Question 1 (see Fig. 1) Two 2×4 m plywood panels, each of weight **60** N, are nailed together. The panels are supported by ball-and-socket joints at **A** and **F** and by the wire **BH**. It is required that the tension in the wire be minimum. Appropriate location of **H (x,y)** in the xy plane and the corresponding minimum tension $\mathbf{T_{min}}$ are to be determined.

(a) Draw the free-body diagram of the panels (combined) and derive the equation(s) of equilibrium, in which the only unknown is the force $\underline{\mathbf{T}}$ (vector). [**10 marks**]

(b) Using vector identities or otherwise, determine $\mathbf{T_{min}}$ and corresponding (x,y). [**10 marks**]

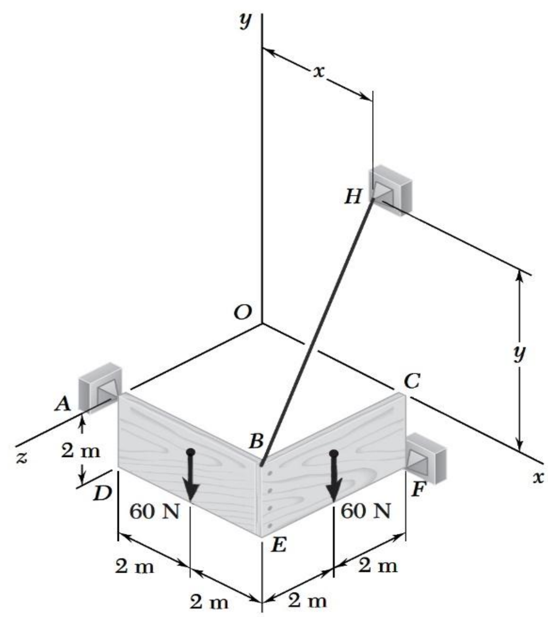


Figure 1 (Question 1)

SOLUTION:

Part (a)

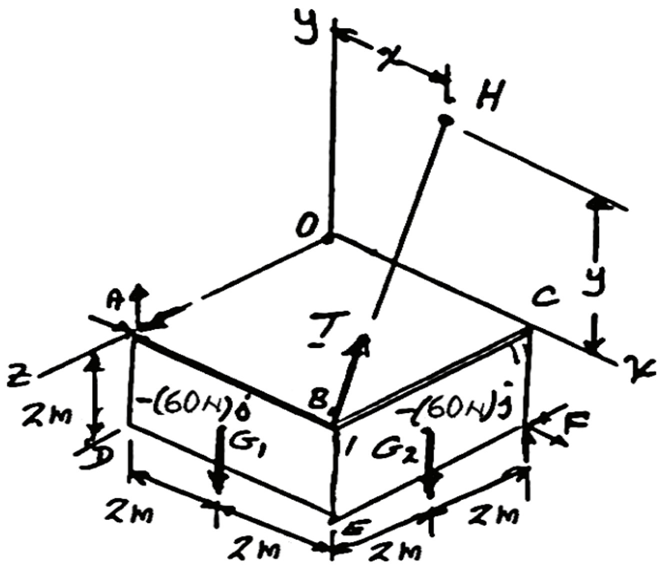


Figure: Free Body Diagram

$\underline{\text{AF}} = 4\hat{\imath} - 2\hat{\jmath} - 4\hat{k}$

$AF = \sqrt{4^2 + 2^2 + 4^2} = 6\ m$

$\hat{\lambda}_{AF} = \frac{\underline{\text{AF}}}{AF} = \frac{1}{3}\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right)$

$\underline{\text{r}}_{G_1/A} = 2\hat{\imath} - \hat{\jmath}$

$\underline{\text{r}}_{G_2/A} = 4\hat{\imath} - \hat{\jmath} - 2\hat{k}$

$\underline{\text{r}}_{B/A} = 4\hat{\imath}$

$$M_{AF}^{G_1} = \hat{\lambda}_{AF} \,.\left(\underline{\text{r}}_{G_1/A} \times \underline{\text{G}}_1\right) = \frac{1}{3}\begin{vmatrix} 2 & -1 & -2 \\ 2 & -1 & 0 \\ 0 & -60 & 0 \end{vmatrix}$$

$$= \frac{1}{3} \times 60 \times (0 + 4) = 80 \text{ N m}$$

$$M_{AF}^{G_2} = \hat{\lambda}_{AF} \,.\left(\underline{\text{r}}_{G_2/A} \times \underline{\text{G}}_2\right) = \frac{1}{3}\begin{vmatrix} 2 & -1 & -2 \\ 4 & -1 & -2 \\ 0 & -60 & 0 \end{vmatrix}$$

$$= \frac{1}{3} \times 60 \times (-4 + 8) = \frac{1}{3} \times 60 \times 4 = 80 \text{ N m}$$

$M_{AF}^{T} = \hat{\lambda}_{AF} \cdot \left(\underline{\mathrm{r}}_{B/A} \times \underline{\mathrm{T}}\right)$

$\sum \underline{\mathrm{M}}_{AF} = 0$

$\Rightarrow \ 80 + 80 + \hat{\lambda}_{AF} \cdot \left(\underline{\mathrm{r}}_{B/A} \times \underline{\mathrm{T}}\right) = 0$

$\hat{\lambda}_{AF} \cdot \left(\underline{\mathrm{r}}_{B/A} \times \underline{\mathrm{T}}\right) = -160$ **(1)**

Part (b)

Using the property of scalar triple product, Eq. (1) can be rewritten as

$\underline{\mathrm{T}} \cdot \left(\underline{\mathrm{r}}_{B/A} \times \hat{\lambda}_{AF}\right) = -160$

Thus, the projection of $\underline{\mathrm{T}}$ on $\left(\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A}\right)$ is constant.

