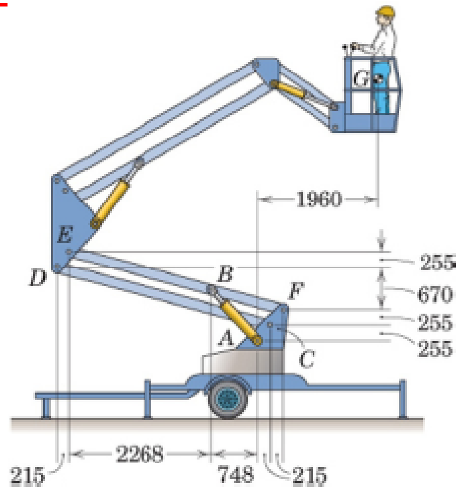
Frames and Machines

A structure is called a Frame or Machine if at least one of its individual members is a <u>multi-force member</u>

- member with 3 or more forces acting, or
- member with 2 or more forces and 1 or more couple acting



Frames: generally stationary and are used to support loads

Machines: contain moving parts and are designed to transmit and alter the effect of forces acting

Multi-force members: the forces in these members in general will not be along the directions of the members

→ methods used in simple truss analysis cannot be used

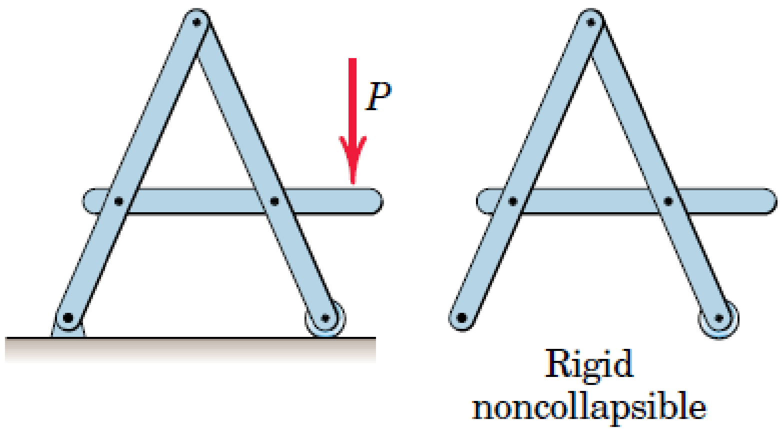
Interconnected Rigid Bodies with Multi-force Members

- Rigid Non-collapsible

–structure constitutes a rigid unit by itself when removed from its supports

–first find all forces external to the structure treated as a single rigid body

–then dismember the structure & consider equilibrium of each part

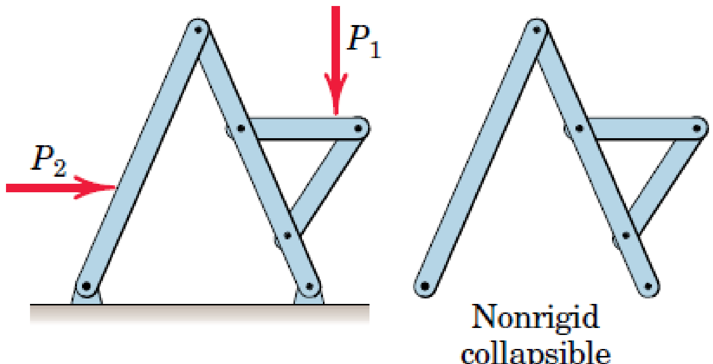


(a)

- Non-rigid Collapsible

–structure is not a rigid unit by itself but depends
on its external supports for rigidity

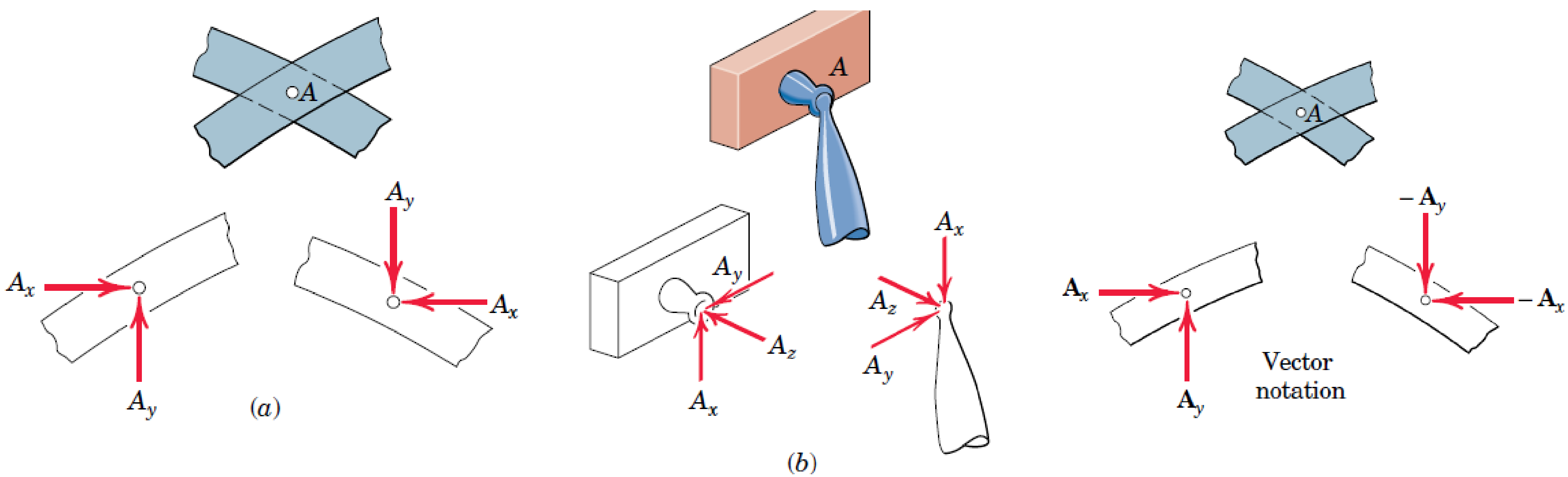
–calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analyse



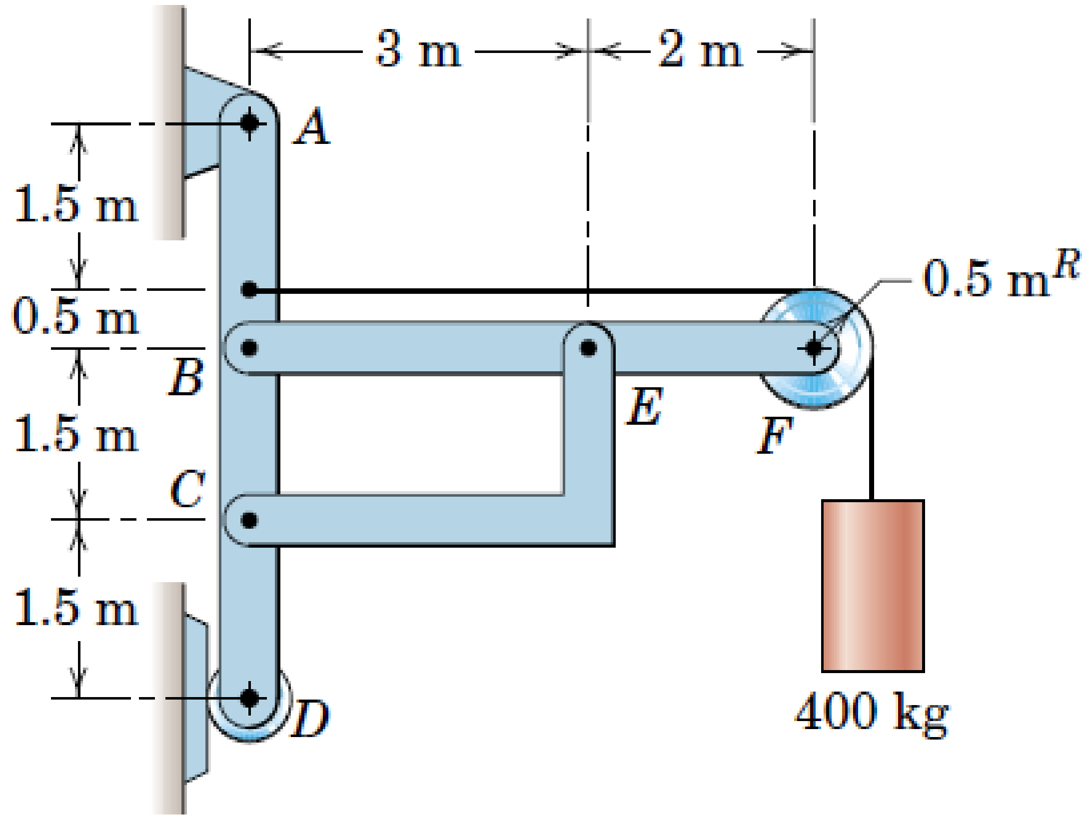
(b)

Free Body Diagrams: Forces of Interactions

- force components must be consistently represented in opposite directions on the separate FBDs (Ex: Pin at A).
- apply action-and-reaction principle (Ex: Ball & Socket at A).
- Vector notation: use plus sign for an action and a minus sign for the corresponding reaction

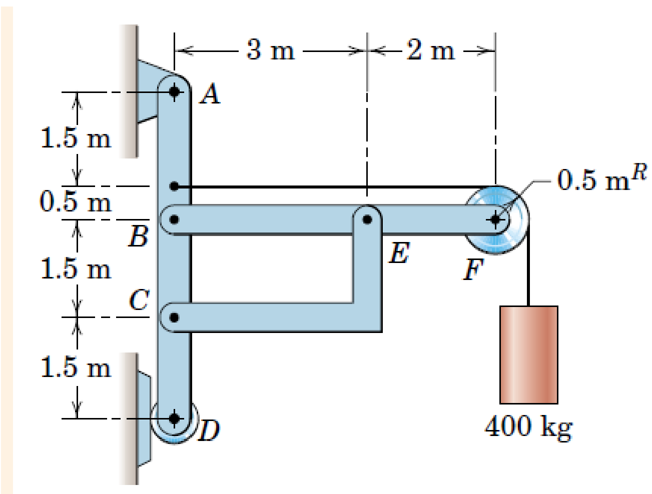


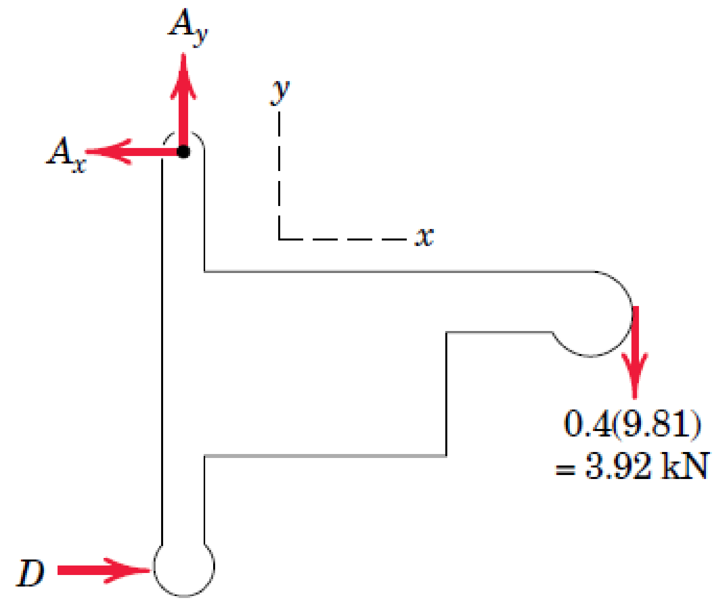
The frame supports the 400-kg load in the manner shown. Neglect the weights of the members compared with the forces induced by the load and compute the horizontal and vertical components of all forces acting on each of the
members.



- The three supporting members which constitute the frame form a rigid assembly that can be analyzed as a single unit

- We also observe that the arrangement of the external supports makes the frame statically determinate

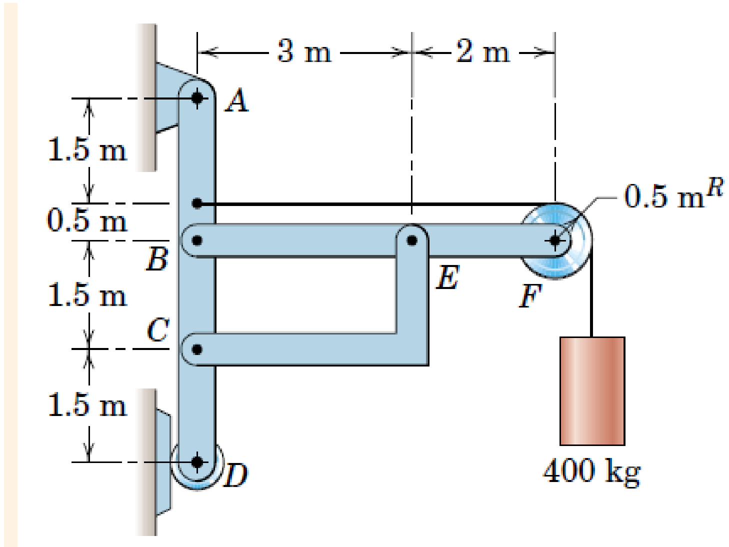


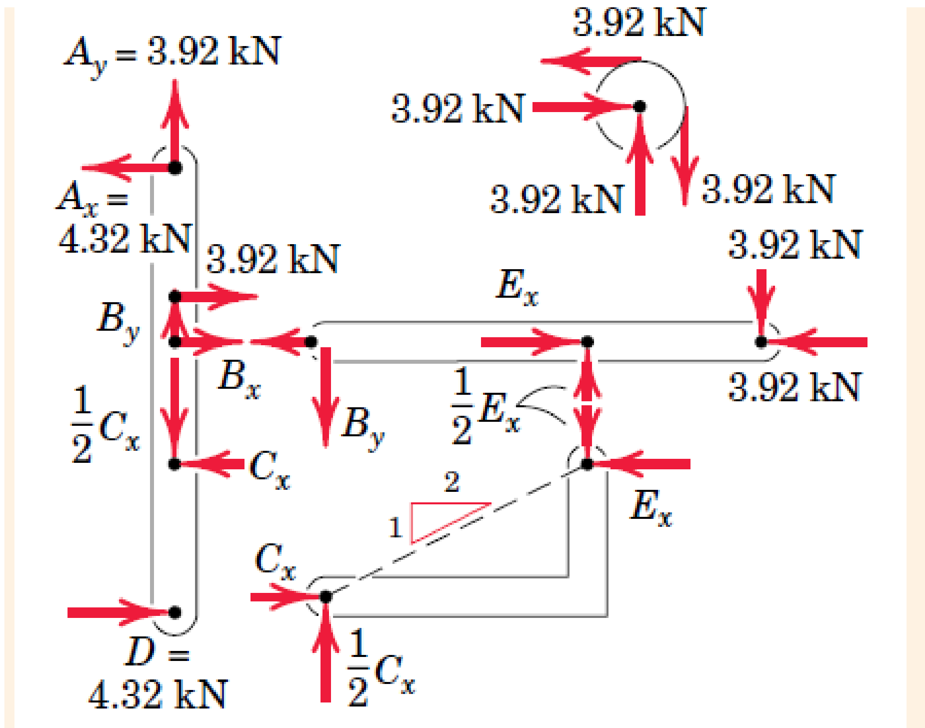


$[\Sigma M_A = 0]$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32$ kN
$[\Sigma F_x = 0]$	$A_x - 4.32 = 0$	$A_x = 4.32$ kN
$[\Sigma F_y = 0]$	$A_y - 3.92 = 0$	$A_y = 3.92$ kN

Dismember the frame and draw a separate free-body diagram of each member

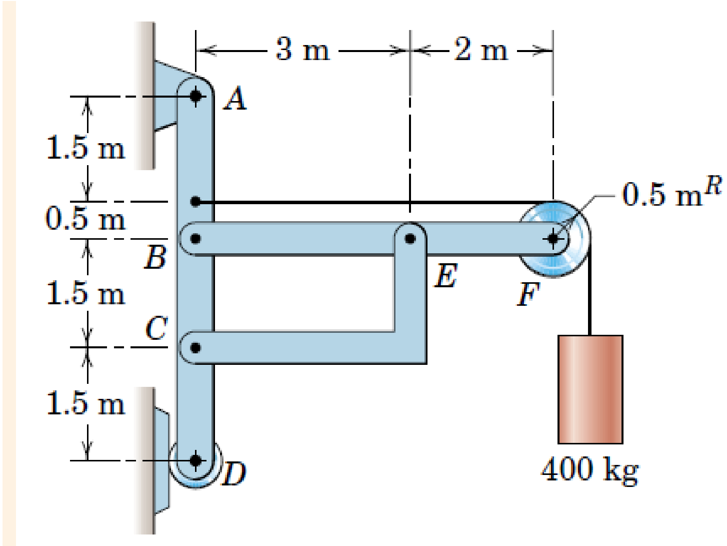
3.92-kN forces exerted by the shaft of the pulley on the member *BF*,

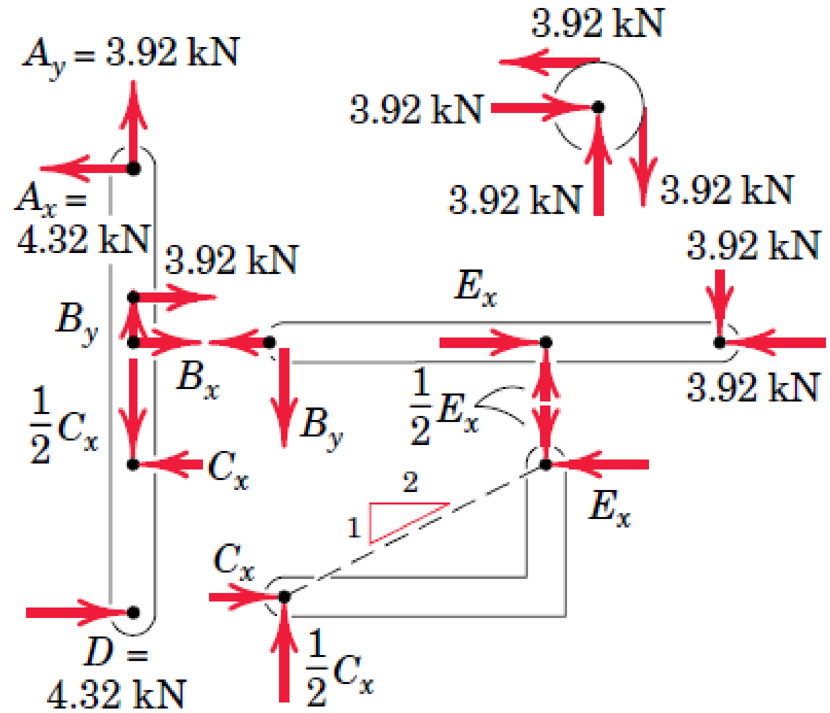




Here we observe that *CE* is a two-force member. The force components on *CE* have equal and opposite reactions, which are shown on *BF* at *E* and on *AD* at *C*.

