Engineering Mechanics: ME101

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Statics: Lecture 11

5rd Feb 2016

Friction

Types of Friction

Applications of Friction in Machines

Wedges

Screws

Journal bearing, thrust bearings

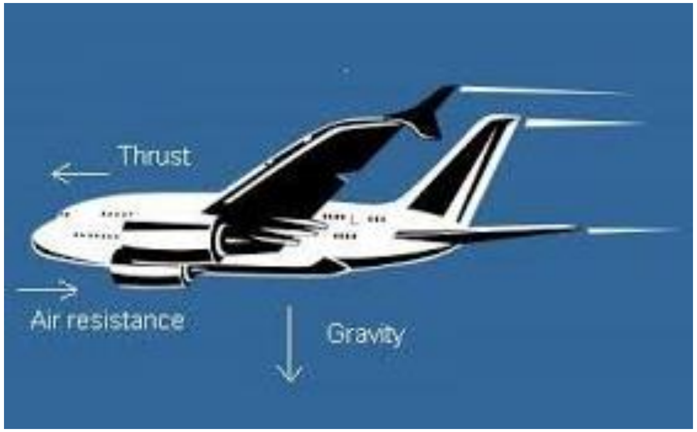
Flexible belts

Rolling Resistance

In this course so far, it has been assumed that all bodies considered have smooth surfaces. Till date, only forces perpendicular to the contact plane can be transferred between two bodies in contact.

This is a proper description of the mechanical behavior if the tangential forces occurring in reality due to the roughness of the surfaces can be neglected.

We will address problems for which this simplification is not valid.



Friction in Driving a Car

- **However, the friction forces are limited in magnitude and will not prevent motion if sufficiently large forces are applied.**

- **The distinction between frictionless and rough is, therefore, a matter of degree.**

- **There are two types of friction: *dry* or *Coulomb friction* and *fluid friction*. Fluid friction applies to lubricated mechanisms. The present discussion is limited to dry friction between nonlubricated surfaces.**

Types of Friction

Friction is the force resisting the relative motion of solid surfaces, fluid layers, and material elements sliding against each other.[1] There are several types of friction:

- **Dry friction** resists relative lateral motion of two solid surfaces in contact. Dry friction is subdivided into *static friction* ("stiction") between non-moving surfaces, and *kinetic friction* between moving surfaces.

- **Fluid friction** describes the friction between layers of a viscous fluid that are moving relative to each other.

- **Lubricated friction** is a case of fluid friction where a lubricant fluid separates two solid surfaces.

- **Skin friction** is a component of drag, the force resisting the motion of a fluid across the surface of a body.

- **Internal friction** is the force resisting motion between the elements making up a solid material while it undergoes deformation.

Limitation of Coulomb Friction

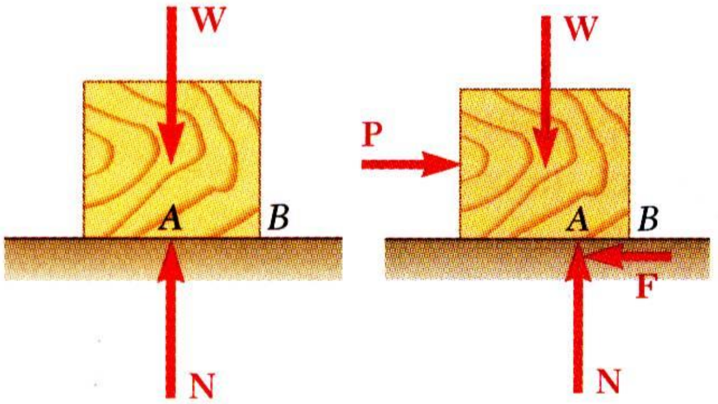
Frictional force is proportional to the applied normal force, independently of the contact area

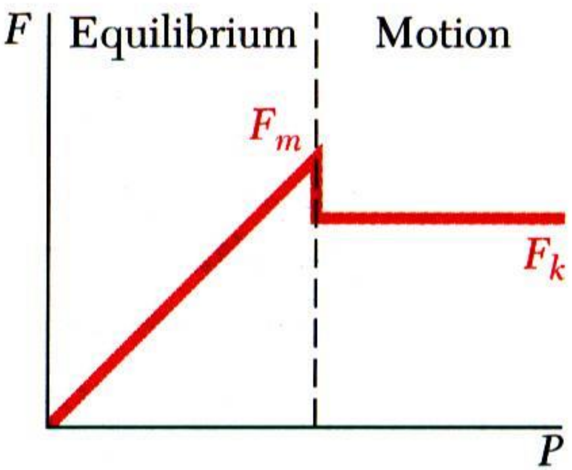
When the surfaces are conjoined, Coulomb friction becomes a very poor approximation (for example, adhesive tape resists sliding even when there is no normal force, or a negative normal force).

In this case, the frictional force may depend strongly on the area of contact. Some drag racing tires are adhesive for this reason.

However, despite the complexity of the fundamental physics behind friction, the relationships are accurate enough to be useful in many applications.

The Laws of Dry Friction. Coefficients of Friction





- Block of weight *W* placed on horizontal surface. Forces acting on block are its weight and reaction of surface *N*.

- Small horizontal force *P* applied to block. For block to remain stationary, in equilibrium, a horizontal component *F* of the surface reaction is required. *F* is a *static-friction force*.

- As *P* increases, the static-friction force *F* increases as well until it reaches a maximum value F_m.

$$F_m = \mu_s N$$

- Further increase in P causes the block to begin to move as *F* drops to a smaller *kinetic-friction force* F_k.

$$F_k = \mu_k N$$

The Laws of Dry Friction. Coefficients of Friction

Table 8.1. Approximate Values of Coefficient of Static Friction for Dry Surfaces

Metal on metal	0.15–0.60
Metal on wood	0.20–0.60
Metal on stone	0.30–0.70
Metal on leather	0.30–0.60
Wood on wood	0.25–0.50
Wood on leather	0.25–0.50
Stone on stone	0.40–0.70
Earth on earth	0.20–1.00
Rubber on concrete	0.60–0.90

- Maximum static-friction force:

$$F_m = \mu_s N$$

- Kinetic-friction force:

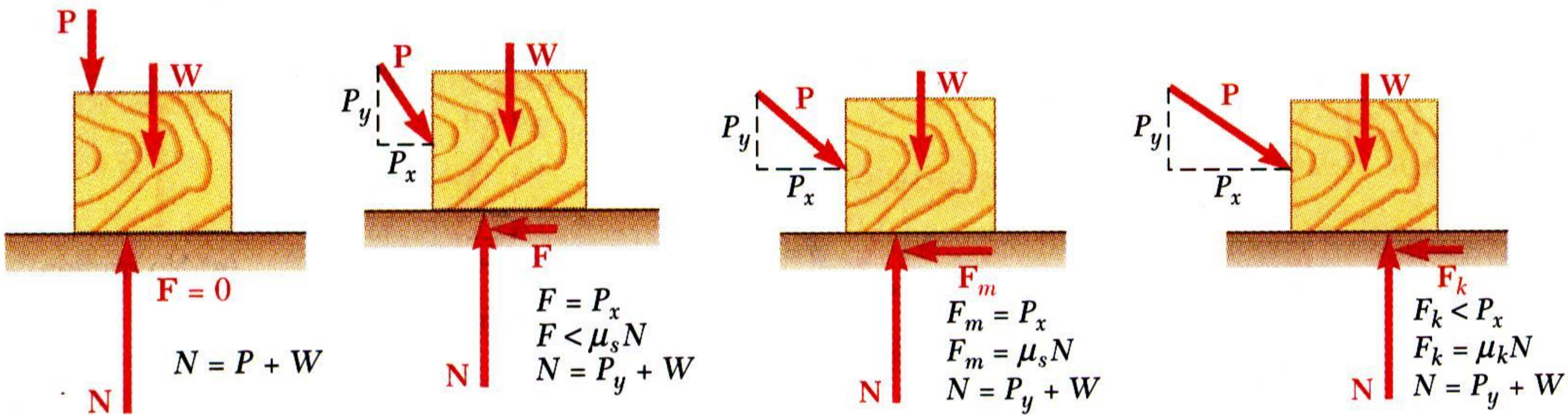
$$F_k = \mu_k N$$

$$\mu_k \cong 0.75 \mu_s$$

- Maximum static-friction force and kinetic-friction force are:
  - proportional to normal force
  - dependent on type and condition of contact surfaces
  - independent of contact area