Hence, $\underline{\mathrm{T}}_{\mathrm{min}}$ is parallel to $\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A}$,

$\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A} = \frac{1}{3}\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) \times 4\hat{\imath}$

$$= \frac{1}{3}\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & -2 \\ 4 & 0 & 0 \end{vmatrix} = \frac{1}{3}\left(-8\hat{\jmath} + 4\hat{k}\right)$$

Magnitude, $\left|\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A}\right| = \frac{1}{3}\sqrt{8^2 + 4^2} = \frac{1}{3}\sqrt{80} = \frac{4}{3}\sqrt{5}$

Corresponding unit vector $= \frac{\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A}}{\left|\hat{\lambda}_{AF} \times \underline{\mathrm{r}}_{B/A}\right|} = \frac{\frac{1}{3}\left(-8\hat{\jmath} + 4\hat{k}\right)}{\frac{4}{3}\sqrt{5}}$

$$= \frac{1}{\sqrt{5}}\left(-2\hat{\jmath} + \hat{k}\right)$$

$\Rightarrow \underline{\mathrm{T}}_{\mathrm{min}} = \frac{T}{\sqrt{5}}\left(-2\hat{\jmath} + \hat{k}\right)$ **(2)**

Eq. **(1)**:

$$\frac{T}{\sqrt{5}}\left(-2\hat{\jmath} + \hat{k}\right) \cdot \left[\frac{1}{3}\left(2\hat{\imath} - \hat{\jmath} - 2\hat{k}\right) \times 4\hat{\imath}\right] = -160$$

$$\Rightarrow \frac{T}{\sqrt{5}}\left(-2\hat{\jmath} + \hat{k}\right) \cdot \frac{1}{3}\left(-8\hat{\jmath} + 4\hat{k}\right) = -160$$

$$\Rightarrow \frac{T}{3\sqrt{5}}(16 + 4) = -160$$

$$\Rightarrow T = -\frac{3\sqrt{5}(160)}{20} = 24\sqrt{5} = 53.67 \text{ N}$$

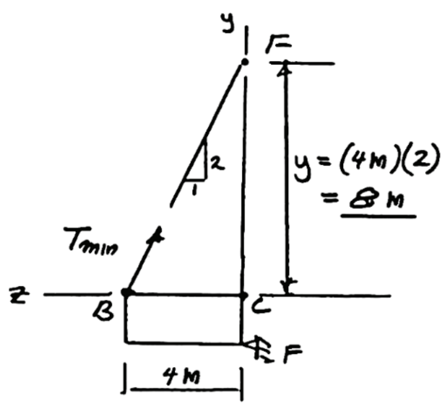
Eq. **(2)**:

$$\underline{T}_{\min} = \frac{T}{\sqrt{5}}\left(-2\hat{j} + \hat{k}\right) = 24\sqrt{5}\left(-2\hat{j} + \hat{k}\right)\frac{1}{\sqrt{5}}$$

$$\Rightarrow \underline{T}_{\min} = (-48\ \mathrm{N})\hat{j} + (24\ \mathrm{N})\hat{k}$$

Since $\underline{T}_{\min}$ has no $\hat{\imath}$ component, wire *BH* is parallel to the *y*-*z* plane.

Hence, $x = 4$ m.



From the diagram, $y = 8.00$ m.

So, the answers are the following.

$x = 4.00$ m

$y = 8.00$ m

$\underline{T}_{min} = 53.7$ N

Alternate Solution for Part (b)

Eq. (1):

$\hat{\lambda}_{AF} \cdot (\underline{r}_{B/A} \times \underline{T}) = -160$

$\underline{T} = T\hat{\lambda}_{BH}$

$$\hat{\lambda}_{BH} = \frac{\overline{BH}}{BH} = \frac{(x-4)\hat{\imath} + y\hat{\jmath} - 4\hat{k}}{[(x-4)^2 + y^2 + 16]^{1/2}}$$

$$\frac{1}{3}\begin{vmatrix} 2 & -1 & -2 \\ 4 & 0 & 0 \\ x-4 & y & -4 \end{vmatrix} \frac{T}{[(x-4)^2 + y^2 + 16]^{1/2}} = -160$$