$[\Sigma M_B = 0]$	$3.92(5) - \frac{1}{2}E_x(3) = 0$	$E_x = 13.08$ kN
$[\Sigma F_y = 0]$	$B_y + 3.92 - 13.08/2 = 0$	$B_y = 2.62$ kN
$[\Sigma F_x = 0]$	$B_x + 3.92 - 13.08 = 0$	$B_x = 9.15$ kN





Member CE

$$C_x = E_x = 13.08 \text{ kN}$$

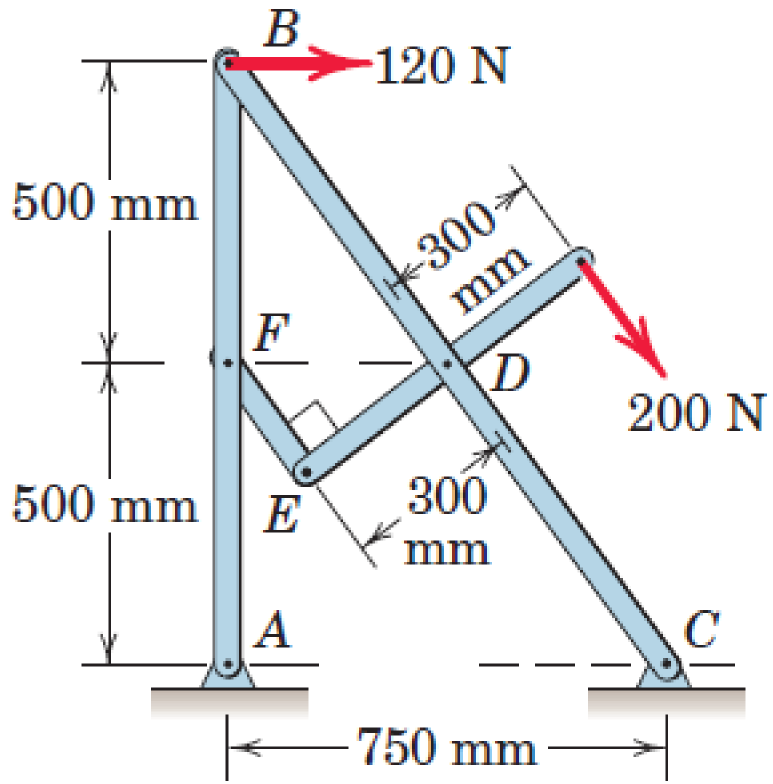
Member AD Checks

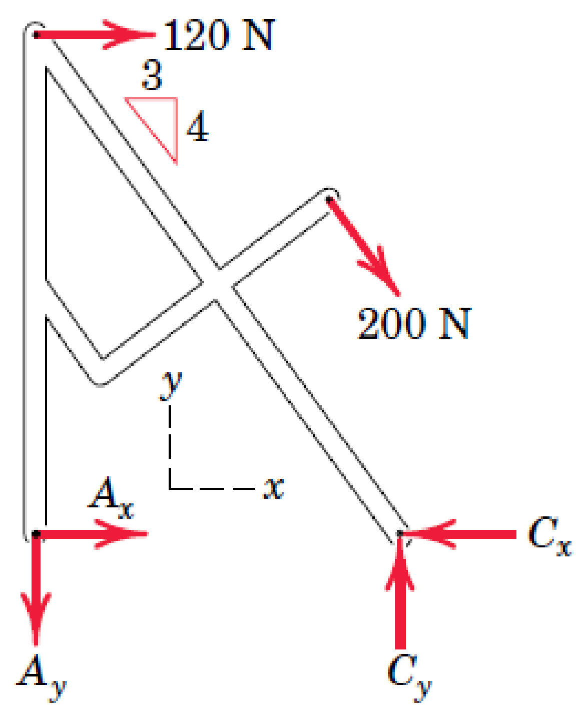
$$[\Sigma M_C = 0] \qquad 4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$$

$$[\Sigma F_x = 0] \qquad 4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$$

$$[\Sigma F_y = 0] \qquad -13.08/2 + 2.62 + 3.92 = 0$$

Neglect the weight of the frame and compute the forces acting on all of its members





$[\Sigma M_C = 0]$ $\quad 200(0.3) + 120(0.1) - 0.750A_y = 0 \quad$ $A_y = 240\text{ N}$

$[\Sigma F_y = 0]$ $\quad C_y - 200(4/5) - 240 = 0 \quad$ $C_y = 400\text{ N}$

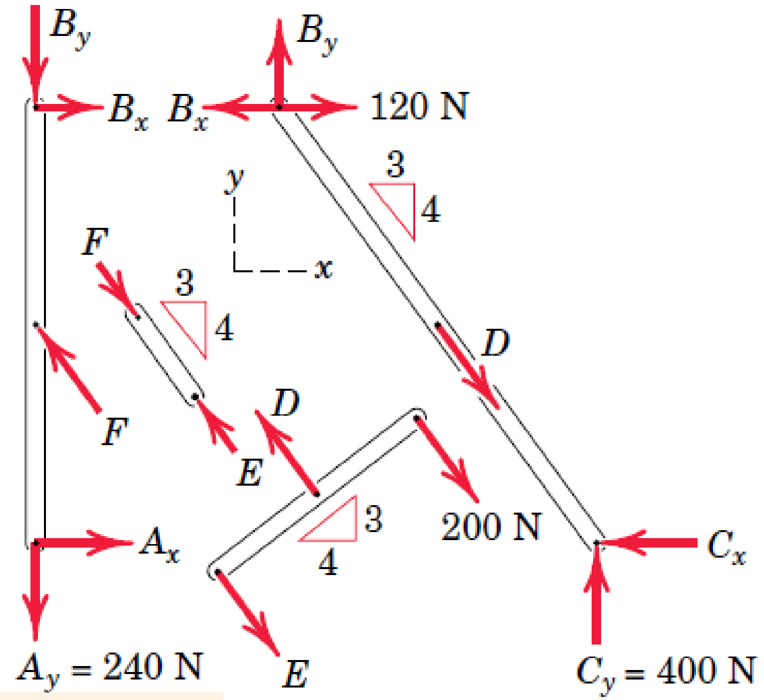
Next we dismember the frame and draw the free-body diagram of each part. Since *EF* is a two-force member, the direction of the force at *E* on *ED* and at *F* on *AB* is known. We assume that the 120-N force is applied to the pin as a part of member *BC*.

Member ED. The two unknowns are easily obtained by

$[\Sigma M_D = 0]$	$200(0.3) - 0.3E = 0$	$E = 200\text{ N}$
$[\Sigma F = 0]$	$D - 200 - 200 = 0$	$D = 400\text{ N}$

Member EF.

Clearly *F* is equal and opposite to *E* with the magnitude of 200 N.



Member AB. Since F is now known, we solve for B_x, A_x, and B_y from

$[\Sigma M_A = 0]$	$200(3/5)(0.5) - B_x(1.0) = 0$	$B_x = 60\text{ N}$
$[\Sigma F_x = 0]$	$A_x + 60 - 200(3/5) = 0$	$A_x = 60\text{ N}$
$[\Sigma F_y = 0]$	$200(4/5) - 240 - B_y = 0$	$B_y = -80\text{ N}$

The minus sign shows that we assigned B_y in the wrong direction.

Member BC. The results for B_x, B_y, and D are now transferred to BC, and the remaining unknown C_x is found from

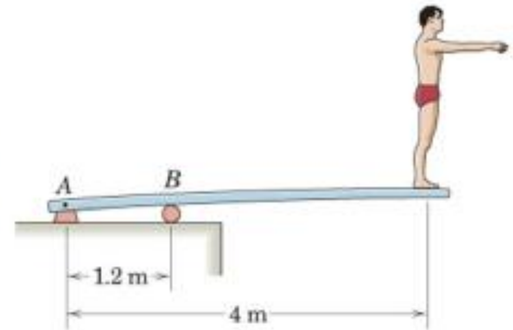
$$[\Sigma F_x = 0] \qquad 120 + 400(3/5) - 60 - C_x = 0 \qquad C_x = 300 \text{ N} \qquad \textit{Ans.}$$

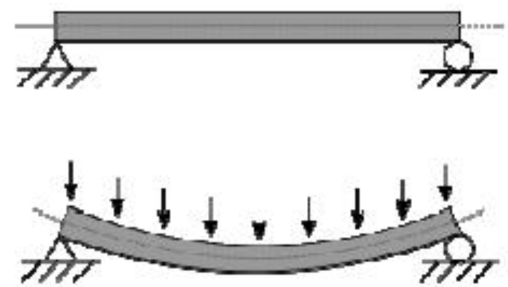
We may apply the remaining two equilibrium equations as a check. Thus,

$$[\Sigma F_y = 0] \qquad 400 + (-80) - 400(4/5) = 0$$

$$[\Sigma M_C = 0] \qquad (120 - 60)(1.0) + (-80)(0.75) = 0$$

Beams





Beams are structural members that offer resistance to bending due to applied load

Beams

• mostly long prismatic bars

– Prismatic: many sided, same section throughout

• non-prismatic beams are also useful

• cross-section of beams much smaller than beam length

• loads usually applied normal to the axis of the bar

• Determination of Load Carrying Capacity of Beams

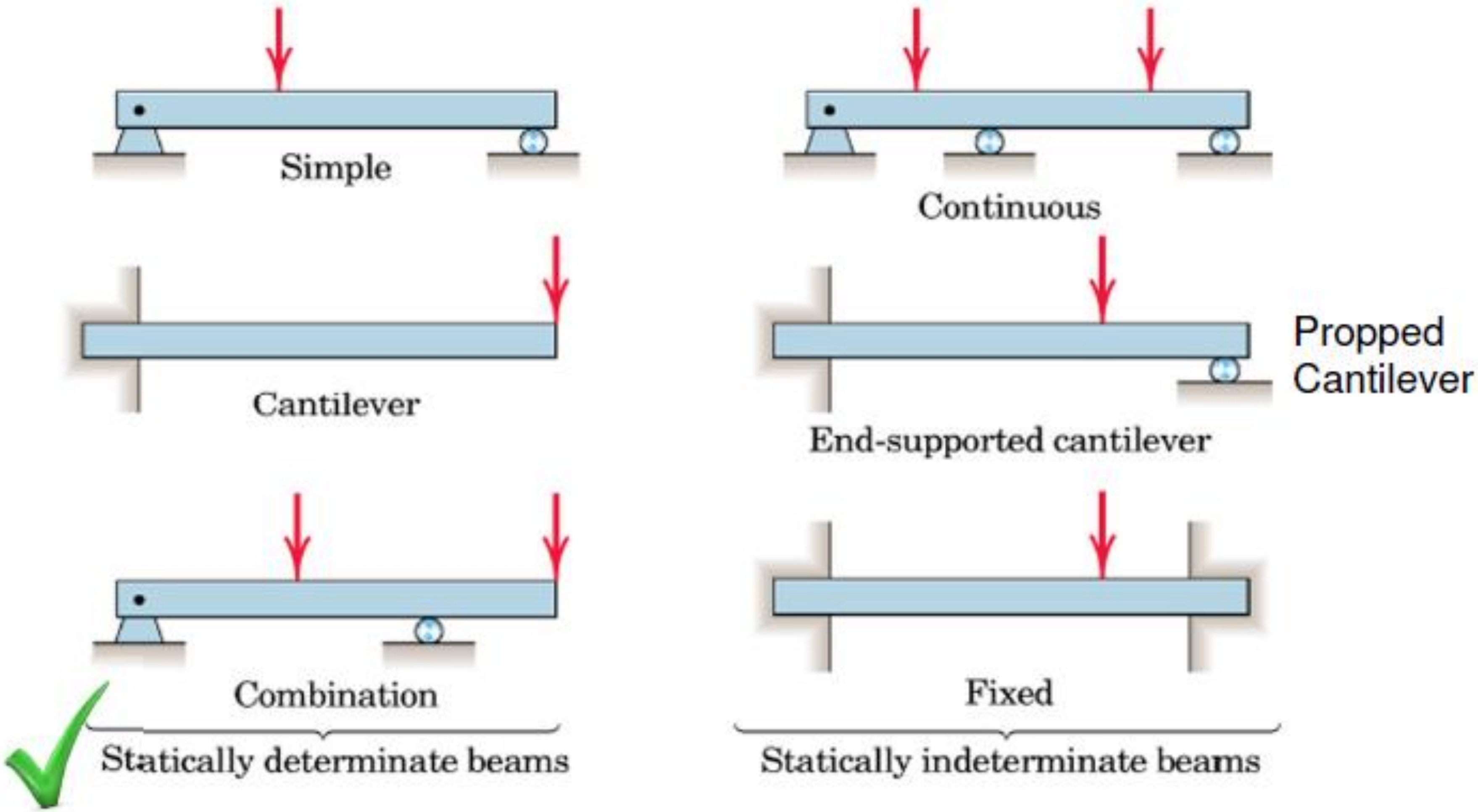
• Statically Determinate Beams

– Beams supported such that their external support reactions can be
calculated by the methods of statics

• Statically Indeterminate Beams

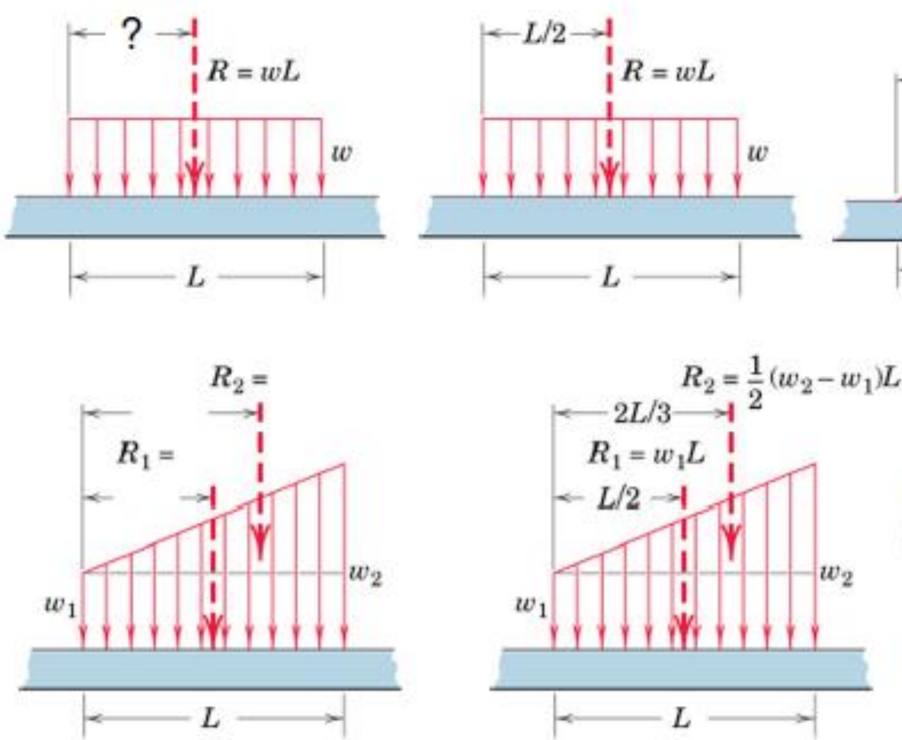
– Beams having more supports than needed to provide equilibrium

- Based on support conditions



Distributed Loads on beams

- Determination of Resultant Force (R) on beam is important

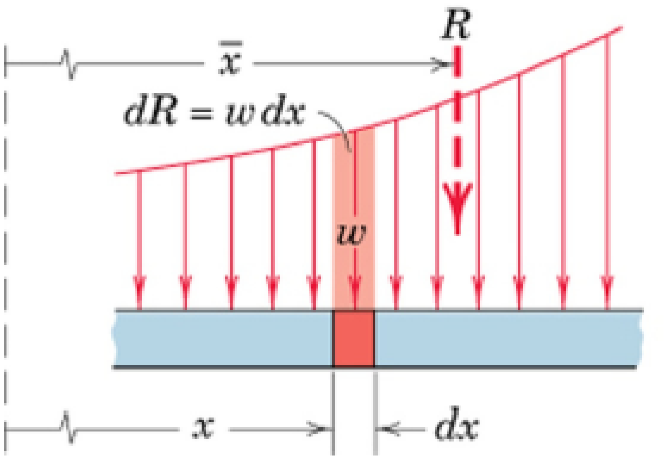


R = area formed by w and length L over which the load is distributed

R passes through centroid of this area

Distributed Loads on beams

- General Load Distribution



Differential increment of force is

$dR = w\,dx$

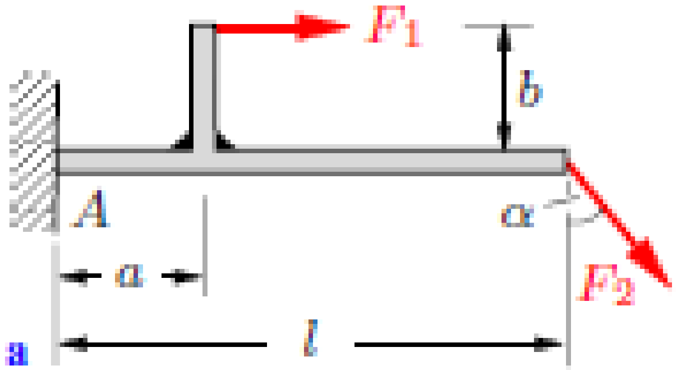
Total load *R* is sum of all the differential forces

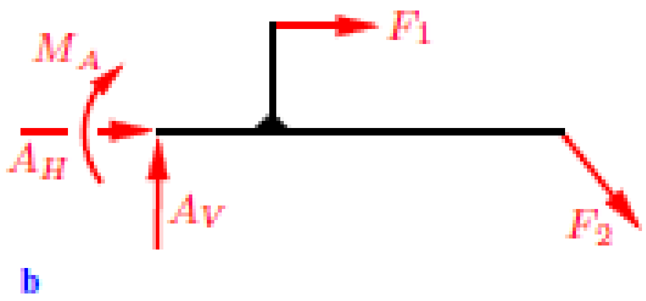
$R = \int w\,dx$ acting at centroid of the area under consideration

$$\bar{x} = \frac{\int xw\,dx}{R}$$

Once *R* is known reactions can be found out from Statics

The clamped beam shown in Fig. a is loaded by the two forces F1 and F2. Determine the reactions at the support



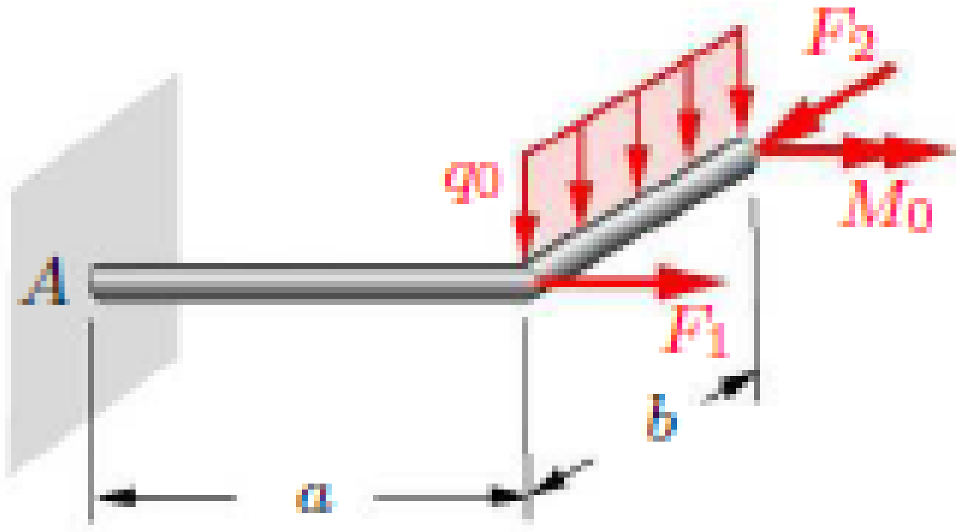


$$\uparrow: \quad A_V - F_2\cos\alpha = 0 \quad \rightarrow \quad \underline{\underline{A_V = F_2\cos\alpha}}\,,$$

$$\rightarrow: \quad A_H + F_1 + F_2\sin\alpha = 0 \quad \rightarrow \quad \underline{\underline{A_H = -(F_1 + F_2\sin\alpha)}}\,,$$