The Laws of Dry Friction. Coefficients of Friction

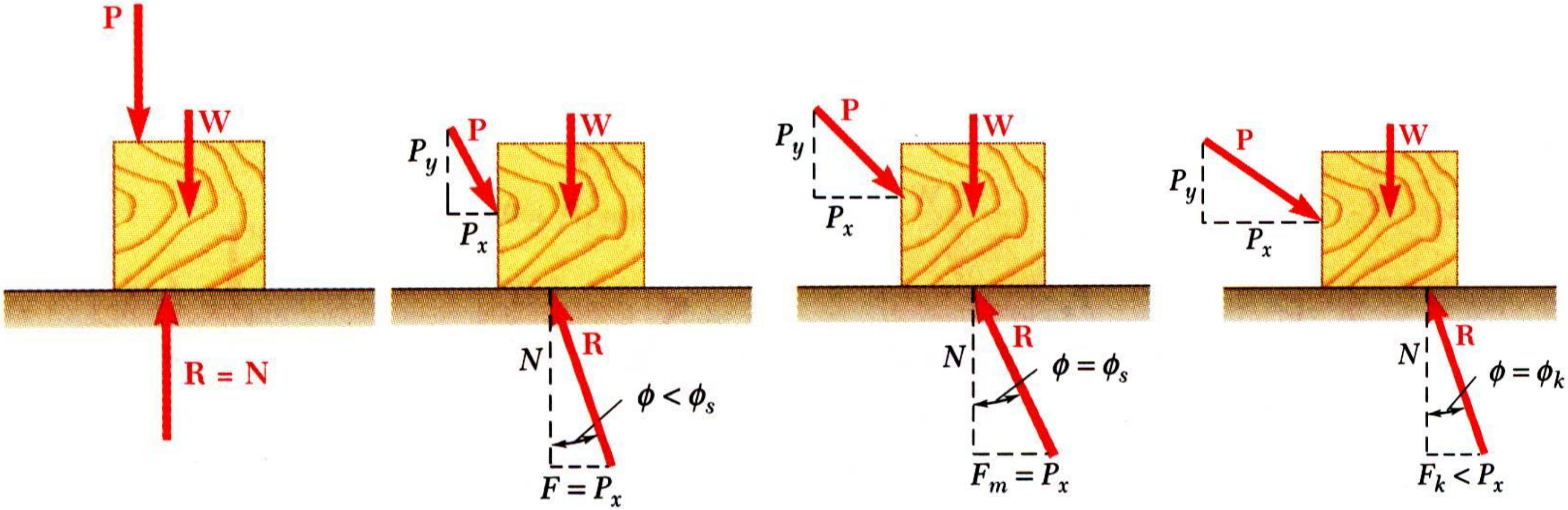
- Four situations can occur when a rigid body is in contact with a horizontal surface:



- No friction, $(P_x = 0)$
- No motion, $(P_x < F_m)$
- Motion impending, $(P_x = F_m)$
- Motion, $(P_x > F_m)$

Angles of Friction

- It is sometimes convenient to replace normal force *N* and friction force *F* by their resultant ***R***:



- No friction
- No motion
- Motion impending
- Motion

$$\tan\phi_s = \frac{F_m}{N} = \frac{\mu_s N}{N}$$

$$\tan\phi_s = \mu_s$$

$$\tan\phi_k = \frac{F_k}{N} = \frac{\mu_k N}{N}$$

$$\tan\phi_k = \mu_k$$

Characteristics of Dry Friction. As a result of *experiments* that pertain to the foregoing discussion, we can state the following rules which apply to bodies subjected to dry friction.

- The frictional force acts *tangent* to the contacting surfaces in a direction *opposed* to the *motion* or tendency for motion of one surface relative to another.

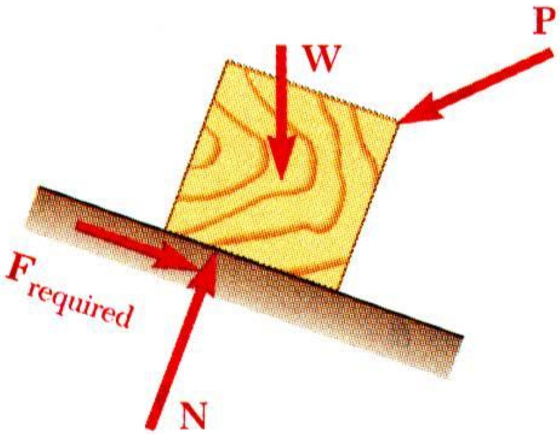
- The maximum static frictional force F_s that can be developed is independent of the area of contact, provided the normal pressure is not very low nor great enough to severely deform or crush the contacting surfaces of the bodies.

- The maximum static frictional force is generally greater than the kinetic frictional force for any two surfaces of contact. However, if one of the bodies is moving with a *very low velocity* over the surface of another, F_k becomes approximately equal to F_s, i.e., $\mu_s \approx \mu_k$.

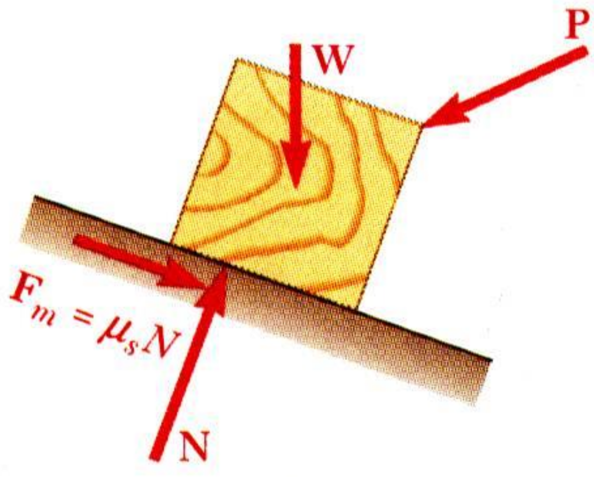
- When *slipping* at the surface of contact is *about to occur*, the maximum static frictional force is proportional to the normal force, such that $F_s = \mu_s N$.

- When *slipping* at the surface of contact is *occurring*, the kinetic frictional force is proportional to the normal force, such that $F_k = \mu_k N$.

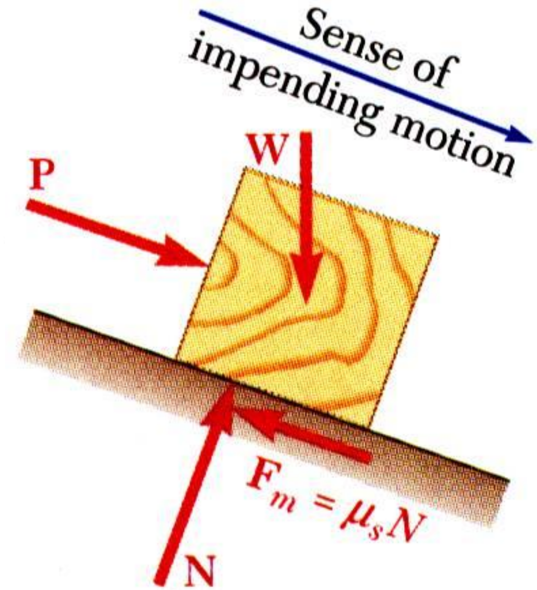
Problems Involving Dry Friction



- All applied forces known
- Coefficient of static friction is known
- Determine whether body will remain at rest or slide

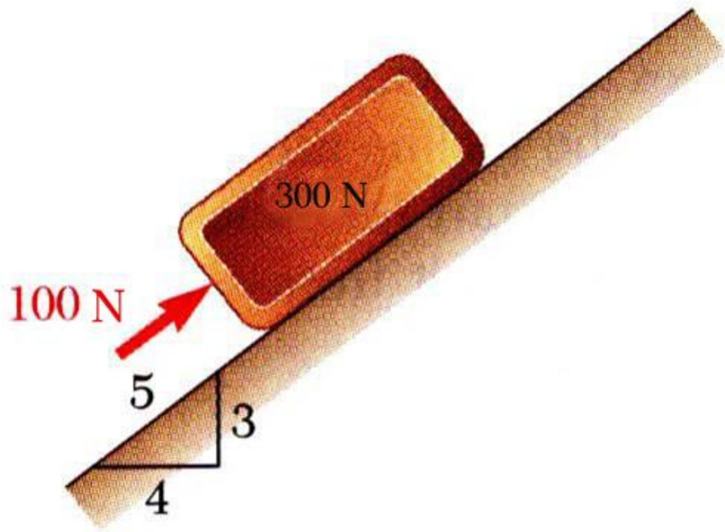


- All applied forces known
- Motion is impending
- Determine value of coefficient of static friction.