$$\Rightarrow -\frac{4}{3}(4+2y)\frac{T}{[(x-4)^2 + y^2 + 16]^{1/2}} = -160$$

$$\Rightarrow T = \frac{60[(x-4)^2 + y^2 + 16]^{1/2}}{(2+y)}$$

$$\Rightarrow \ T^2 = \frac{3600[(x-4)^2 + y^2 + 16]}{(y+2)^2}$$

$$= 3600\, f(x,y)$$

where $f(x,y) = \frac{[(x-4)^2+y^2+16]}{(y+2)^2}$

$$f_x(x,y) = \frac{2(x-4)}{(y+2)^2}$$

$$f_x(x,y) = 0$$

$$\Rightarrow \frac{2(x-4)}{(y+2)^2} = 0$$

$$\Rightarrow x - 4 = 0$$

$$\Rightarrow x = 4$$

$$f_y(x,y) = \frac{2y}{(y+2)^2} - \frac{2[(x-4)^2 + y^2 + 16]}{(y+2)^3}$$

$$= \frac{2}{(y+2)^3}[y(y+2) - \{(x-4)^2 + y^2 + 16\}]$$

$$= \frac{2}{(y+2)^3}\{2y - (x-4)^2 - 16\}$$

Putting $f_y(x,y) = 0$

$$\Rightarrow \frac{2}{(y+2)^3}\{2y - (x-4)^2 - 16\} = 0$$

$$\Rightarrow 2y - (x-4)^2 - 16 = 0$$

Putting $x = 4$ gives

$y = 8$

Thus, the critical point is $(4,8)$

Now, $T = \frac{60[(x-4)^2 + y^2 + 16]^{1/2}}{y+2}$

$$\therefore \ T_{min} = \frac{60[(4-4)^2 + 8^2 + 16]^{1/2}}{8+2}$$

$$\Rightarrow T_{min} = 53.76 \ N$$

So, the answers are the following.

$x = 4.00$ m

$y = 8.00$ m

$T_{min} = 53.7$ N

Confirmatory Test:

Now, $f_{xx}(x,y) = \frac{2}{(y+2)^2}$

Thus, $f_{xx}(4,8) = \frac{2}{10^2} = 0.02$

And $f_{yy}(x,y) = -\frac{6}{(y+2)^4}\{2y - (x-4)^2 - 16\} + \frac{2}{(y+2)^3}(2)$

$f_{yy}(4,8) = 0 + \frac{4}{10^3} = 0.004$

Again, $f_{xy}(x,y) = -\frac{4(x-4)}{(y+2)^3}$

So, $f_{xy}(4,8) = 0$

Now, $D = \left[f_{xx} \cdot f_{yy} - {f_{xy}}^2\right]_{(4,8)}$

$= 0.02 \times 0.004 - 0$

$\Rightarrow D = 8 \times 10^{-5} > 0$

Also, $f_{xx}(4,8) > 0$

Thus, $\mathrm{T} = \mathrm{T_{min}}$ at $(\mathrm{x},\mathrm{y}) = (4,8)$

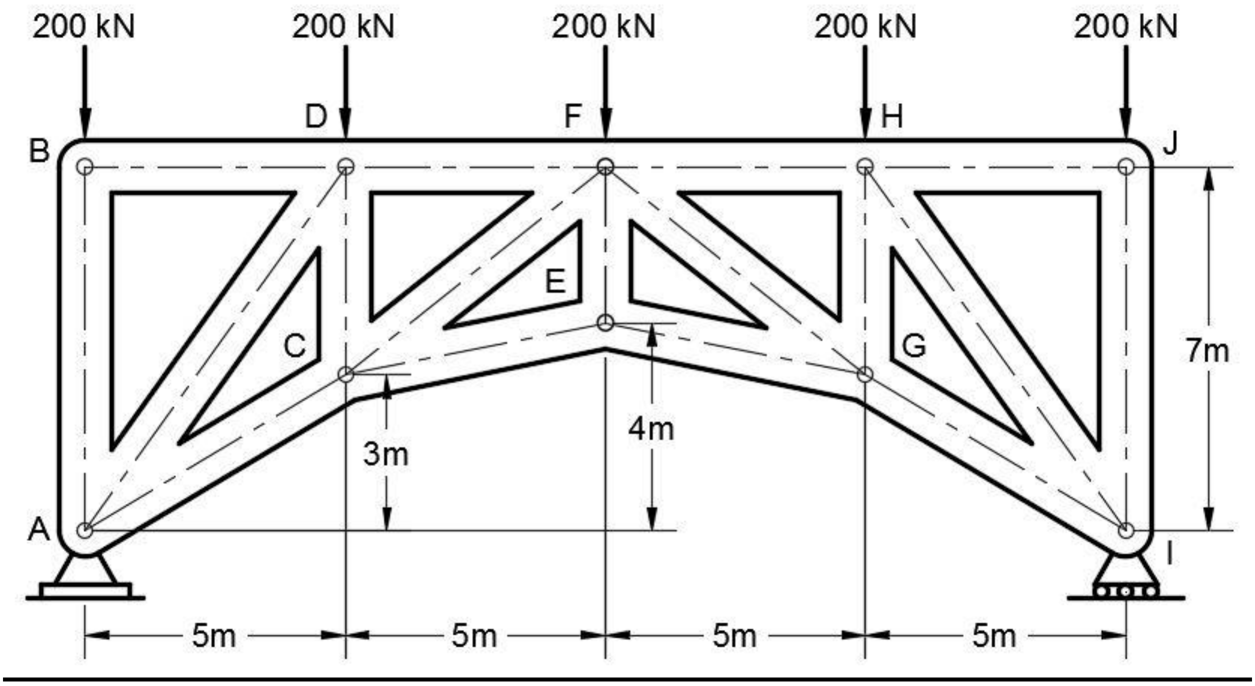


Figure 2.

Question-2:

Find out the support reactions and the forces in the members DF, CE & CF of the truss shown in Figure-1, with proper labels (tension or compression) using the methods of sections?

Solution:

From the entire structure, we find the reactions at A.

$$\sum \boldsymbol{F_x}: \quad A_x = 0$$

$$\sum \boldsymbol{M_i}: \ (200kN)(5m) + (200kN)(10m) + (200kN)(15m) + (200kN)(20m) - A_y(20m) = 0 \Rightarrow A_y = 500kN$$

Now we cut through $DF, CF \ and \ CE$ and use the left section.