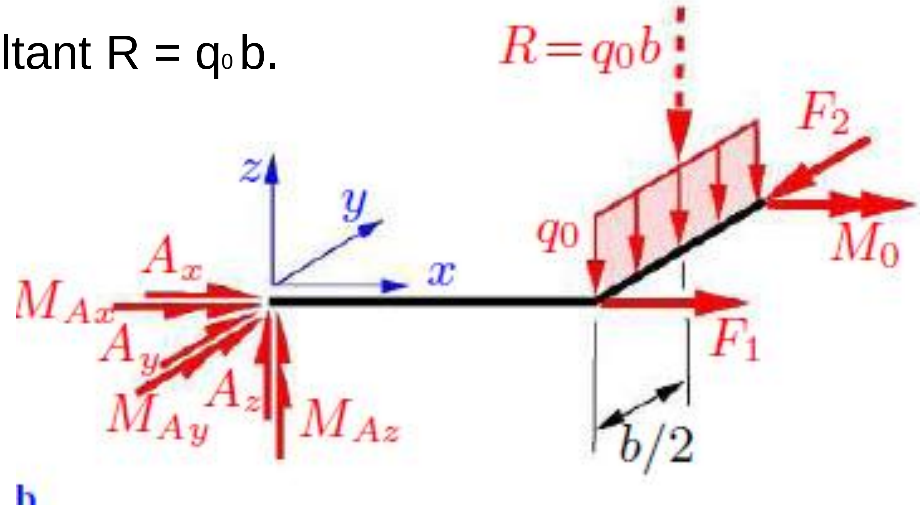
$$\overset{\curvearrowright}{A}: \quad M_A + b\,F_1 + l\,F_2\cos\alpha = 0 \quad \rightarrow \quad \underline{\underline{M_A = -(b\,F_1 + l\,F_2\cos\alpha)}}.$$

The rectangular lever which is clamped at A (Fig. 5.12a) is loaded by the line load q_0, two forces F_1, F_2 and the moment M_0. Determine the support reactions



a

The line load can be replaced by its resultant R = q_0 b.



$$\sum F_{ix} = 0: \;\; A_x + F_1 = 0 \quad \rightarrow \quad \underline{\underline{A_x = -F_1}}\,,$$

$$\sum F_{iy} = 0: \;\; A_y - F_2 = 0 \quad \rightarrow \quad \underline{\underline{A_y = F_2}}\,,$$

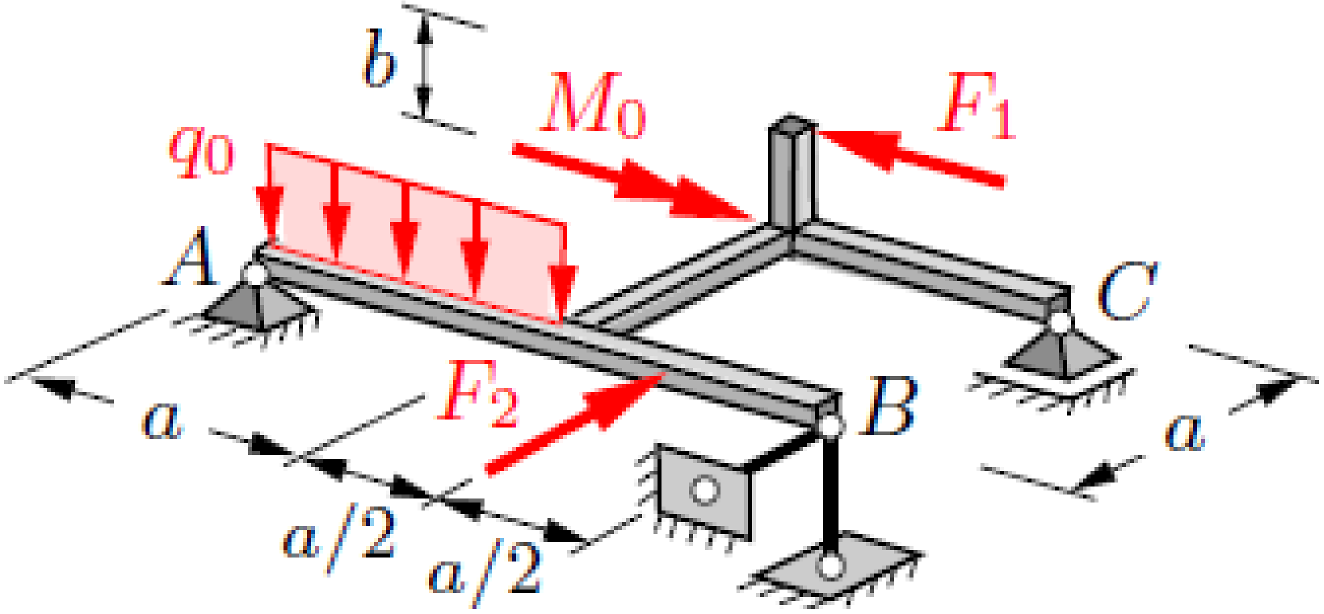
$$\sum F_{iz} = 0: \;\; A_z - q_0\, b = 0 \quad \rightarrow \quad \underline{\underline{A_z = q_0\, b}}\,,$$

$$\sum M_{ix}^{(A)} = 0: \;\; M_{Ax} + M_0 - \frac{b}{2}(q_0\, b) = 0 \rightarrow \quad \underline{\underline{M_{Ax} = \frac{q_0\, b^2}{2} - M_0}}\,,$$

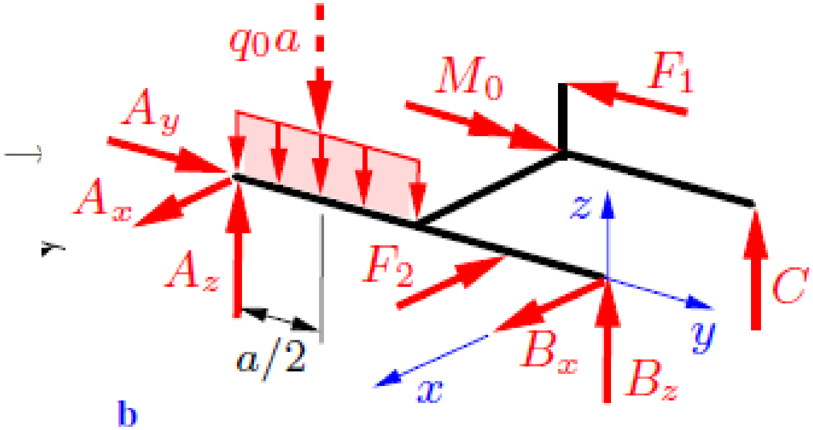
$$\sum M_{iy}^{(A)} = 0: \;\; M_{Ay} + a(q_0\, b) = 0 \quad \rightarrow \quad \underline{\underline{M_{Ay} = -q_0\, a\, b}}\,,$$

$$\sum M_{iz}^{(A)} = 0: \;\; M_{Az} - a\, F_2 = 0 \quad \rightarrow \quad \underline{\underline{M_{Az} = a\, F_2}}\,.$$

A spatial frame is supported at A, B and C. It is loaded by the line load q0, the forces F1, F2 and the moment M0. Determine the support reactions.



$$\sum F_{ix} = 0: \quad A_x + B_x - F_2 = 0\,,$$
$$\sum F_{iy} = 0: \quad A_y - F_1 = 0 \quad \rightarrow$$
$$\sum F_{iz} = 0: \quad A_z + B_z + C - q_0\,a = 0\,.$$



$$\sum M_{ix}^{(B)} = 0: -2\,a\,A_z + \frac{3}{2}a\,(q_0\,a) + b\,F_1 = 0 \rightarrow \quad \underline{\underline{A_z = \frac{3}{4}q_0\,a + \frac{b}{2\,a}F_1}}\,,$$

$$\sum M_{iy}^{(B)} = 0: \quad a\,C + M_0 = 0 \quad \rightarrow \quad \underline{\underline{C = -\frac{1}{a}M_0}}\,,$$

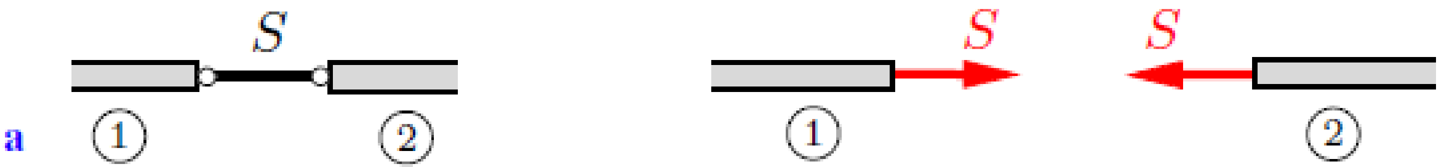
$$\sum M_{iz}^{(B)} = 0: \quad 2\,a\,A_x + a\,F_1 - \frac{a}{2}F_2 = 0 \quad \rightarrow \quad \underline{\underline{A_x = -\frac{1}{2}F_1 + \frac{1}{4}F_2}}\,.$$

With the results for A_x, A_z and C, (a) and (b) yield

$$\underline{\underline{B_x}} = -A_x + F_2 = \underline{\underline{\frac{1}{2}F_1 + \frac{3}{4}F_2}}\,,$$

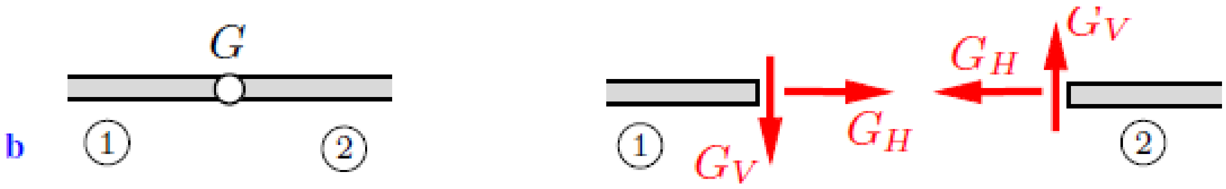
$$\underline{\underline{B_z}} = q_0\,a - A_z - C = \underline{\underline{\frac{1}{4}q_0\,a - \frac{b}{2\,a}F_1 + \frac{1}{a}M_0}}\,.$$

The strut transfers only one single force S in its axial direction



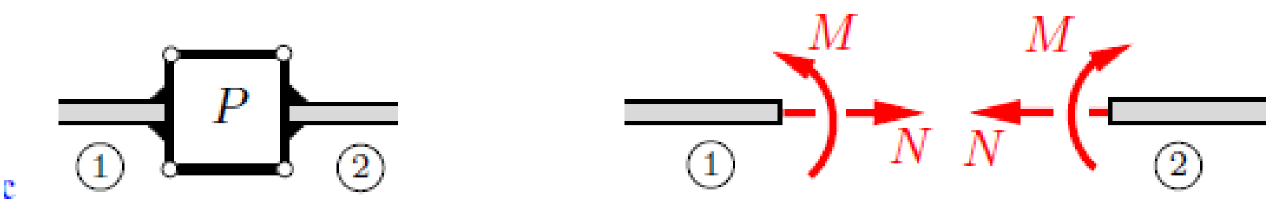
Joint reactions is v = 1

A hinge can transfer a force in an arbitrary direction, i.e. the force components GH and GV .



Joint reactions is v = 2

The parallel motion prevents a relative rotation and a relative displacement in the horizontal direction of the connected bodies



Joint reactions is v = 2

Three equilibrium conditions can be formulated for each body of the structure.

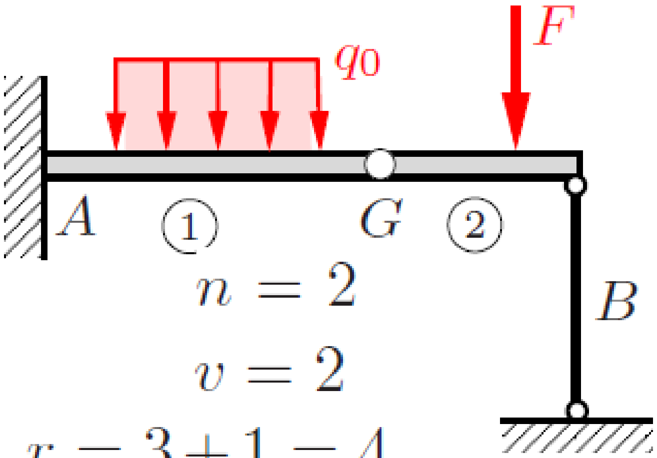
Therefore, there are in total 3 n equations if the structure consists of n bodies.

Let r be the number of support reactions and v be the number of transferred joint reactions.

The necessary condition for statically determinacy is that the number of equations and the number of unknowns are equal:

$$r + v = 3\,n$$

if the structure is rigid, this condition is sufficient for statical determinacy

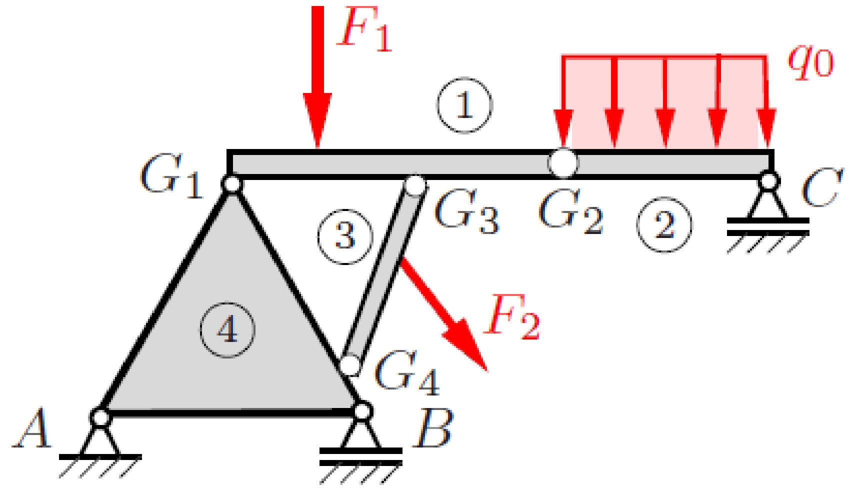


$$n = 2$$

$$v = 2$$

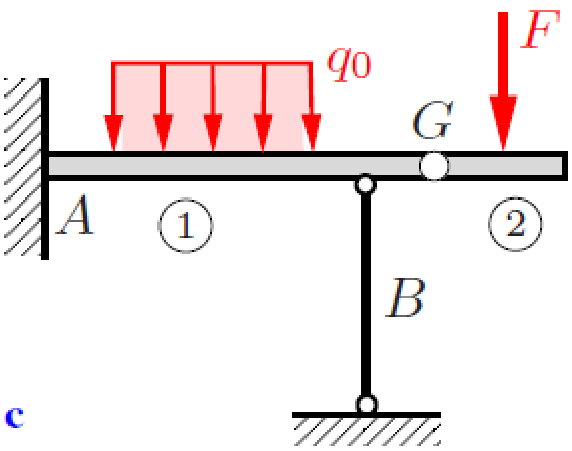
$$r = 3 + 1 = 4$$

$$4 + 2 = 3 \cdot 2$$

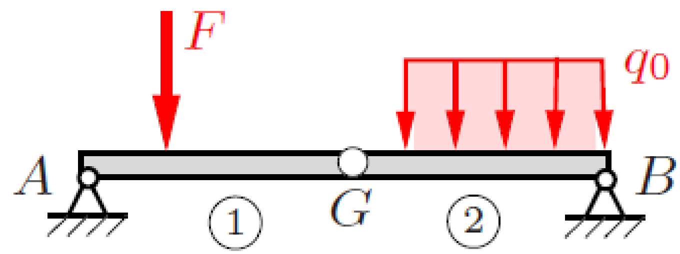


b

three beams ① - ③ and the disk ④:



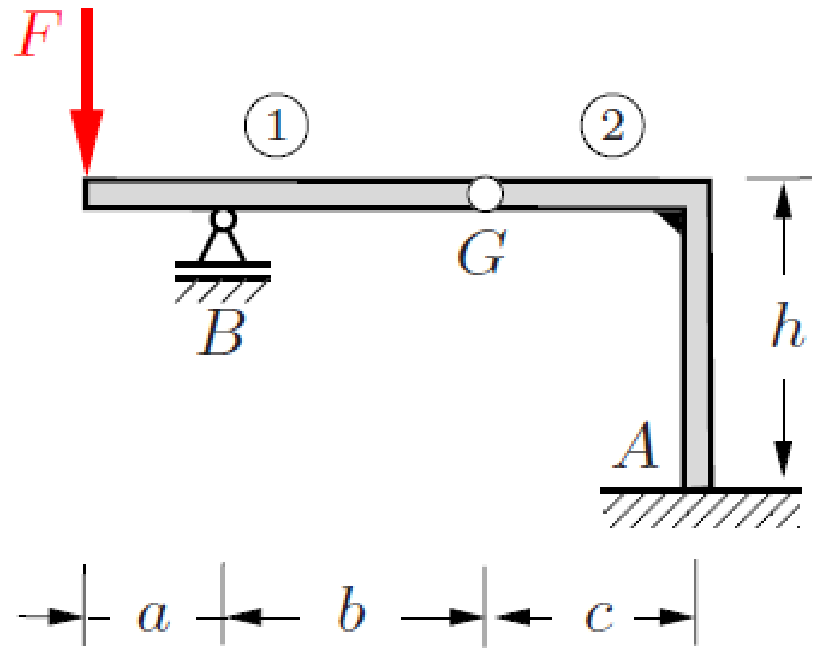
c



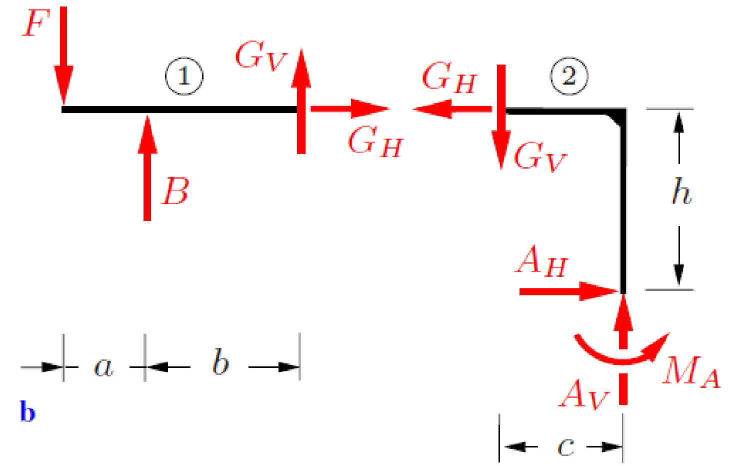
d

Fig. 5.15

The structure shown in consists of a beam 1 and the angled part 2 , which are connected by the hinge G. The angled part is clamped at A and the beam is supported at B. The system is loaded by the force F.