- Coefficient of static friction is known
- Motion is impending
- Determine magnitude or direction of one of the applied forces

Sample Problem 8.1

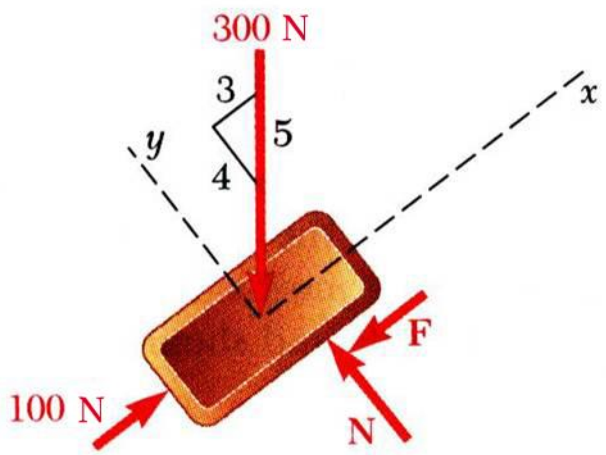


A 100 N force acts as shown on a 300 N block placed on an inclined plane. The coefficients of friction between the block and plane are $\mu_s = 0.25$ and $\mu_k = 0.20$. Determine whether the block is in equilibrium and find the value of the friction force.

SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.
- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.
- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

Sample Problem 8.1



SOLUTION:

- Determine values of friction force and normal reaction force from plane required to maintain equilibrium.

$$\sum F_x = 0: \qquad 100\text{ N} - \tfrac{3}{5}(300\text{ N}) - F = 0$$

$$F = -80\text{ N}$$

$$\sum F_y = 0: \qquad N - \tfrac{4}{5}(300\text{ N}) = 0$$

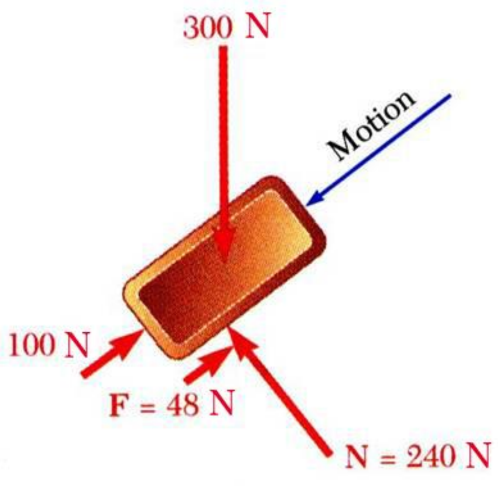
$$N = 240\text{ N}$$

- Calculate maximum friction force and compare with friction force required for equilibrium. If it is greater, block will not slide.

$$F_m = \mu_s N \qquad F_m = 0.25(240\text{ N}) = 48\text{ N}$$

The block will slide down the plane.

Sample Problem 8.1

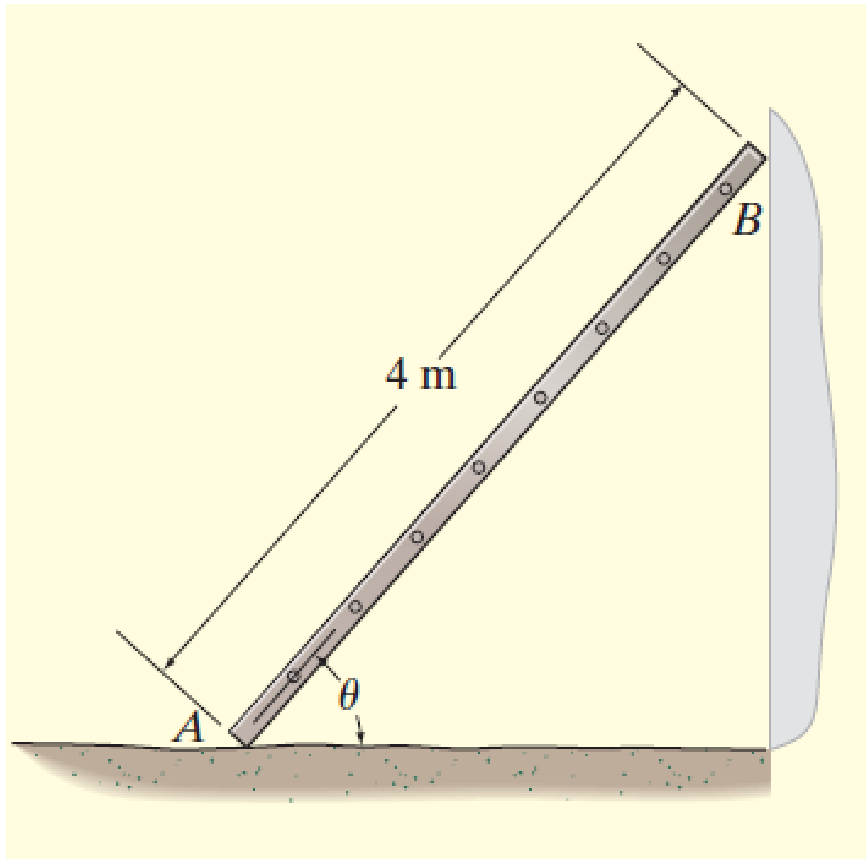


- If maximum friction force is less than friction force required for equilibrium, block will slide. Calculate kinetic-friction force.

$$F_{actual} = F_k = \mu_k \mathrm{N}$$
$$= 0.20(240\ \mathrm{N})$$

$$\boxed{F_{actual} = 48\ \mathrm{N}}$$

The uniform 10-kg ladder in Fig. *a* rests against the smooth wall at *B* , and the end *A* rests on the rough horizontal plane for which the coefficient of static friction is = 0.3 . Determine the angle of inclination u of the ladder and the normal reaction at *B* if the ladder is on the verge of slipping



(a)

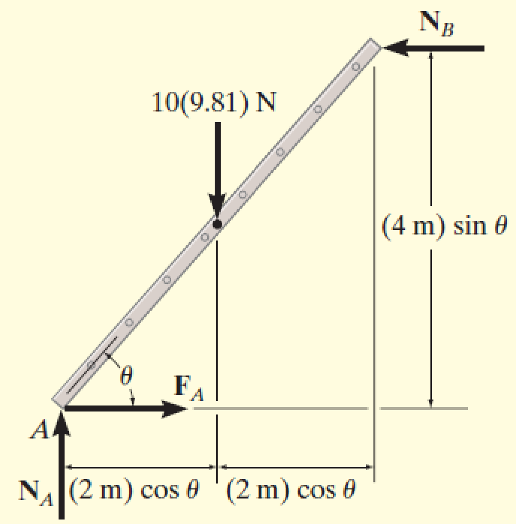
Equations of Equilibrium and Friction. Since the ladder is on the verge of slipping, then $F_A = \mu_s N_A = 0.3N_A$. By inspection, N_A can be obtained directly.

$$+\uparrow \Sigma F_y = 0; \qquad N_A - 10(9.81)\text{ N} = 0 \qquad N_A = 98.1\text{ N}$$

Using this result, $F_A = 0.3(98.1\text{ N}) = 29.43\text{ N}$. Now N_B can be found.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 29.43\text{ N} - N_B = 0$$

$$N_B = 29.43\text{ N} = 29.4\text{ N} \qquad \textit{Ans.}$$



(b)

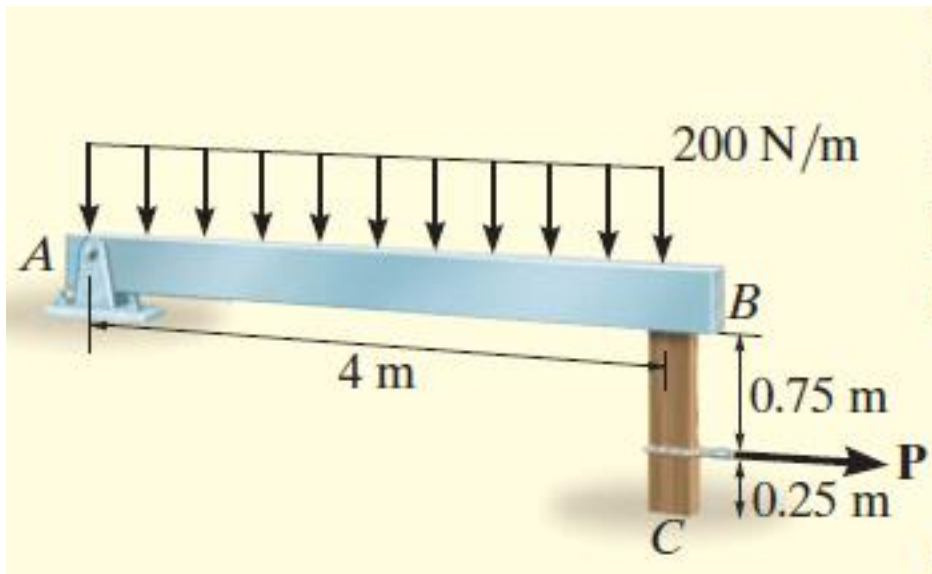
Finally, the angle θ can be determined by summing moments about point A.

