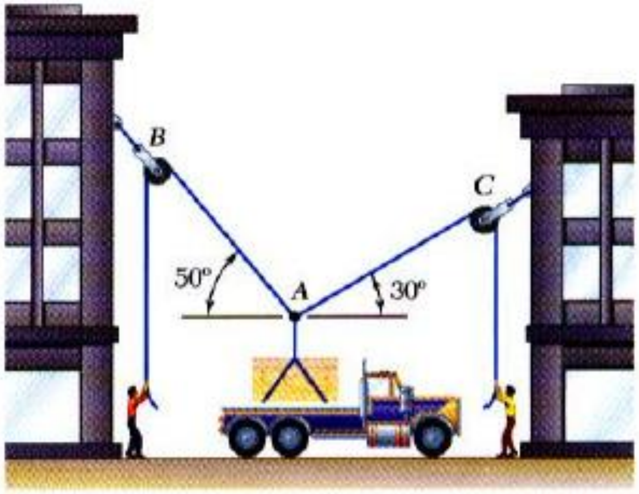
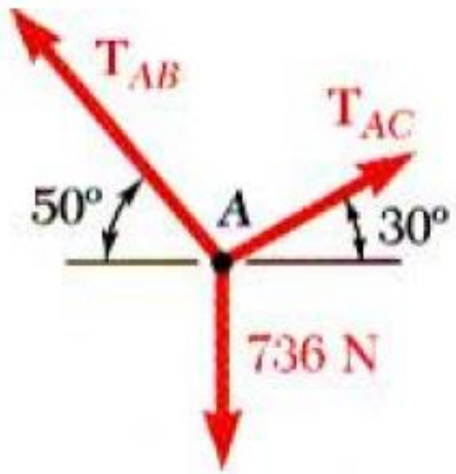
Rigid Body Equilibrium

Free-Body Diagrams



Space Diagram: A sketch showing the physical conditions of the problem.



Free-Body Diagram: A sketch showing only the forces on the selected particle.

Rigid Body Equilibrium

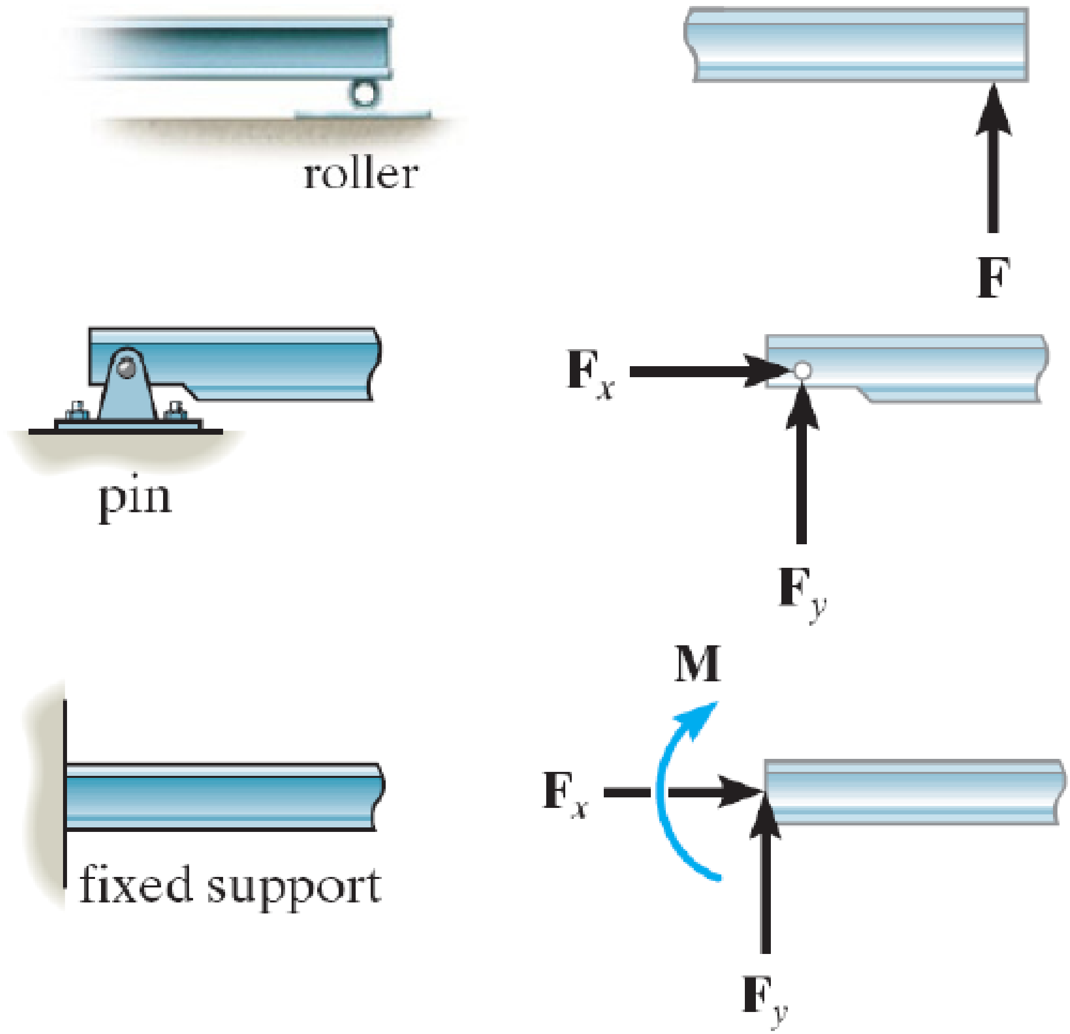
Support Reactions

Prevention of

Translation or

Rotation of a body

Restraints

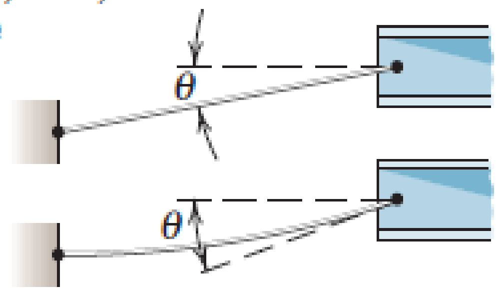


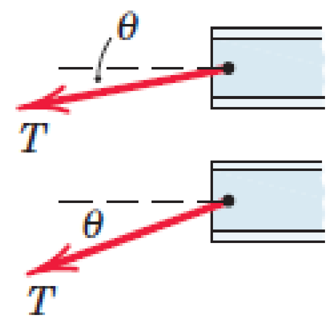
MODELING THE ACTION OF FORCES IN TWO-DIMENSIONAL ANALYSIS

1. Flexible cable, belt, chain, or rope

Weight of cable negligible

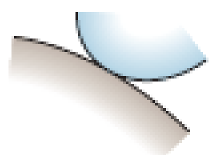
Weight of cable not negligible

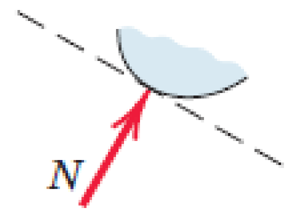




Force exerted by a flexible cable is always a tension away from the body in the direction of the cable.

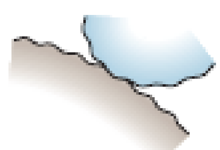
2. Smooth surfaces

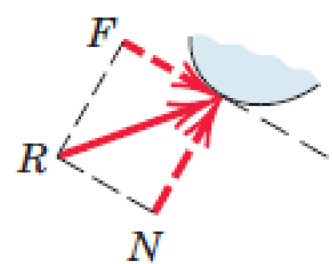




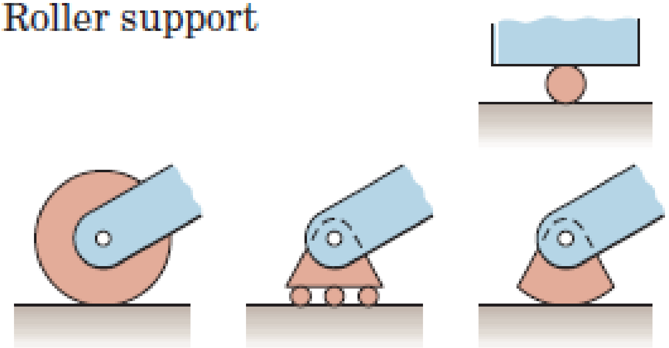
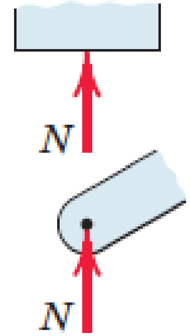
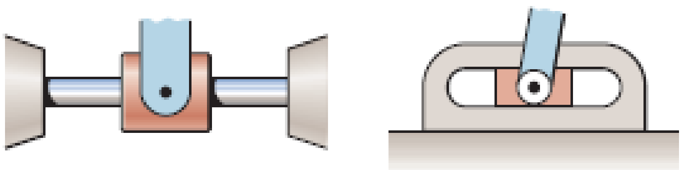
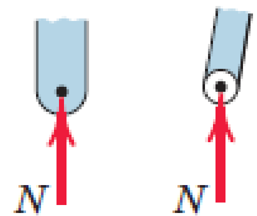
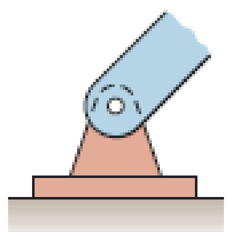
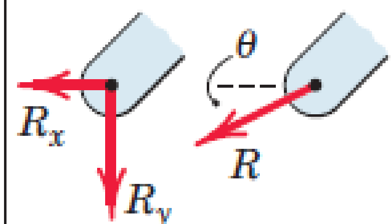
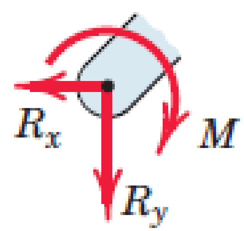
Contact force is compressive and is normal to the surface.

3. Rough surfaces





Rough surfaces are capable of supporting a tangential compo-nent F (frictional force) as well as a normal component N of the resultant

4. Roller support 	 Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.
5. Freely sliding guide 	 Collar or slider free to move along smooth guides; can support force normal to guide only.
6. Pin connection 	Pin free to turn  Pin not free to turn  A freely hinged pin connection is capable of supporting a force in any direction in the plane normal to the pin axis. We may either show two components R_x and R_y or a magnitude R and direction θ. A pin not free to turn also supports a couple M.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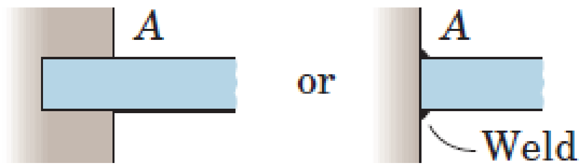
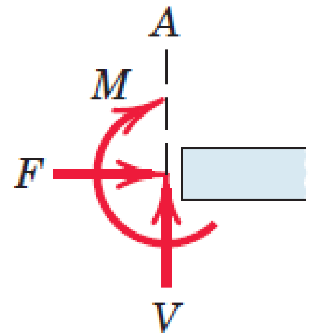
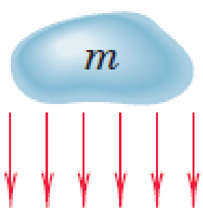
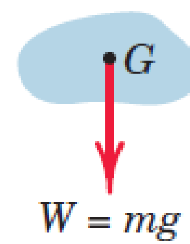
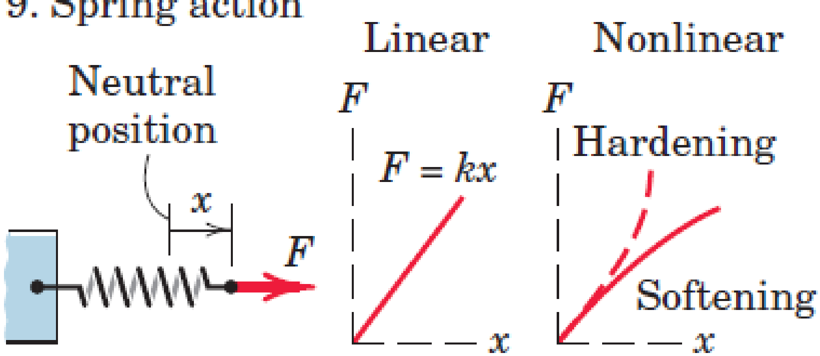
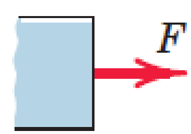
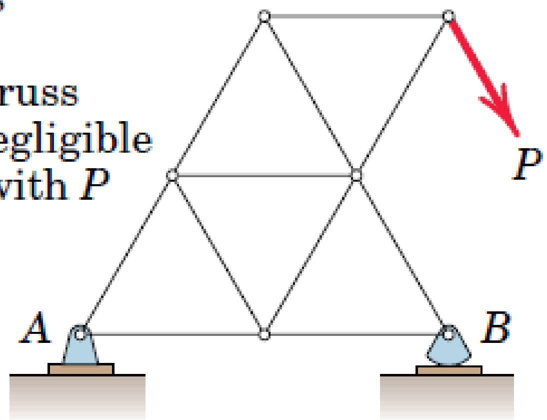
7. Built-in or fixed support 	 A built-in or fixed support is capable of supporting an axial force F, a transverse force V (shear force), and a couple M (bending moment) to prevent rotation.
8. Gravitational attraction 	 The resultant of gravitational attraction on all elements of a body of mass m is the weight $W = mg$ and acts toward the center of the earth through the center mass G.
9. Spring action 	 Spring force is tensile if spring is stretched and compressive if compressed. For a linearly elastic spring the stiffness k is the force required to deform the spring a unit distance.

Steps for Free-Body Diagrams

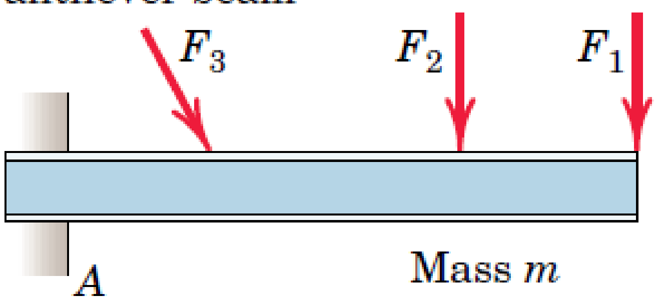
1. Make an unequivocal decision as to which system (a body or collection of bodies) in equilibrium is to be analyzed.
2. Isolate the system in question from all contacting bodies by drawing its *free-body diagram* showing *all* forces and couples acting *on* the isolated system from external sources.
3. Observe the principle of action and reaction (Newton's third law) when assigning the sense of each force.
4. Label all forces and couples, known and unknown.
5. Choose and label reference axes, always choosing a right-handed set when vector notation is used (which is usually the case for three-dimensional analysis).
6. Check the adequacy of the constraints (supports) and match the number of unknowns with the number of available independent equations of equilibrium.

1. Plane truss

Weight of truss assumed negligible compared with P

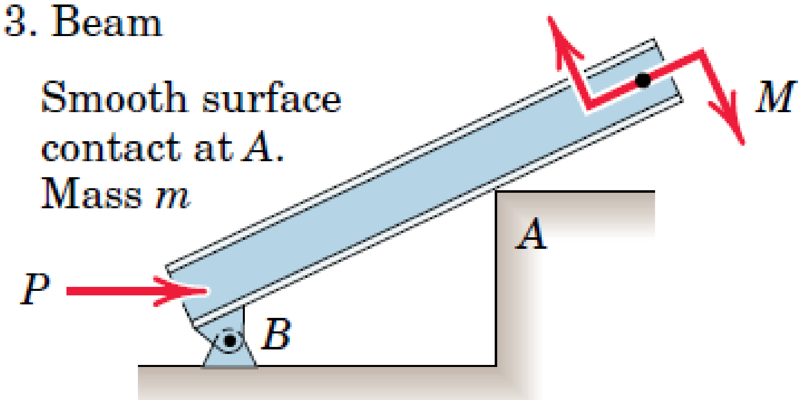


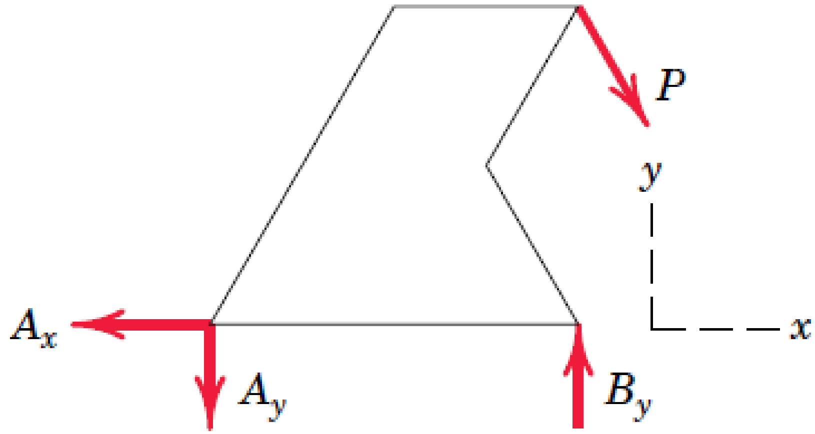
2. Cantilever beam

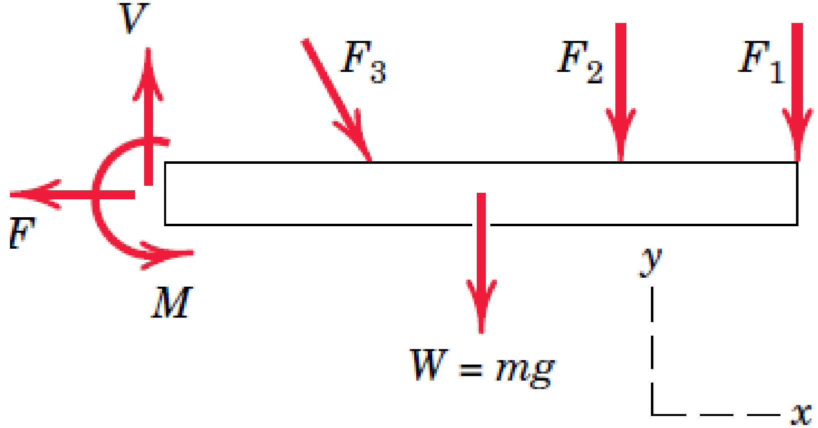


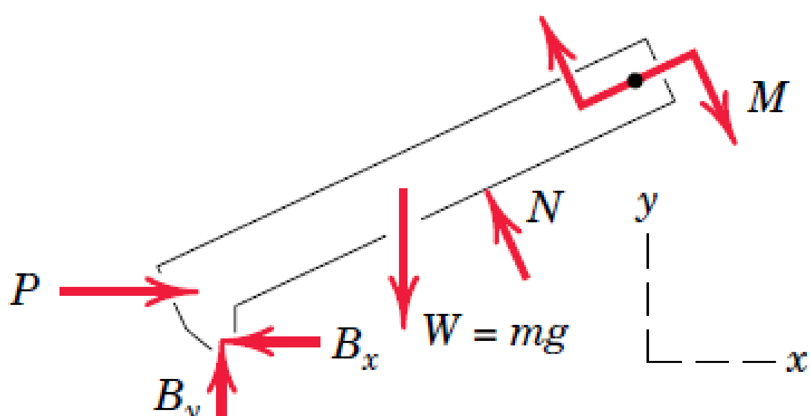
3. Beam

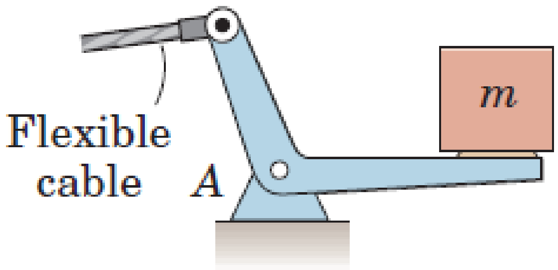
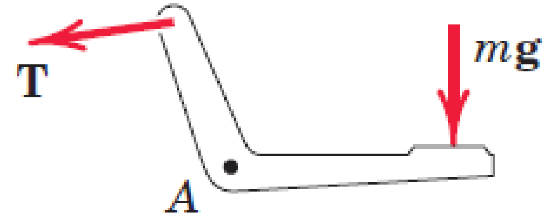
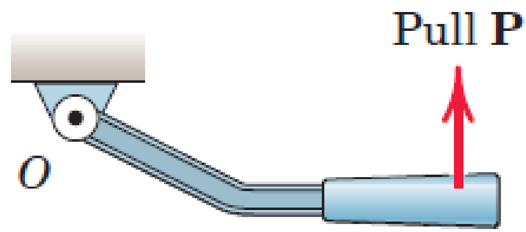
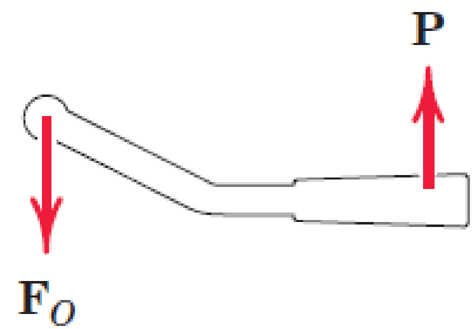
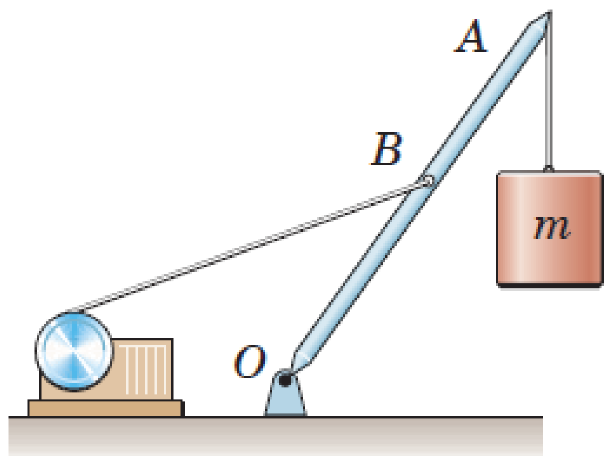
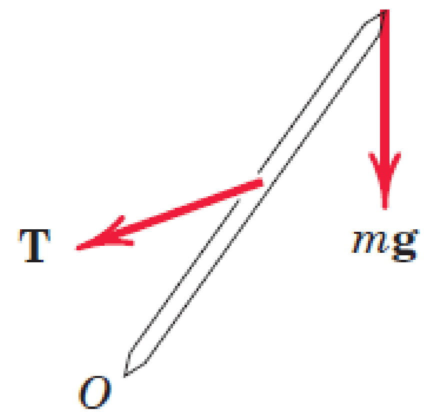
Smooth surface contact at A.
Mass m