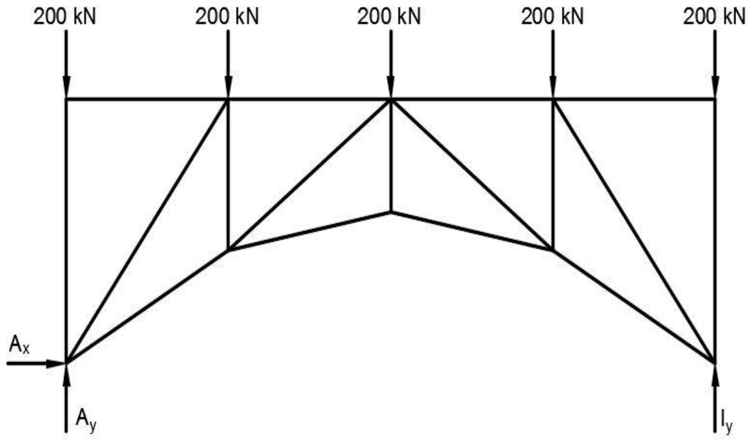
$$\sum \boldsymbol{M_c}: \ (200kN)(5m) - A_y(5m) + A_x(3m) - F_{DF}(4m) = 0$$

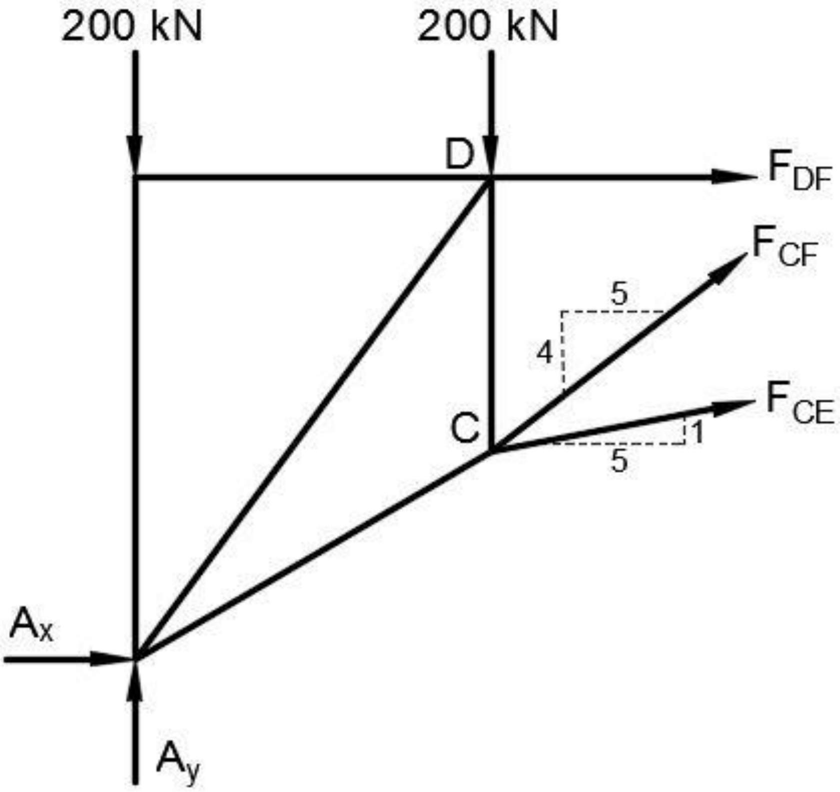
$$\boldsymbol{F_{DF}} = -375 \ kN$$

$$\sum \boldsymbol{M_F}: \ (200kN)(10m) + (200kN)(5m) - A_y(10m) + A_x(7m) + \frac{5}{\sqrt{26}} F_{CE}(4m) - \frac{1}{\sqrt{26}} F_{CE}(5m) = 0 \Rightarrow \boldsymbol{F_{CE}} = 680kN$$

$$\sum \boldsymbol{F_x}:\ A_x + F_{DF} + \frac{5}{\sqrt{26}} F_{CE} + \frac{5}{\sqrt{41}} F_{CF} = 0$$

$F_{CF} = 374\ kN$





Question 3:

For the beam ABC shown in Figure. 3 (The member DB is a two-force member), answer the following questions:

a) Mention the sign convention for shear force and bending moment
b) Find out the expressions of the shear force and bending moment as a function of distance **x** considered from the support A

c) Draw the shear force and the bending moment diagrams. Find out the location of maximum bending moment.

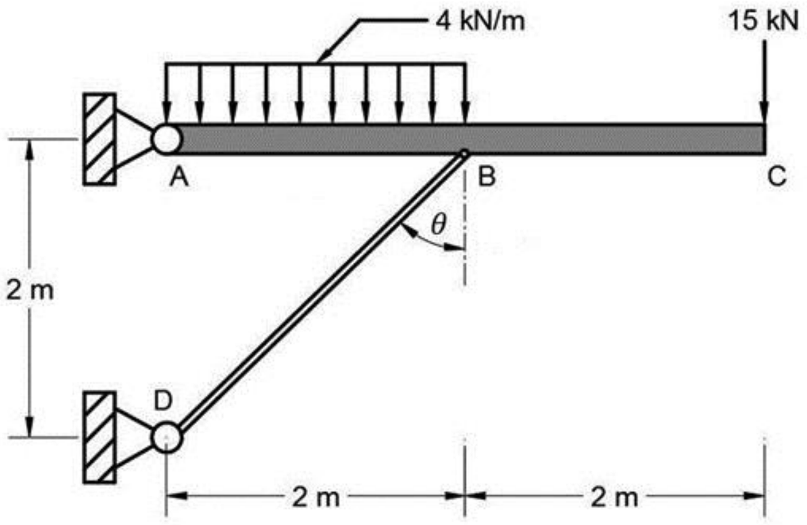
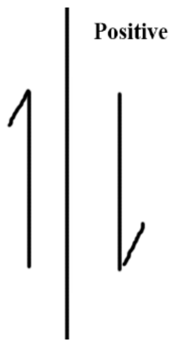