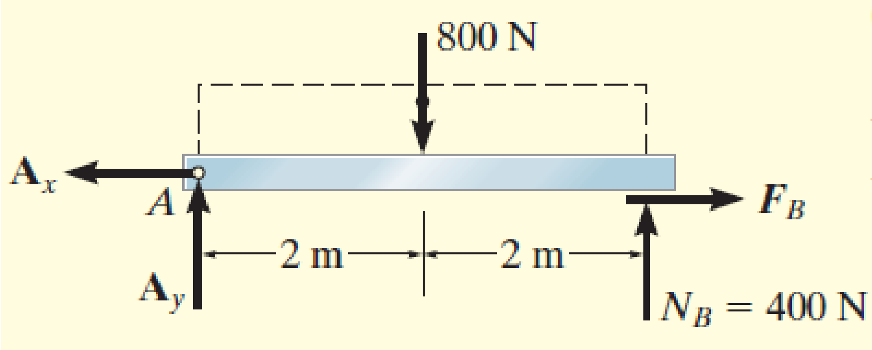
$$\circlearrowleft + \Sigma M_A = 0; \qquad (29.43\text{ N})(4\text{ m})\sin\theta - [10(9.81)\text{ N}](2\text{ m})\cos\theta = 0$$

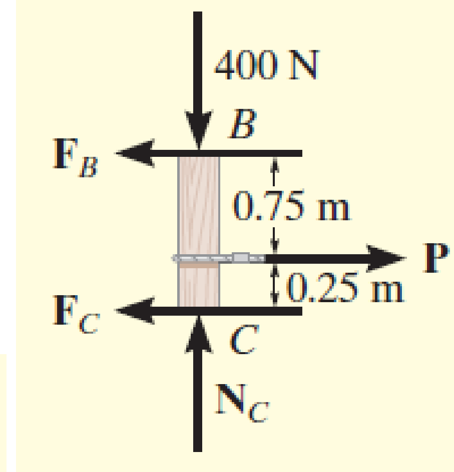
$$\frac{\sin\theta}{\cos\theta} = \tan\theta = 1.6667$$

$$\theta = 59.04^\circ = 59.0^\circ \qquad \textit{Ans.}$$

Beam AB is subjected to a uniform load of 200 N/m and is supported at B by post BC, Fig. 8–10a. If the coefficients of static friction at B and C are $\mu_B = 0.2$ and $\mu_C = 0.5$, determine the force **P** needed to pull the post out from under the beam. Neglect the weight of the members and the thickness of the beam.







Equations of Equilibrium and Friction.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_B - F_C = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 400\text{ N} = 0 \qquad (2)$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -P(0.25\text{ m}) + F_B(1\text{ m}) = 0 \qquad (3)$$

(Post Slips at *B* and Rotates about C.) This requires $F_C \leq \mu_C N_C$ and

$$F_B = \mu_B N_B; \qquad F_B = 0.2(400\text{ N}) = 80\text{ N}$$

Using this result and solving Eqs. 1 through 3, we obtain

N

$$P = 320\text{ N}$$
$$F_C = 240\text{ N}$$
$$N_C = 400\text{ N}$$

Since $F_C = 240\text{ N} > \mu_C N_C = 0.5(400\text{ N}) = 200\text{ N}$, slipping at C occurs. Thus the other case of movement must be investigated.

(Post Slips at C and Rotates about B.) Here $F_B \leq \mu_B N_B$ and

$$F_C = \mu_C N_C; \qquad F_C = 0.5N_C \tag{4}$$

Solving Eqs. 1 through 4 yields

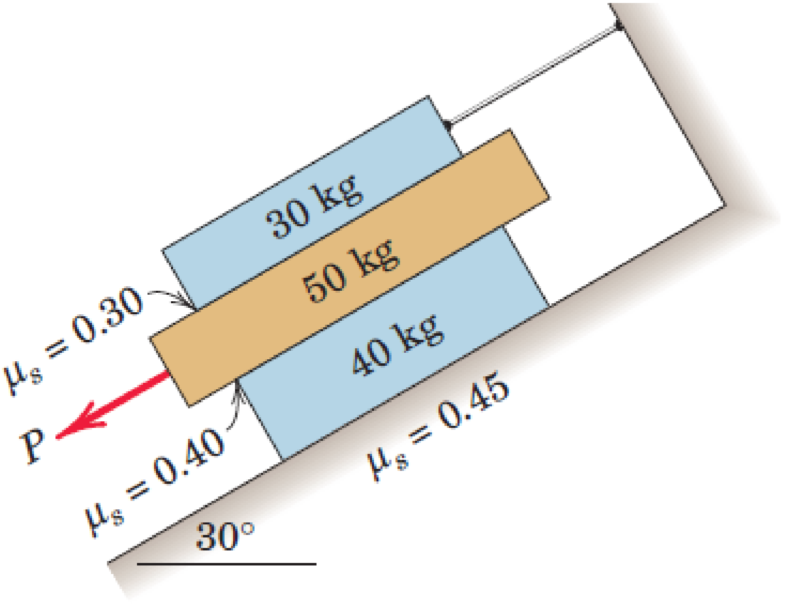
$$P = 267 \text{ N} \qquad \textit{Ans.}$$
$$N_C = 400 \text{ N}$$
$$F_C = 200 \text{ N}$$
$$F_B = 66.7 \text{ N}$$

Obviously, this case occurs first since it requires a *smaller* value for P.

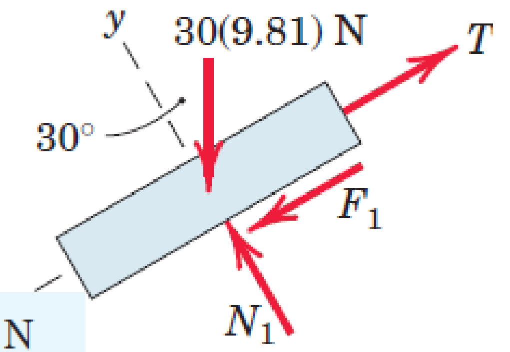
The three flat blocks are positioned on the 30 incline as shown, and a force *P* parallel to the incline is applied to the middle block. The upper block is prevented from moving by a wire which attaches it to the fixed support. The coefficient of static friction for each of the three pairs of mating surfaces is shown.

Determine the maximum value which *P* may have before any slipping takes place.