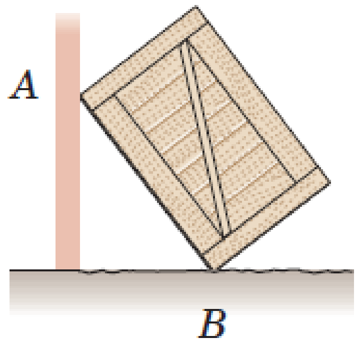
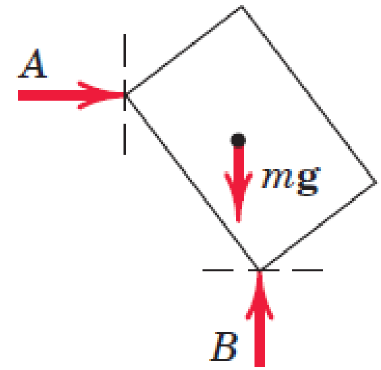
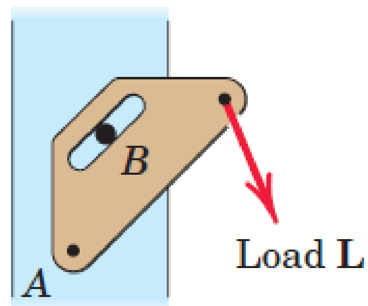
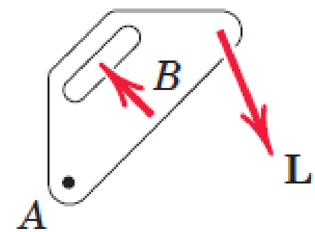


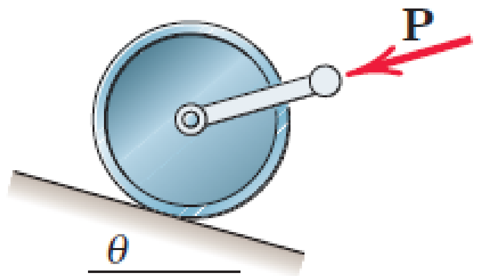
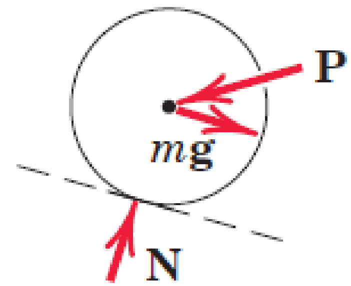
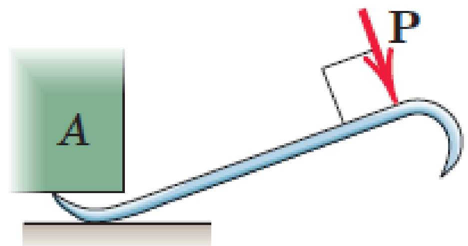
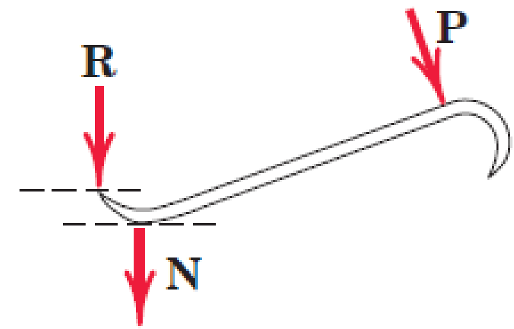
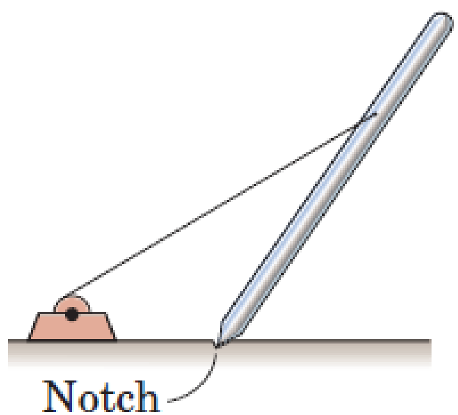
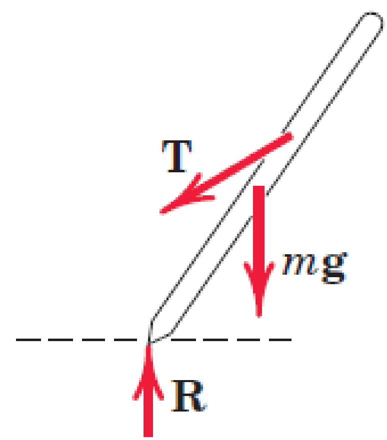

P
y
x
A_x
A_y
B_y

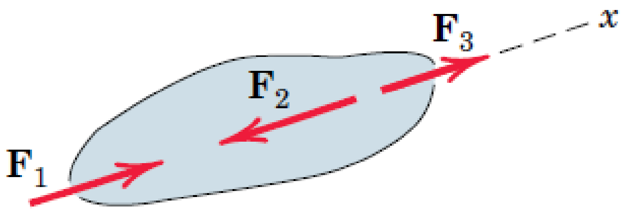
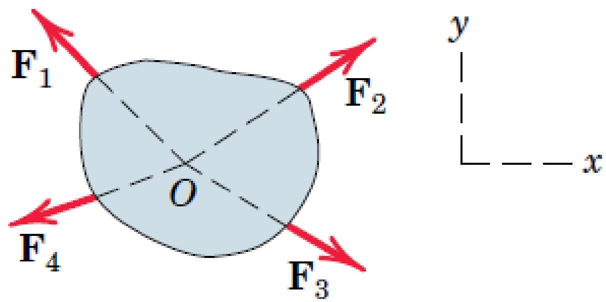
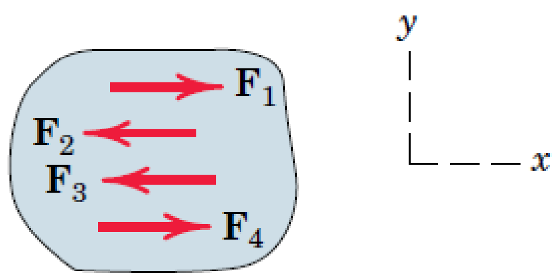
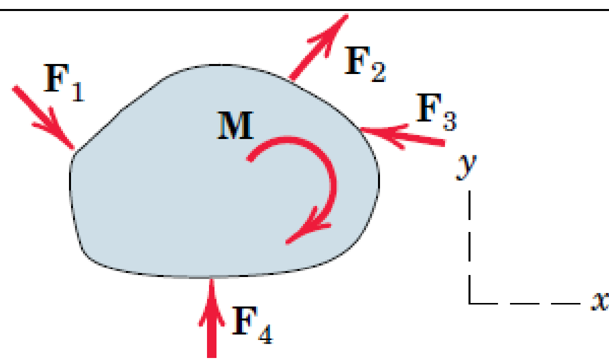

V
F_3
F_2
F_1
F
M
y
W = mg
x

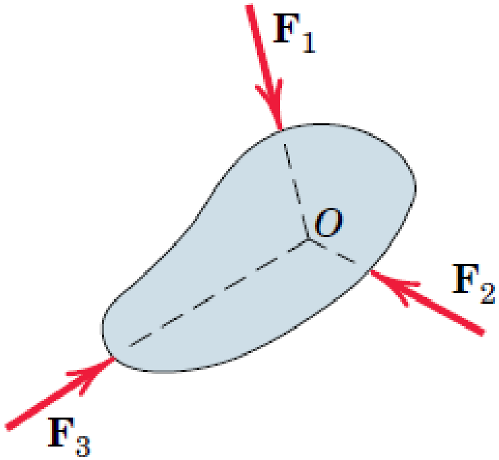

M
N
y
P
B_x
W = mg
x
B_y

<table>
<tr><th></th><th>Body</th><th>Incomplete FBD</th></tr>
<tr><td>1. Bell crank supporting mass m with pin support at A.</td><td>
</td><td>
</td></tr>
<tr><td>2. Control lever applying torque to shaft at O.</td><td>
</td><td>
</td></tr>
<tr><td>3. Boom OA, of negligible mass compared with mass m. Boom hinged at O and supported by hoisting cable at B.</td><td>
</td><td>
</td></tr>
</table>

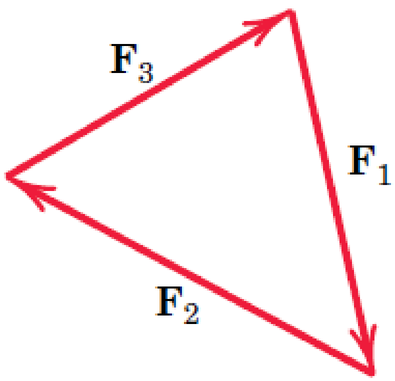
4. Uniform crate of mass m leaning against smooth vertical wall and supported on a rough horizontal surface.		
5. Loaded bracket supported by pin connection at A and fixed pin in smooth slot at B.		

	Body	Wrong or Incomplete FBD
1. Lawn roller of mass m being pushed up incline θ .		
2. Prybar lifting body A having smooth horizontal surface. Bar rests on horizontal rough surface.		
3. Uniform pole of mass m being hoisted into position by winch. Horizontal supporting surface notched to prevent slipping of pole.		

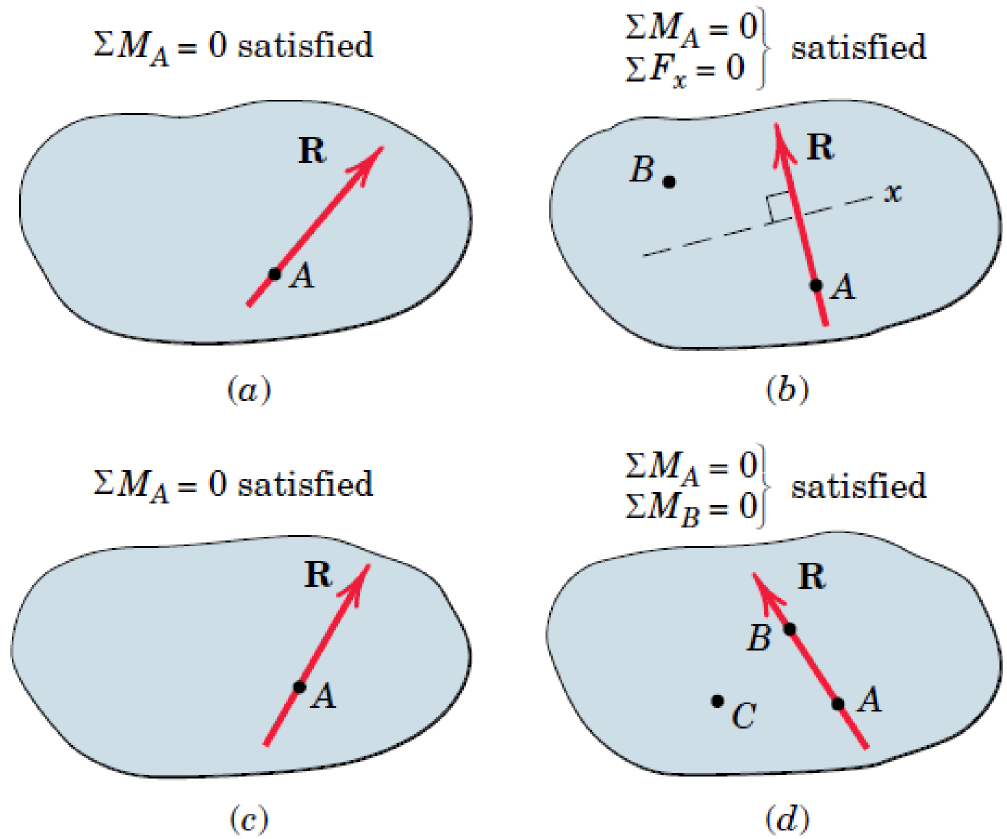
CATEGORIES OF EQUILIBRIUM IN TWO DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Collinear		$\Sigma F_x = 0$
2. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$
3. Parallel		$\Sigma F_x = 0 \qquad \Sigma M_z = 0$
4. General		$\Sigma F_x = 0 \qquad \Sigma M_z = 0$ $\Sigma F_y = 0$



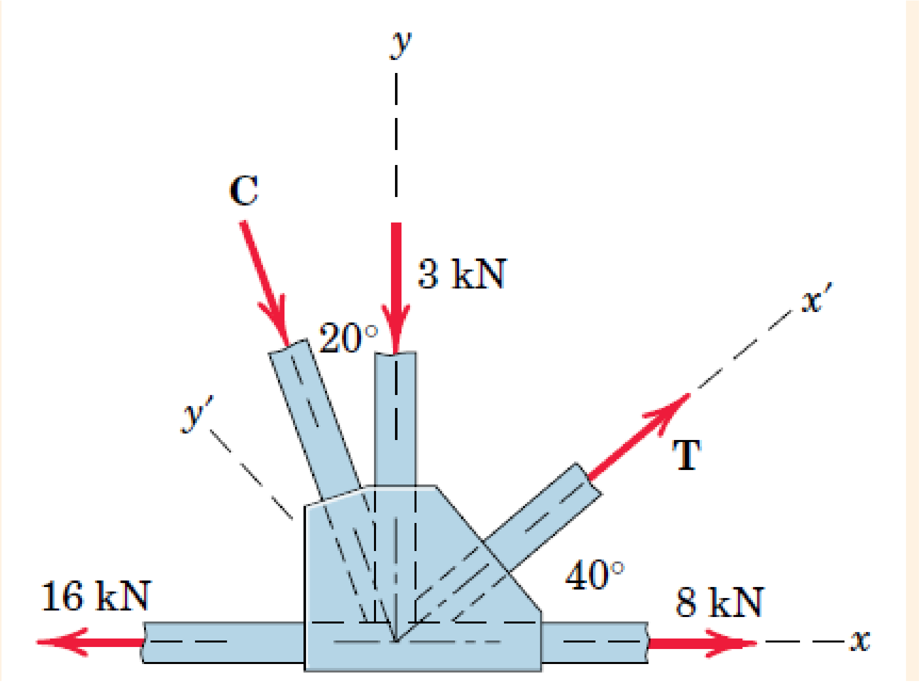
(a) Three-force member



(b) Closed polygon satisfies $\Sigma\mathbf{F} = \mathbf{0}$



3.1 Determine the magnitudes of the forces **C** and **T**, which, along with the other three forces shown, act on the bridge-truss joint



Solution 3.1

Solution I (scalar algebra). For the x-y axes as shown we have

$$[\Sigma F_x = 0] \qquad 8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

$$0.766T + 0.342C = 8 \qquad (a)$$

$$[\Sigma F_y = 0] \qquad T\sin 40^\circ - C\cos 20^\circ - 3 = 0$$

$$0.643T - 0.940C = 3 \qquad (b)$$

Simultaneous solution of Eqs. (a) and (b) produces

$$T = 9.09 \text{ kN} \qquad C = 3.03 \text{ kN} \qquad \textit{Ans.}$$

Solution III (vector algebra). With unit vectors **i** and **j** in the x- and y-directions, the zero summation of forces for equilibrium yields the vector equation

$$[\Sigma \mathbf{F} = \mathbf{0}] \qquad 8\mathbf{i} + (T\cos 40^\circ)\mathbf{i} + (T\sin 40^\circ)\mathbf{j} - 3\mathbf{j} + (C\sin 20^\circ)\mathbf{i} - (C\cos 20^\circ)\mathbf{j} - 16\mathbf{i} = \mathbf{0}$$

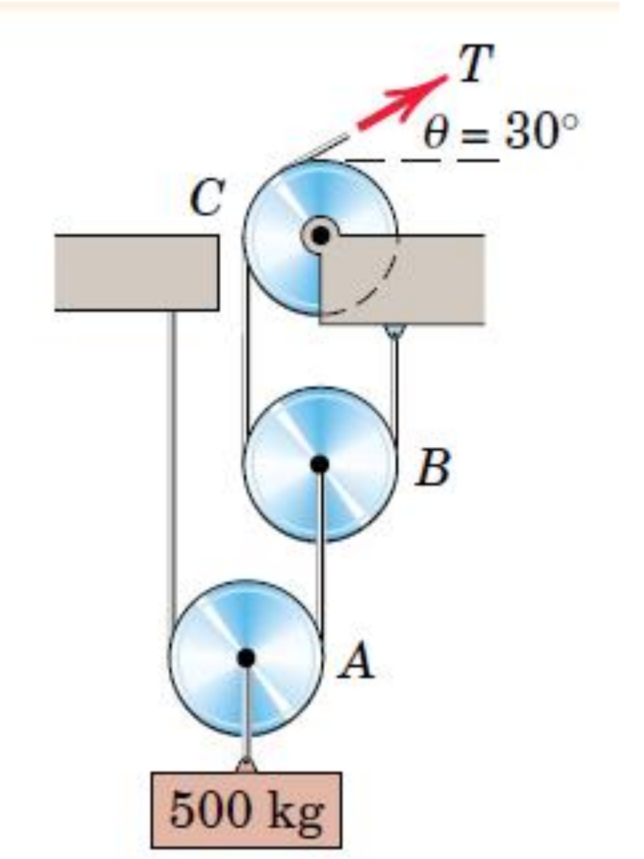
Equating the coefficients of the **i**- and **j**-terms to zero gives

$$8 + T\cos 40^\circ + C\sin 20^\circ - 16 = 0$$

$$T\sin 40^\circ - 3 - C\cos 20^\circ = 0$$

which are the same, of course, as Eqs. (a) and (b), which we solved above.

Calculate the tension T in the cable which supports the 500-kg mass with the pulley arrangement shown. Each pulley is free to rotate about its bearing, and the weights of all parts are small compared with the load. Find the magnitude of the total force on the bearing of pulley C

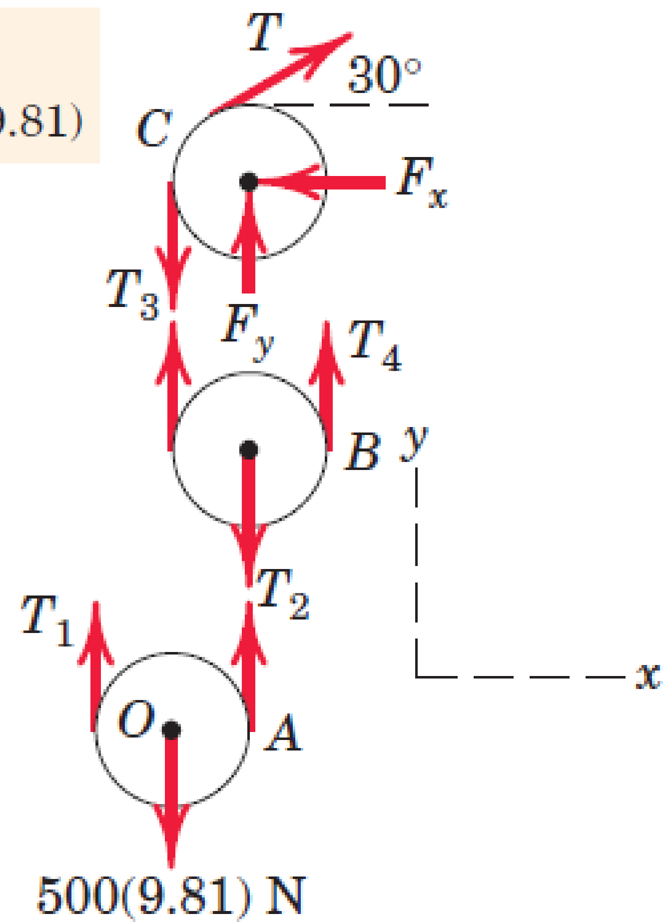


The free-body diagram of each pulley is drawn in its relative position to the others. We begin with pulley *A*, which includes the only known force. With the unspecified pulley radius designated by *r*, the equilibrium of moments about its center *O* and the equilibrium of forces in the vertical direction require

$$[\Sigma M_O = 0] \qquad T_1 r - T_2 r = 0 \qquad T_1 = T_2$$

$$[\Sigma F_y = 0] \qquad T_1 + T_2 - 500(9.81) = 0 \qquad 2T_1 = 500(9.81)$$

$$T_1 = T_2 = 2450 \text{ N}$$



From the example of pulley *A* we may write the equilibrium of forces on pulley *B* by inspection as

$$T_3 = T_4 = T_2/2 = 1226 \text{ N}$$

For pulley *C* the angle 30 in no way affects the moment of *T* about the center of the pulley, so that moment equilibrium requires

$$T = T_3 \qquad \text{or} \qquad T = 1226 \text{ N}$$

Equilibrium of the pulley in the x- and y-directions requires

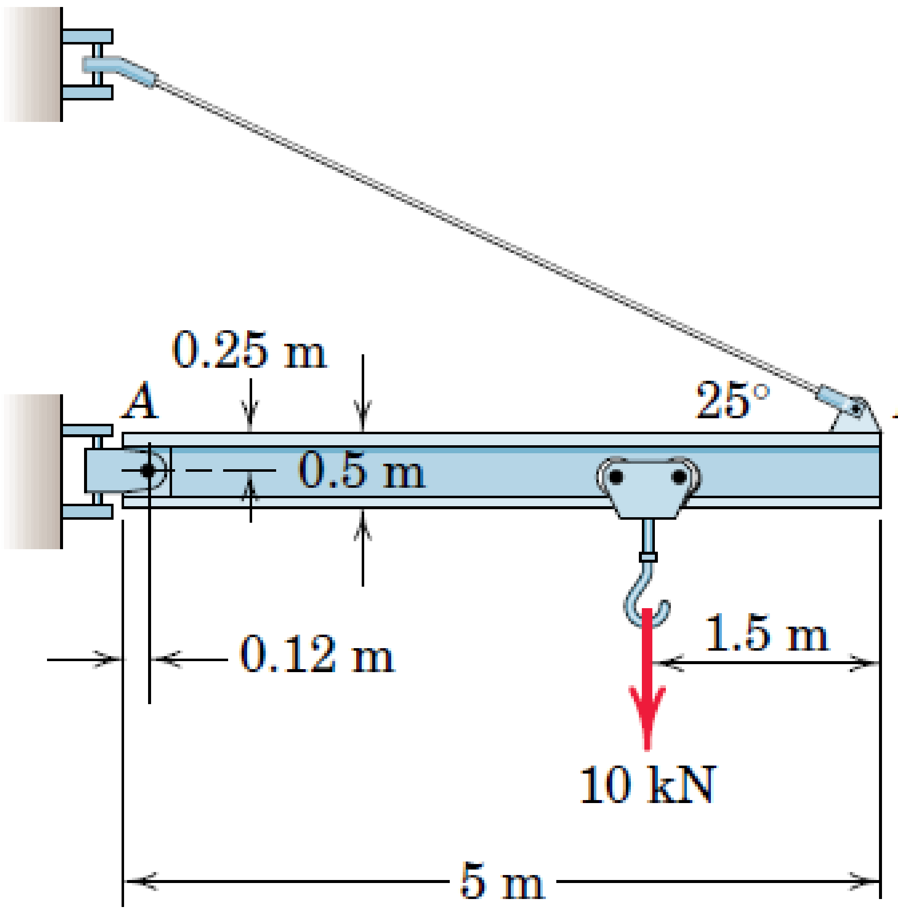
$$[\Sigma F_x = 0] \qquad 1226 \cos 30° - F_x = 0 \qquad F_x = 1062 \text{ N}$$

$$[\Sigma F_y = 0] \qquad F_y + 1226 \sin 30° - 1226 = 0 \qquad F_y = 613 \text{ N}$$

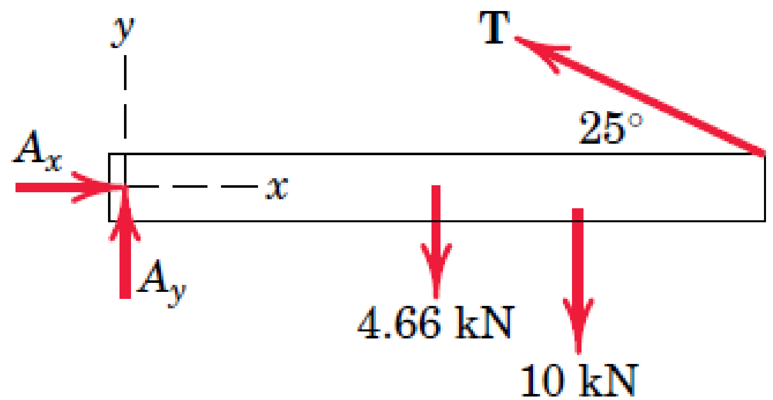
$$\left[F = \sqrt{F_x{}^2 + F_y{}^2}\right] \qquad F = \sqrt{(1062)^2 + (613)^2} = 1226 \text{ N}$$

Clearly the radius *r* does not influence the results. Once we have analyzed a simple pulley, the results should be perfectly clear by inspection.