The equilibrium conditions for body (1) yield



$$\rightarrow: \quad \underline{\underline{G_H = 0}},$$

$$\overset{\curvearrowleft}{G}: \quad (a+b)F - b\,B = 0 \quad \rightarrow \quad \underline{\underline{B = \frac{a+b}{b}F}},$$

$$\overset{\curvearrowleft}{B}: \quad a\,F + b\,G_V = 0 \quad \rightarrow \quad \underline{\underline{G_V = -\frac{a}{b}F}}.$$

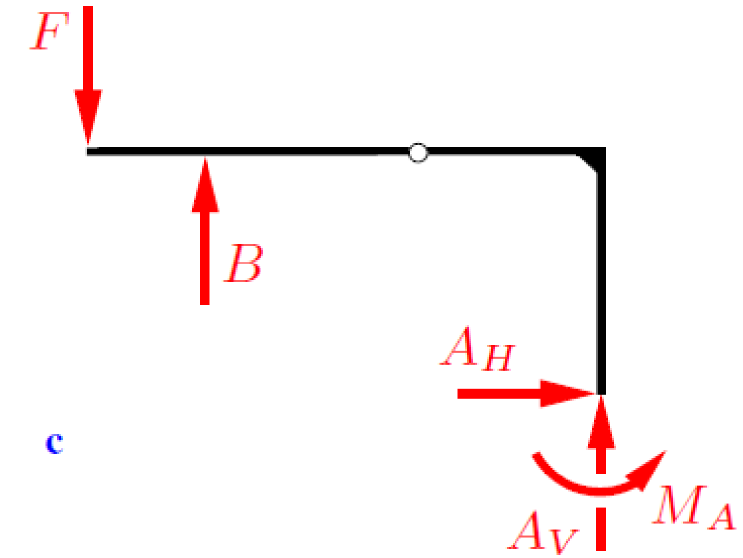
From the equilibrium conditions for body 2 in conjunction with the results for GH and GV , we obtain

$$\uparrow: \quad -G_V + A_V = 0 \quad \rightarrow \quad \underline{\underline{A_V}} = G_V = \underline{\underline{-\frac{a}{b}F}},$$

$$\rightarrow: \quad -G_H + A_H = 0 \quad \rightarrow \quad \underline{\underline{A_H}} = G_H = \underline{\underline{0}},$$

$$\overset{\curvearrowleft}{A}: \quad M_A + h\,G_H + c\,G_V = 0 \quad \rightarrow \quad \underline{\underline{M_A}} = -h\,G_H - c\,G_V = \underline{\underline{\frac{a\,c}{b}F}}.$$

As a check, the equilibrium conditions are applied to the complete system, where the hinge G is assumed to be frozen:



$$\uparrow: \quad -F + B + A_V = 0 \quad \rightarrow \quad -F + \frac{a+b}{b}F - \frac{a}{b}F = 0\,,$$

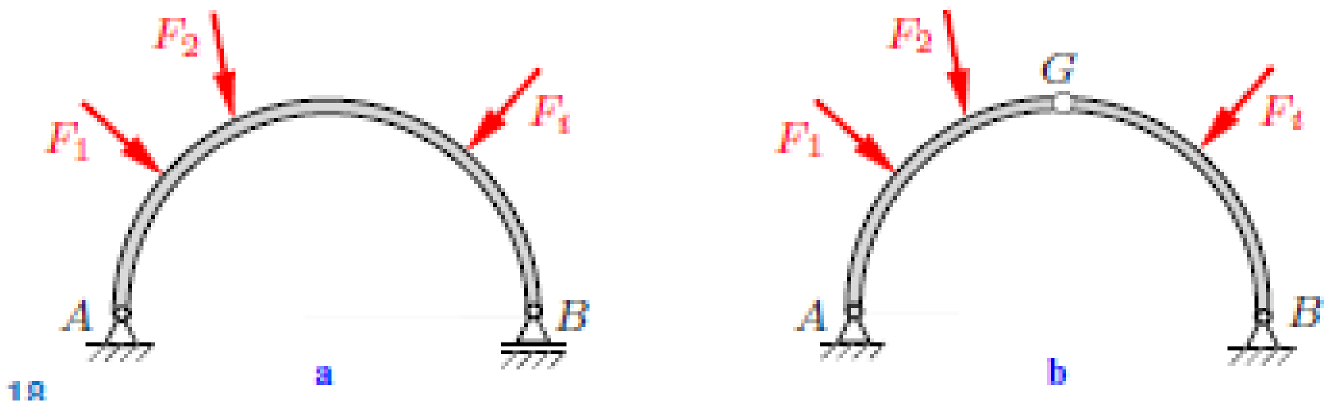
$$\rightarrow: \quad A_H = 0\,,$$

$$\overset{\curvearrowleft}{B}: \quad a\,F + M_A + h\,A_H + (b+c)A_V = 0$$

$$\rightarrow \quad a\,F + \frac{a\,c}{b}F - (b+c)\frac{a}{b}F = 0\,.$$

Three-Hinged Arch

The arch shown in Fig. a is statically determinate because it is immobile and in total three support reactions exist at A and B. In a real construction, the arch AB is not rigid but deforms under applied loads. If B is a roller support, this may lead to a large deformation that cannot be tolerated.



Such a displacement is prevented if A and B are designed as hinged supports. As a consequence, the statical determinacy of the structure gets lost. However, statical determinacy can be reestablished if an additional hinge G is introduced at an arbitrary location (Fig. b). Such a structure is called a three-hinged arch. It consists of n = 2 bodies connected by the hinge G, which transfers v = 2 joint reactions. Since the supports A and B transfer r = 2 + 2 = 4 support reactions, the condition for statical determinacy s fulfilled: 4 + 2 = 3 · 2. Therefore, taking the immobility of the structure into account, the three-hinged arch is statically determinate.

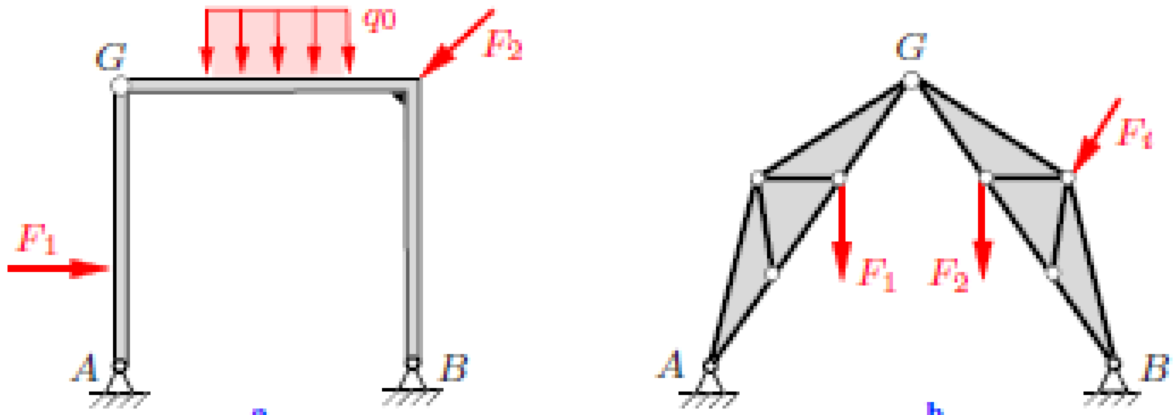
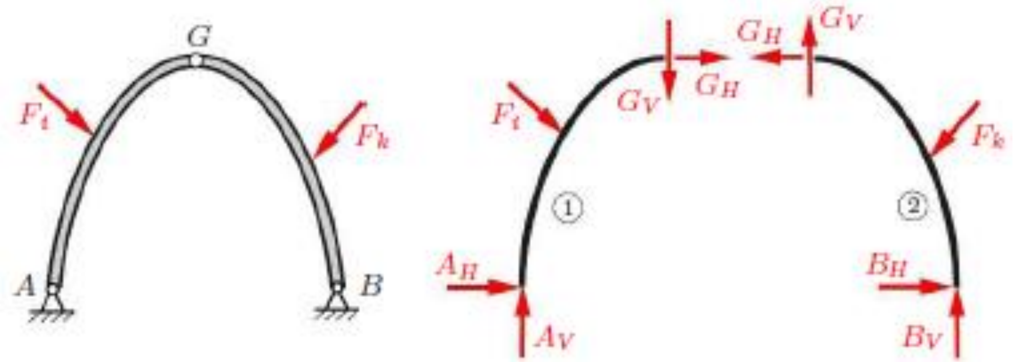


Fig. 5.10

The two bodies of a three-hinged arch need not necessarily be arch shaped. An arbitrary structure consisting of two bodies connected by a hinge and supported by two hinged supports (in total three hinges) is also called a three-hinged arch from now on.

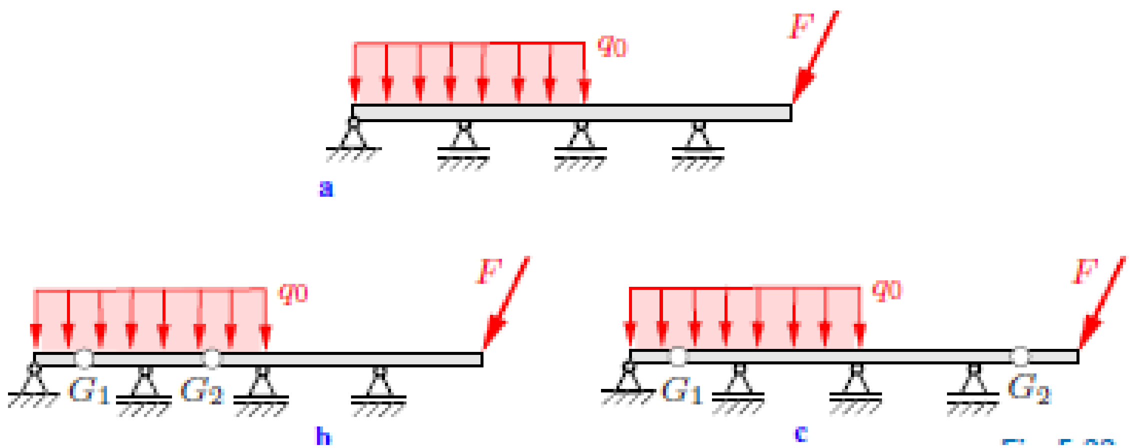
In order to calculate the forces at the supports and the hinge, we isolate the two bodies 1 and 2 and apply the equilibrium conditions to each body. From the 2 ·3 = 6 equations,



Hinged Beam

Structures with a wide span width are necessarily often supported by more than two supports.

Structures with a wide span width are necessarily often supported by more than two supports. As an example, consider the beam shown in Fig. a. Since r = 5, the system is statically indeterminate with two degrees of statical. Therefore, the calculation of the support reactions solely from the equilibrium conditions is impossible.



A statically determinate multi-body structure can be obtained if the continuous beam is divided into several parts by introducing an appropriate number of hinges. Such a structure is called a hinged beam.

If the number of these hinges is g, the continuous beam is divided into $n = g + 1$ parts. Since each hinge transfers two force components, the number of joint reactions is $v = 2g$. Therefore, the necessary condition for statical determinacy takes the form

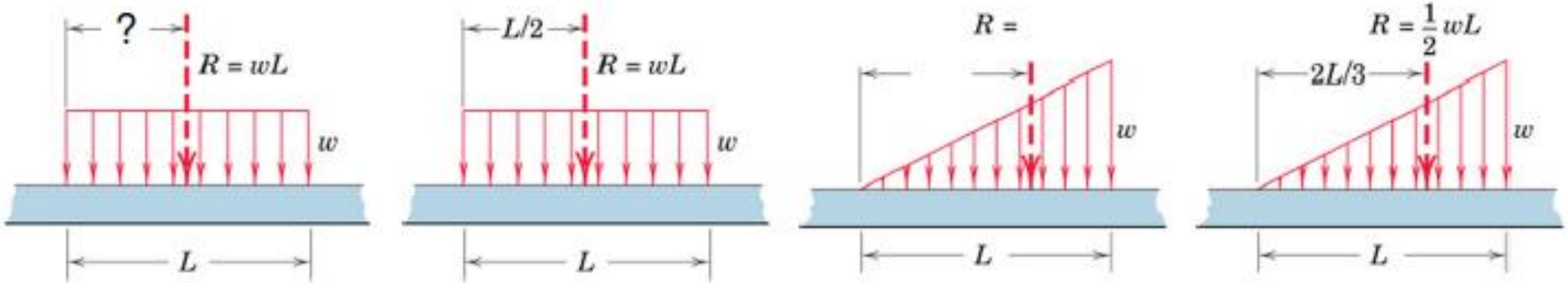
$r + v = 3n \rightarrow r+2g = 3(g+1)$

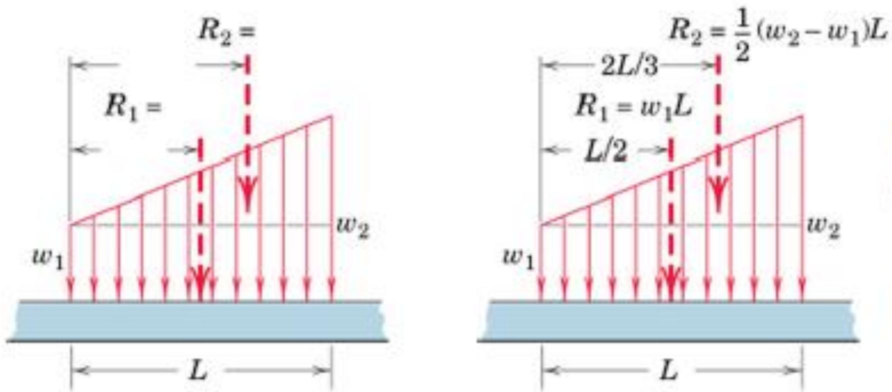
Thus, the necessary number of hinges is given by

$g = r - 3$.

Distributed Loads on beams

- Determination of Resultant Force (R) on beam is important



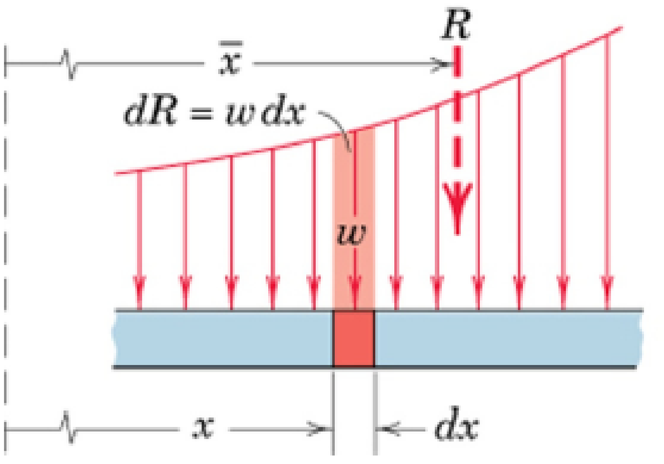


R = area formed by w and length L over which the load is distributed

R passes through centroid of this area

Distributed Loads on beams

- General Load Distribution



Differential increment of force is

$dR = w\ dx$

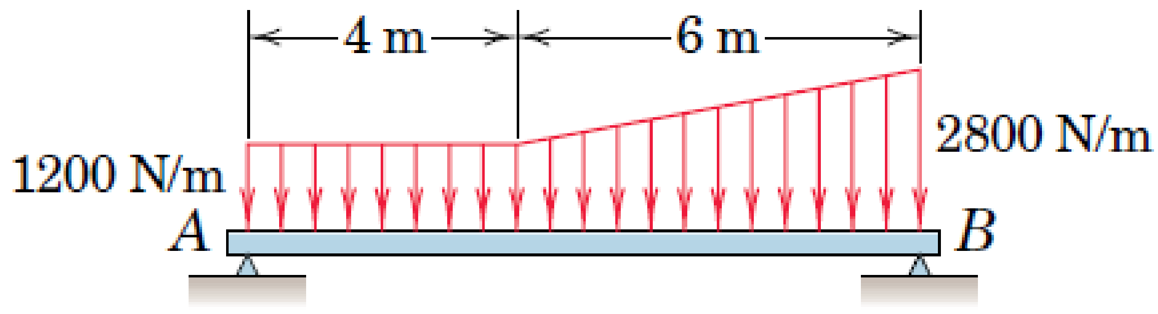
Total load *R* is sum of all the differential forces

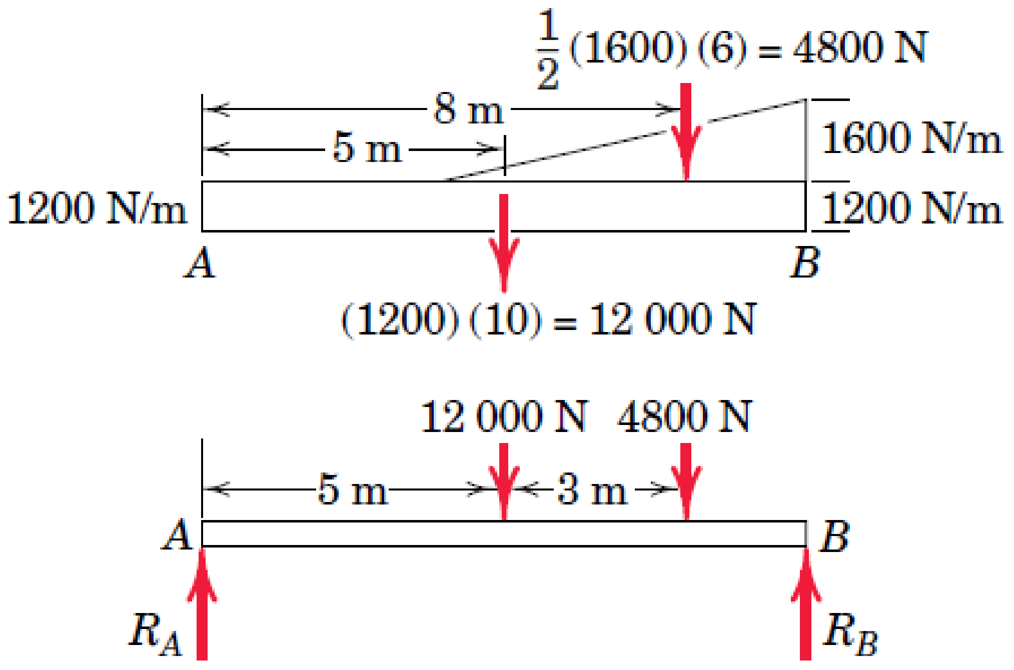
$R = \int w\,dx$ acting at centroid of the area under consideration

$$\bar{x} = \frac{\int xw\,dx}{R}$$

Once *R* is known reactions can be found out from Statics

Determine the equivalent concentrated load(s) and external reactions for the simply supported beam which is subjected to the distributed load shown.