The free-body diagram of each block is drawn. The friction forces are assigned in the directions to oppose the relative motion which would occur if no friction were present.
There are two possible conditions for impending motion.
Either the 50-kg block slips and the 40-kg block remains in place, or
the 50- and 40-kg blocks move together with slipping occurring between the 40-kg block and the incline



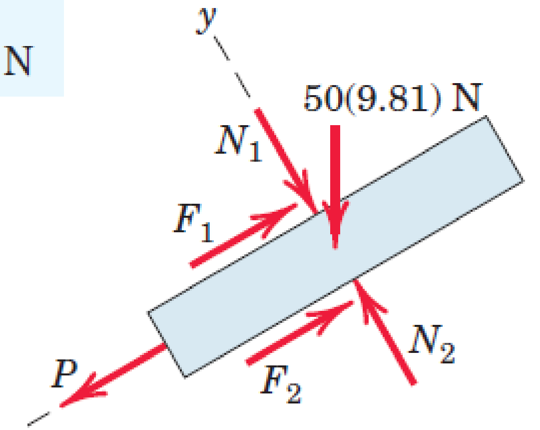
The normal forces, which are in the *y*-direction, may be determined without reference to the friction forces, which are all in the *x*-direction



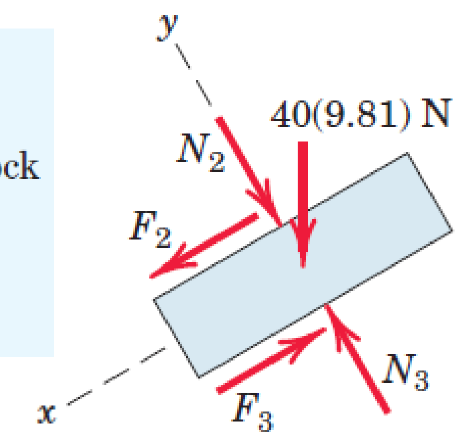
$[\Sigma F_y = 0]$ (30-kg) $N_1 - 30(9.81)\cos 30° = 0$ $N_1 = 255$ N

(50-kg) $N_2 - 50(9.81)\cos 30° - 255 = 0$ $N_2 = 680$ N

(40-kg) $N_3 - 40(9.81)\cos 30° - 680 = 0$ $N_3 = 1019$ N



We will assume arbitrarily that only the 50-kg block slips, so that the 40-kg block remains in place. Thus, for impending slippage at both surfaces of the 50-kg block, we have



$[F_{max} = \mu_s N]$ $F_1 = 0.30(255) = 76.5$ N $F_2 = 0.40(680) = 272$ N

The assumed equilibrium of forces at impending motion for the 50-kg block gives

$[\Sigma F_x = 0]$ $P - 76.5 - 272 + 50(9.81)\sin 30° = 0$ $P = 103.1$ N

We now check on the validity of our initial assumption. For the 40-kg block with $F2$ =272 N the friction force $F3$ would be given by

$$[\Sigma F_x = 0] \qquad 272 + 40(9.81)\sin 30° - F_3 = 0 \qquad F_3 = 468 \text{ N}$$

But the maximum possible value of

F_3 is $F_3 = \mu_s N_3 = 0.45(1019) = 459$ N.

Thus, 468 N cannot be supported and our initial assumption was wrong. We conclude, therefore, that slipping occurs first between the 40-kg block and the incline. With the corrected value $F3$ 459 N, equilibrium of the 40-kg block for its impending motion requires

$$[\Sigma F_x = 0] \qquad F_2 + 40(9.81)\sin 30° - 459 = 0 \qquad F_2 = 263 \text{ N}$$

Equilibrium of the 50-kg block gives, finally,

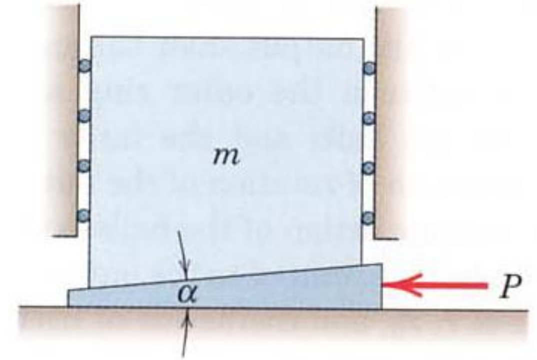
$$[\Sigma F_x = 0] \qquad P + 50(9.81)\sin 30° - 263 - 76.5 = 0$$

$$P = 93.8 \text{ N} \qquad \textit{Ans.}$$

Thus, with $P = 93.8$ N, motion impends for the 50-kg and 40-kg blocks as a unit.

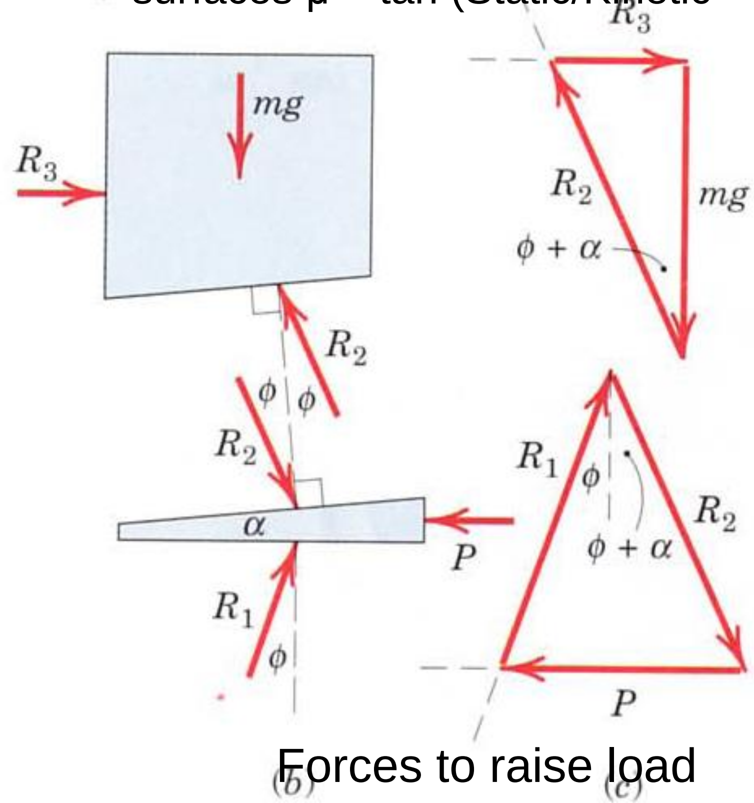
Wedge Problem

- Simple machines used to raise heavy loads.
- Force required to lift block is significantly less than block weight.
- Friction prevents wedge from sliding out.
- Want to find minimum force *P* to raise block.



Coefficient of Friction for each pair of surfaces μ = tan (Static/Kinetic

FBDs:
Reactions are inclined at an angle from their respective normals and are in the direction opposite to the motion.
Force vectors acting on each body can also be shown.
R_2 is first found from upper diagram since mg is known. Then P can be found out from the lower diagram since R_2 is known.



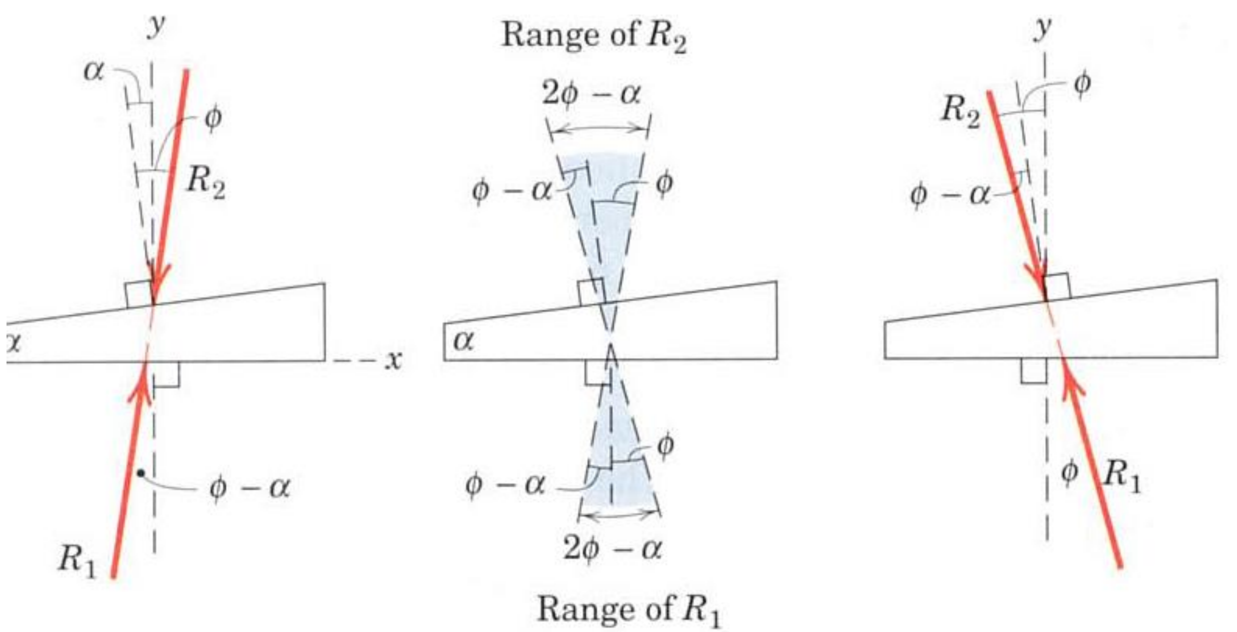
Forces to raise load

P is removed and wedge remains in place

Equilibrium of wedge requires that the equal reactions R_1 and R_2 be collinear.
In the figure, wedge angle α is taken to be less than Impending slippage at the upper surface Impending slippage at the lower surface