Determine the magnitude *T* of the tension in the supporting cable and the magnitude of the force on the pin at *A* for the jib crane shown. The beam *AB* is a standard 0.5-m I-beam with a mass of 95 kg per meter of length



The weight of the beam is 95(103)(5)9.81 4.66 kN and acts through its center



$[\Sigma M_A = 0]$ $\quad (T\cos 25°)0.25 + (T\sin 25°)(5 - 0.12)$
$\qquad - 10(5 - 1.5 - 0.12) - 4.66(2.5 - 0.12) = 0$

from which $\quad T = 19.61 \text{ kN}$ *Ans.*

Equating the sums of forces in the *x*- and *y*-directions to zero gives

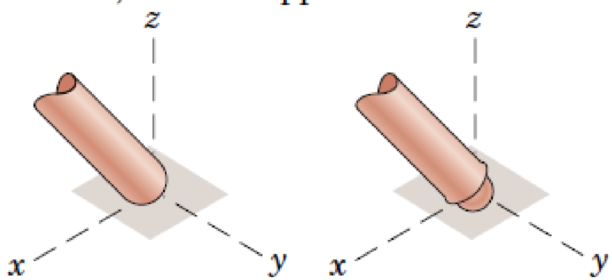
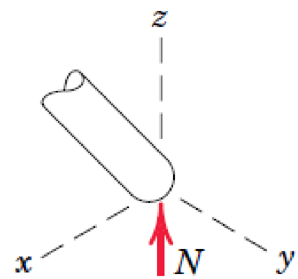
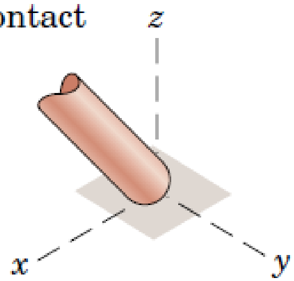
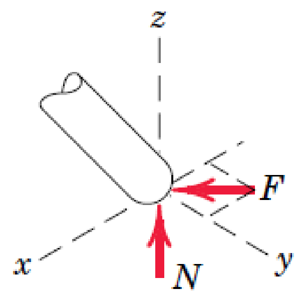
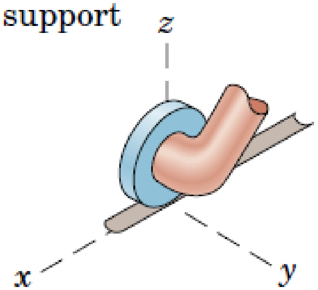
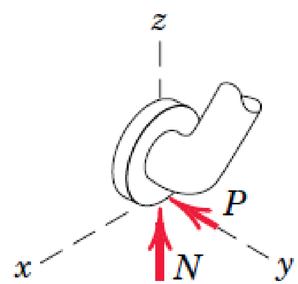
$[\Sigma F_x = 0]$ $\quad A_x - 19.61\cos 25° = 0 \quad A_x = 17.77 \text{ kN}$

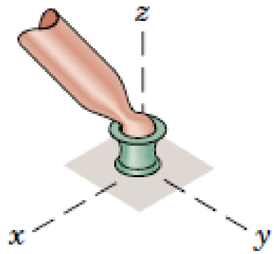
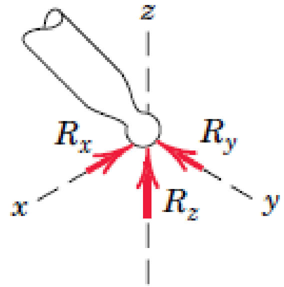
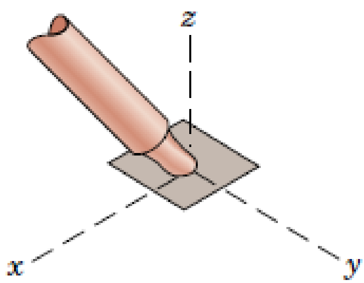
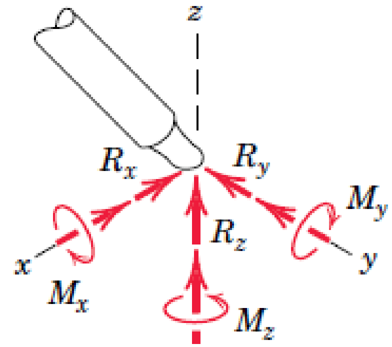
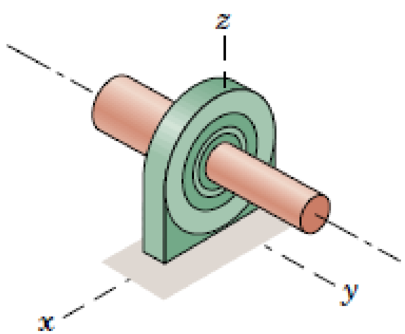
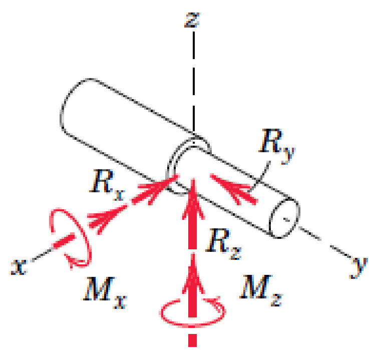
$[\Sigma F_y = 0]$ $\quad A_y + 19.61\sin 25° - 4.66 - 10 = 0 \quad A_y = 6.37 \text{ kN}$

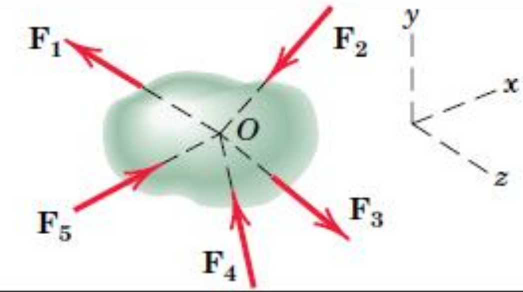
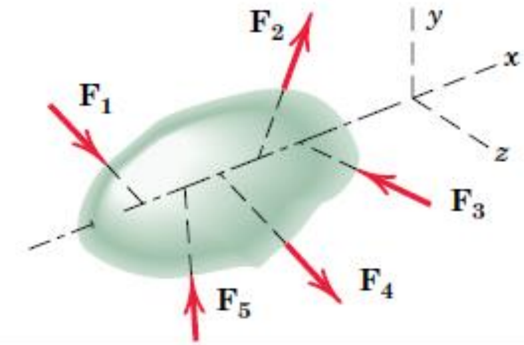
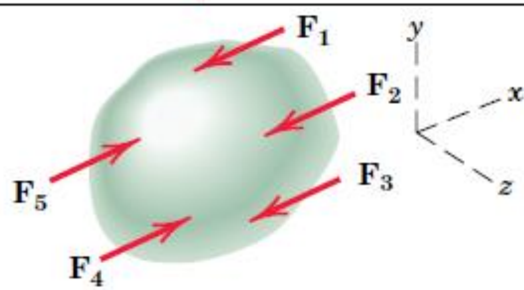
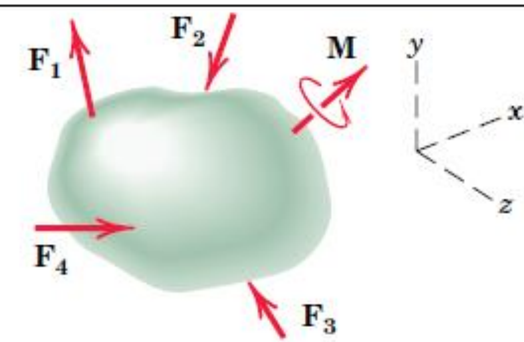
$[A = \sqrt{A_x^{\,2} + A_y^{\,2}}]$ $\quad A = \sqrt{(17.77)^2 + (6.37)^2} = 18.88 \text{ kN}$ *Ans.*

The equilibrium of structural components such as these shell-like panels is an issue both during and after construction. This structure will be used to house airships.

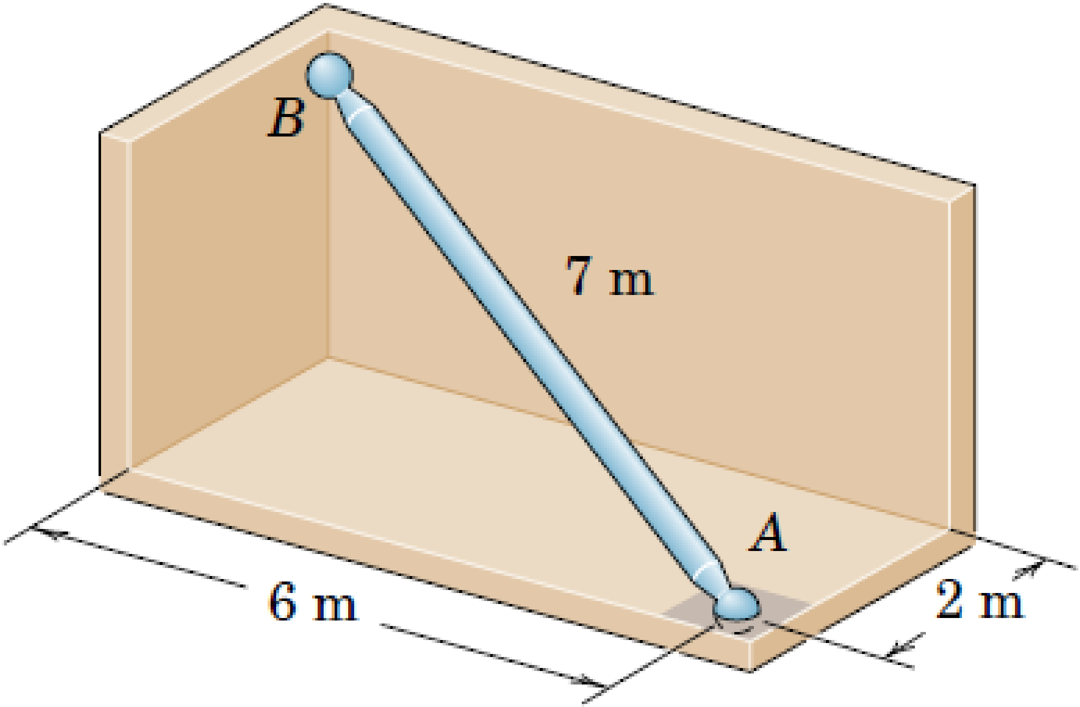
MODELING THE ACTION OF FORCES IN THREE-DIMENSIONAL ANALYSIS

Type of Contact and Force Origin	Action on Body to Be Isolated
1. Member in contact with smooth surface, or ball-supported member 	 Force must be normal to the surface and directed toward the member.
2. Member in contact with rough surface 	 The possibility exists for a force F tangent to the surface (friction force) to act on the member, as well as a normal force N.
3. Roller or wheel support with lateral constraint 	 A lateral force P exerted by the guide on the wheel can exist, in addition to the normal force N.

4. Ball-and-socket joint  	 A ball-and-socket joint free to pivot about the center of the ball can support a force **R** with all three components.
5. Fixed connection (embedded or welded)  	 In addition to three components of force, a fixed connection can support a couple **M** represented by its three components.
6. Thrust-bearing support  	 Thrust bearing is capable of supporting axial force R_y as well as radial forces R_x and R_z. Couples M_x and M_z must, in some cases, be assumed zero in order to provide statical determinacy.

CATEGORIES OF EQUILIBRIUM IN THREE DIMENSIONS		
Force System	Free-Body Diagram	Independent Equations
1. Concurrent at a point		$\Sigma F_x = 0$ $\Sigma F_y = 0$ $\Sigma F_z = 0$
2. Concurrent with a line		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_z = 0$ $\Sigma F_z = 0$
3. Parallel		$\Sigma F_x = 0$ $\quad \Sigma M_y = 0$ $\Sigma M_z = 0$
4. General		$\Sigma F_x = 0$ $\quad \Sigma M_x = 0$ $\Sigma F_y = 0$ $\quad \Sigma M_y = 0$ $\Sigma F_z = 0$ $\quad \Sigma M_z = 0$

The uniform 7-m steel shaft has a mass of 200 kg and is supported by a ball and-socket joint at *A* in the horizontal floor. The ball end *B* rests against the smooth vertical walls as shown. Compute the forces exerted by the walls and the floor on the ends of the shaft



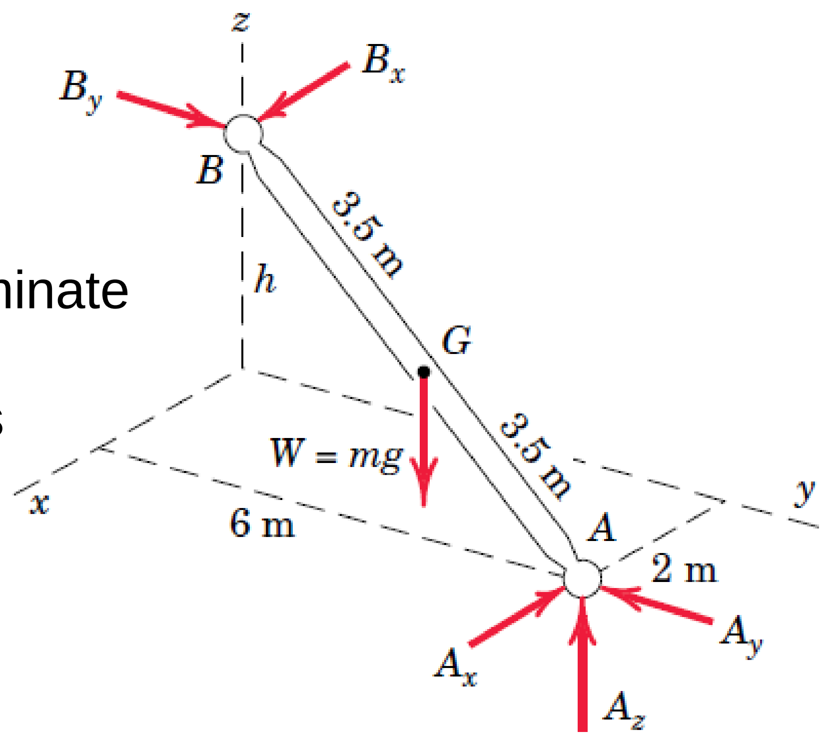
The free-body diagram of the shaft is first drawn where the contact forces acting on the shaft at *B* are shown normal to the wall surfaces.

In addition to the weight

$$W = mg = 200(9.81) = 1962 \text{ N},$$

The vertical position of *B*

$$7 = \sqrt{2^2 + 6^2 + h^2},\ h = 3 \text{ m}$$



We will use *A* as a moment center to eliminate reference to the forces at *A*. The position vectors needed to compute the moments about *A* are

$$\mathbf{r}_{AG} = -1\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k} \text{ m} \qquad \text{and} \qquad \mathbf{r}_{AB} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k} \text{ m}$$

$[\Sigma \mathbf{M}_A = \mathbf{0}]$ $\qquad \mathbf{r}_{AB} \times (\mathbf{B}_x + \mathbf{B}_y) + \mathbf{r}_{AG} \times \mathbf{W} = \mathbf{0}$

$$(-2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}) \times (B_x\mathbf{i} + B_y\mathbf{j}) + (-\mathbf{i} - 3\mathbf{j} + 1.5\mathbf{k}) \times (-1962\mathbf{k}) = \mathbf{0}$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2 & -6 & 3 \\ B_x & B_y & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & -3 & 1.5 \\ 0 & 0 & -1962 \end{vmatrix} = \mathbf{0}$$

$$(-3B_y + 5890)\mathbf{i} + (3B_x - 1962)\mathbf{j} + (-2B_y + 6B_x)\mathbf{k} = \mathbf{0}$$

Equating the coefficients of **i**, **j**, and **k** to zero and solving give

$$B_x = 654 \text{ N} \qquad \text{and} \qquad B_y = 1962 \text{ N}$$

The forces at A are easily determined by

$[\Sigma \mathbf{F} = \mathbf{0}]$ $\qquad (654 - A_x)\mathbf{i} + (1962 - A_y)\mathbf{j} + (-1962 + A_z)\mathbf{k} = \mathbf{0}$

and $\qquad A_x = 654 \text{ N} \qquad A_y = 1962 \text{ N} \qquad A_z = 1962 \text{ N}$

Finally $\qquad A = \sqrt{A_x{}^2 + A_y{}^2 + A_z{}^2}$

$$= \sqrt{(654)^2 + (1962)^2 + (1962)^2} = 2850 \text{ N}$$

Scalar solution. Evaluating the scalar moment equations about axes through A parallel, respectively, to the x- and y-axes, gives

$$[\Sigma M_{A_x} = 0] \qquad 1962(3) - 3B_y = 0 \qquad B_y = 1962 \text{ N}$$

$$[\Sigma M_{A_y} = 0] \qquad -1962(1) + 3B_x = 0 \qquad B_x = 654 \text{ N}$$

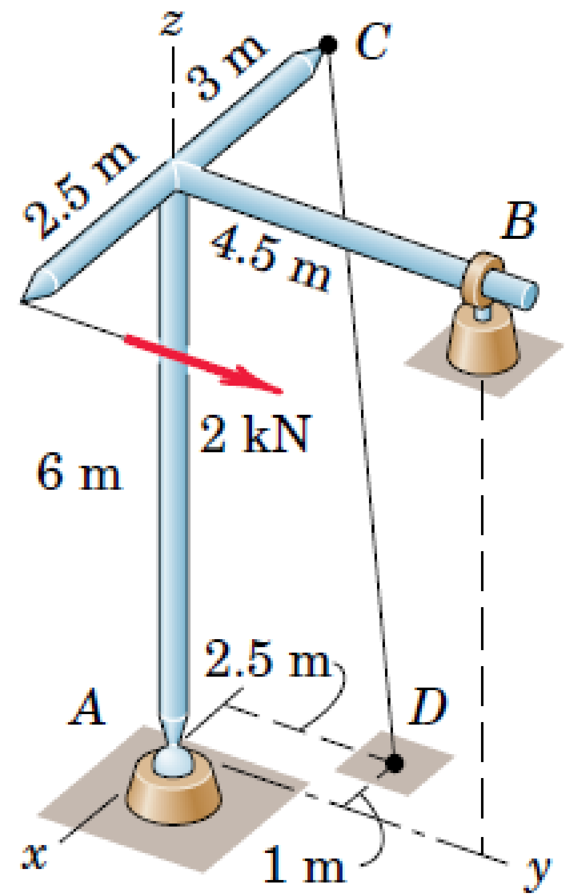
The force equations give, simply,

$$[\Sigma F_x = 0] \qquad -A_x + 654 = 0 \qquad A_x = 654 \text{ N}$$

$$[\Sigma F_y = 0] \qquad -A_y + 1962 = 0 \qquad A_y = 1962 \text{ N}$$

$$[\Sigma F_z = 0] \qquad A_z - 1962 = 0 \qquad A_z = 1962 \text{ N}$$

The welded tubular frame is secured to the horizontal *x*-*y* plane by a socket joint at *A* and receives support from the loose-fitting ring at *B*. Under the action of the 2-kN load, rotation about a line from *A* to *B* is prevented by the cable *CD*, and the frame is stable in the position shown. Neglect the weight of the frame compared with the applied load and determine the tension *T* in the cable, the reaction at the ring, and the reaction components at *A*.