Figure 3

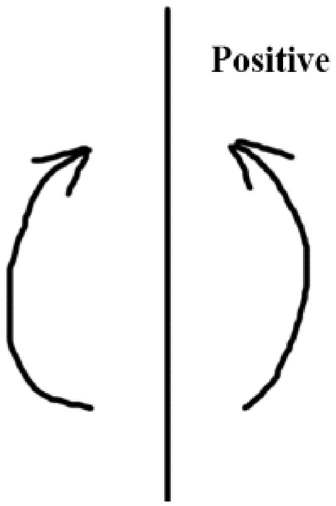
SOLUTION

a.) Sign Conventions

Shear sign convention

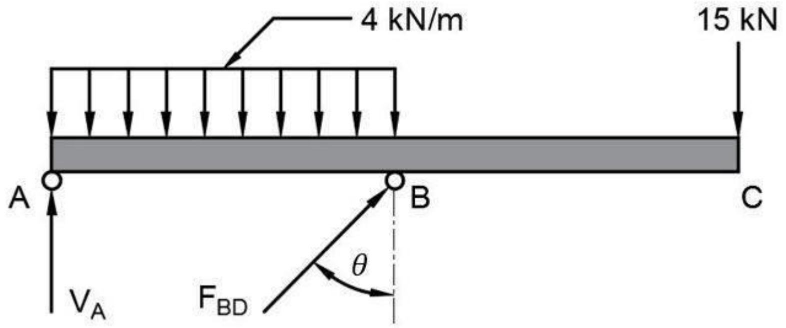


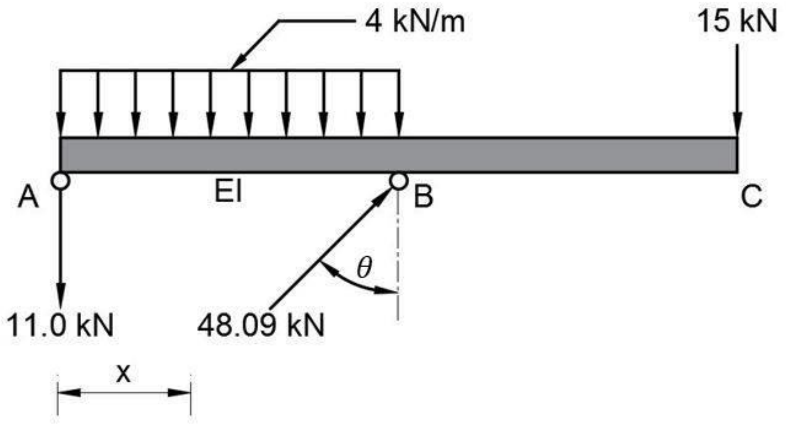
BMD sign convention



Sagging positive and Hogging negative

b.) SFD Functions:





1) At $x = 0$

   SHEAR FORCE = $-\ 11\ kN$

2) For $0 < x \leq 2$:

   SHEAR FORCE = $-\ 11 - 4x\ kN$

3) For $2 < x \leq 4$:

   SHEAR FORCE = $-\ 15\ kN$

<u>BMD Functions:</u>

1) At $x = 0$

   BENDING MOMENT = $0\ kNm$

2) For $0 < x \leq 2$:

   BENDING MOMENT = $-\ 11x - 2x^2\ kNm$

3) For $2 < x \leq 4$ (x measured from the right end):