Slippage must occur at both surfaces simultaneously In order for the wedge to slide out of its space, Else, the wedge is Self-Locking

Range of angular positions of R_1 and R_2 for which the wedge will remain in place is shown in figure (b)



(a) Slipping impending at upper surface
(b) Range of $R_1 = R_2$ for no slip
(c) Slipping impending at lower surface

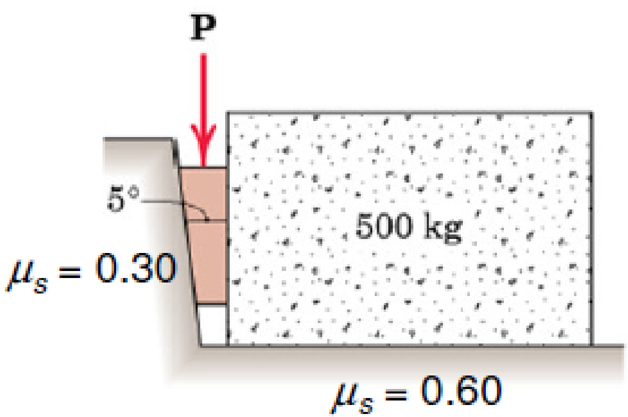
Example: Wedge

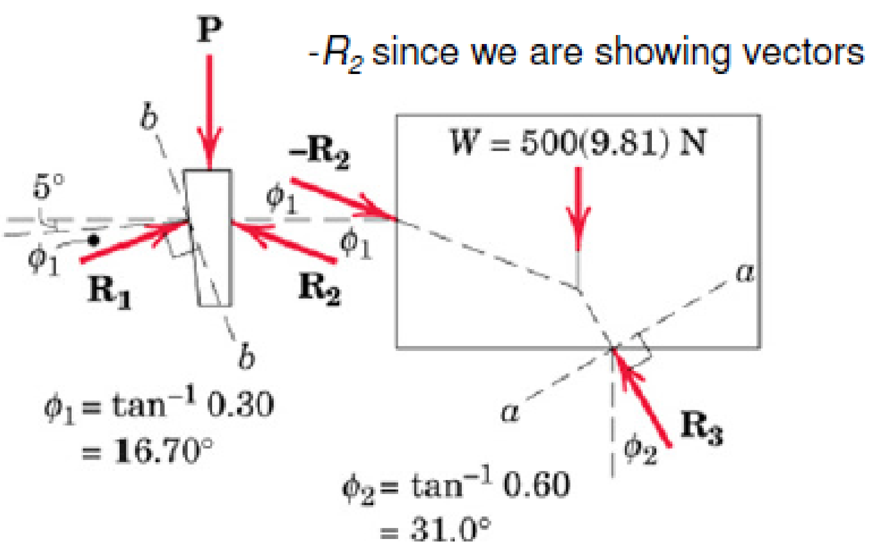
Coefficient of Static Friction for both pairs of wedge = 0.3

Coefficient of Static Friction between block and horizontal surface = 0.6

Find the least P required to move the block

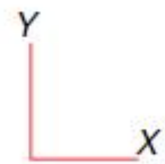
Solution: Draw FBDs

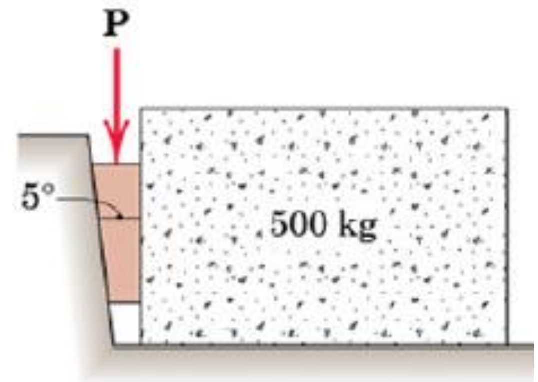




Solution: $W = 500\text{x}9.81 = 4905$ N

Three ways to solve





Method 1:

Equilibrium of FBD of the Block

$\sum F_X = 0$

$R_2 \cos \phi_1 = R_3 \sin \phi_2 \rightarrow R_2 = 0.538 R_3$

$\sum F_Y = 0$

$4905 + R_2 \sin \phi_1 = R_3 \cos \phi_2 \rightarrow R_3 = 6970$ N

$\rightarrow R_2 = 3750$ N

Equilibrium of FBD of the Wedge

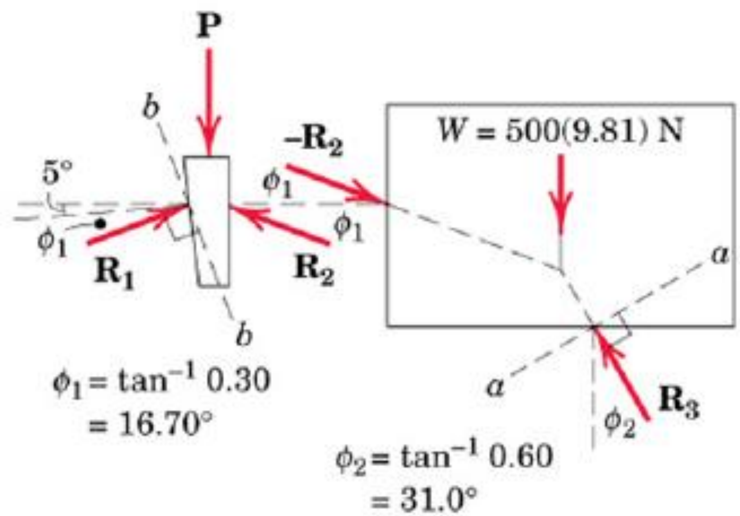
$\sum F_X = 0$

$R_2 \cos \phi_1 = R_1 \cos(\phi_1 + 5) \rightarrow R_1 = 3871$ N

$\sum F_Y = 0$

$R_1 \sin(\phi_1 + 5) + R_2 \sin \phi_1 = P$

$\rightarrow P = 2500$ N



Method 2:

Using Equilibrium equations along reference axes *a-a* and *b-b*

→ No need to solve simultaneous equations

Angle between R_2 and *a-a* axis = 16.70+31.0 = 47.7°

Equilibrium of Block:

$$[\Sigma F_a = 0] \quad 500(9.81)\sin 31.0^\circ - R_2 \cos 47.7^\circ = 0$$

$$R_2 = 3750 \text{ N}$$

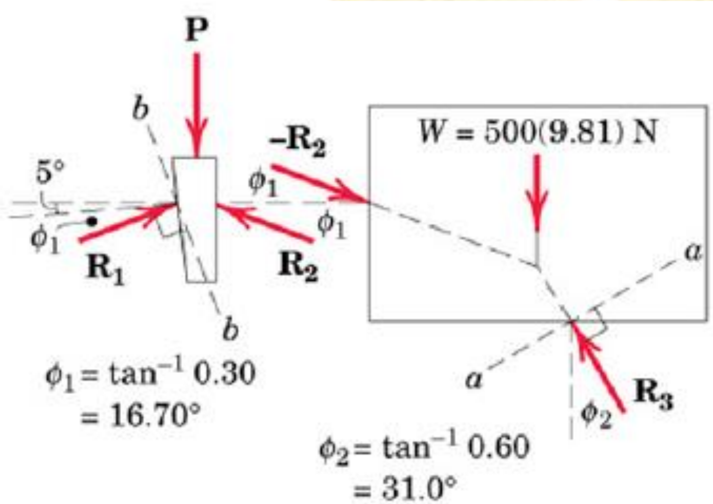
Equilibrium of Wedge:

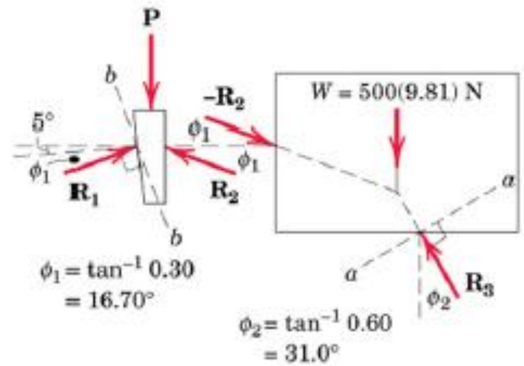
Angle between R_2 and *b-b* axis = $90-(2\Phi_1+5)$ = 51.6°

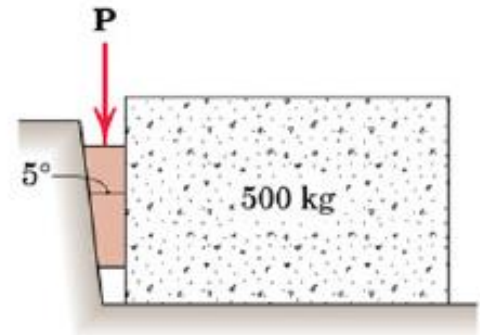
Angle between *P* and *b-b* axis = Φ_1+5 = 21.7°

$$[\Sigma F_b = 0] \quad 3750 \cos 51.6^\circ - P \cos 21.7^\circ = 0$$

$$P = 2500 \text{ N}$$

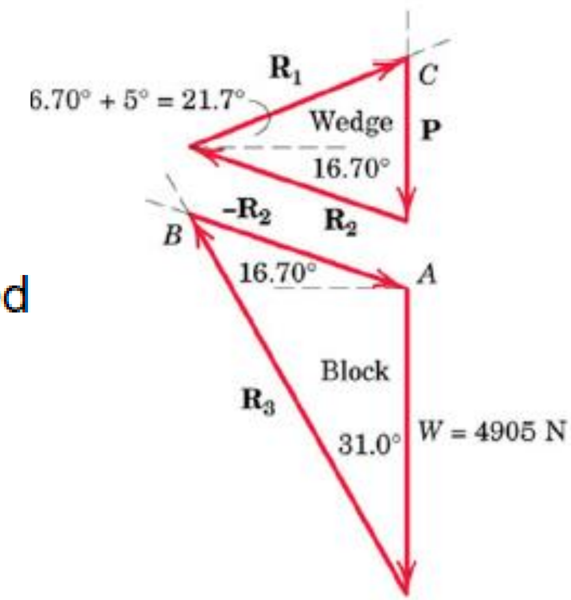






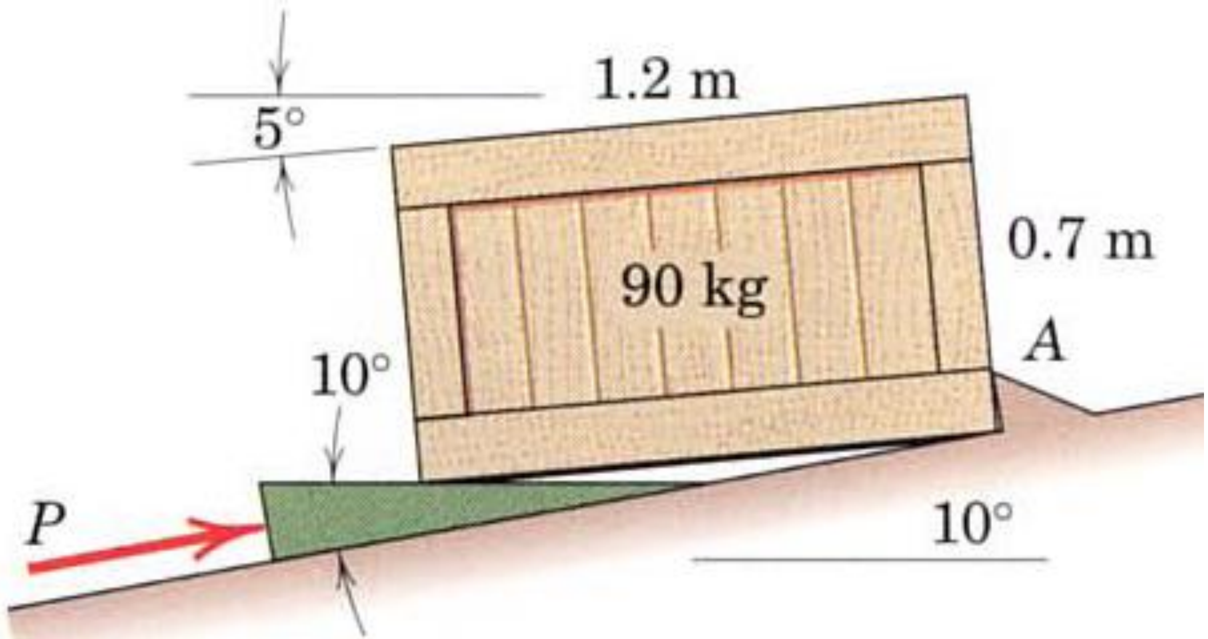
Method 3:
Graphical solution using vector polygons

Starting with equilibrium of the block:
W is known, and directions of R_2 and R_3 are known
→ Magnitudes of R_2 and R_3 can be determined graphically



Similarly, construct vector polygon for the wedge from known magnitude of R_2, and known directions of R_2, R_1, *and* P.
→ Find out the magnitude of P graphically

Determine the force P required to force the 10° wedge under the 90-kg uniform crate which rests against the small stop at A. The coefficient of friction for all surfaces is 0.40.

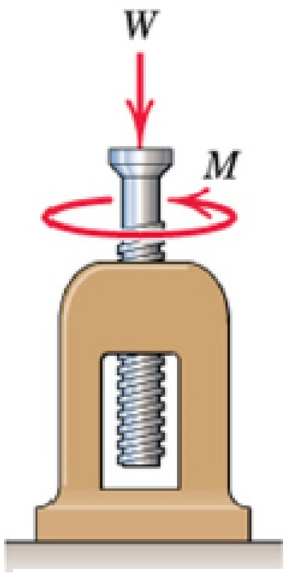


Screw

Square Threaded Screws

- Used for fastening and for transmitting power or motion
- Square threads are more efficient
- Friction developed in the threads largely determines

the action of the screw

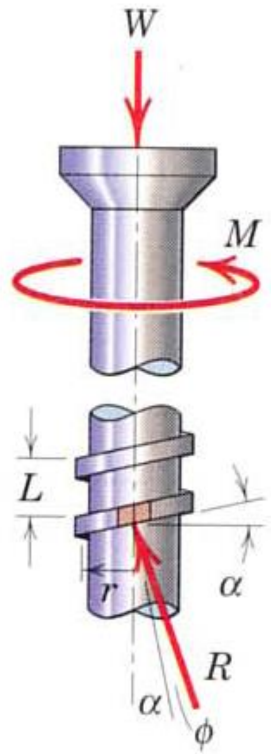


FBD of the Screw: R exerted by the thread of the jack frame on a small portion of the screw thread is shown

Lead = L = advancement per revolution

L = Pitch – for single threaded screw
L = 2xPitch – for double threaded screw (twice advancement per revolution)

Pitch = axial distance between adjacent threads on a helix or screw
Mean Radius = $\boldsymbol{r}$; α = Helix Angle



Similar reactions exist on all segments of the screw threads

Analysis similar to block on inclined plane since friction force does not depend on area of contact.