The direction of *AB* is specified by the unit

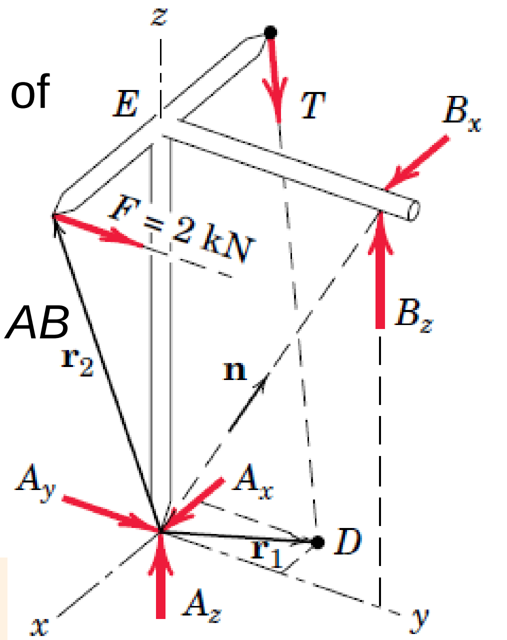
$$\text{vector } \mathbf{n} = \frac{1}{\sqrt{6^2 + 4.5^2}}(4.5\mathbf{j} + 6\mathbf{k}) = \frac{1}{5}(3\mathbf{j} + 4\mathbf{k}).$$

The moment of **T** about *AB* is the component in the direction of *AB* of the vector moment about the point *A*

$$\mathbf{r}_1 \times \underline{\mathbf{T}} \cdot \mathbf{n}$$

and equals Similarly the moment of the applied load *F* about *AB*

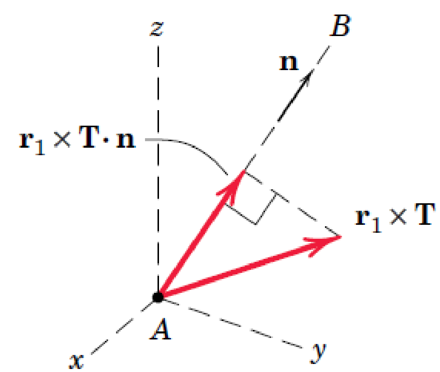
$$\mathbf{r}_2 \times \mathbf{F} \cdot \mathbf{n}.$$



With $CD = \sqrt{46.2}$ m, the vector expressions for $\mathbf{T}$, $\mathbf{F}$, $\mathbf{r}_1$, and $\mathbf{r}_2$ are

$$\mathbf{T} = \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \qquad \mathbf{F} = 2\mathbf{j} \text{ kN}$$

$$\mathbf{r}_1 = -\mathbf{i} + 2.5\mathbf{j} \text{ m} \qquad \mathbf{r}_2 = 2.5\mathbf{i} + 6\mathbf{k} \text{ m}$$



The moment equation now becomes

$$[\Sigma M_{AB} = 0] \quad (-\mathbf{i} + 2.5\mathbf{j}) \times \frac{T}{\sqrt{46.2}}(2\mathbf{i} + 2.5\mathbf{j} - 6\mathbf{k}) \cdot \tfrac{1}{5}(3\mathbf{j} + 4\mathbf{k})$$

$$+ (2.5\mathbf{i} + 6\mathbf{k}) \times (2\mathbf{j}) \cdot \tfrac{1}{5}(3\mathbf{j} + 4\mathbf{k}) = \mathbf{0}$$

Completion of the vector operations gives

$$-\frac{48T}{\sqrt{46.2}} + 20 = 0 \qquad T = 2.83 \text{ kN} \qquad \textit{Ans.}$$

and the components of T become

$$T_x = 0.833 \text{ kN} \qquad T_y = 1.042 \text{ kN} \qquad T_z = -2.50 \text{ kN}$$

We may find the remaining unknowns by moment and force summations as follows:

$[\Sigma M_z = 0]$	$2(2.5) - 4.5B_x - 1.042(3) = 0$	$B_x = 0.417$ kN	*Ans.*
$[\Sigma M_x = 0]$	$4.5B_z - 2(6) - 1.042(6) = 0$	$B_z = 4.06$ kN	*Ans.*
$[\Sigma F_x = 0]$	$A_x + 0.417 + 0.833 = 0$	$A_x = -1.250$ kN	*Ans.*
$[\Sigma F_y = 0]$	$A_y + 2 + 1.042 = 0$	$A_y = -3.04$ kN	*Ans.*
$[\Sigma F_z = 0]$	$A_z + 4.06 - 2.50 = 0$	$A_z = -1.556$ kN	*Ans.*

Structures:

Difference between trusses, frames and beams, Assumptions followed in the analysis of structures; 2D truss; Method of joints; Method of section;

Structural Analysis

Engineering Structure

An engineering structure is any connected system of members built to support or transfer forces and to safely withstand the loads applied to it.

Structures can be classified according to their geometrical shape and the loads acting on them

Bar or a Rod: A slender structural element (cross-sectional dimensions much smaller than its length) that is loaded solely in the axial direction (tension or compression)

Beam: If the same geometrical object is subjected to a load perpendicular to its axis

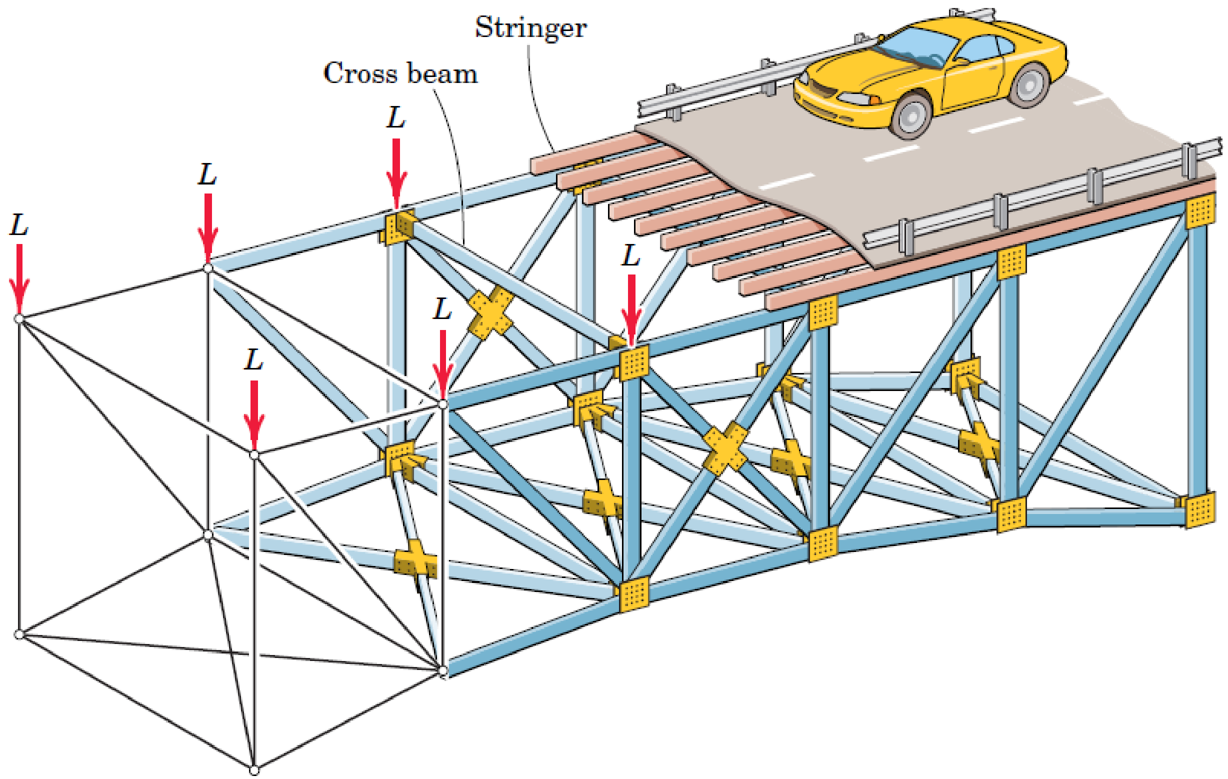
A curved beam is usually designated as an arch.

Structures consisting of inclined, rigidly joined beams are called frames

A plane structure with a thickness much smaller than its characteristic in-plane length is called a disk if it is solely loaded in its plane, e.g., by in-plane forces

If the same geometrical structure is loaded perpendicularly to its mid-plane it is called a plate.

If such a structure is curved it is a shell



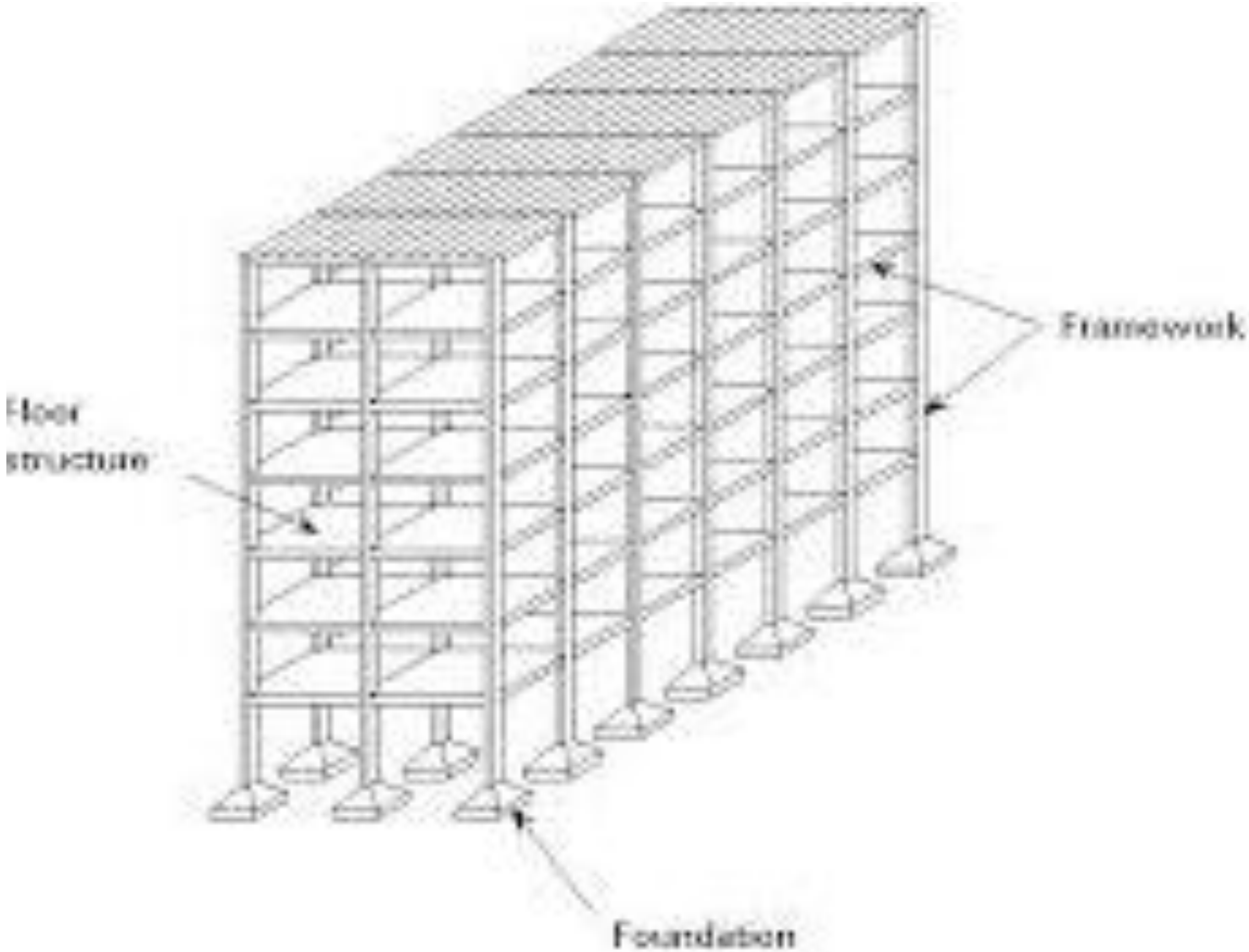


Figure 1 Main structural components of a multi-storey building

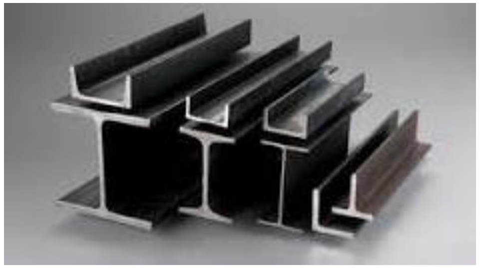
A truss is a structure composed of slender members that are connected at their ends by joints. The truss is one of the most important structures in engineering applications

A framework composed of members joined at their ends to form a rigid structure is called a *truss*.

Bridges, roof supports, derricks, and other such structures are common examples of trusses

Structural members commonly used are I-beams, channels, angles, bars, and special shapes which are fastened together at their ends by welding, riveted connections, or large bolts or pins

When the members of the truss lie essentially in a single plane, the truss is called a *plane truss*

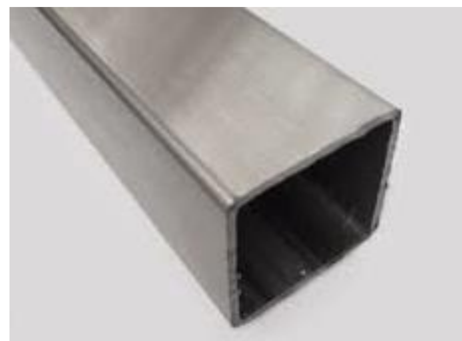


I-Beam

Channel

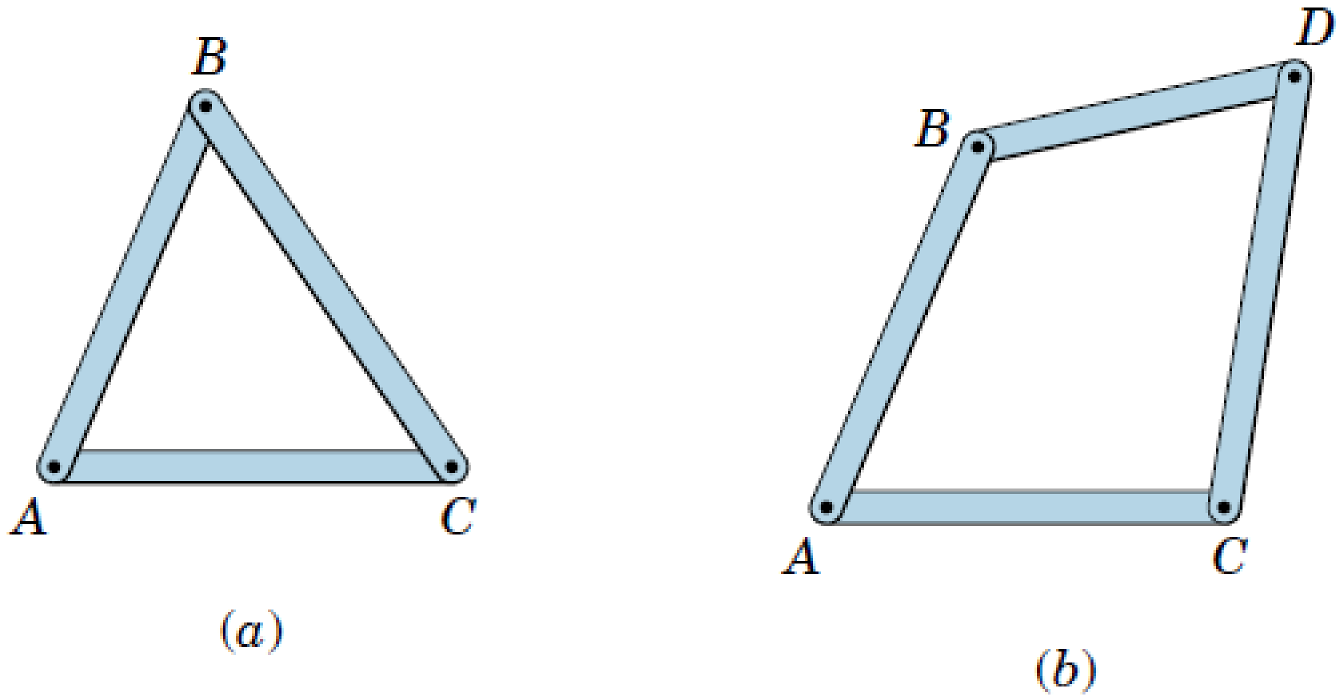
Angle

Bar



Box

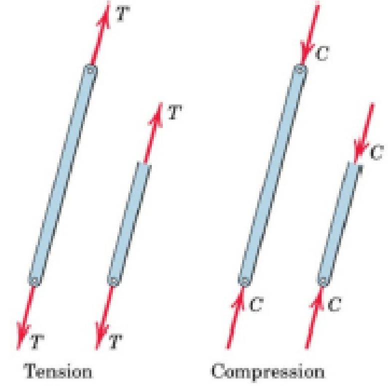
wire



The basic element of a plane truss is the triangle. Three bars joined by pins at their ends, Fig. *a*, constitute a rigid frame. The term *rigid* is used to mean non-collapsible and also to mean that deformation of the members due to induced internal strains is negligible.

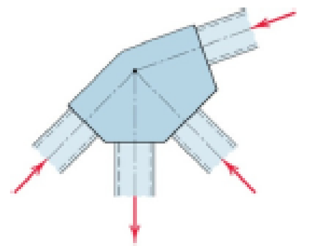
On the other hand, four or more bars pin-jointed to form a polygon of as many sides constitute a nonrigid frame

Structural Analysis: Plane Truss



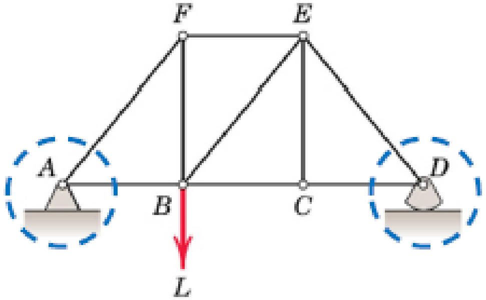
Basic Assumptions in Truss Analysis

- All members are two-force members.
- Weight of the members is small compared with the force it supports (weight may be considered at joints)
- No effect of bending on members even if weight is considered



- External forces are applied at the pin connections
- Welded or riveted connections
- Pin Joint if the member centerlines are concurrent at the joint

Common Practice in Large Trusses



- Roller/Rocker at one end. Why?
  - to accommodate deformations due to temperature changes and applied loads.
  - otherwise it will be a statically indeterminate truss

Structural Analysis: Plane Truss

Truss Analysis: Method of Joints

- Finding forces in members

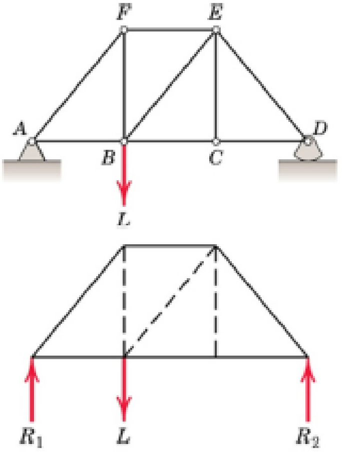
Method of Joints: Conditions of equilibrium are satisfied for the forces at each joint

– Equilibrium of concurrent forces at each joint

– only two independent equilibrium equations are involved

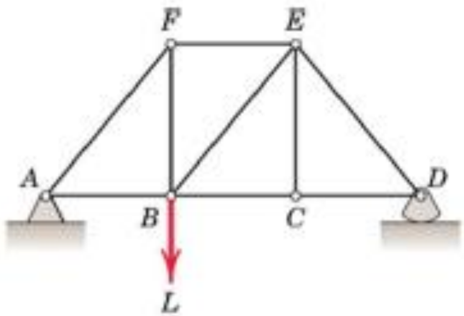
Steps of Analysis

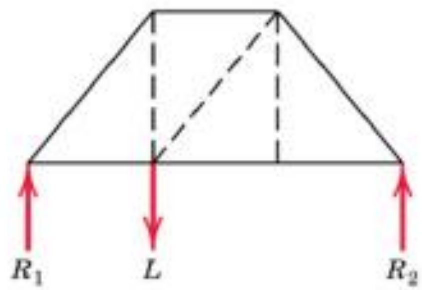
1. Draw Free Body Diagram of Truss
2. Determine external reactions by applying equilibrium equations to the whole truss
3. Perform the force analysis of the remainder of the truss by Method of Joints

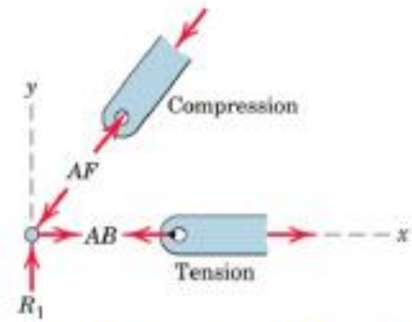


Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.