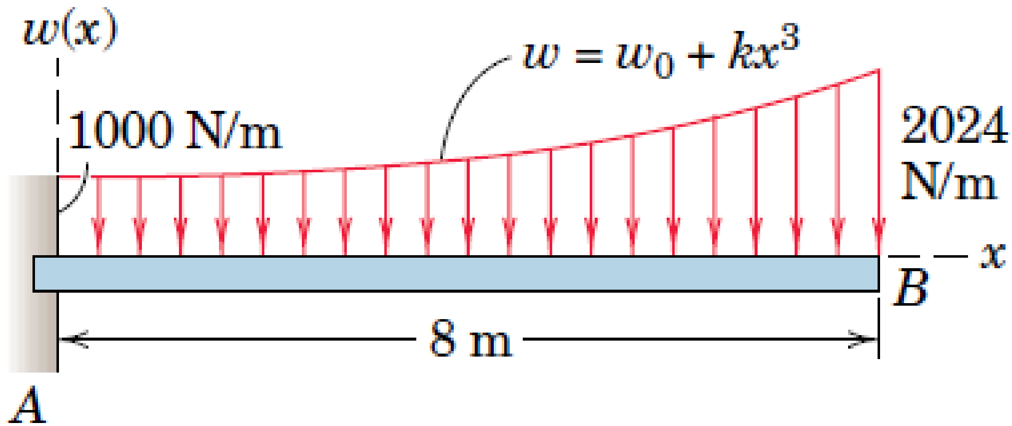
$[\Sigma M_A = 0]$ $$12\,000(5) + 4800(8) - R_B(10) = 0$$

$$R_B = 9840 \text{ N or } 9.84 \text{ kN}$$

$[\Sigma M_B = 0]$ $$R_A(10) - 12\,000(5) - 4800(2) = 0$$

$$R_A = 6960 \text{ N or } 6.96 \text{ kN}$$

Determine the reaction at the support A of the loaded cantilever beam.



Solution. The constants in the load distribution are found to be $w_0 = 1000$ N/m and $k = 2$ N/m^4. The load R is then

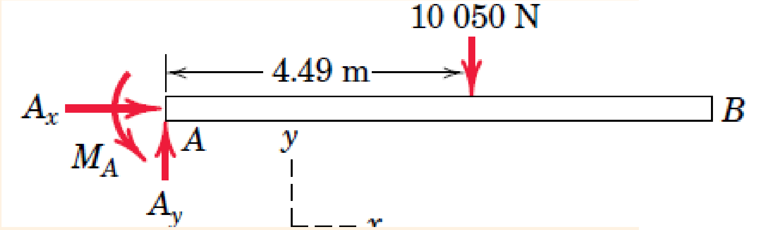
$$R = \int w\,dx = \int_0^8 (1000 + 2x^3)\,dx = \left(1000x + \frac{x^4}{2}\right)\Bigg|_0^8 = 10\,050 \text{ N}$$

The x-coordinate of the centroid of the area is found by

$$\bar{x} = \frac{\int xw\,dx}{R} = \frac{1}{10\,050}\int_0^8 x(1000 + 2x^3)\,dx$$

$$= \frac{1}{10\,050}\left(500x^2 + \tfrac{2}{5}x^5\right)\Big|_0^8 = 4.49 \text{ m}$$

From the free-body diagram of the beam, we have



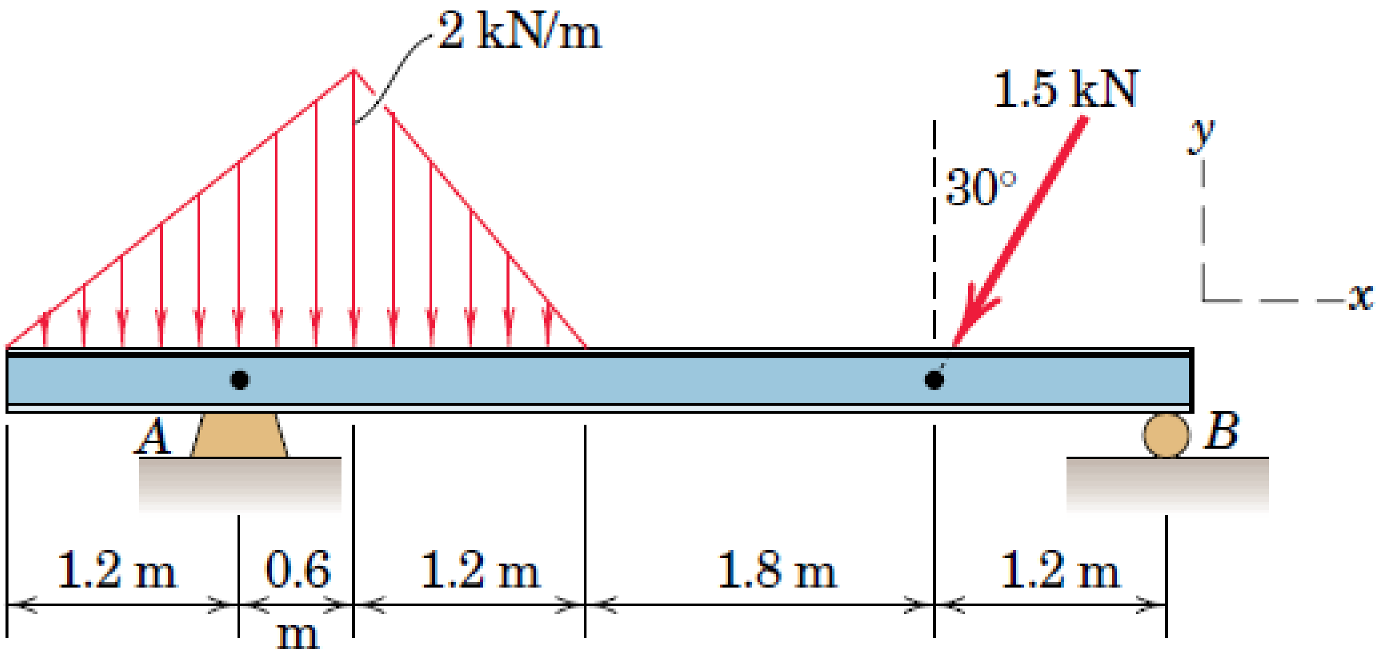
$[\Sigma M_A = 0]$ $\qquad M_A - (10\,050)(4.49) = 0$

$$M_A = 45\,100 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

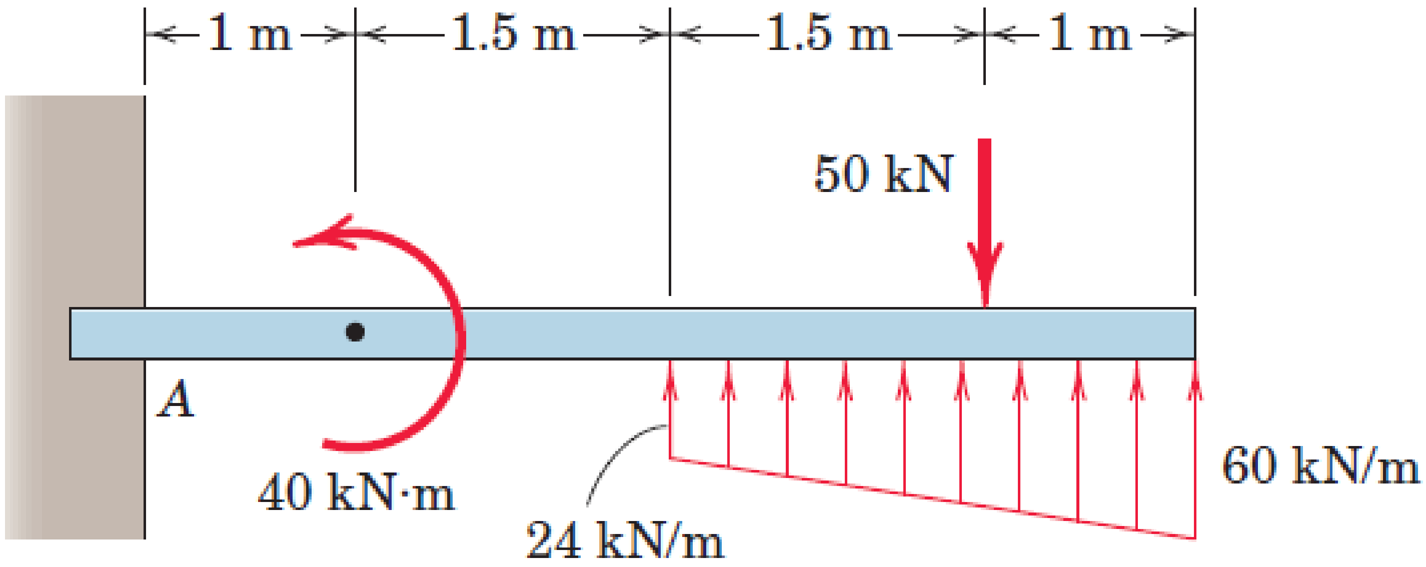
$[\Sigma F_y = 0]$ $\qquad A_y = 10\,050 \text{ N} \qquad$ *Ans.*

Note that $A_x = 0$ by inspection.

Determine the reactions at *A* and *B* for the beam subjected to a combination of distributed and point loads.



Determine the force and moment reactions at *A* for the beam which is subjected to the load combination shown.



Beam internal stresses.

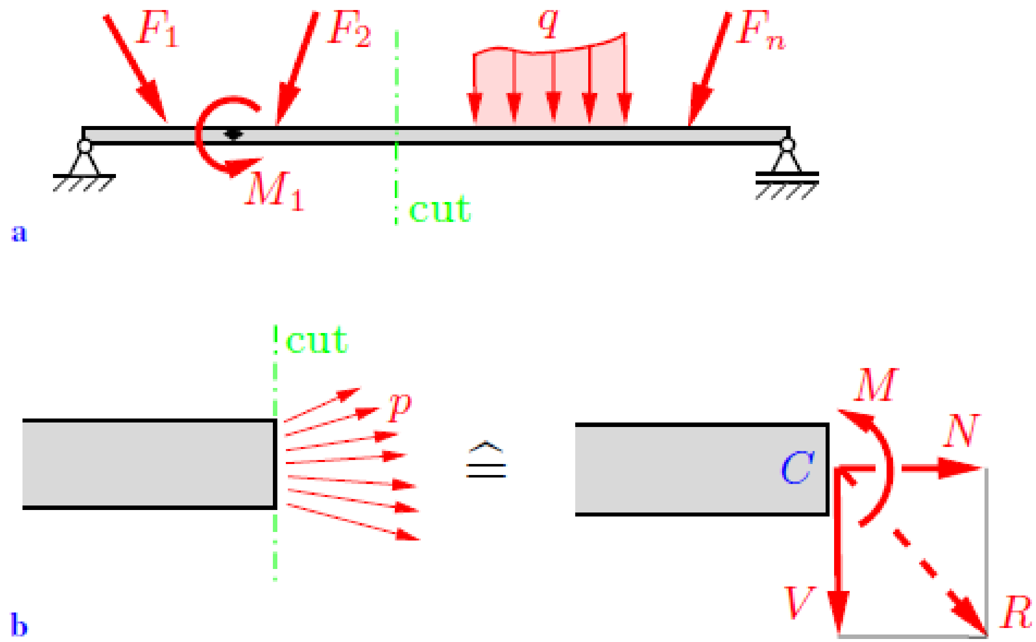
Beams are among the most important elements in structural engineering. In these slides, it is explained how the internal forces in a beam can be made accessible to calculation

Internal forces are important

To determine the load-bearing capacity of a beam

To compute the area of the cross-section required to sustain a given load

To compute the deformation

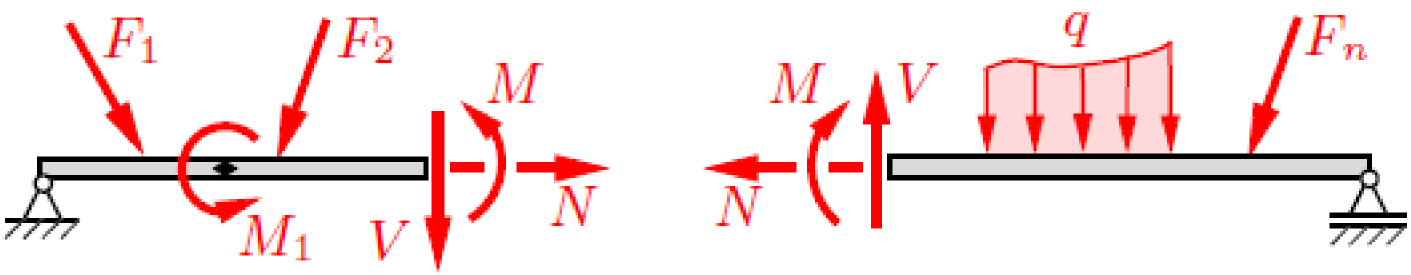


The quantities N, V and M are called the stress resultants

In particular,

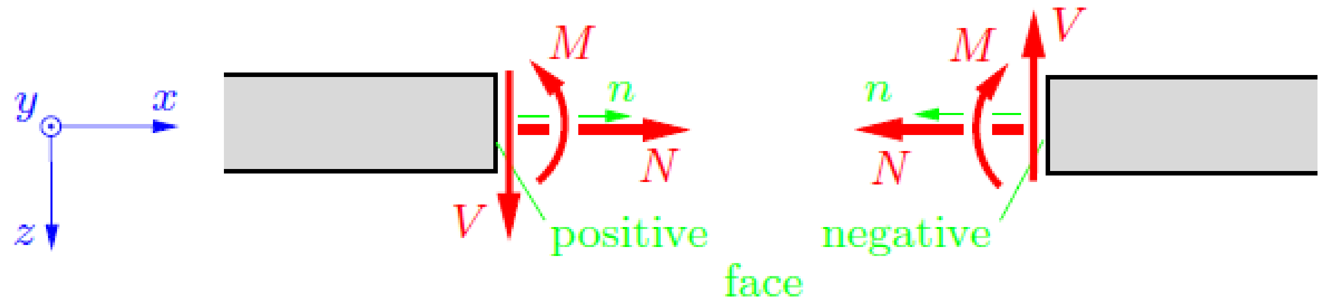
N is called the normal force,
V is the shear force and
M is the bending moment.

In order to determine the stress resultants, the beam may be divided by a cut into two segments (method of sections). A free body diagram of each part of the beam will include all of the forces acting on the respective part, i.e., the applied loads (forces and couples), the support reactions and the stress resultants acting at the cut sections.

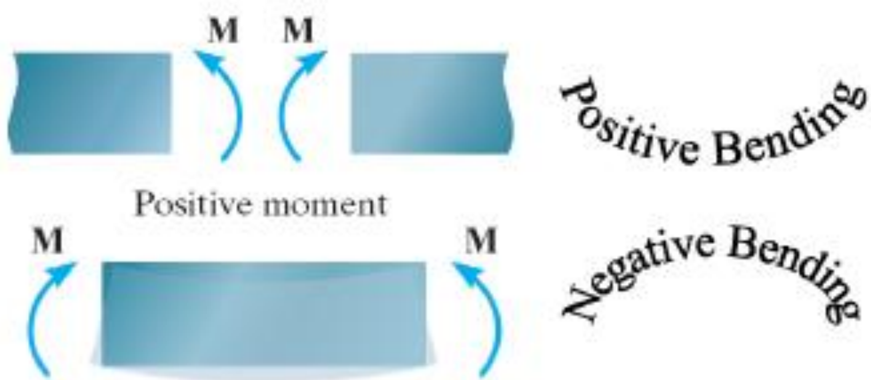


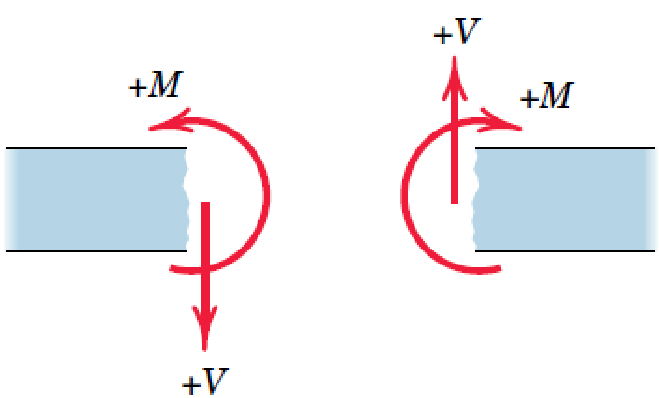
Sign Convention

Positive stress resultants at a *positive (negative)* face point in the *positive (negative)* directions of the coordinates.



Here, the bending moment M has to be interpreted as a moment vector pointing in the direction of the y-axis (positive direction according to the right-hand rule).





Shear Force and bending moment diagram

$$\rightarrow: \quad A_H - F_H = 0 \quad \rightarrow \quad A_H = F_H\,,$$

$$\overset{\curvearrowleft}{A}: \quad l\,B - a\,F_V = 0 \quad \rightarrow \quad B = \frac{a}{l}\,F_V\,,$$

$$\overset{\curvearrowleft}{B}: \quad -l\,A_V + b\,F_V = 0 \quad \rightarrow \quad A_V = \frac{b}{l}\,F_V\,.$$

$$\rightarrow: \quad A_H - F_H = 0 \quad \rightarrow \quad A_H = F_H\,,$$

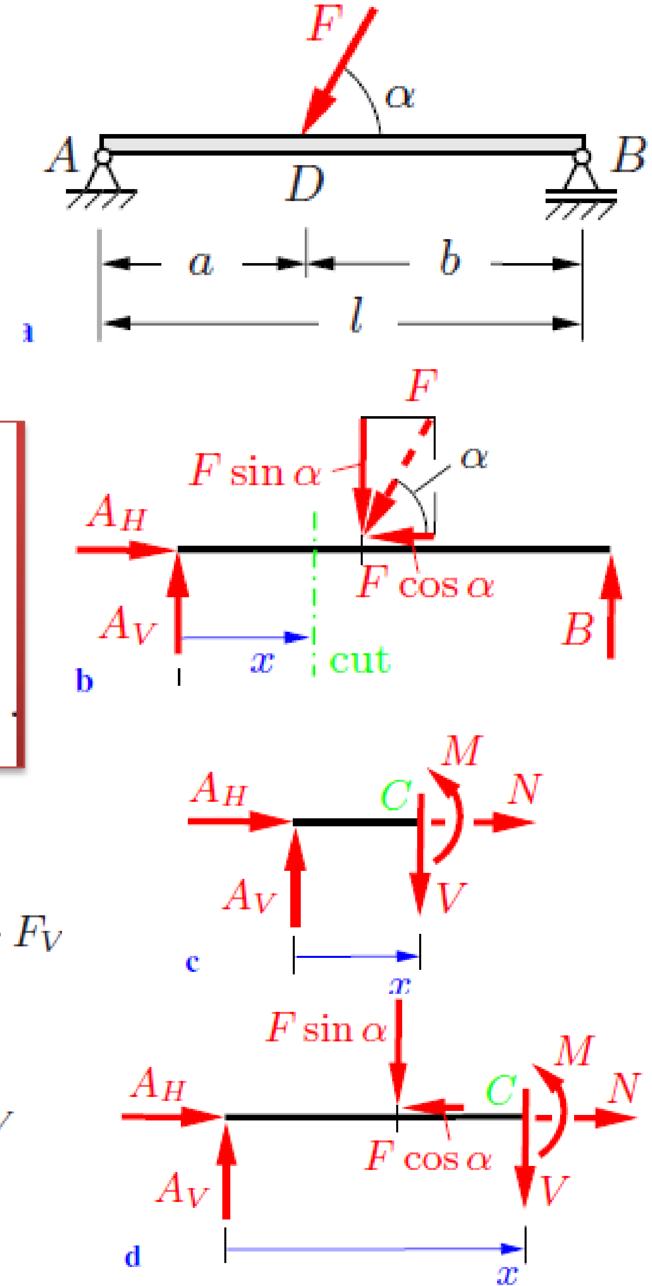
$$\overset{\curvearrowleft}{A}: \quad l\,B - a\,F_V = 0 \quad \rightarrow \quad B = \frac{a}{l}\,F_V\,,$$

$$\overset{\curvearrowleft}{B}: \quad -l\,A_V + b\,F_V = 0 \quad \rightarrow \quad A_V = \frac{b}{l}\,F_V\,.$$

$$\rightarrow: \quad A_H - F_H + N = 0 \rightarrow N = F_H - A_H = 0\,,$$

$$\uparrow: \quad A_V - F_V - V = 0 \rightarrow V = A_V - F_V = \frac{b-l}{l}\,F_V = -\frac{a}{l}\,F_V = -B\,,$$

$$\overset{\curvearrowright}{C}: \quad x\,A_V - (x-a)F_V - M = 0 \rightarrow M = x\,A_V - (x-a)F_V = \left(1 - \frac{x}{l}\right)a\,F_V\,.$$