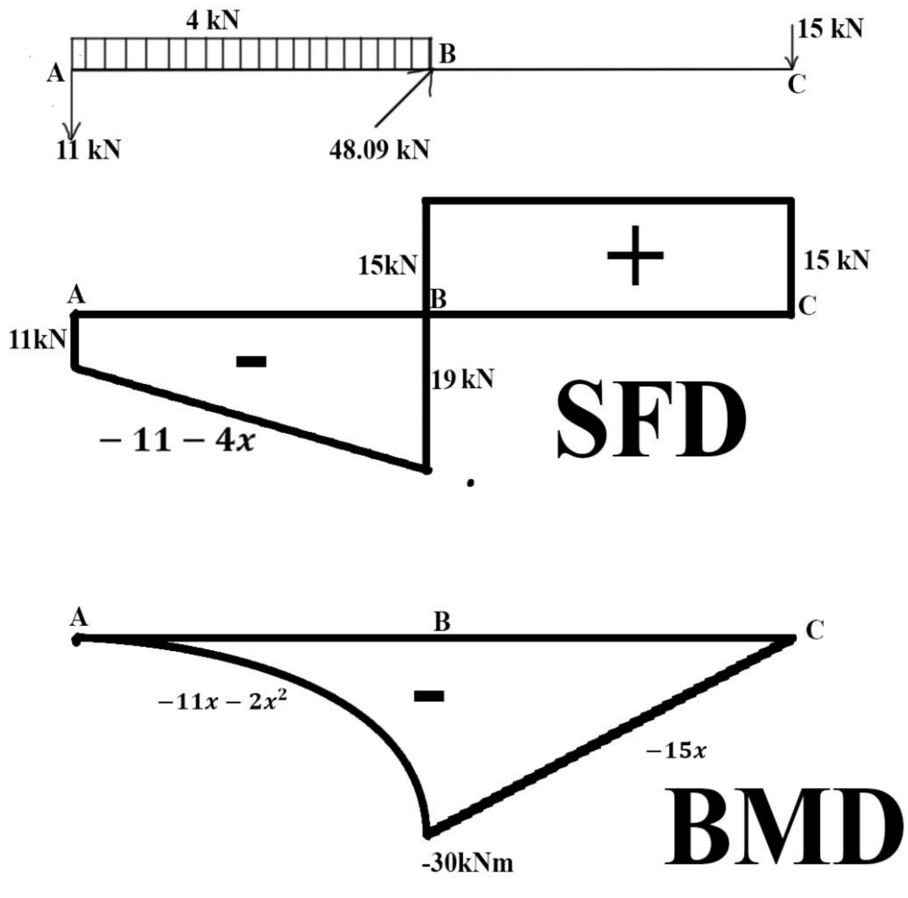
   BENDING MOMENT = $-\ 15x\ kNm$

c.) To find maximum bending moment, we have to differentiate the BMD Function wrt x, and equate it to zero –

$\frac{d}{dx}(-11x - 2x^2) = 0$ → $-11 - 4x = 0$ → x = $-2.75\ m$

This means that the maxima lies outside the region of $0 < x \leq 2$, and hence, the curve is steeply falling in this region. Thus, the maximum bending moment (absolute maximum, i.e. minimum in this context) will occur at the end of the region ($x = 2$)

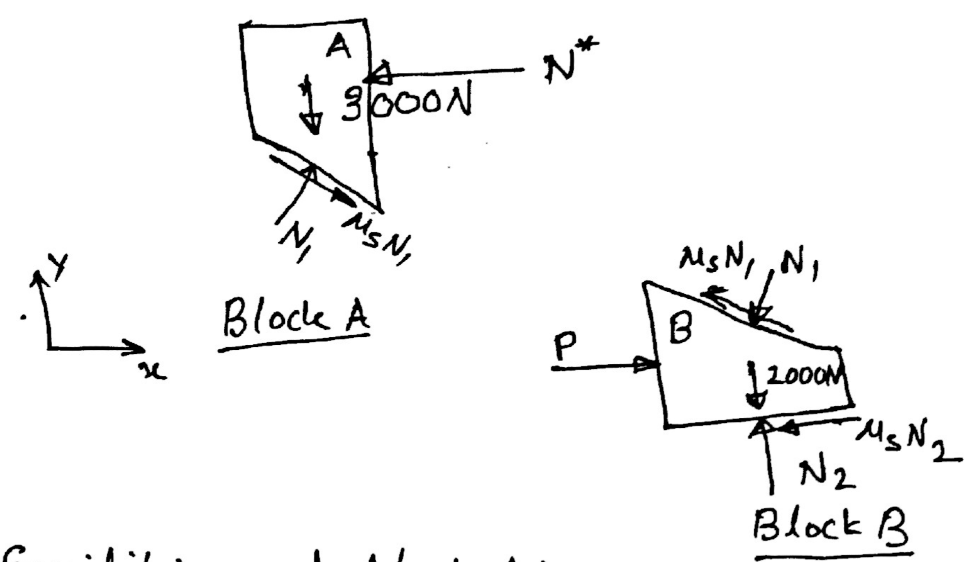
Maximum Bending Moment = $-15 * (2) = -30\ kNm$



Q-4

Case I : If block B moves to the right:

Let P be the applied force on B. Free body diagrams of blocks are



Equilibrium of block A:

$\sum F_y = 0 \Rightarrow -3000 + N_1 \cos 30 - \mu_s N_1 \sin 30 = 0$

or $N_1(\cos 30 - \mu_s \sin 30) = 3000$

$N_1 = \dfrac{3000}{\cos 30 - 0.12 \sin 30} = \dfrac{3000}{0.806} = 3721.97\,N$

Equilibrium of block B:

$F_y = 0 \Rightarrow -N_1 \cos 30 + \mu_s N_1 \sin 30 - 2000 + N_2 = 0$

or $N_2 = 2000 + N_1(\cos 30 - \mu_s \sin 30)$

$= 2000 + 3721.97 \times 0.806$

$N_2 = 5000\,N$

$F_x = 0 \Rightarrow P - \mu_s N_2 - \mu_s N_1 \cos 30 - N_1 \sin 30 = 0$

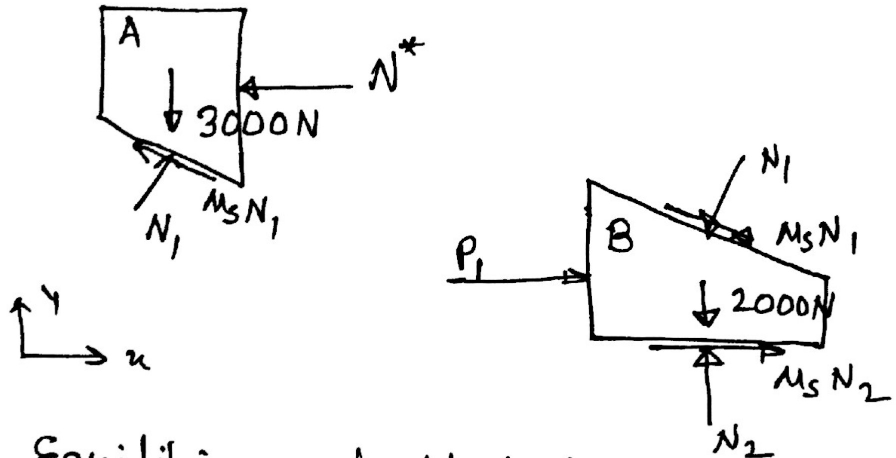
or $P = 0.12 \times 5000 + N_1(0.12 \cos 30 + \sin 30)$

$P = 2847.78\,N$

The given force 400 N is less than the required force to move block B to the right. Hence block B will not move to the right.