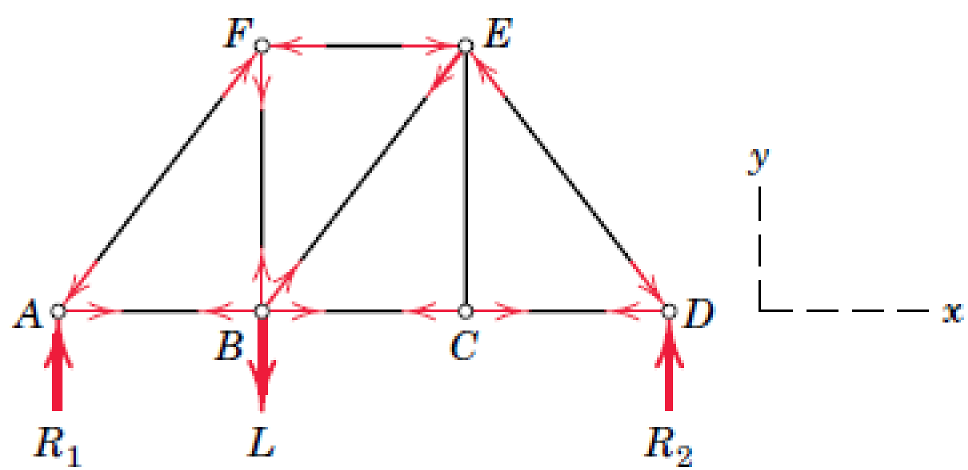
FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

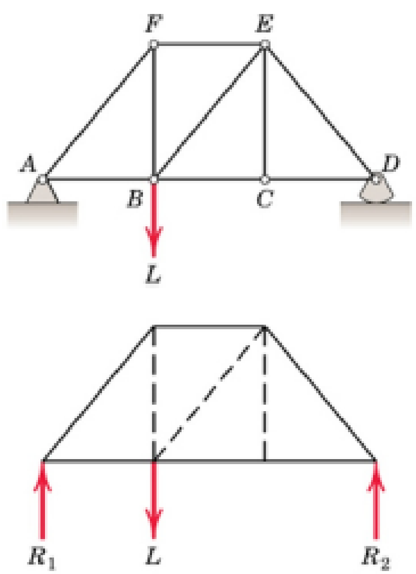
Magnitude of AF from $\Sigma F_y = 0$

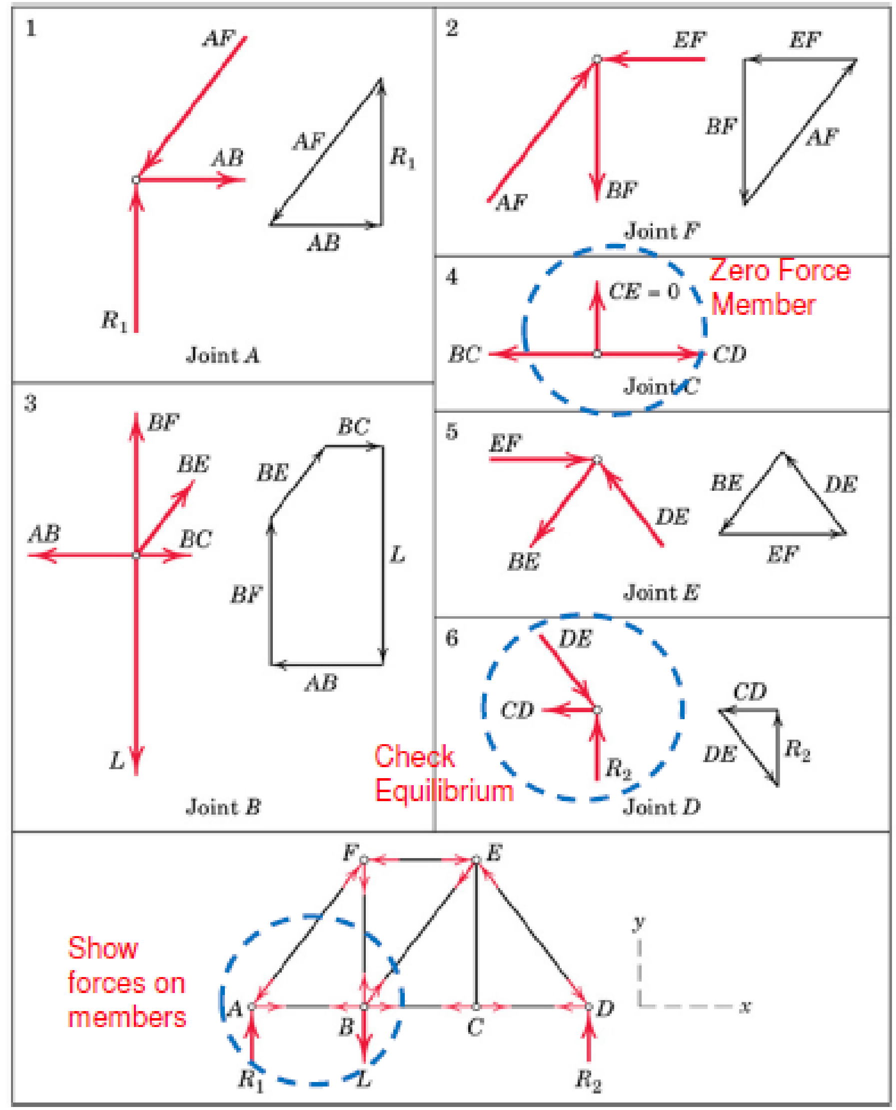
Magnitude of AB from $\Sigma F_x = 0$

Analyze joints F, B, C, E, & D in that



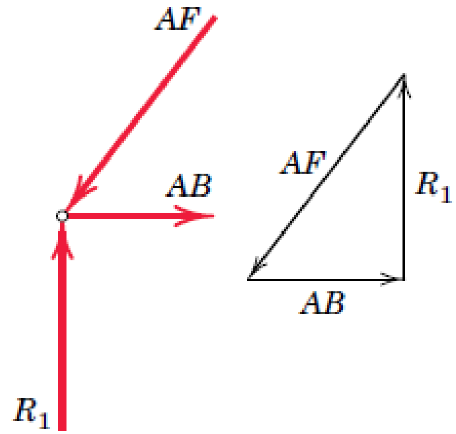
Method of Joints





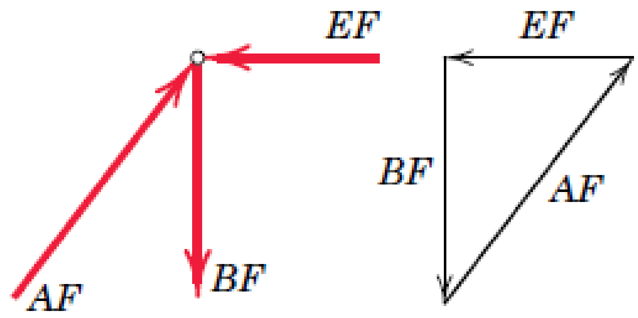
- Negative force if assumed sense is incorrect

1



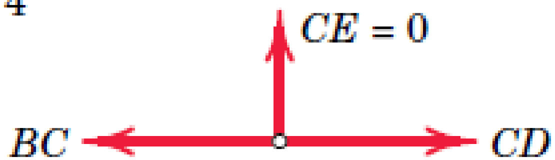
Joint A

2



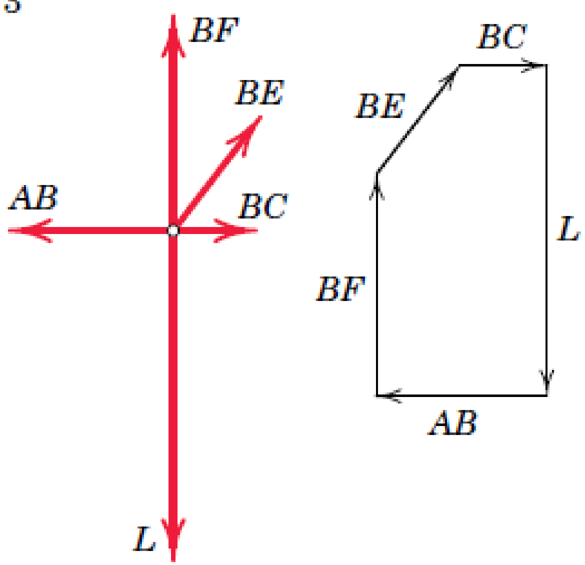
Joint F

4



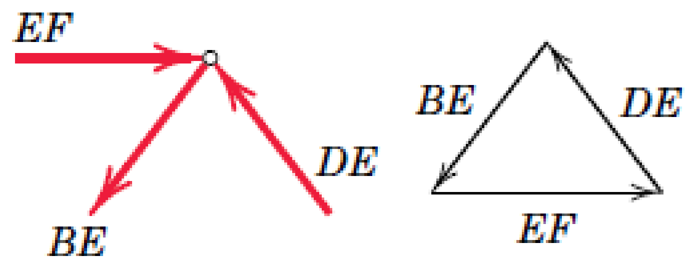
Joint C

3



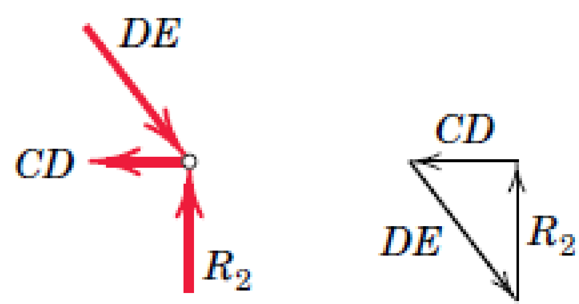
Joint B

5



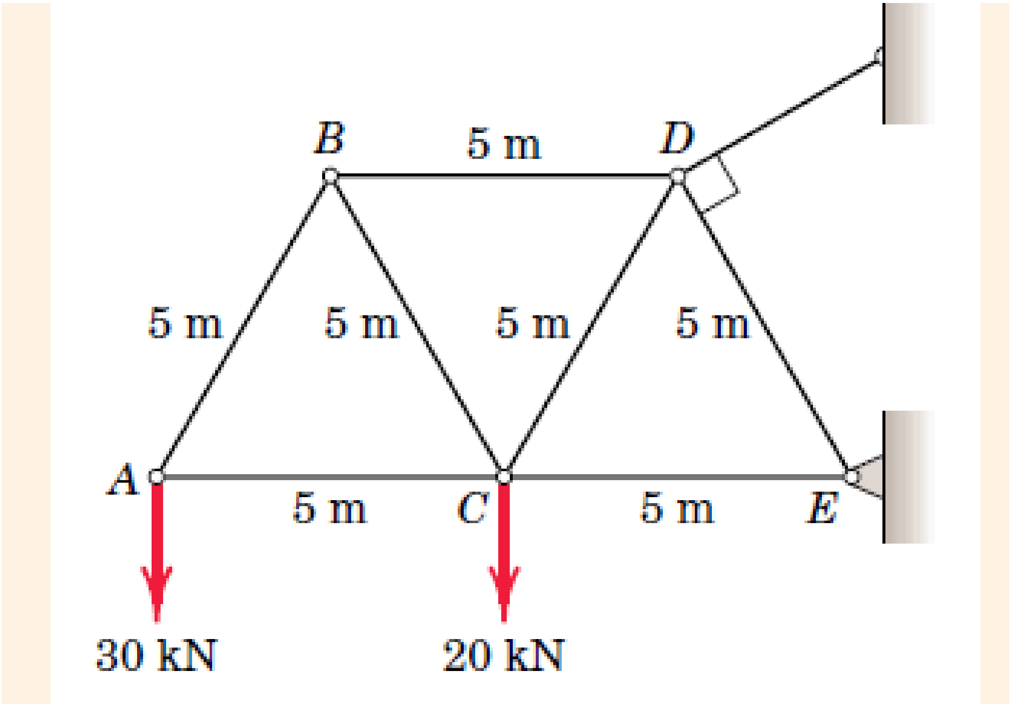
Joint E

6

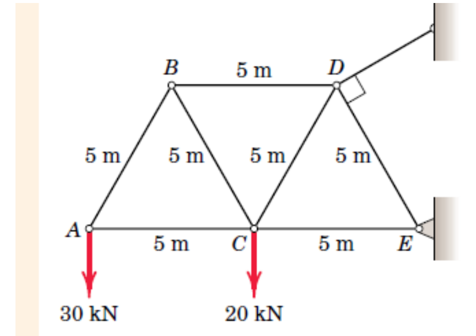


Joint D

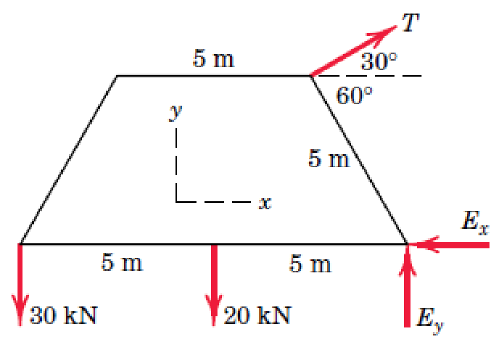
Compute the force in each member of the loaded cantilever truss by the method of joints.



The first step will be to compute the external forces at *D* and *E* from the free-body diagram of the truss as a whole.

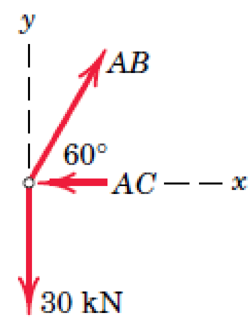


$[\Sigma M_E = 0]$	$5T - 20(5) - 30(10) = 0$	$T = 80\text{ kN}$
$[\Sigma F_x = 0]$	$80 \cos 30^\circ - E_x = 0$	$E_x = 69.3\text{ kN}$
$[\Sigma F_y = 0]$	$80 \sin 30^\circ + E_y - 20 - 30 = 0$	$E_y = 10\text{ kN}$



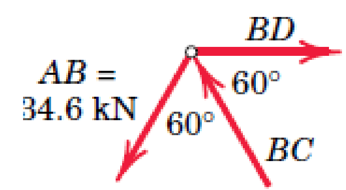
Next we draw free-body diagrams showing the forces acting on each of the connecting pins. The correctness of the assigned directions of the forces is verified when each joint is considered in sequence.

$[\Sigma F_y = 0]$	$0.866AB - 30 = 0$	$AB = 34.6\text{ kN } T$
$[\Sigma F_x = 0]$	$AC - 0.5(34.6) = 0$	$AC = 17.32\text{ kN } C$

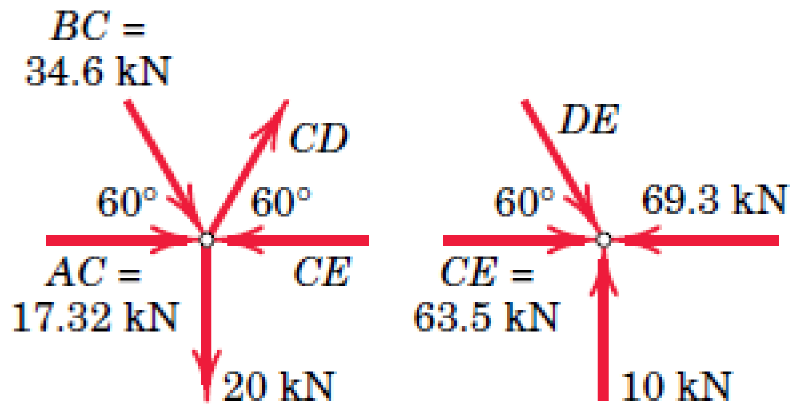


Joint *A*

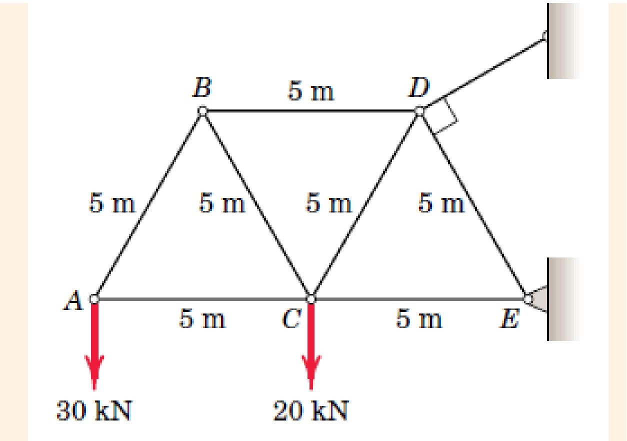
$[\Sigma F_y = 0]$	$0.866BC - 0.866(34.6) = 0$	$BC = 34.6\text{ kN } C$
$[\Sigma F_x = 0]$	$BD - 2(0.5)(34.6) = 0$	$BD = 34.6\text{ kN } T$



Joint *B*



Joint C Joint E



$[\Sigma F_y = 0]$ $\quad 0.866CD - 0.866(34.6) - 20 = 0$

$CD = 57.7 \text{ kN } T$

$[\Sigma F_x = 0]$ $\quad CE - 17.32 - 0.5(34.6) - 0.5(57.7) = 0$

$CE = 63.5 \text{ kN } C$

Finally, from joint E there results

$[\Sigma F_y = 0]$ $\quad 0.866DE = 10 \qquad DE = 11.55 \text{ kN } C$

and the equation $\Sigma F_x = 0$ checks.

Statically Determinate Trusses

A structure is called statically determinate if the support reactions can be calculated from the three equilibrium conditions

When more number of members/supports are present than are needed to prevent collapse/stability

→ Statically Indeterminate Truss

- cannot be analyzed using equations of equilibrium alone!
- additional members or supports which are not necessary for maintaining the equilibrium configuration → Redundant

Internal and External Redundancy

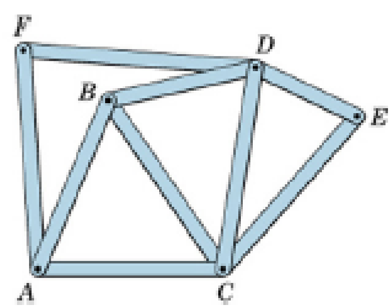
Extra Supports than required → External Redundancy

– Degree of indeterminacy from available equilibrium equations

Extra Members than required → Internal Redundancy

(truss must be removed from the supports to calculate internal redundancy)

– Is this truss statically determinate internally?



Truss is statically determinate internally if $m + 3 = 2j$

$m = 2j - 3$ m is number of members, and j is number of joints in truss

Internal Redundancy or
Degree of Internal Static Indeterminacy

Extra Members than required → Internal Redundancy

Equilibrium of each joint can be specified by two scalar force equations → 2j equations for a truss with "j" number of joints
→ Known Quantities

For a truss with "m" number of two force members, and maximum 3 unknown support reactions → Total Unknowns = m + 3
("m" member forces and 3 reactions for externally determinate truss)

Therefore:

$m + 3 = 2j$ → Statically Determinate Internally

$m + 3 > 2j$ → Statically Indeterminate Internally

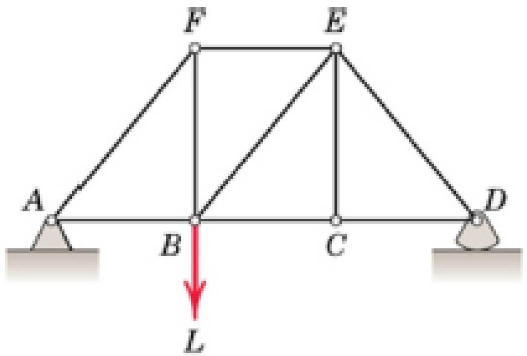
$m + 3 < 2j$ → Unstable Truss

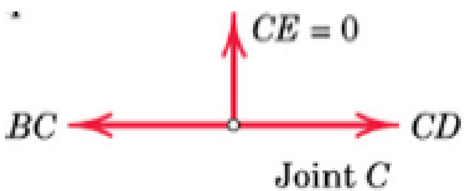
A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss

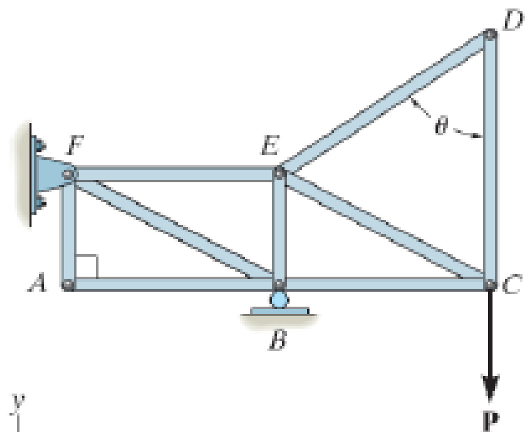
Why to Provide Redundant Members?