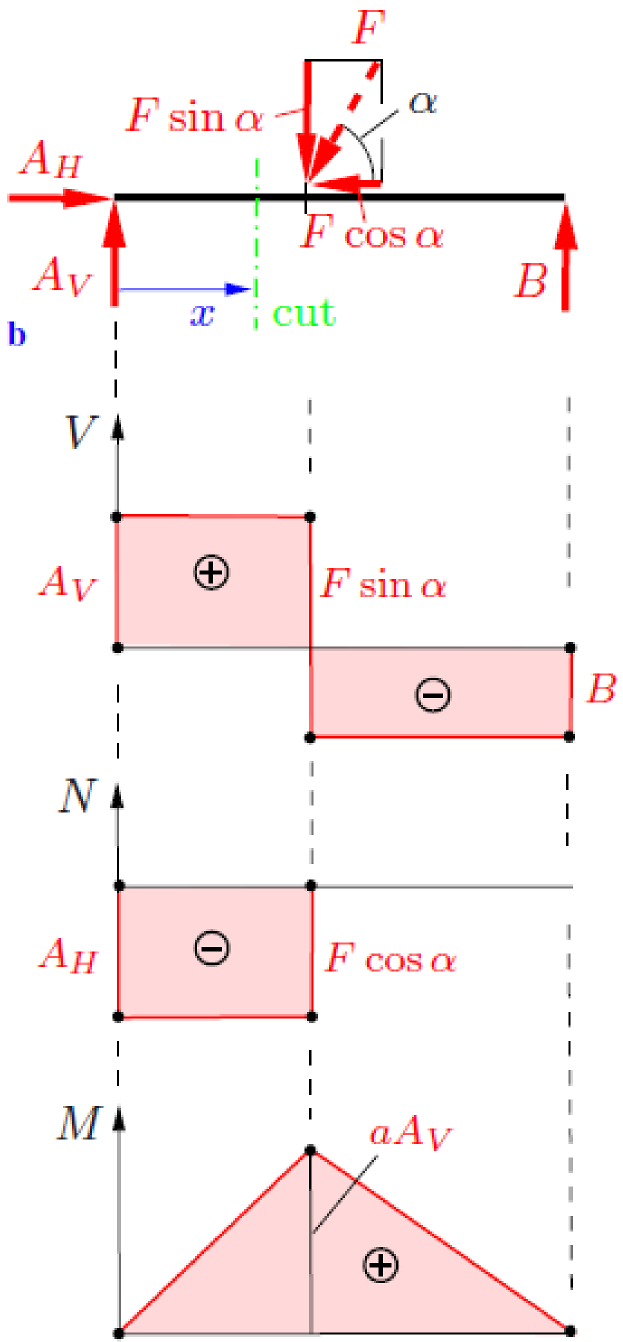


These graphs are called the shear-force, normal-force and bending-moment diagrams, respectively.

The shear-force and normal-force diagrams display a jump discontinuity at point D (point of application of the external force F).

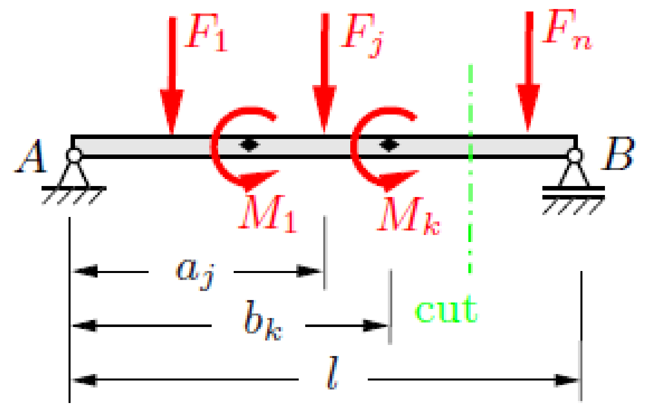
The bending-moment diagram shows a slope discontinuity (kink) at D. The maximum bending moment is located at D.

It is usually the most important value in the design of a beam



Relationship between stress and loading

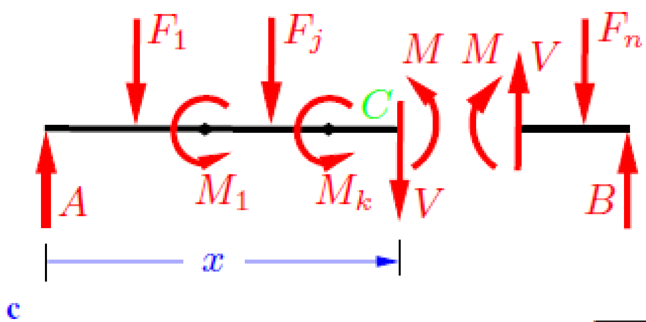
(A) Concentrated Load and moments

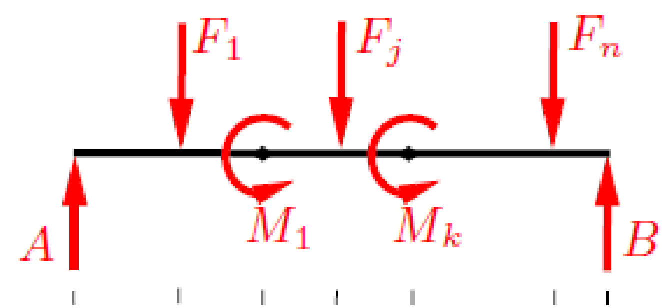


$$\overset{\curvearrowleft}{A}:\ l\,B - \sum a_i\,F_i + \sum M_i = 0 \ \rightarrow\ B = \frac{1}{l}\left[\sum a_i\,F_i - \sum M_i\right]$$

$$\overset{\curvearrowleft}{B}:\ -l\,A + \sum (l - a_i)F_i + \sum M_i = 0$$

$$\rightarrow\ A = \frac{1}{l}\left[\sum (l - a_i)F_i + \sum M_i\right]$$





$$\uparrow:\quad A - \sum F_i - V = 0\,,$$

$$\overset{\curvearrowleft}{C}:\quad -x\,A + \sum (x - a_i)F_i + \sum M_i + M = 0\,,$$

$$V = A - \sum F_i\,,$$

$$M = x\,A - \sum (x - a_i)F_i - \sum M_i$$

Relationship between stress and loading

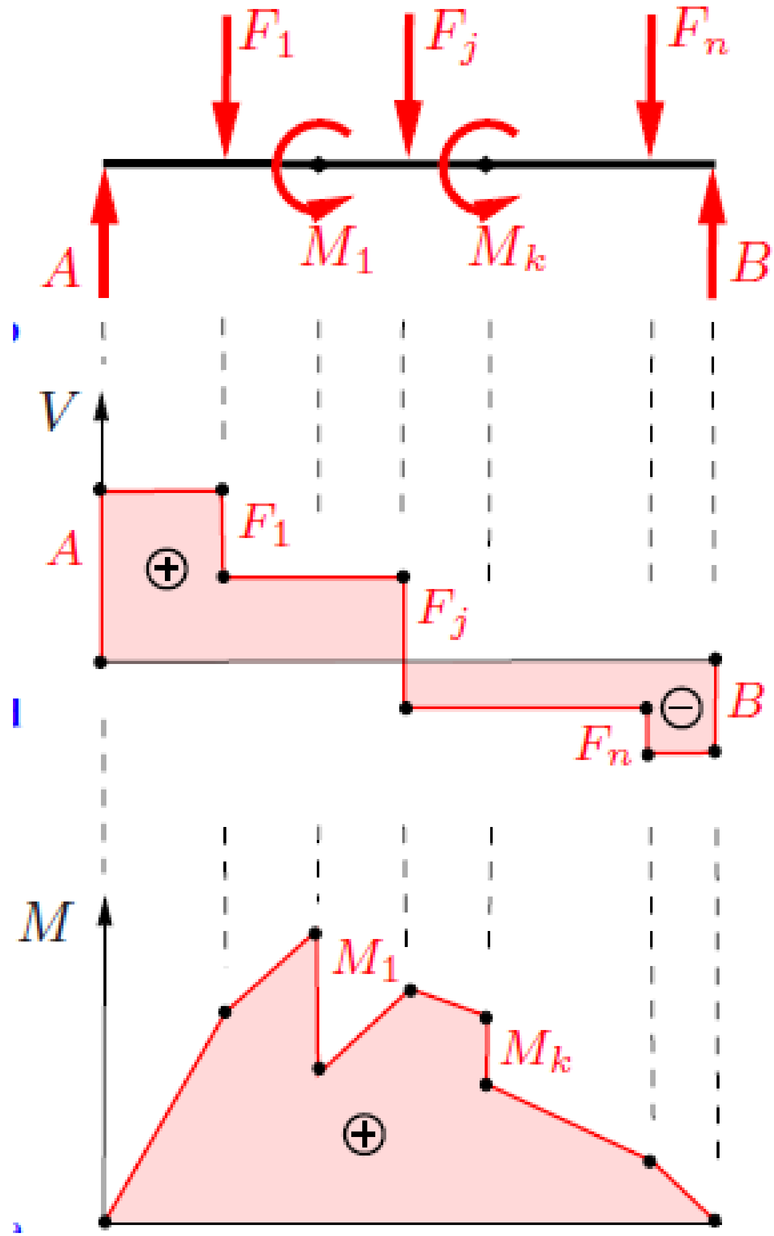
$$V = A - \sum F_i \,,$$

$$M = x\,A - \sum (x - a_i)F_i - \sum M_i$$

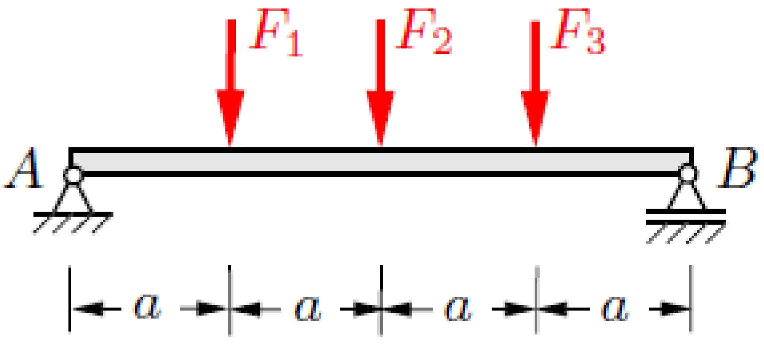
A relationship exists between the bending moment and the shear force

$$\frac{\mathrm{d}M}{\mathrm{d}x} = A - \sum F_i = V$$

The slopes of the straight lines in the bending-moment diagram are thus given by the corresponding values of the shear force



The simply supported beam in Fig. is subjected to the three forces F1 = F, F2 = 2F and F3 = −F



$$\overset{\curvearrowleft}{A}: \quad -a\,F - 2\,a\,2\,F + 3\,a\,F + 4\,a\,B = 0 \quad \rightarrow \quad B = \frac{1}{2}\,F\,,$$

$$\overset{\curvearrowleft}{B}: \quad -4\,a\,A + 3\,a\,F + 2\,a\,2\,F - a\,F = 0 \quad \rightarrow \quad A = \frac{3}{2}\,F\,.$$

$$V = A = 3F/2 \qquad \text{for} \quad 0 < x < a\,,$$

$$V = A - F = F/2 \qquad \text{for} \quad a < x < 2\,a\,,$$

$$V = A - F - 2\,F = -\,3F/2 \qquad \text{for} \quad 2\,a < x < 3\,a\,,$$

$$V = A - F - 2\,F + F = -\,F/2 \qquad \text{for} \quad 3\,a < x < 4\,a\,.$$

$$M = xA = \tfrac{3}{2}\,xF \qquad \text{for} \quad 0 \le x \le a\,,$$

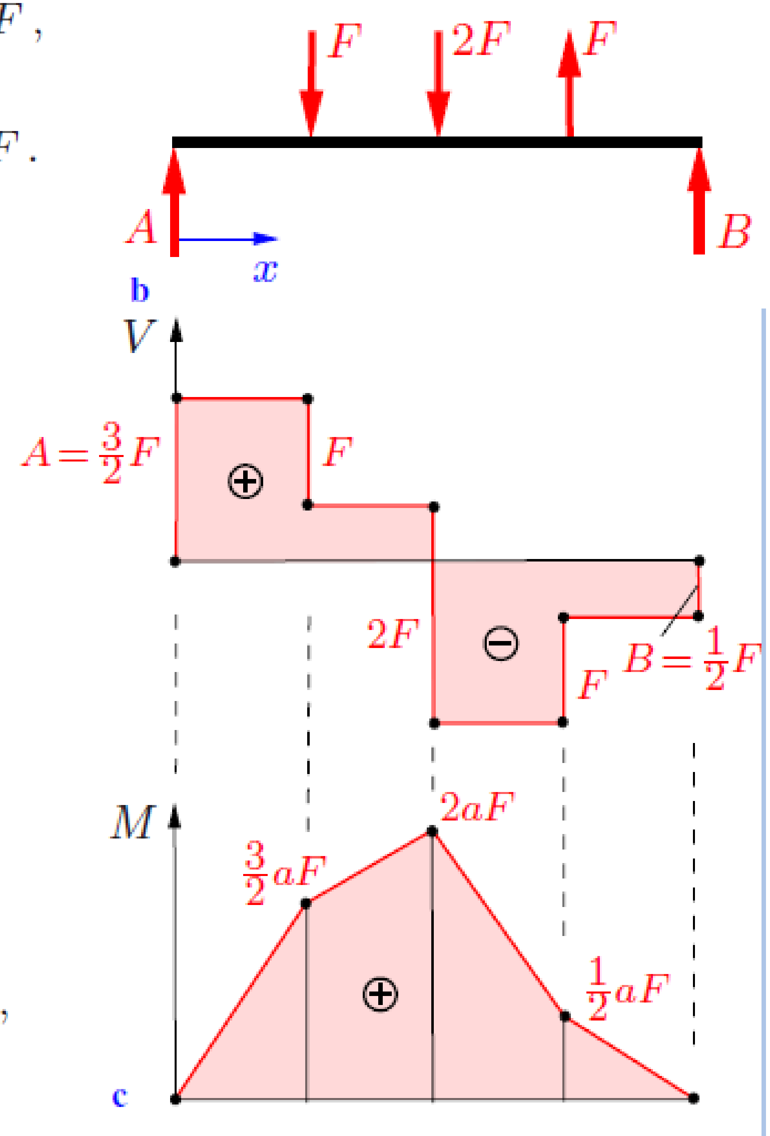
$$M = xA - (x-a)F = (a + \tfrac{1}{2}\,x)F \qquad \text{for} \quad a \le x \le 2a\,,$$

$$M = xA - (x-a)F - (x-2a)2F = (5a - \tfrac{3}{2}\,x)F$$

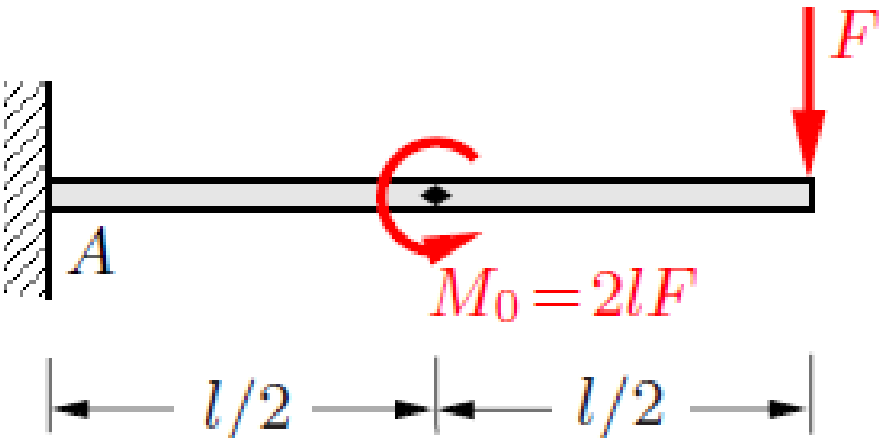
$$\text{for} \quad 2a \le x \le 3a\,,$$

$$M = xA - (x-a)F - (x-2a)2F + (x-3a)F$$

$$= (2a - \tfrac{1}{2}\,x)F \qquad \text{for} \quad 3a \le x \le 4a\,.$$

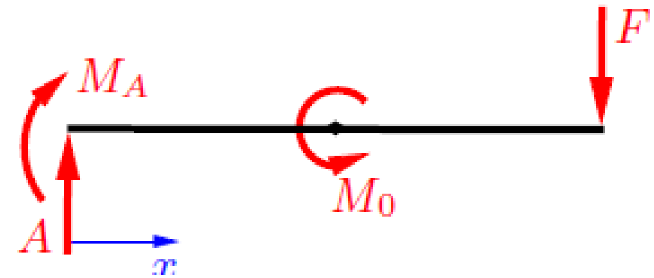


Determine the shear-force and bending-moment diagrams for the cantilever beam



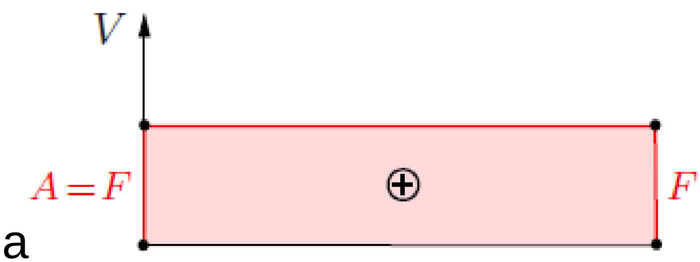
$$\uparrow: \quad A - F = 0 \qquad \rightarrow \qquad A = F\,,$$

$$\overset{\curvearrowleft}{A}: \quad -M_A + M_0 - l\,F = 0 \quad \rightarrow \quad M_A = M_0 - l\,F = l\,F\,.$$

