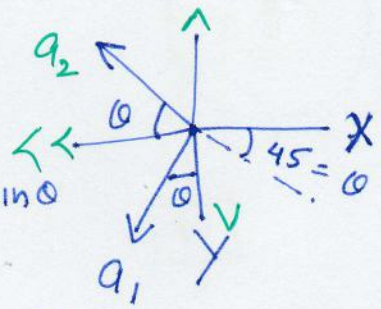
a_1 = Tangential acceleration

a_2 = Normal acceleration

$$a_1 = L/2\,\alpha = \frac{L}{2}\ddot{\theta}$$

$$a_2 = L/2\,\omega^2 = \frac{L}{2}(\dot{\theta})^2$$

Now Component along



$$\bar{a}_x = -a_2\cos\theta - a_1\sin\theta$$

$$\bar{a}_y = a_1\cos\theta - a_2\sin\theta$$

Now $(\dot{\theta}, \ddot{\theta})$ → unknown, A_x, A_y

Four Unknown A_x, A_y, $\dot{\theta}$, $\ddot{\theta}$

~~Taking Moment around~~ Moment of momentum Equation for any position θ

$$W\frac{L}{2}\cos\theta = I\ddot{\theta} \Rightarrow \frac{1}{3}\frac{W}{g}L^2\ddot{\theta}$$

$$\ddot{\theta} = \frac{3g}{2L}\cos\theta$$

Integrating both side w.r.t time.

$$\dot{\theta} = \frac{3g}{2L}\sin\theta + C_1$$

For $\theta = 0$, $C_1 = 0$

$$\dot{\theta} = \frac{3g}{2L}\sin\theta$$

Now Applying Newton 2nd law.

$$A_x = m\bar{a}_x$$

$$W - A_y = m\bar{a}_y$$

After solving

$$A_x = -1.124\,W$$

$$A_y = 1.375\,W$$