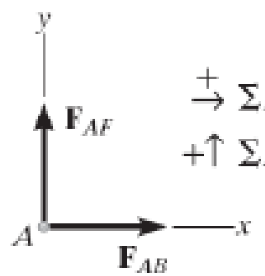
- To maintain alignment of two members during construction
- To increase stability during construction
- To maintain stability during loading (Ex: to prevent buckling of compression members)
- To provide support if the applied loading is changed
- To act as backup members in case some members fail or require strengthening
- Analysis is difficult but possible

Zero Force Members



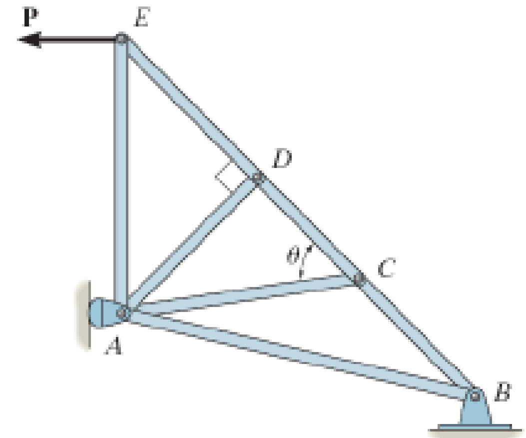


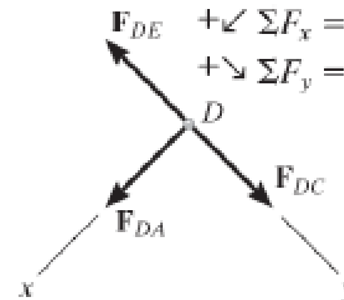




$\xrightarrow{+} \Sigma F_x = 0;\ \boxed{F_{AB} = 0}$

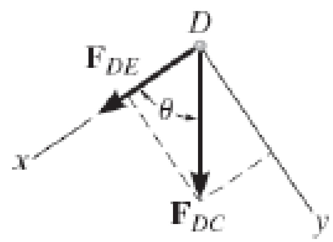
$+\uparrow \Sigma F_y = 0;\ \boxed{F_{AF} = 0}$





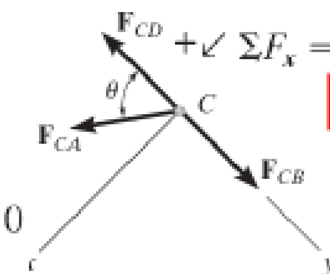
$+\swarrow \Sigma F_x = 0;\ \boxed{F_{DA} = 0}$

$+\searrow \Sigma F_y = 0;\ F_{DC} = F_{DE}$



$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ \boxed{F_{DC} = 0}$ since $\sin\theta \neq 0$

$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ \boxed{F_{DE} = 0}$



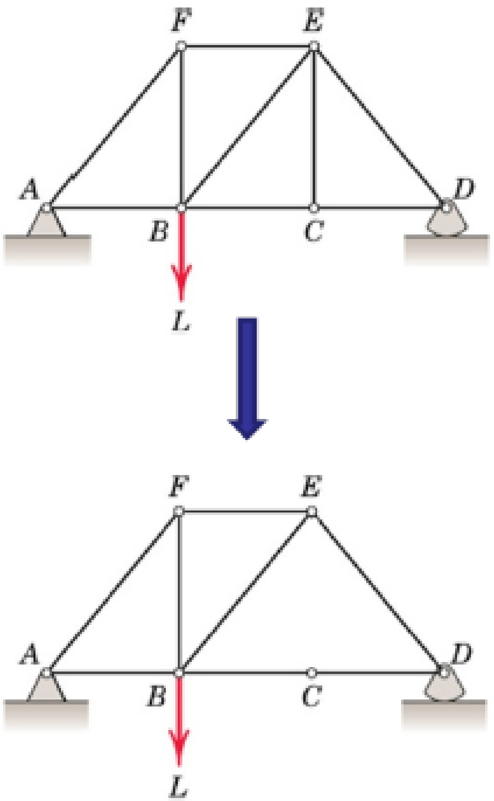
$+\swarrow \Sigma F_x = 0;\ F_{CA} \sin\theta = 0$

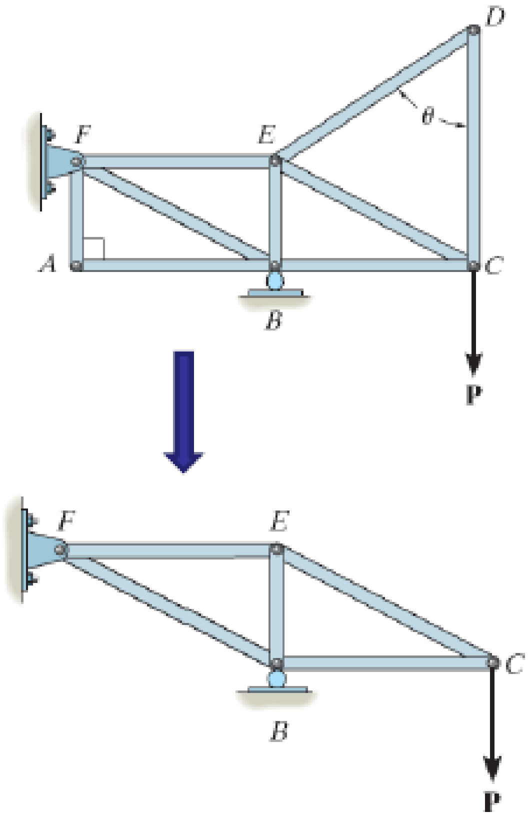
$\boxed{F_{CA} = 0}$

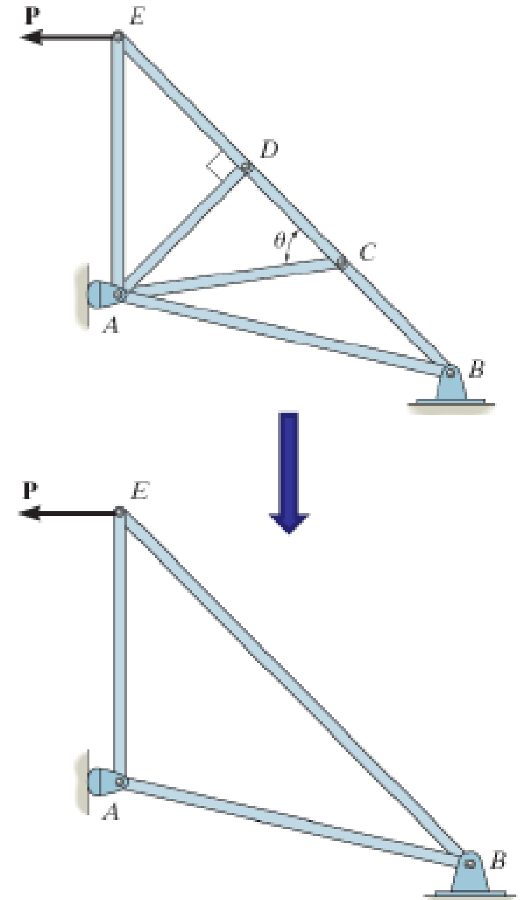
since $\sin\theta \neq 0$

$+\searrow \Sigma F_y = 0;$

$F_{CB} = F_{CD}$


F
E
A
D
B
C
L

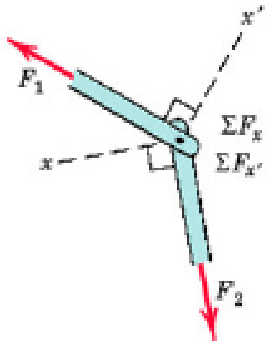

D
θ
F
E
A
C
B
P


P
E
D
θ
C
A
B

Zero Force Members: Conditions

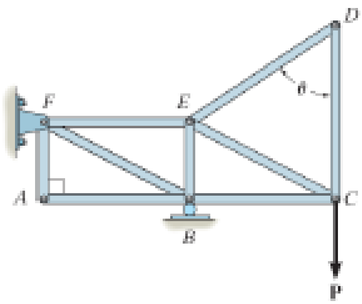
If only two non-collinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members

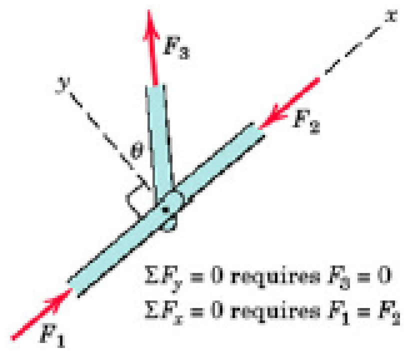
If three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint

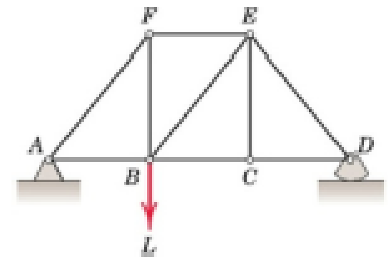


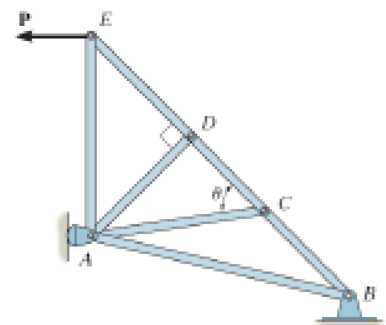
$\Sigma F_x = 0$ requires $F_1 = 0$

$\Sigma F_{x'} = 0$ requires $F_2 = 0$



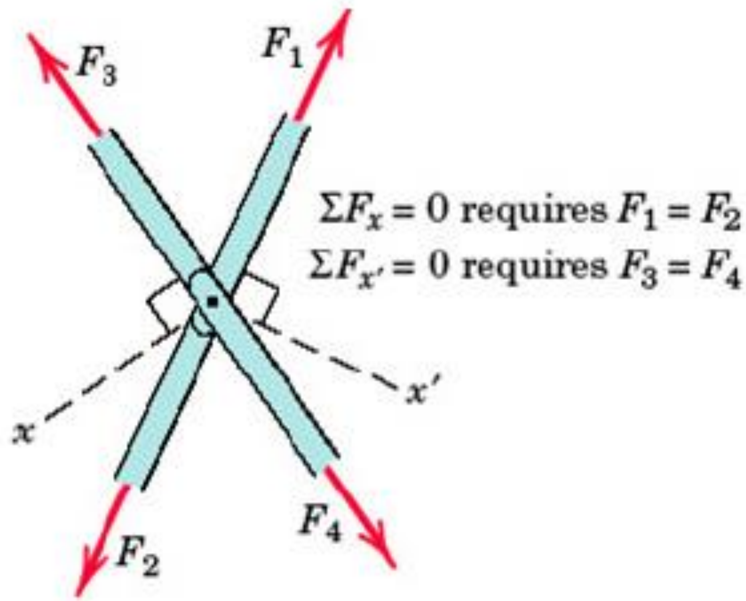




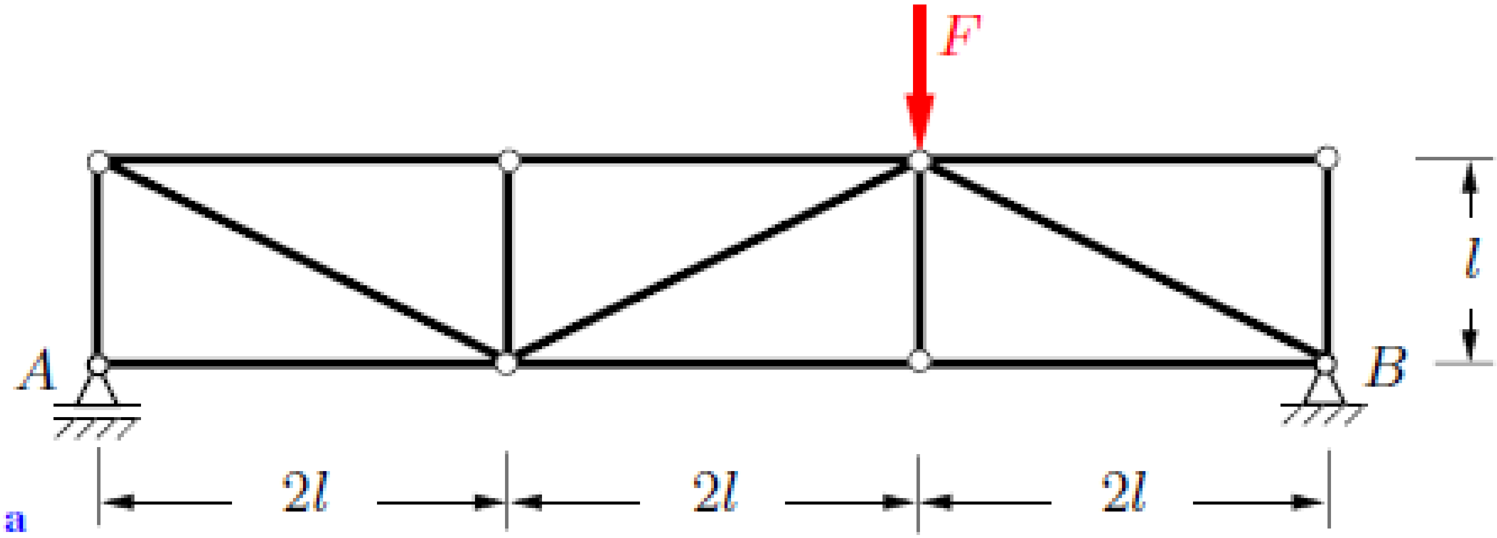


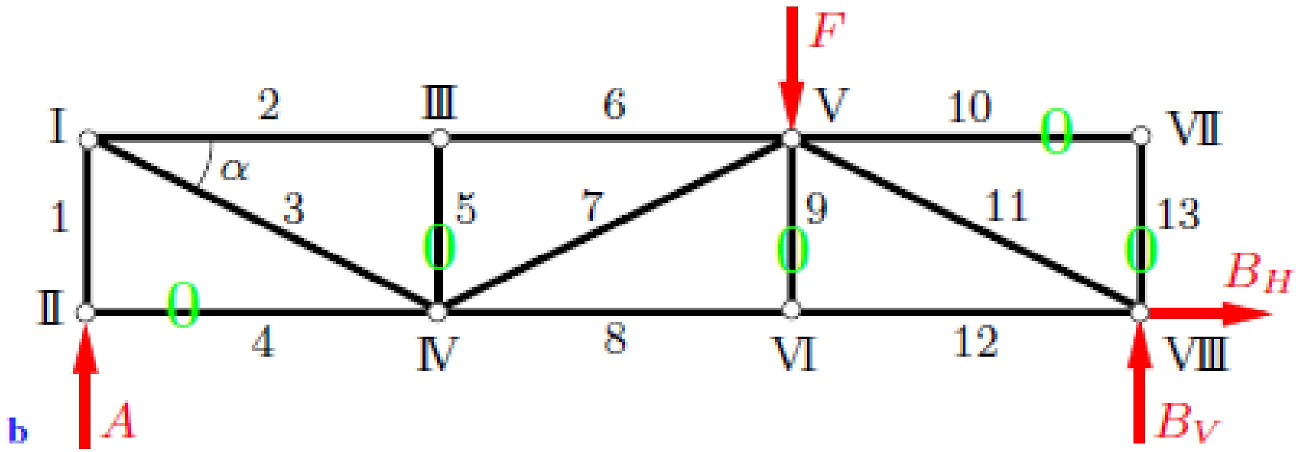
Special Condition

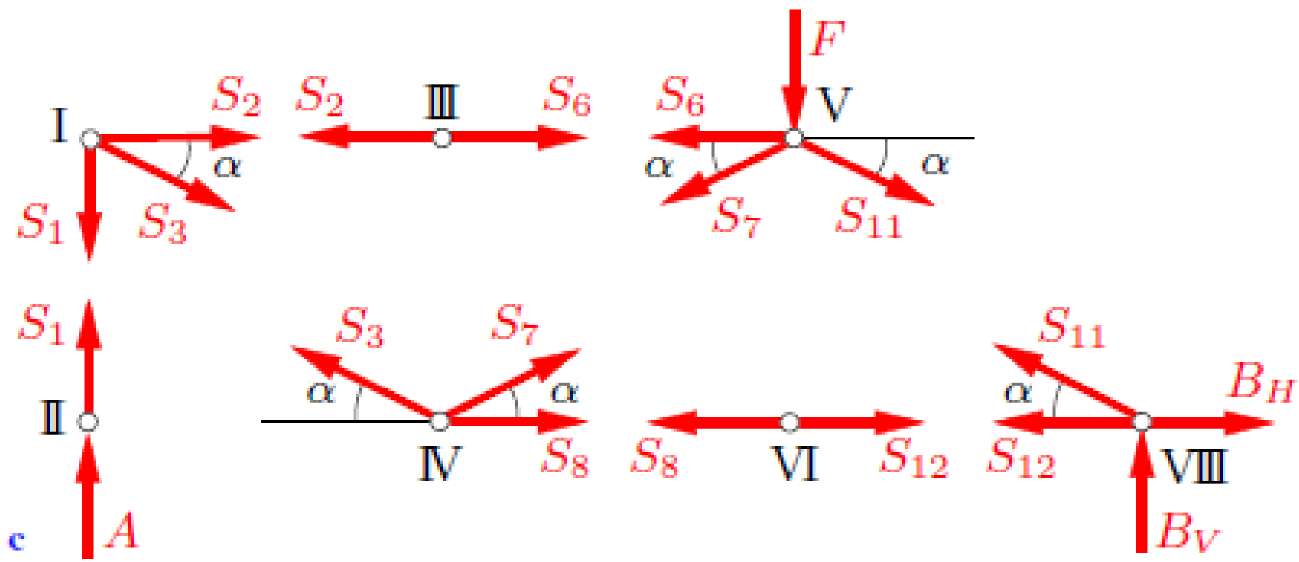
When two pairs of collinear members are joined as shown in figure, the forces in each pair must be equal and opposite.



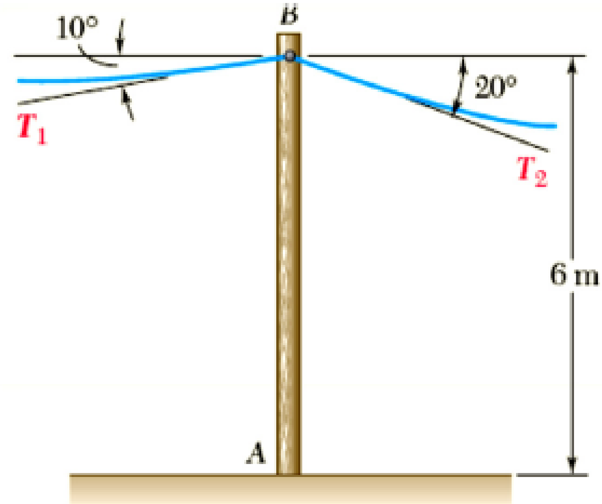
The truss shown in Fig. a is loaded by an external force F. Determine the forces at the supports and in the members of the truss




F
I
2
III
6
V
10
0
VII
α
1
3
0
5
7
9
0
11
13
0
B_H
II
0
4
IV
8
VI
12
VIII
b
A
B_V


F
S_2
S_2
III
S_6
S_6
V
I
α
α
α
S_1
S_3
S_7
S_{11}
S_1
S_3
S_7
S_{11}
B_H
α
α
α
II
IV
S_8
S_8
VI
S_{12}
S_{12}
VIII
c
A
B_V

Example problem 3

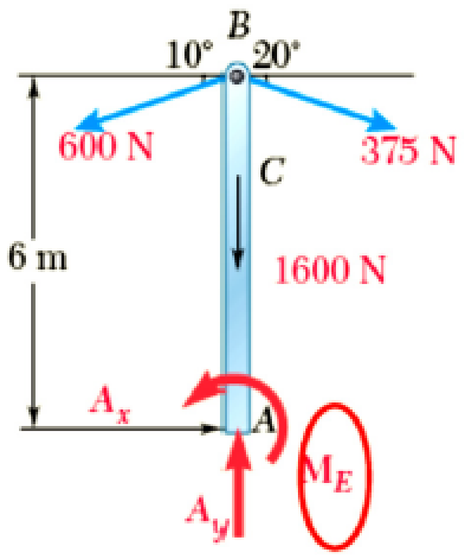


A 6-m telephone pole of 1600-N used to support the wires. Wires $T_1 = 600$ N and $T_2 = 375$ N.

Determine the reaction at the fixed end A.

SOLUTION:

- Create a free-body diagram for the telephone cable.
- Solve 3 equilibrium equations for the reaction force components and couple at A.



- Create a free-body diagram for the frame and cable.

- Solve 3 equilibrium equations for the reaction force components and couple.

$$\sum F_x = 0: \quad A_x + (375\text{N})\cos 20° - (600\text{N})\cos 10° = 0$$

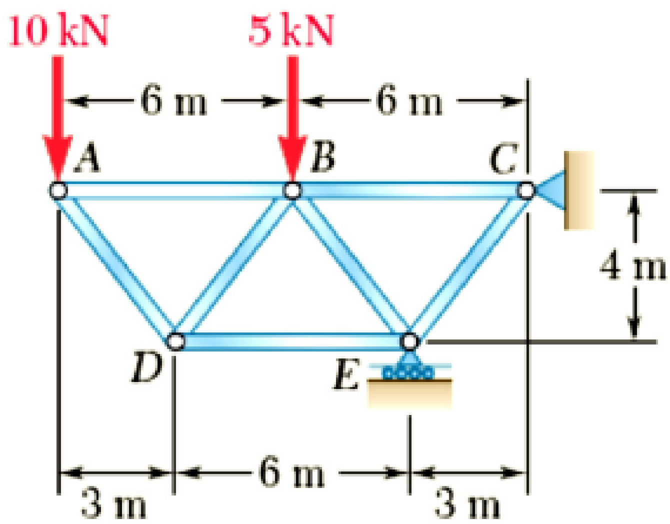
$$\boxed{A_x = +238.50\ \text{N}}$$

$$\sum F_y = 0: \quad A_y - 1600\text{N} - (600\text{N})\sin 10° - (375\ \text{N})\sin 20° = 0$$

$$\boxed{A_y = +1832.45\ \text{N}}$$

$$\sum M_A = 0: \ M_A + (600\text{N})\cos 10°(6m) - (375\text{N})\cos 20°$$
$$(6\text{m}) = 0$$

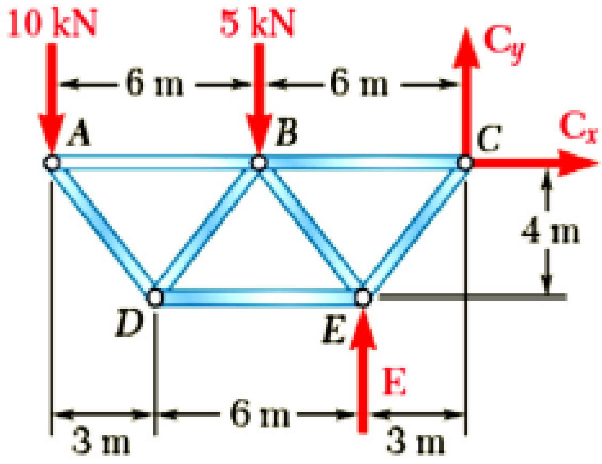
$$\boxed{M_A = +1431.00\text{N.m}}$$



Using the method of joints, determine the force in each member of the truss.

SOLUTION:

- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at *E* and *C*.
- Joint *A* is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.
- In succession, determine unknown member forces at joints *D*, *B*, and *E* from joint equilibrium requirements.
- All member forces and support reactions are known at joint *C*. However, the joint equilibrium requirements may be applied to check the results.



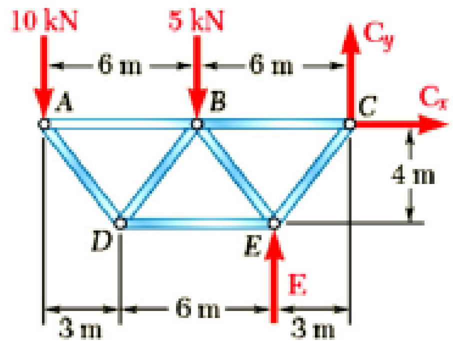
SOLUTION:

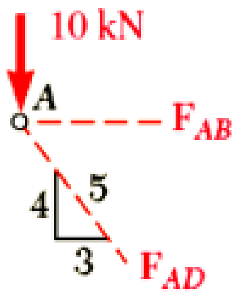
- Based on a free-body diagram of the entire truss, solve the 3 equilibrium equations for the reactions at E and C.