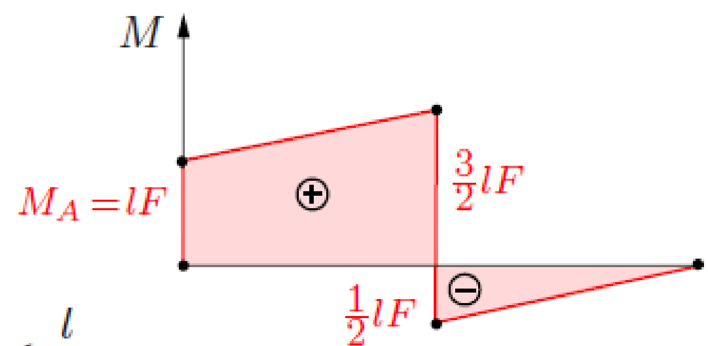


The shear force follows from the equilibrium conditions of the forces in the vertical direction

$$\underline{\underline{V}} = A = \underline{\underline{F}} \quad \text{for} \quad 0 < x < l\,.$$



Because of the couple M_0 at the center of the beam, two regions of x must be considered when the bending moment is calculated. Accordingly, we pass a cut in the region given by 0 < x < l/2 and another one in the span l/2 < x < l. The equilibrium of the moments yields



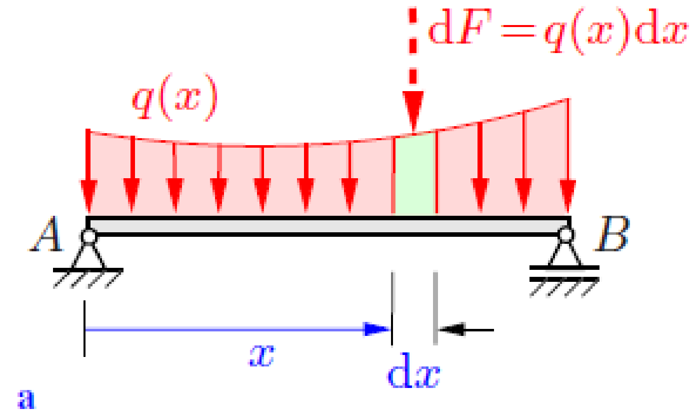
$$\underline{\underline{M}} = M_A + x\,A = \underline{\underline{(l+x)F}} \qquad \text{for} \quad 0 < x < \frac{l}{2}\,,$$

$$\underline{\underline{M}} = M_A + x\,A - M_0 = \underline{\underline{(x-l)F}} \quad \text{for} \quad \frac{l}{2} < x \le l\,.$$

Relationship between stress and loading

(B) Uniform Distributed loads

The conditions of equilibrium yield

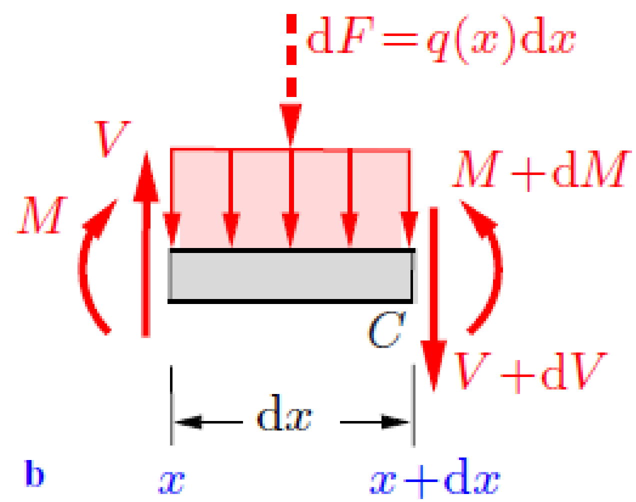


$$\uparrow: \quad V - q\,\mathrm{d}x - (V + \mathrm{d}V) = 0 \quad \rightarrow \quad q\,\mathrm{d}x + \mathrm{d}V =$$

$$\stackrel{\curvearrowleft}{C}: \quad -M - \mathrm{d}x\,V + \frac{\mathrm{d}x}{2}\,q\,\mathrm{d}x + M + \mathrm{d}M = 0$$

$$\rightarrow \quad -V\,\mathrm{d}x + \mathrm{d}M + \frac{1}{2}\,q\,\mathrm{d}x \cdot \mathrm{d}x = 0\,.$$

From first Equation $\boxed{\dfrac{\mathrm{d}V}{\mathrm{d}x} = -q\,.}$



Thus, the slope of the shear-force diagram is equal to the negative intensity of the applied loading

The term in second equation containing dx · dx is “small of higher order” compared with dx or dM. Therefore, it vanishes in the limit dx → 0 and moment equation reduces to

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V$$

The derivative of the bending moment with respect to x is equal to the shear force

Relation between bending moment and load

$$\frac{\mathrm{d}^2M}{\mathrm{d}x^2} = -q$$

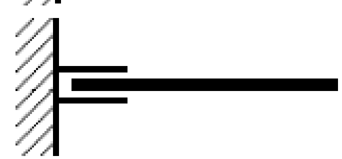
q	V	M
0	constant	linear
constant	linear	quadratic parabola
linear	quadratic parabola	cubic parabola

The relations may also be used to quantitatively determine the stress resultants for a given load q(x). If we integrate above eq. we obtain

$$\frac{\mathrm{d}V}{\mathrm{d}x} = -q\,. \qquad V = -\int q\,\mathrm{d}x + C_1\,,$$

$$\frac{\mathrm{d}M}{\mathrm{d}x} = V \qquad M = \int V\,\mathrm{d}x + C_2\,.$$

The constants of integration C1 and C2 can be calculated if the functions for the stress resultants are evaluated at positions of x where the values of V or M are known

support		V	M
pin		$\neq 0$	0
parallel motion		0	$\neq 0$
sliding sleeve		$\neq 0$	$\neq 0$
fixed end		$\neq 0$	$\neq 0$
free end		0	0

$$V = -\,q_0\,x + C_1 \qquad M = -\,\frac{1}{2}\,q_0\,x^2 + C_1\,x + C_2$$

a) $M(0) = 0\,,$

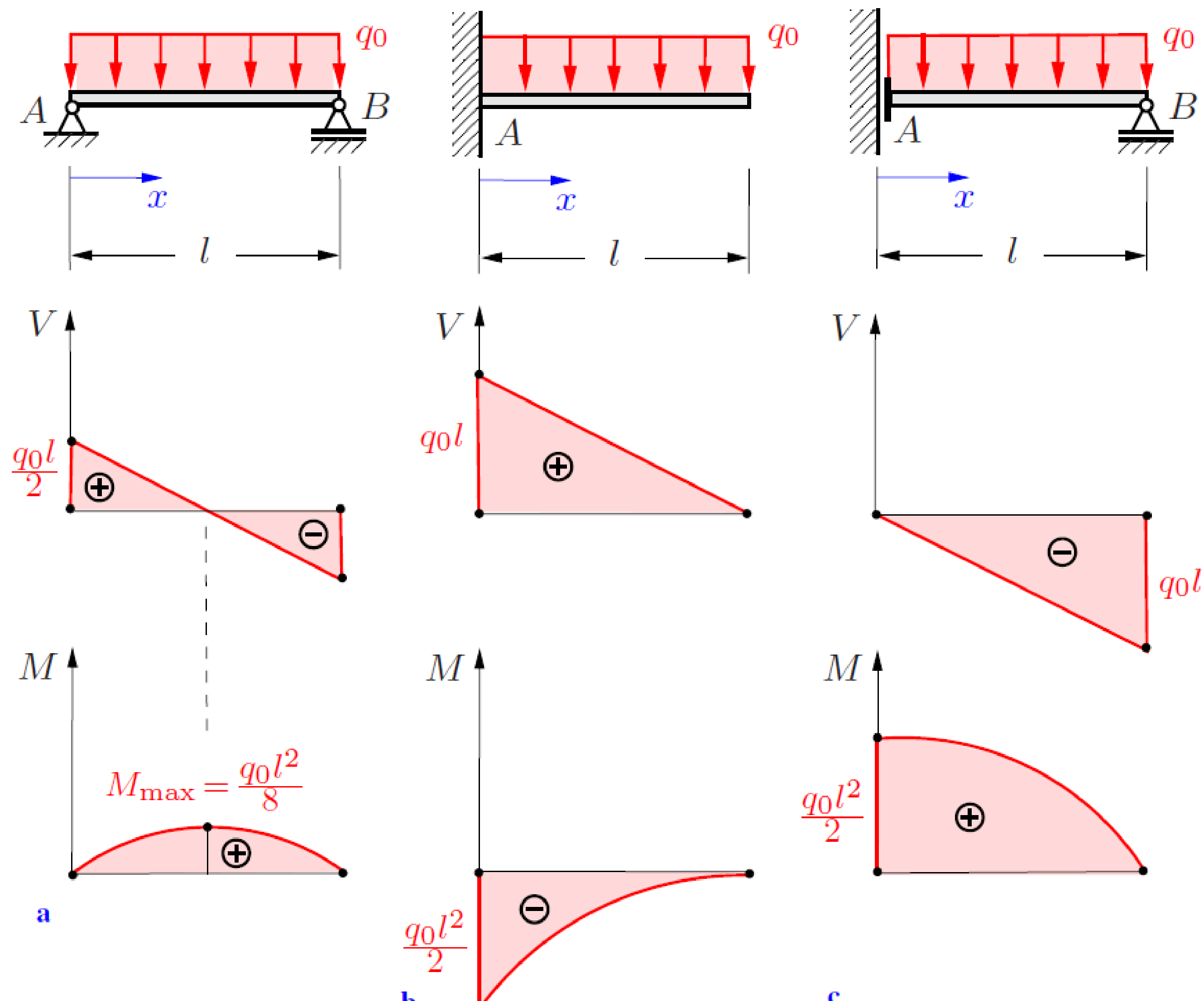
$M(l) = 0\,,$

b) $V(l) = 0\,,$

$M(l) = 0$

c) $V(0) = 0\,,$

$M(l) = 0$



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$$\text{a) } 0 = C_2\,, \qquad \text{b) } 0 = -\,q_0\,l + C_1\,, \quad \text{c) } 0 = C_1\,,$$

$$\text{a), b), c)} \quad 0 = -\,\tfrac{1}{2}\,q_0\,l^2 + C_1\,l + C_2$$

$$\rightarrow \quad \begin{cases} C_1 = \tfrac{1}{2}\,q_0\,l\,, \\ C_2 = 0\,, \end{cases} \qquad \begin{cases} C_1 = q_0\,l\,, \\ C_2 = -\,\tfrac{1}{2}\,q_0\,l^2\,, \end{cases} \qquad \begin{cases} C_1 = 0\,, \\ C_2 = \tfrac{1}{2}\,q_0\,l^2 \end{cases}$$

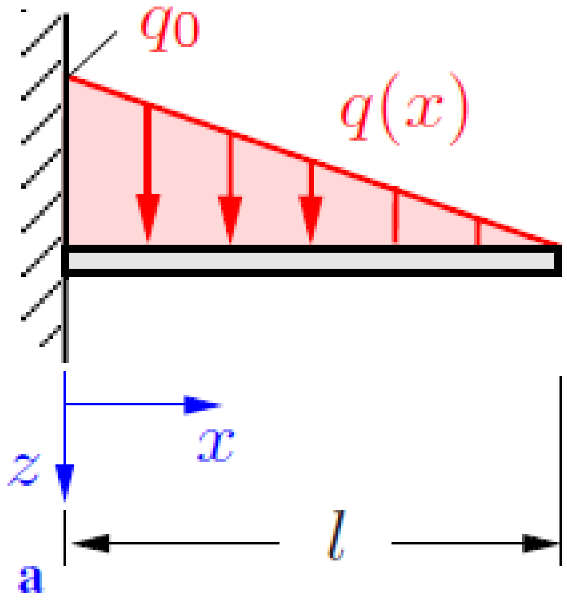
$$\text{a) } V = \frac{1}{2}\,q_0\,l\left(1 - 2\,\frac{x}{l}\right), \qquad \text{b) } V = q_0\,l\left(1 - \frac{x}{l}\right),$$

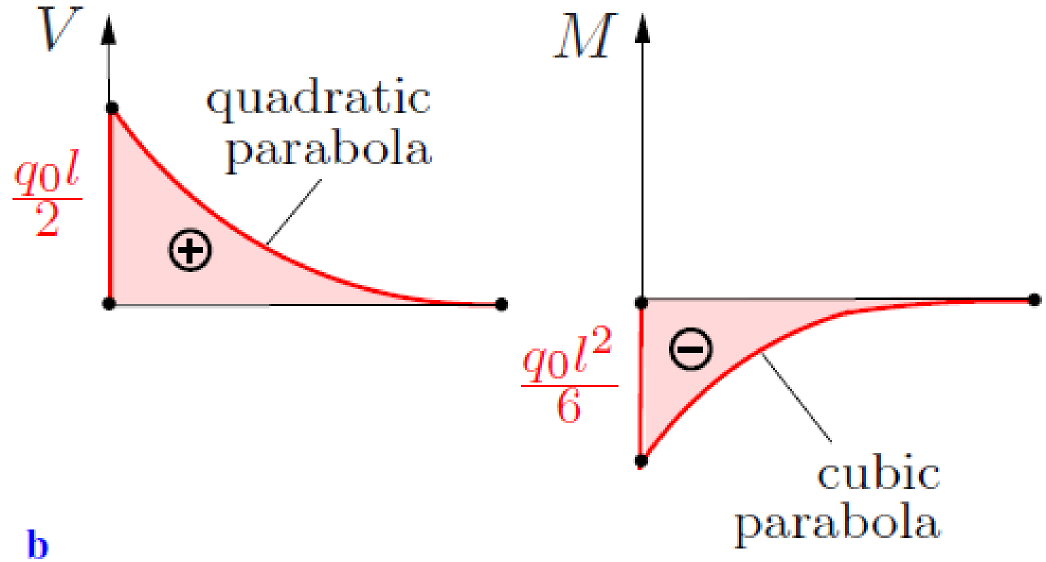
$$M = \frac{1}{2}\,q_0\,l^2\,\frac{x}{l}\left(1 - \frac{x}{l}\right), \qquad M = -\,\frac{1}{2}\,q_0\,l^2\left(1 - \frac{x}{l}\right)^2$$

$$\text{c) } V = -\,q_0\,x\,,$$

$$M = \frac{1}{2}\,q_0\,l^2\left[1 - \left(\frac{x}{l}\right)^2\right].$$

A simply supported beam is subjected to a concentrated force and a triangular line load

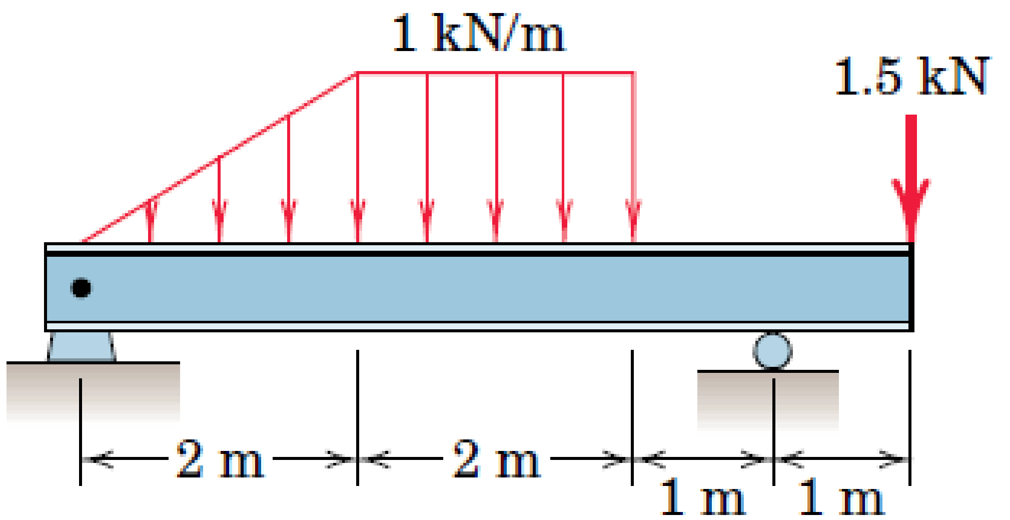




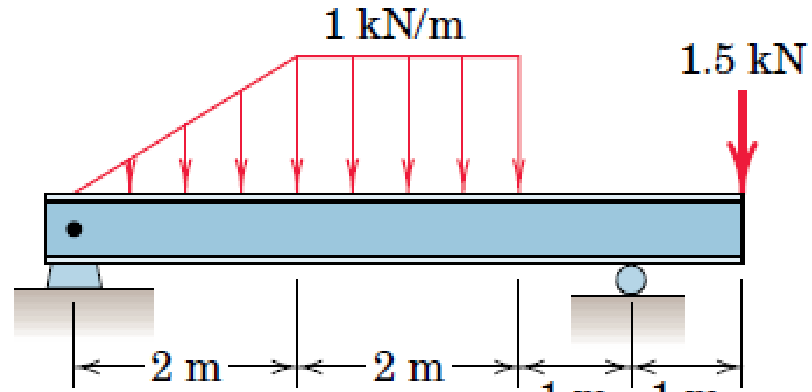
b

$$\underline{\underline{V(x) = \frac{1}{2}\, q_0\, l \left(1 - \frac{x}{l}\right)^2}}\,, \quad \underline{\underline{M(x) = -\,\frac{1}{6}\, q_0\, l^2 \left(1 - \frac{x}{l}\right)^3}}\,,$$

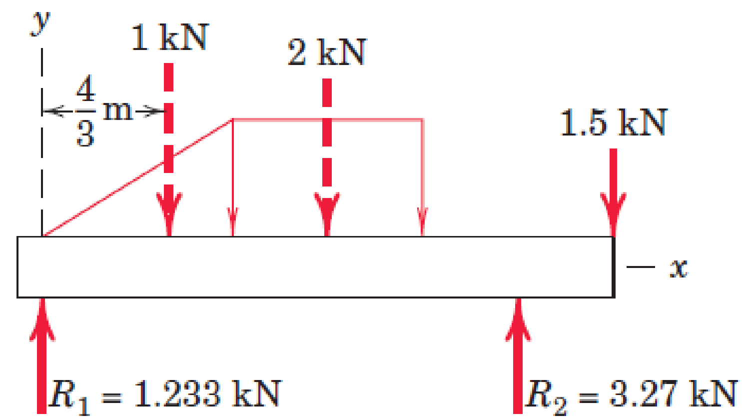
Draw the shear-force and bending-moment diagrams for the loaded beam and determine the maximum moment *M* and its location *x* from the left end



The support reactions are most easily obtained by considering the resultants of the distributed loads as shown on the free-body diagram of the beam as a whole

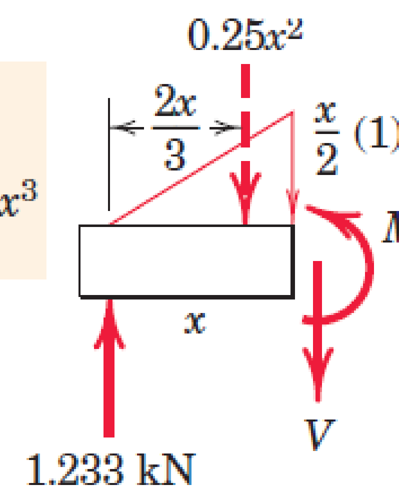


The first interval of the beam is analyzed from the free-body diagram of the section for 0 *to* 2 m. A summation of vertical forces and a moment summation about the cut section yield