Case II: If block B moves to the left:

Let applied force on the block B be P_1. Free body diagrams of blocks are



Equilibrium of block A:

$F_y = 0 \Rightarrow -3000 + N_1 \cos 30 + \mu_s N_1 \sin 30 = 0$

or $N_1 = \dfrac{3000}{\cos 30 + 0.12 \times \sin 30} = \dfrac{3000}{0.926} = 3239.65\,N$

Equilibrium of block B:

$F_y = 0 \Rightarrow N_2 - 2000 - N_1 \cos 30 - \mu_s N_1 \sin 30 = 0$

or $N_2 = 2000 + N_1 \times (\cos 30 + \mu_s \sin 30)$

$= 2000 + 3239.65 \times 0.926$

$N_2 = 5000\,N$

$F_x = 0 \Rightarrow P_1 + \mu_s N_2 + \mu_s N_1 \cos 30 - N_1 \sin 30 = 0$

or $P_1 = N_1(\sin 30 - \mu_s \cos 30) - \mu_s N_2$

$= 3239.65 \times 0.396 - 0.12 \times 5000$

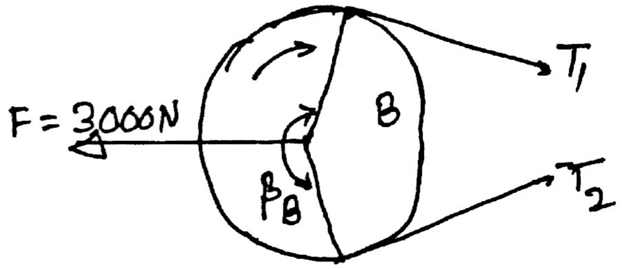
$P_1 = 683.15\,N$

As the applied force (given) 400N is less than the required force to move (stop) block B to the left, the block B will move to the left.

The blocks are not in equilibrium.

Q-5

Consider pulley B.



We have

$$\beta_B = 180 + 2 \times 8.63$$

or $\beta_B = 197.26°$

For impending friction, we have

$$\frac{T_1}{T_2} = e^{\mu_s \beta_B} = e^{(\mu_s \times \frac{197.26}{180} \times \pi)} \Rightarrow \frac{T_1}{T_2} = e^{3.44 \mu_s} \quad \text{—— (1)}$$

We have moment relation

$$(T_1 - T_2) \cdot 8 = 750 \Rightarrow T_1 - T_2 = 937.5 \quad \text{—(2)}$$

$F_x = 0$ for the pulley gives

$$(T_1 + T_2) \cos(8.63) = 3000$$

or $T_1 + T_2 = 3034.355$ —— (3)

Addition of equations (2) and (3) gives

$$2T_1 = 3971.855 \Rightarrow T_1 = 1985.93 \text{ N}$$

substituting T_1 in equation (2) and solving for T_2, we get

$$T_2 = 1985.93 - 937.5 = 1048.43$$

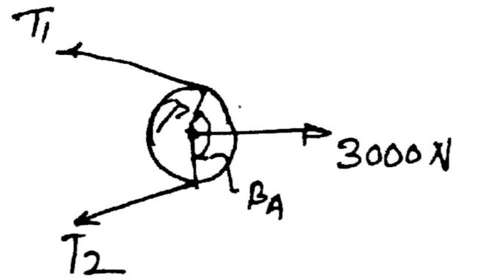
substituting T_1 and T_2 in eqn (1) we get

$$\frac{1985.93}{1048.43} = e^{3.44 \mu_s}$$

or $\mu_s = \frac{1}{3.44} \ln\left(\frac{1985.93}{1048.43}\right) = 0.186$

$$\boxed{\mu_s = 0.186}$$

Consider pulley A



$\beta_A = 180 - 2 \times 8.63$

$\beta_A = 162.74^\circ$

we have for impending slippage.

$$\frac{T_1}{T_2} = e^{\left(\frac{162.74}{180} \times \pi\right)\mu_s} = e^{2.84\,\mu_s}$$