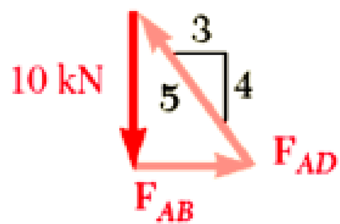
$$\sum M_C = 0$$
$$= (10\text{ kN})(12\text{ m}) + (5\text{ kN})(6\text{ m}) - E(3\text{ m})$$
$$\boxed{E = 50\text{kN} \uparrow}$$

$$\sum F_x = 0 = C_x \qquad \boxed{C_x = 0}$$

$$\sum F_y = 0 = -10\text{kN} - 5\text{ kN} + 50\text{ kN} + C_y$$
$$\boxed{C_y = 35\text{ kN} \downarrow}$$



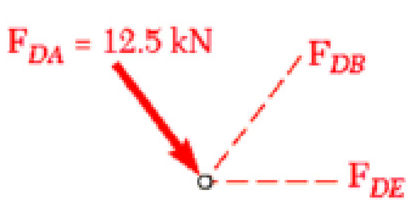


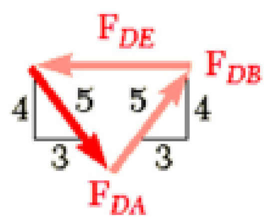


- Joint A is subjected to only two unknown member forces. Determine these from the joint equilibrium requirements.

$$\frac{10\text{ kN}}{4} = \frac{F_{AB}}{3} = \frac{F_{AD}}{5}$$

$$\boxed{\begin{aligned} F_{AB} &= 7.5\text{ kN } T \\ F_{AD} &= 12.5\text{ kN } C \end{aligned}}$$



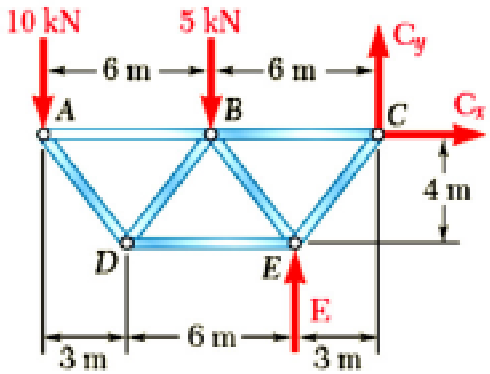


- There are now only two unknown member forces at joint D.

$$F_{DB} = F_{DA}$$

$$F_{DE} = 2\left(\tfrac{3}{5}\right)F_{DA}$$

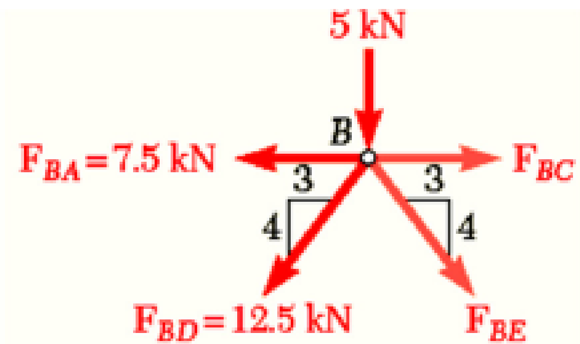
$$\boxed{\begin{aligned} F_{DB} &= 12.5\text{ kN } T \\ F_{DE} &= 15\text{ kN } C \end{aligned}}$$



- There are now only two unknown member forces at joint B. Assume both are in tension.

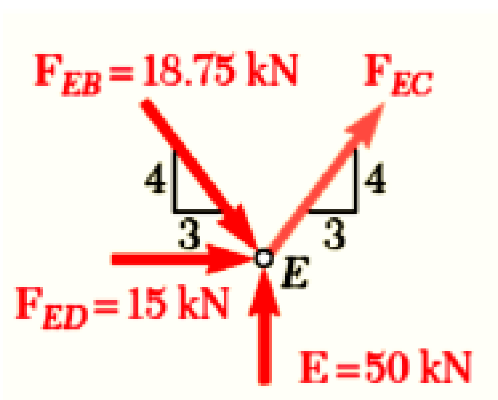
$$\sum F_y = 0 = -5\text{kN} - \tfrac{4}{5}(12\text{kN}) - \tfrac{4}{5}F_{BE}$$

$$F_{BE} = -18.75\text{ kN} \qquad \boxed{F_{BE} = 18.75\text{ kN } C}$$



$$\sum F_x = 0 = F_{BC} - 7.5\text{kN} - \tfrac{3}{5}(12.5\text{kN}) - \tfrac{3}{5}(18.75)$$

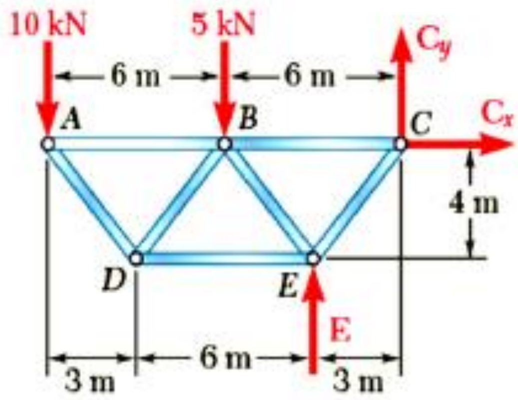
$$F_{BC} = +26.25\text{ kN} \qquad \boxed{F_{BC} = 26.25\text{ kN } T}$$



- There is one unknown member force at joint *E*. Assume the member is in tension.

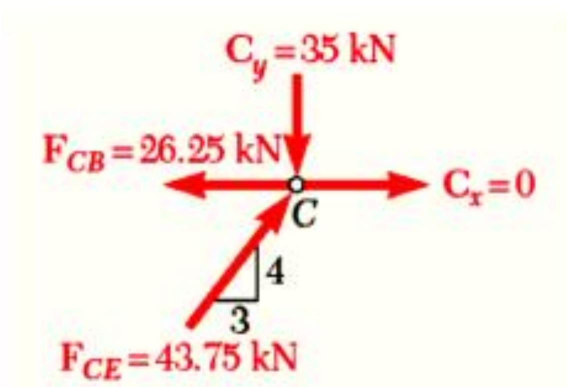
$$\sum F_x = 0 = \tfrac{3}{5}F_{EC} + 15\text{kN} + \tfrac{3}{5}(18.75\text{kN})$$

$$F_{EC} = -43.75\text{ kN} \qquad \boxed{F_{EC} = 43.75\text{ kN } C}$$



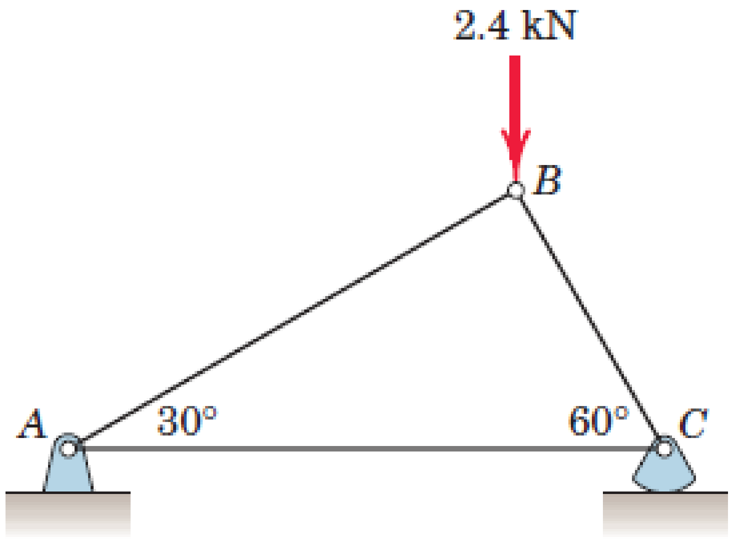
- All member forces and support reactions are known at joint C. However, the joint equilibrium requirements may be applied to check the results.

$$\sum F_x = -26.25 + \tfrac{3}{5}(43.75) = 0 \quad \text{(checks)}$$
$$\sum F_y = -35 + \tfrac{4}{5}(43.75) = 0 \quad \text{(checks)}$$

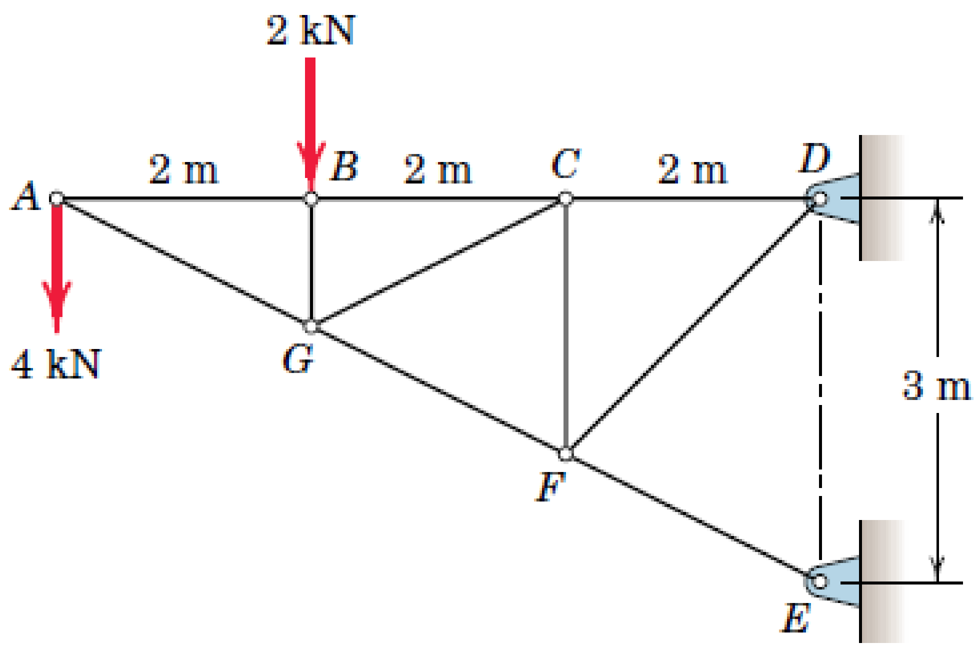


Assignment of Method of joints

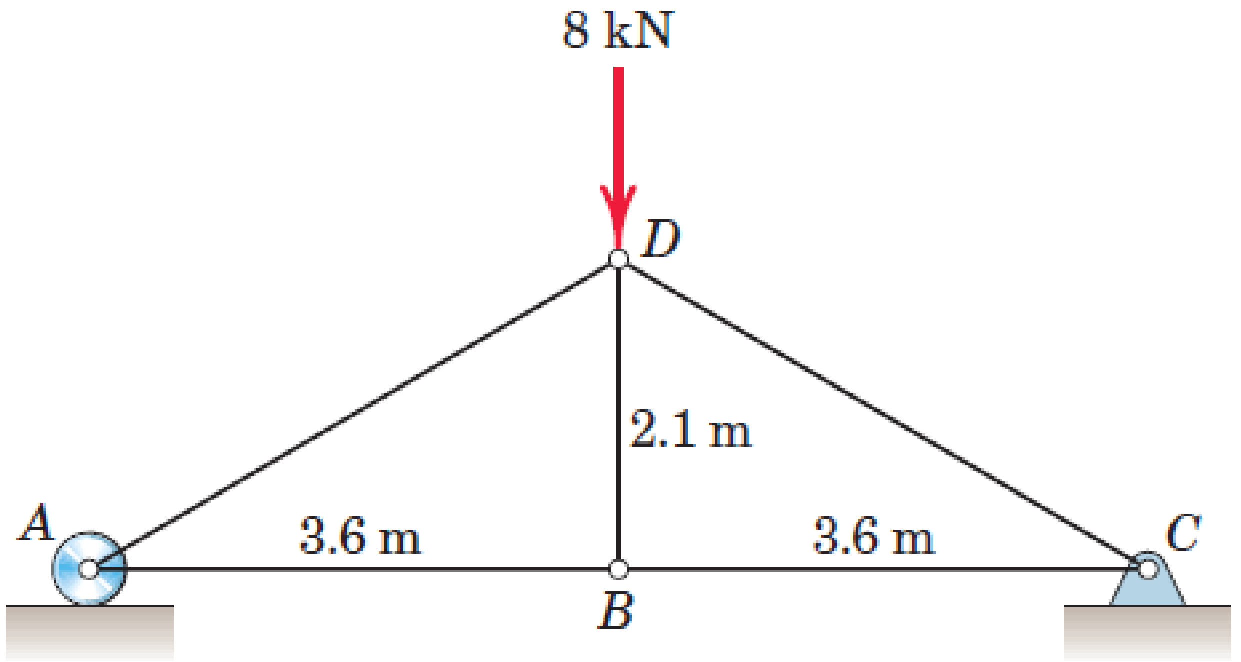
Determine the force in each member of the loaded truss. Explain why knowledge of the lengths of the members is unnecessary.



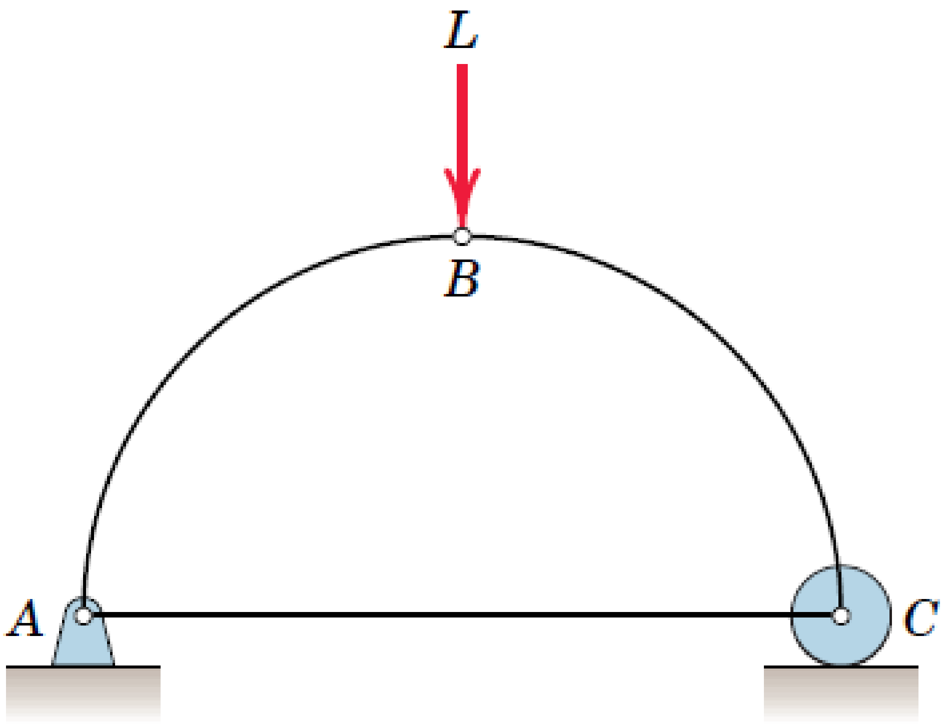
Calculate the forces in members *CG* and *CF* for the truss shown.



Determine the force in each member of the loaded truss. Identify any zero-force members by inspection.



Determine the force in member *AC* of the loaded truss. The two quarter-circular members act as two force members



Engineering Mechanics: ME101

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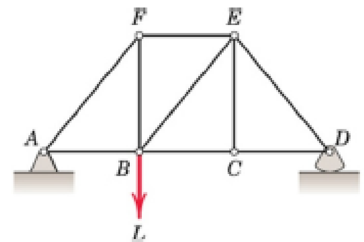
Department of Mechanical Engineering
Indian Institute of Technology Guwahati

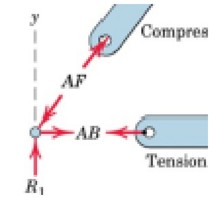
D Block : Room No 201 : Tel: 3434

Statics: Lecture 6

21th Jan 2016

Structural Analysis: Plane Truss





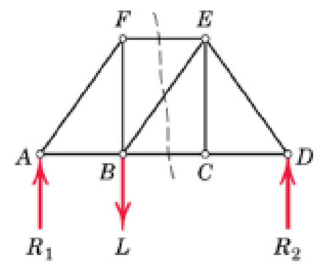
Method of Joints: only two of three equilibrium equations were applied at each joint because the procedures involve concurrent forces at each joint

→Calculations from joint to joint

→More time and effort required

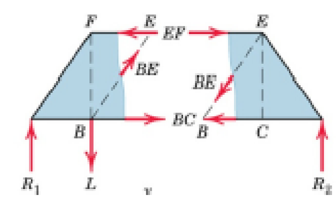
Method of Sections

Take advantage of the 3rd or moment equation of equilibrium by selecting an entire section of truss



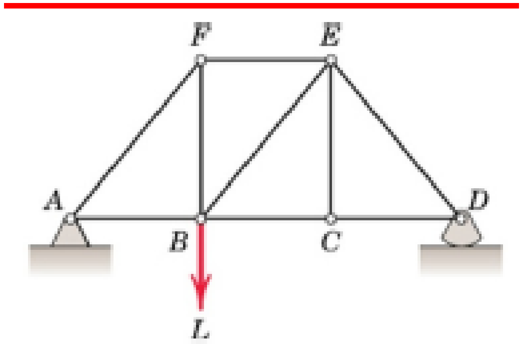
→Equilibrium under non-concurrent force system

→Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations



Method of Sections

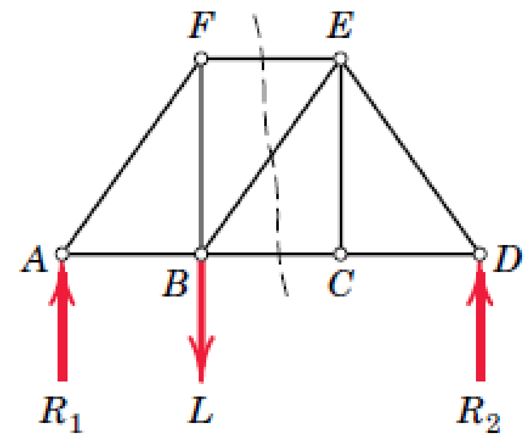
Find out the reactions from equilibrium of whole truss



To find force in member BE

Cut an imaginary section (dotted line)

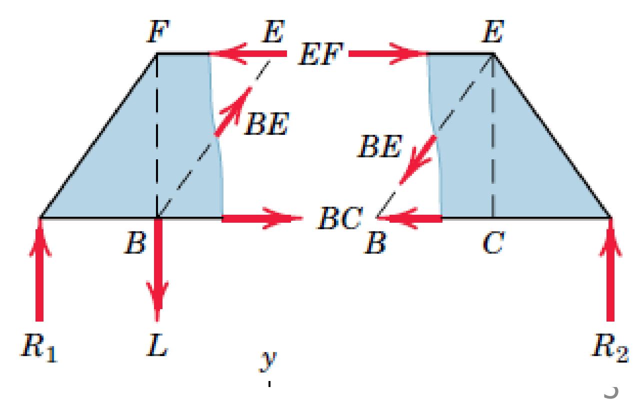
Each side of the truss section should remain in equilibrium



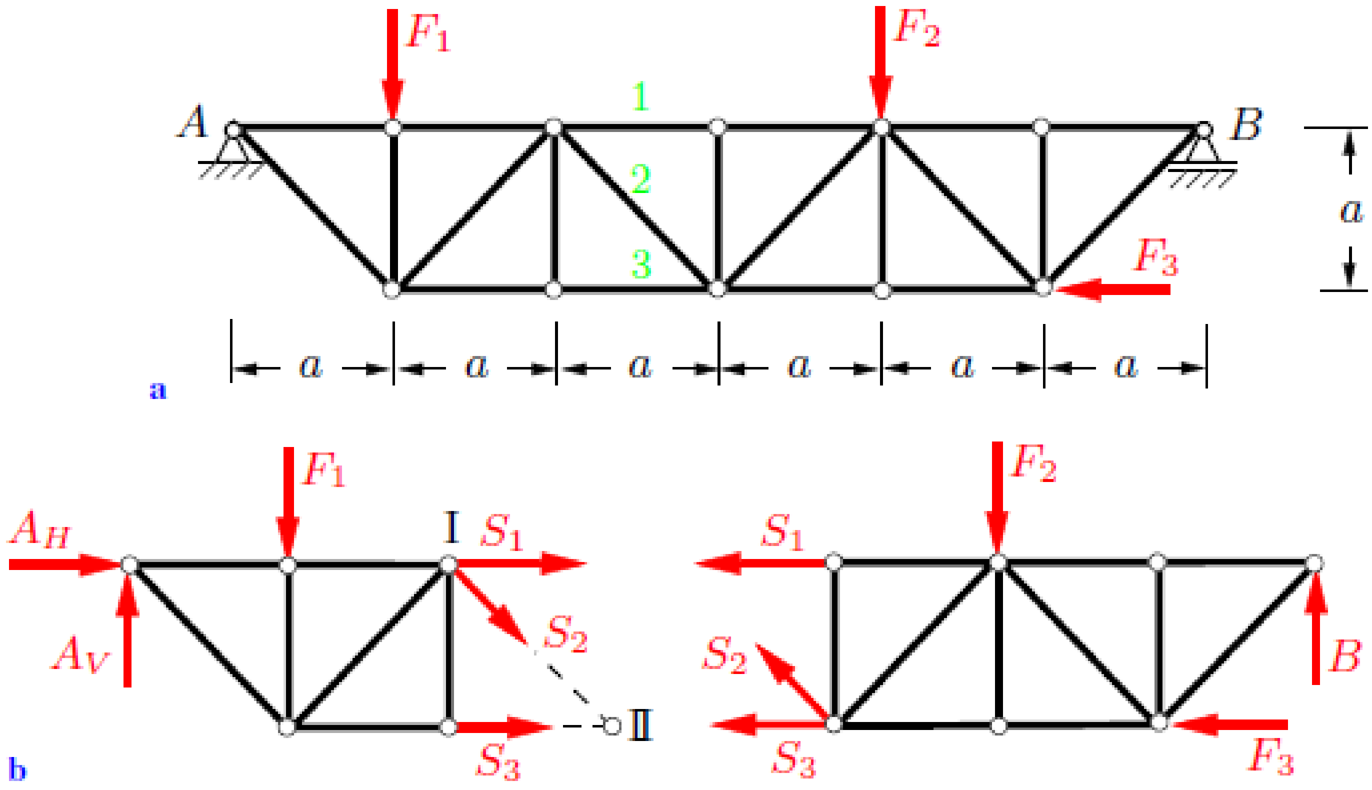
For calculating force on member EF, take moment about B