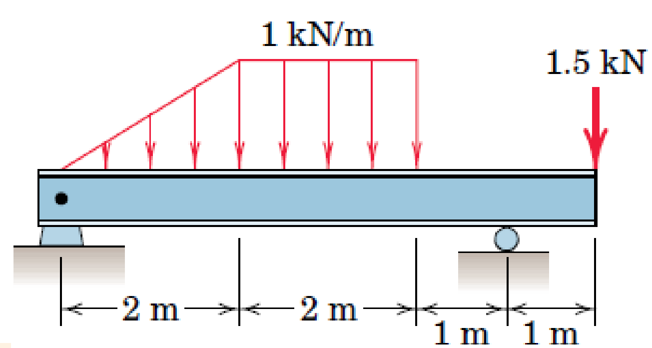


$[\Sigma F_y = 0]$ $\qquad V = 1.233 - 0.25x^2$

$[\Sigma M = 0]$ $\qquad M + (0.25x^2)\dfrac{x}{3} - 1.233x = 0 \qquad M = 1.233x - 0.0833x^3$



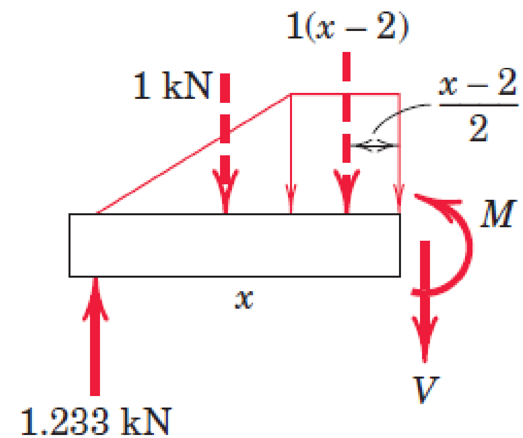
From the free-body diagram of the section for which 2 *to* 4 m, equilibrium in the vertical direction and a moment sum about the cut section give



$$[\Sigma F_y = 0] \qquad V + 1(x-2) + 1 - 1.233 = 0 \qquad V = 2.23 - x$$

$$[\Sigma M = 0] \qquad M + 1(x-2)\frac{x-2}{2} + 1[x - \tfrac{2}{3}(2)] - 1.233x = 0$$

$$M = -0.667 + 2.23x - 0.50x^2$$

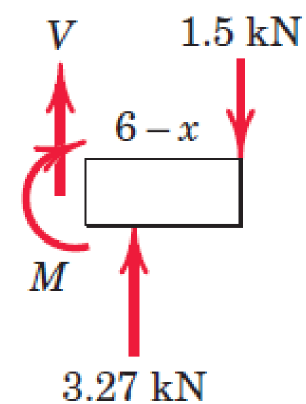


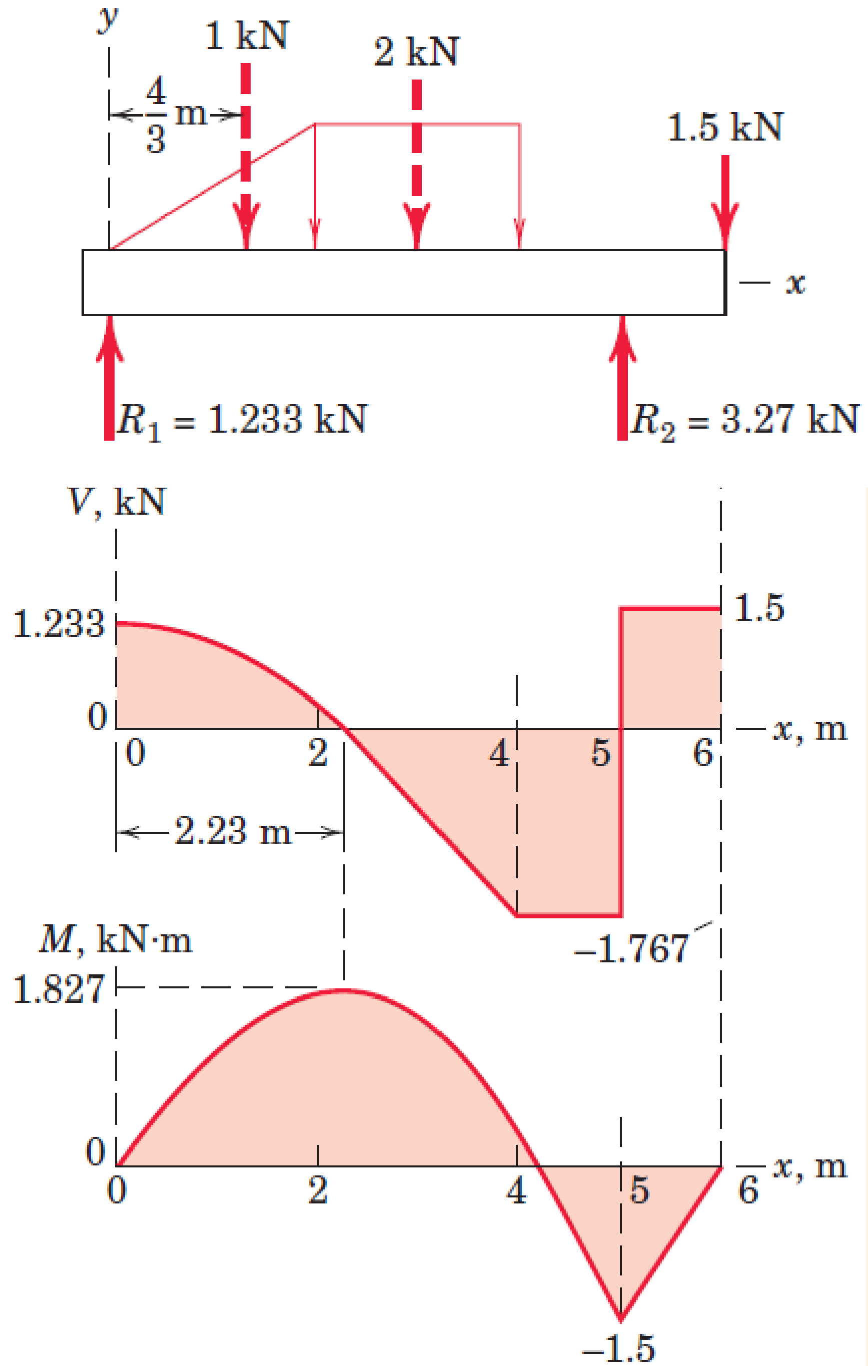
From the free-body diagram of the section for which 4 *to* 5 m, equilibrium in the vertical direction and a moment sum about the cut section give

.

$$V = -1.767 \text{ kN} \qquad \text{and} \qquad M = 7.33 - 1.767x$$

The last interval may be analyzed by inspection. The shear is constant at 1.5 kN, and the moment follows a straight-line relation beginning with zero at the right end of the beam



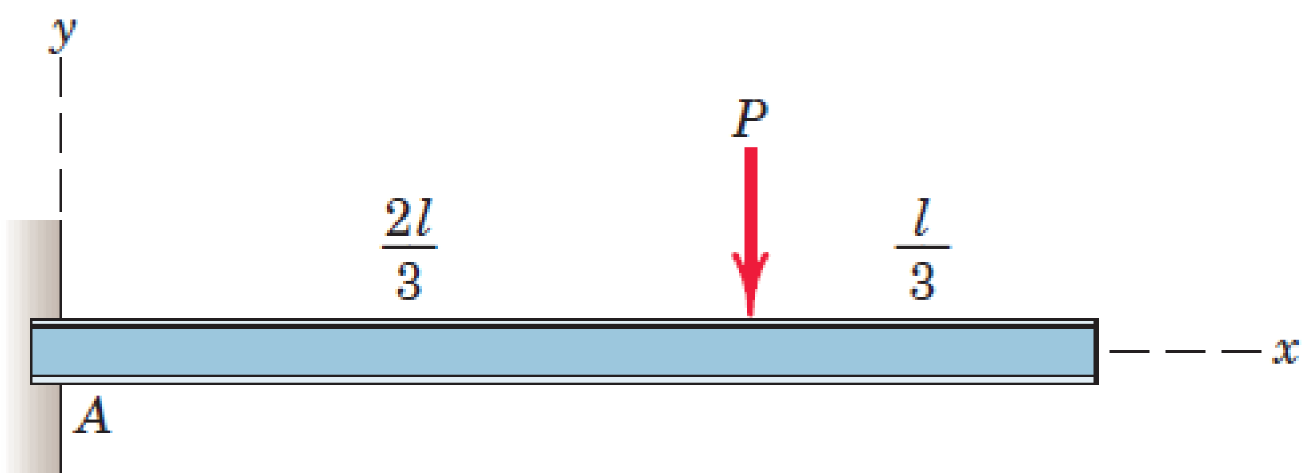


y
1 kN
2 kN
4/3 m
1.5 kN
x
R1 = 1.233 kN
R2 = 3.27 kN
V, kN
1.233
1.5
0
x, m
0
2
4
5
6
2.23 m
M, kN·m
–1.767
1.827
0
x, m
0
2
4
5
6
–1.5

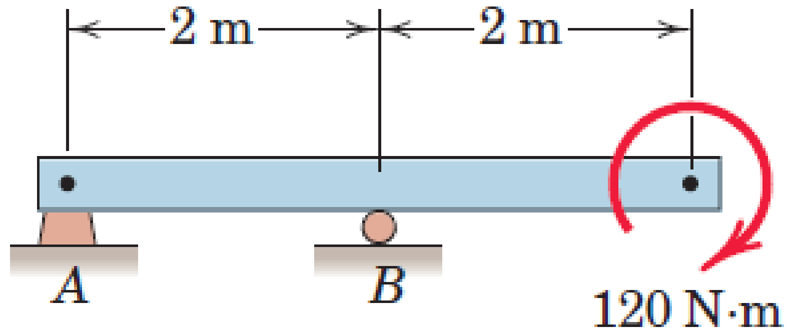
Assignments-question

5/117 Determine the shear-force and bending-moment distributions produced in the beam by the concentrated load. What are the values of the shear and moment at $x = l/2$?

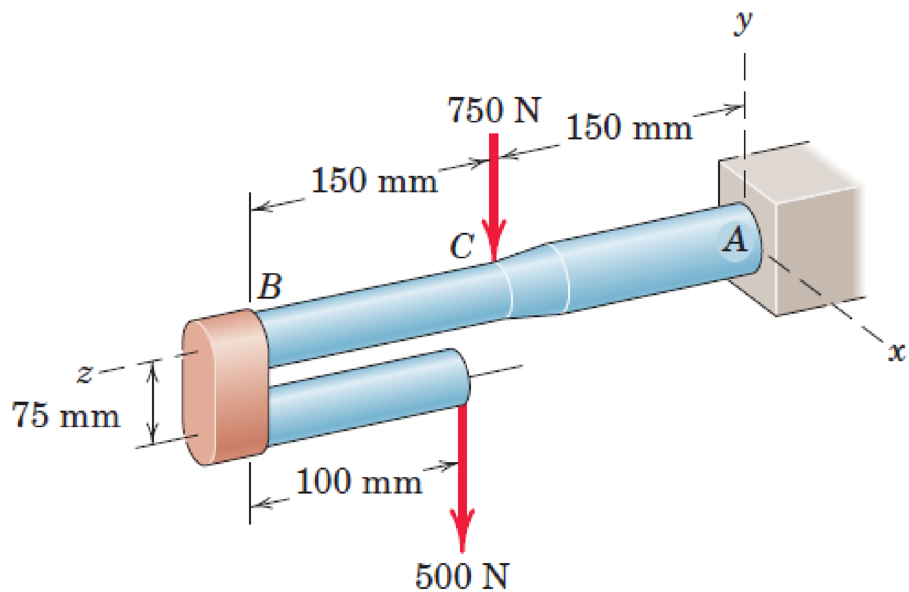
$$Ans.\ V = P,\ M = -\frac{Pl}{6}$$



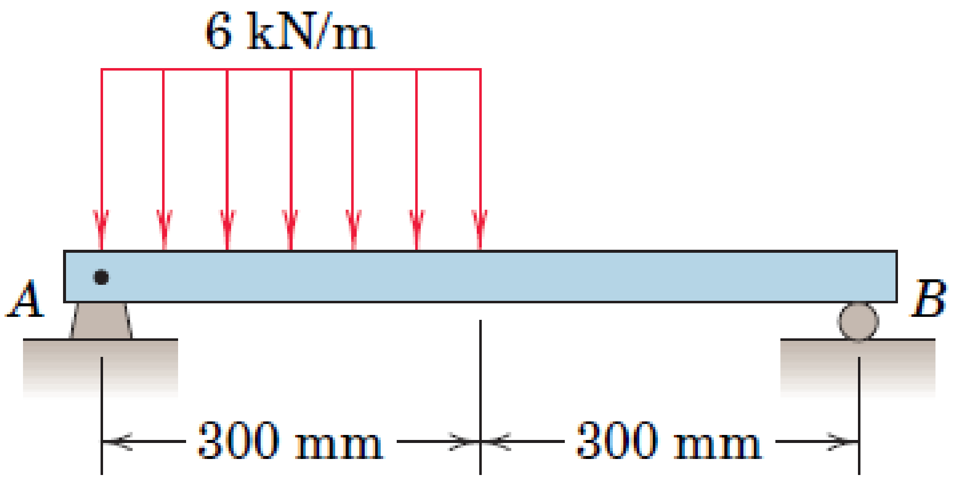
Q1. Draw the shear and moment diagrams for the beam subjected to the end couple. What is the moment *M* at a section 0.5 m to the right of *B*?



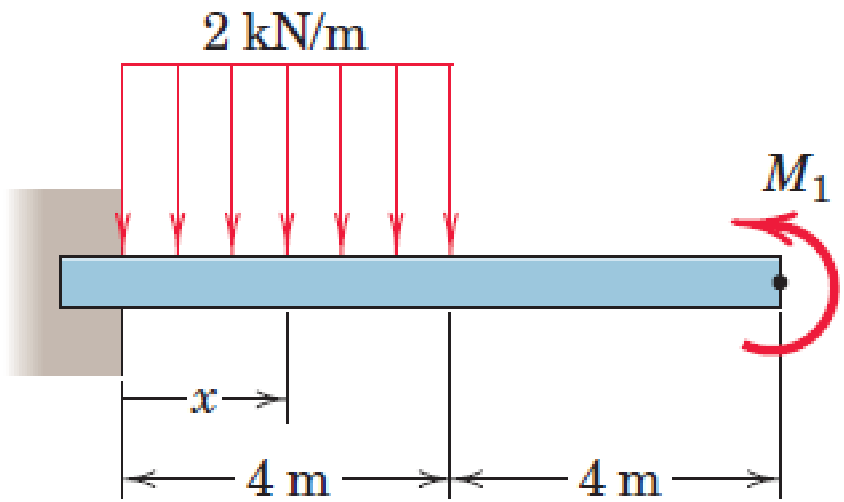
Q2. Construct the bending-moment diagram for the cantilevered shaft *AB* of the rigid unit shown.



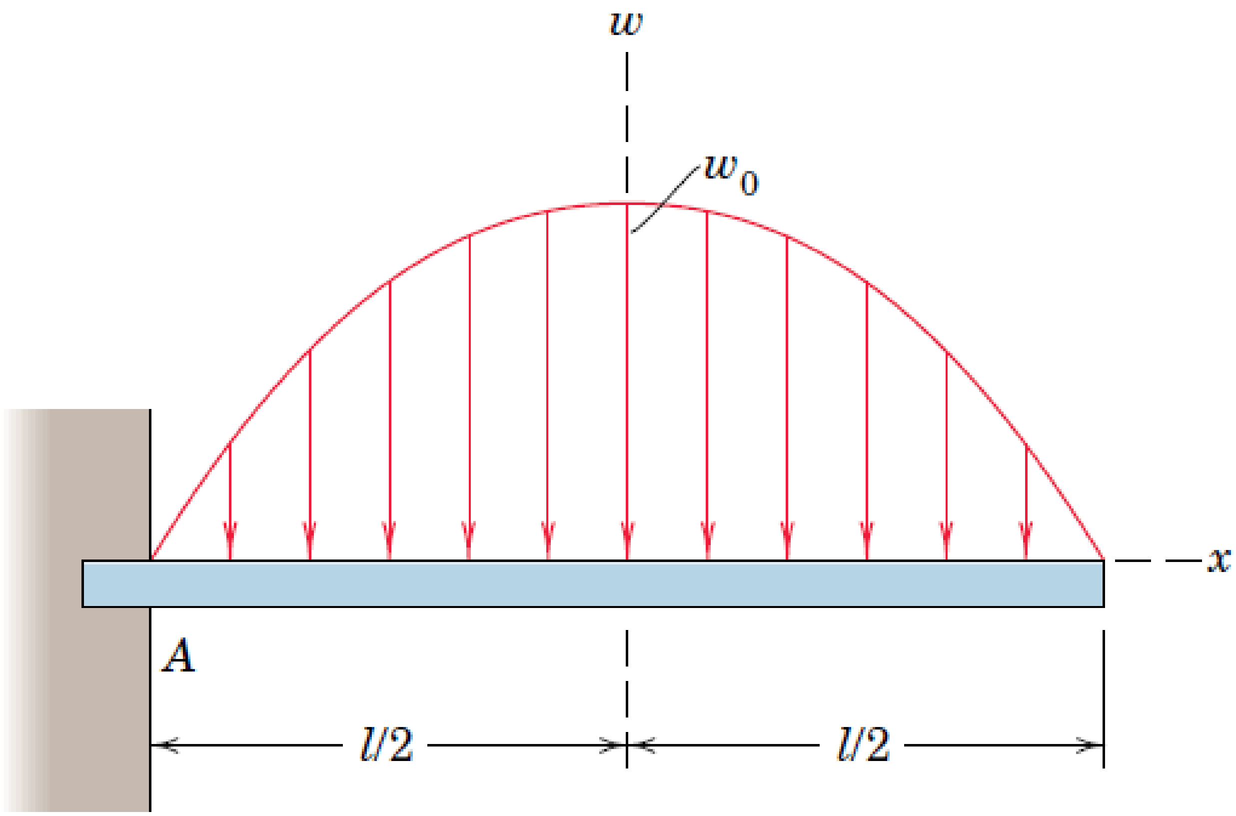
Determine the shear *V* and moment *M* at a section of the loaded beam 200 mm to the right of *A*.



Draw the shear and moment diagrams for the loaded cantilever beam where the end couple *M*1 is adjusted so as to produce zero moment at the fixed end of the beam. Find the bending moment *M* at *x* 2 m.



Derive expressions for the shear *V* and moment *M* in terms of *x* for the cantilever beam of Prob. 5/108 shown again here



$$Ans.\ V = w_0\left(\frac{l}{3} - x + \frac{4x^3}{3l^2}\right)$$

$$M = w_0\left(-\frac{l^2}{16} + \frac{xl}{3} - \frac{x^2}{2} + \frac{x^4}{3l^2}\right)$$

▶5/144 The curved cantilever beam in the form of a quarter-circular arc supports a load of w N/m applied along the curve of the beam on its upper surface. Determine the magnitudes of the torsional moment T and bending moment M in the beam as functions of θ.

$$\textit{Ans. } T = wr^2\left(\frac{\pi}{2} - \theta - \cos\theta\right)$$
$$M = wr^2(1 - \sin\theta)$$