T_1 and T_2 will be the same as for the case of pulley B. substituting T_1 and T_2 in the above expression

$$\frac{1985.93}{1048.43} = e^{2.84\,\mu_s}$$

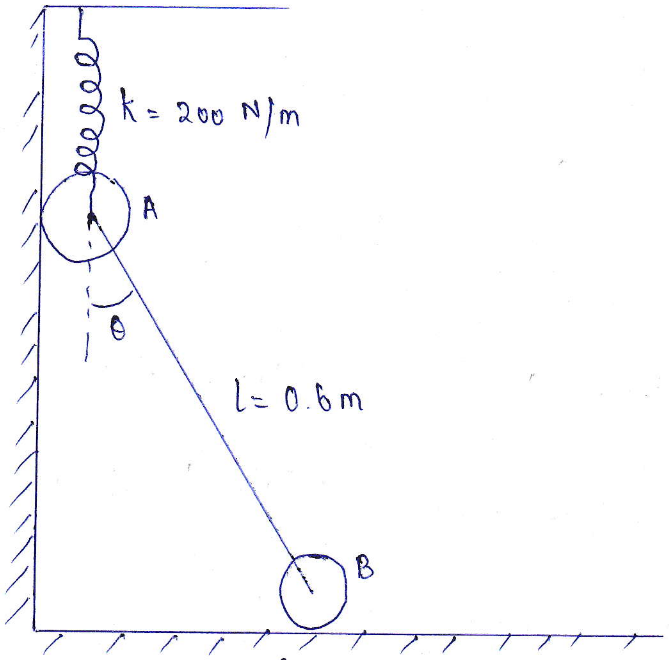
or $$\mu_s = \frac{1}{2.84} \ln\left(\frac{1985.93}{1048.43}\right)$$

$$\boxed{\mu_s = 0.225}$$

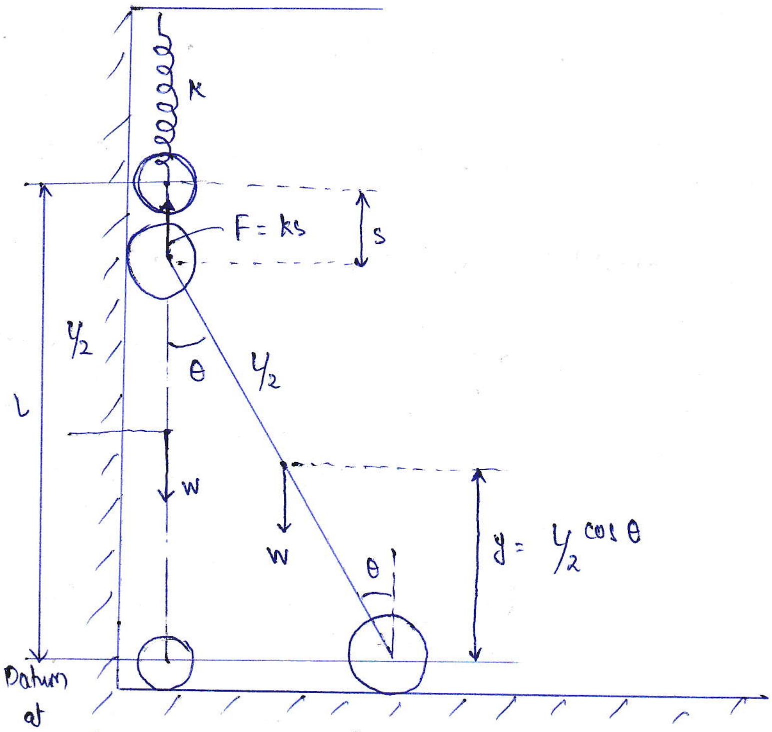
Minimum static coefficient of friction for applied force and moment without slippage to run the system is the maximum of the two values obtained for pulley A and B.

$\mu_s = 0.225$ for both the pulley

Question :6 (Solution)



(Fig 1)



(Fig 2)

When link is located in the arbitrary position θ,

Increase in P.E due to stretching of spring = $V_e = \frac{1}{2} ks^2$

Decrease in P.E due to weight = $V_g = W.y$.

Total P.E = $V_e + V_g = \frac{1}{2} ks^2 + Wy = (V)$

$l = s + l\cos\theta$ (fig 2)

$\Rightarrow s = L(1-\cos\theta)$

& $y = \left(\frac{l}{2}\right)\cos\theta$.

$$\therefore \quad V = \frac{1}{2} kl^2 (1-\cos\theta)^2 + W\left(\frac{l}{2}\cos\theta\right)$$

Equilibrium position: $\frac{dV}{d\theta} = kl^2(1-\cos\theta)\sin\theta - \frac{Wl}{2}\sin\theta = 0$

$$\therefore \quad l\left[kl(1-\cos\theta) - \frac{W}{2}\right]\sin\theta = 0$$

Thus this equation is satisfied provided

$\sin\theta = 0 \quad \therefore \theta = 0°$.

or $\theta = \cos^{-1}\left(1 - \frac{W}{2kL}\right) = \cos^{-1}\left[1 - \frac{10 \cdot (9.81)}{2 \cdot 200 \cdot (0.6)}\right] \simeq 53.8°$

Stability

$$\frac{d^2V}{d\theta^2} = kl^2(1-\cos\theta)\cos\theta + kl^2\sin\theta\sin\theta - \frac{Wl}{2}\cos\theta$$

$$= kl^2(\cos\theta - \cos 2\theta) - \frac{Wl}{2}\cos\theta$$

Substituting values for constants, with $\theta = 0^\circ$ & $\theta = 53.8^\circ$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=0^\circ} = 200(0.6)^2(\cos 0^\circ - \cos 0^\circ) - \frac{10(9.81)(0.6)}{2}\cos 0^\circ$$

$= -29.4$ which is < 0 (unstable equilibrium at $\theta = 0^\circ$)

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta=53.8^\circ} = 200(0.6)^2(\cos 53.8^\circ - \cos 107.6^\circ) - \frac{10(9.81)(0.6)}{2}\cos 53.8^\circ$$

$= 46.9 > 0$ (stable equilibrium at $\theta = 53.8^\circ$)