Take moment about E, for calculating force BC

Now apply $\sum F_y = 0$ to obtain forces on the members BE



- Principle: If a body is in equilibrium, then any part of the body is also in equilibrium.
- Forces in few particular member can be directly found out quickly without solving each joint of the truss sequentially
- Method of Sections and Method of Joints can be conveniently combined
- A section need not be straight.
- More than one section can be used to solve a given problem



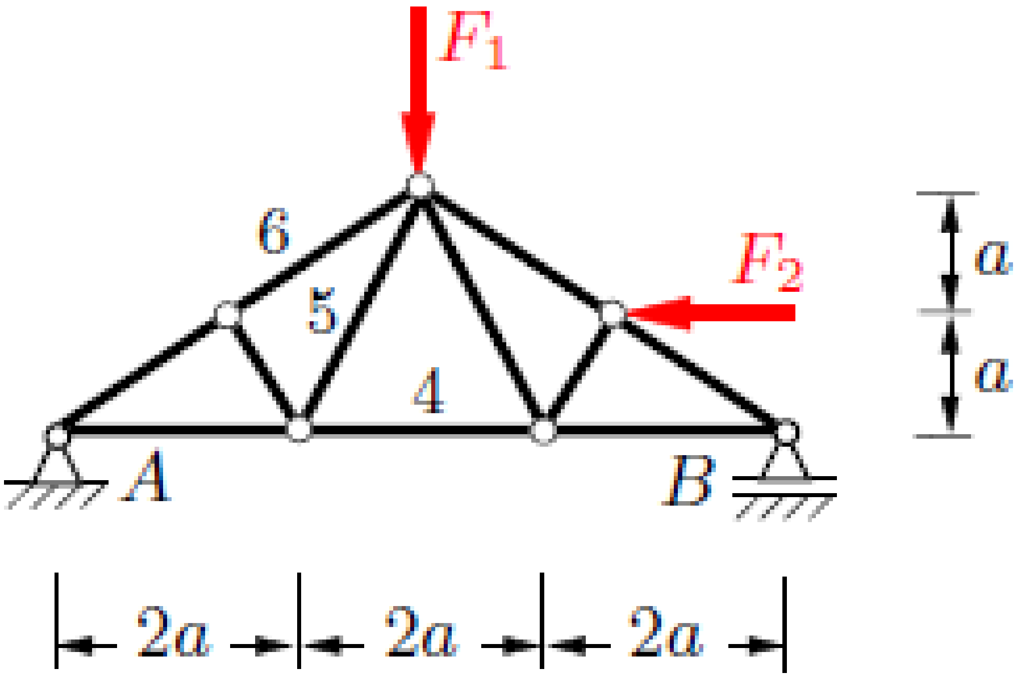
$$\overset{\curvearrowleft}{\mathrm{I}}: \quad -2\,a\,A_V + a\,F_1 + a\,S_3 = 0 \qquad \rightarrow \qquad S_3 = 2\,A_V - F_1\,,$$

$$\overset{\curvearrowleft}{\mathrm{II}}: \quad -3\,a\,A_V - a\,A_H + 2\,a\,F_1 - a\,S_1 = 0$$

$$\rightarrow \qquad S_1 = 2\,F_1 - 3\,A_V - A_H\,,$$

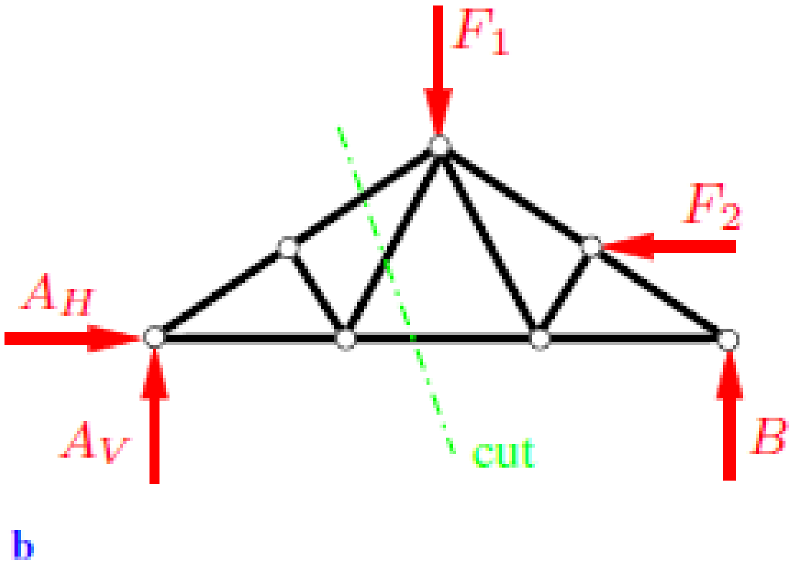
$$\uparrow: \quad A_V - F_1 - \frac{1}{2}\sqrt{2}\,S_2 = 0 \qquad \rightarrow \qquad S_2 = \sqrt{2}\,(A_V - F_1)\,.$$

A truss is loaded by two forces, F1 = 2F and F2 = F, as shown in Fig. a. Determine the force S4.



a

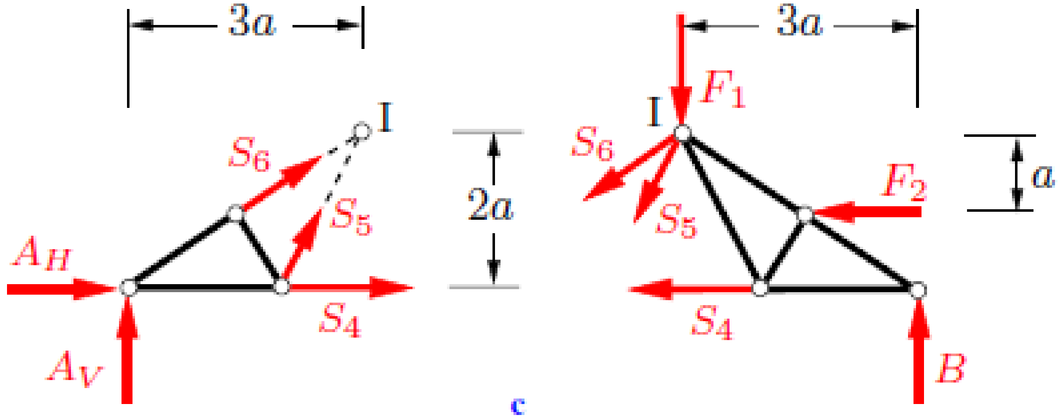
First, we determine the forces at the supports. Applying the equilibrium conditions to the free-body diagram of the whole truss yields



$$\overset{\curvearrowleft}{A}: \; -3\,a\,F_1 + a\,F_2 + 6\,a\,B = 0 \quad \rightarrow \quad B = \frac{3\,F_1 - F_2}{6} = \frac{5}{6}\,F\,,$$

$$\overset{\curvearrowleft}{B}: \; -6\,a\,A_V + 3\,a\,F_1 + a\,F_2 = 0 \quad \rightarrow \quad A_V = \frac{3\,F_1 + F_2}{6} = \frac{7}{6}\,F\,,$$

$$\rightarrow: \quad A_H - F_2 = 0 \quad \rightarrow \quad A_H = F_2 = F\,.$$



The unknown force S_4 follows from the moment equation about point I (intersection of the action lines of the forces (S_5 and S_6) of the free-body diagram on the left-hand side of

$$\overset{\curvearrowleft}{\mathrm{I}}: \quad 2\,a\,S_4 + 2\,a\,A_H - 3\,a\,A_V = 0$$

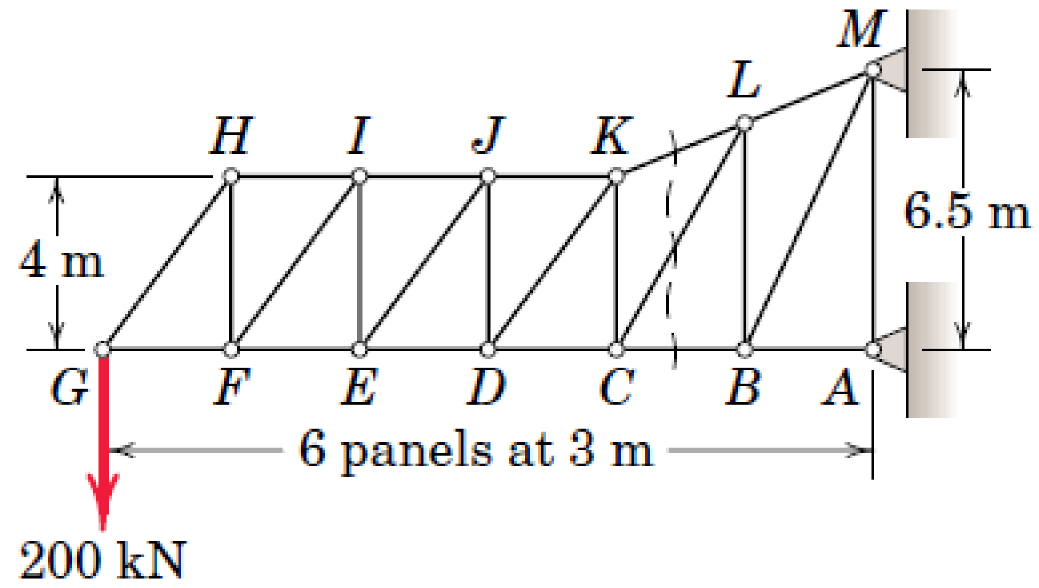
$$\rightarrow \quad \underline{\underline{S_4}} = \frac{1}{2}(3\,A_V - 2\,A_H) = \underline{\underline{\frac{3}{4}\,F}}\,.$$

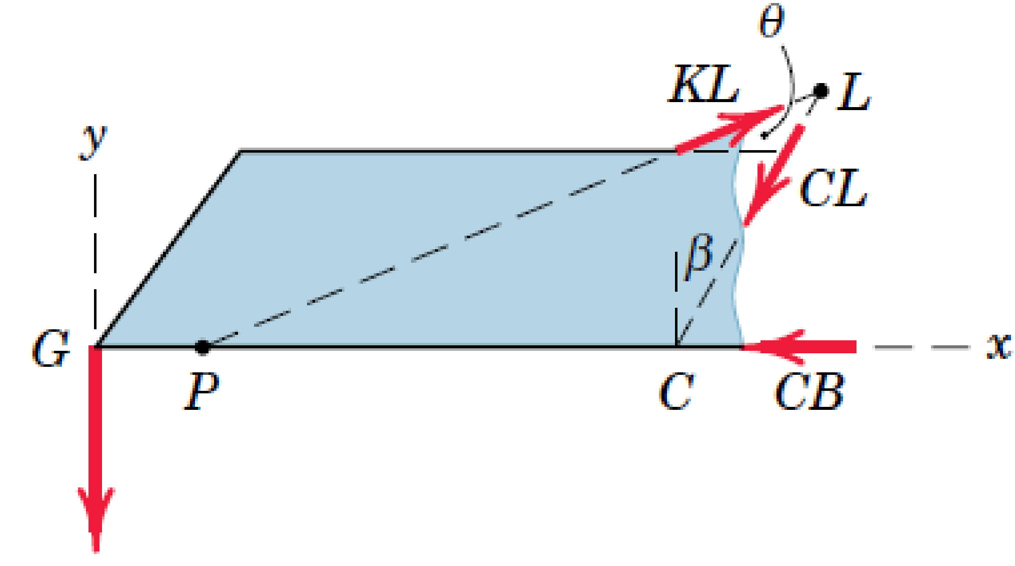
The corresponding moment equation for the free-body diagram on the right-hand side may be used as a check

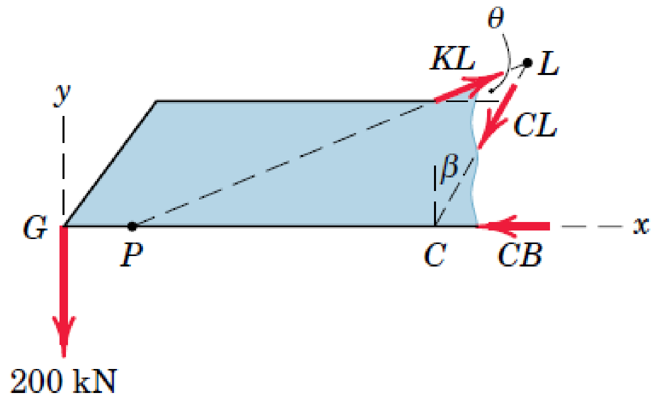
$$\overset{\curvearrowleft}{\mathrm{I}}: \quad -2\,a\,S_4 + 3\,a\,B - a\,F_2 = 0$$

$$\rightarrow \quad S_4 = \frac{1}{2}(3\,B - F_2) = \frac{3}{4}\,F\,.$$

Calculate the forces induced in members *KL*, *CL*, and *CB* by the 200-kN load on the cantilever truss







② Summing moments about L requires finding the moment arm $\overline{BL} = 4 + (6.5 - 4)/2 = 5.25$ m. Thus,

$[\Sigma M_L = 0]$ $\quad 200(5)(3) - CB(5.25) = 0 \quad CB = 571 \text{ kN } C$ *Ans.*

Next we take moments about C, which requires a calculation of cos θ. From the given dimensions we see $\theta = \tan^{-1}(5/12)$ so that $\cos\theta = 12/13$. Therefore,

$[\Sigma M_C = 0]$ $\quad 200(4)(3) - \frac{12}{13}KL(4) = 0 \quad KL = 650 \text{ kN } T$ *Ans.*

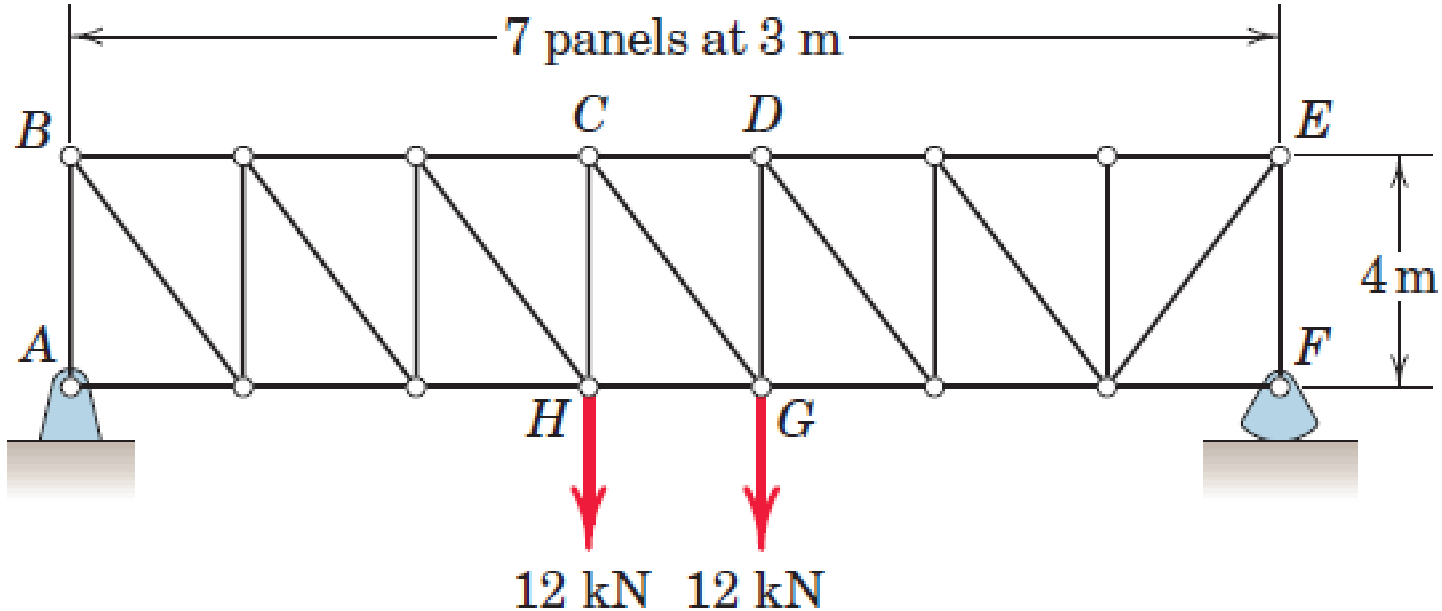
Finally, we may find CL by a moment sum about P, whose distance from C is given by $\overline{PC}/4 = 6/(6.5 - 4)$ or $\overline{PC} = 9.60$ m. We also need β, which is given by $\beta = \tan^{-1}(\overline{CB}/\overline{BL}) = \tan^{-1}(3/5.25) = 29.7°$ and $\cos\beta = 0.868$. We now have

③ $[\Sigma M_p = 0]$ $\quad 200(12 - 9.60) - CL(0.868)(9.60) = 0$

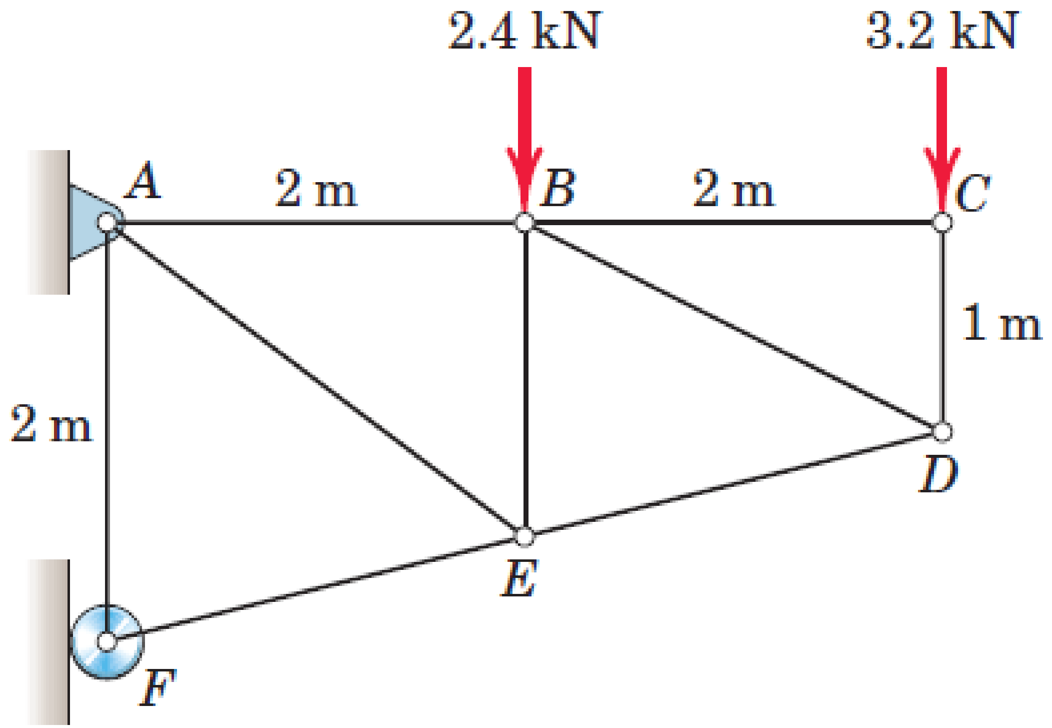
$CL = 57.6 \text{ kN } C$ *Ans.*

Assignment of method of Sections

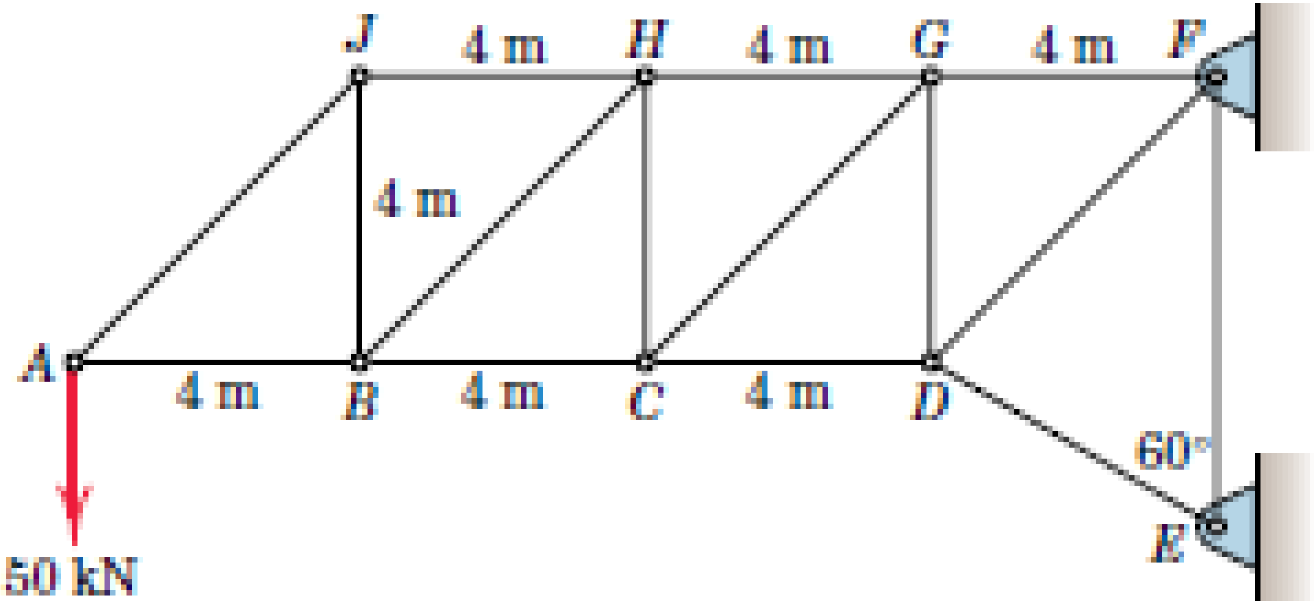
Determine the forces in members *CG* and *GH*.



Determine the force in member *AE* of the loaded truss



4/32 Determine the forces in members *GH* and *CG* for the truss loaded and supported as shown. Does the statical indeterminacy of the supports affect your calculation?



4/35 Determine the forces in members *DE* and *DL*.

Ans. $DE = 24$ kN T, $DL = 33.9$ kN C