Thread of base can be "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

If M is just sufficient to turn the screw → Motion Impending
Angle of friction = ϕ (made by R with the axis normal to the thread)
→ $\tan\phi = \mu$

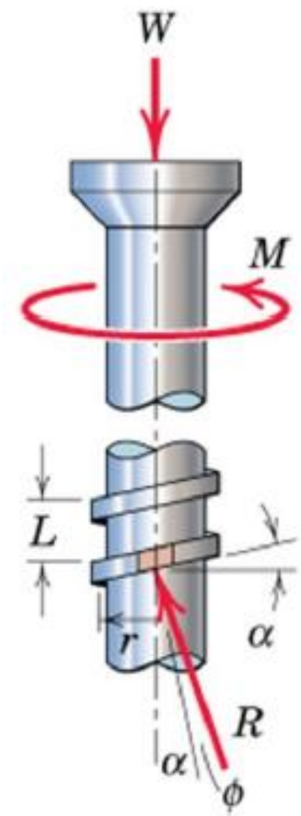
Moment of R @ vertical axis of screw = $R\sin(\alpha+\phi)r$
→ Total moment due to all reactions on the thread = $\sum R\sin(\alpha+\phi)r$
→ Moment Equilibrium Equation for the screw:

$$\rightarrow M = [r\sin(\alpha + \phi)] \sum R$$

Equilibrium of forces in the axial direction: $W = \sum R\cos(\alpha + \phi)$

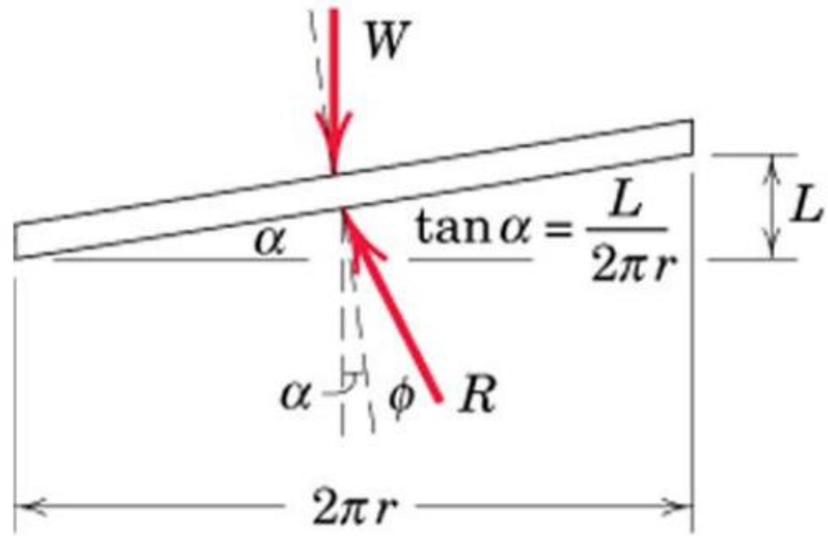
$$\rightarrow W = [\cos(\alpha + \phi)] \sum R$$

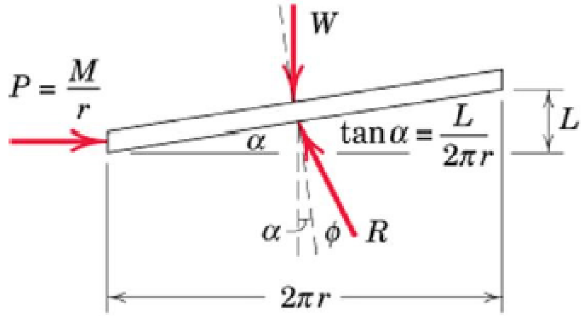
Finally → $\mathbf{M = W\, r \tan(\alpha + \phi)}$



Helix angle α can be determined by unwrapping the thread of the screw for one complete turn

$$\alpha = \tan^{-1} (L/2\pi r)$$





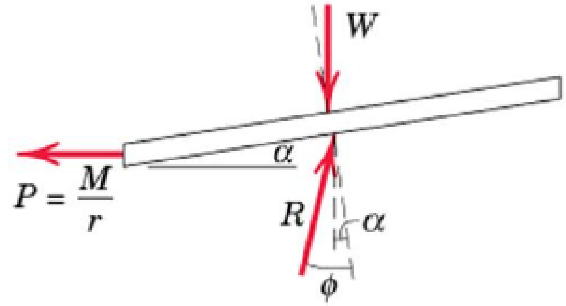
To Raise Load

Equivalent force required to push the movable thread up the fixed incline is:
$P = M/r$

From Equilibrium:

$M = W\, r \tan(\alpha + \phi)$

If M is removed: the screw will remain in place and be self-locking provided $\alpha<\phi$ and will be on the verge of unwinding if $\alpha=\phi$



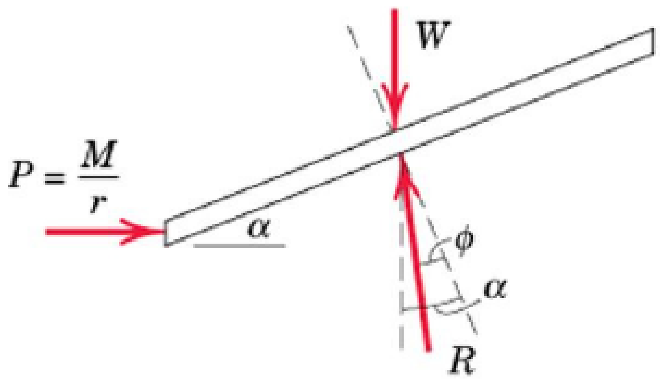
To Lower Load ($\alpha<\phi$)

To lower the load by unwinding the screw, We must reverse the direction of M as long as $\alpha<\phi$

From Equilibrium:

$M = W\, r \tan(\phi - \alpha)$

→ This is the moment required to unwind the screw



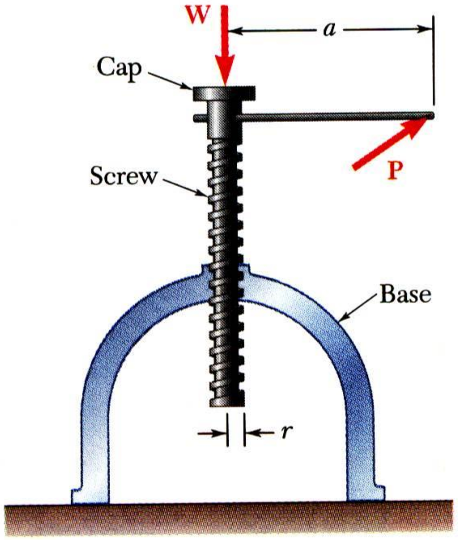
To Lower Load ($\alpha > \phi$)

If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$$M = W\, r \tan(\alpha - \phi)$$

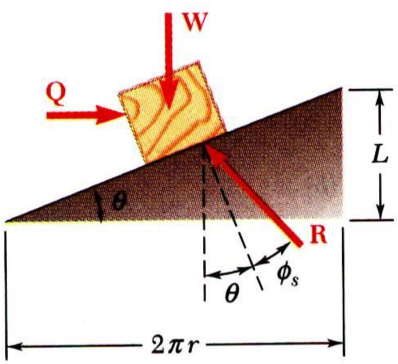
Square-Threaded Screws

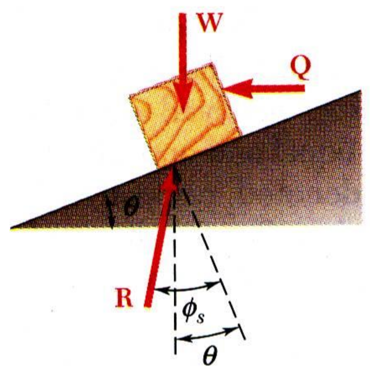


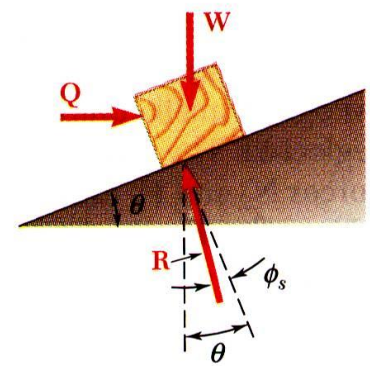
- Square-threaded screws frequently used in jacks, presses, etc. Analysis similar to block on inclined plane. Recall friction force does not depend on area of contact.

- Thread of base has been "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

- Moment of force Q is equal to moment of force P. $Q = Pa/r$





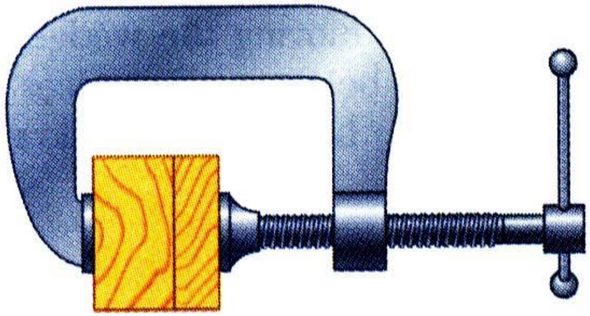


- Impending motion upwards. Solve for Q.

- $\phi_s > \theta$, Self-locking, solve for Q to lower load.

- $\phi_s > \theta$, Non-locking, solve for Q to hold load.

Sample Problem 8.5



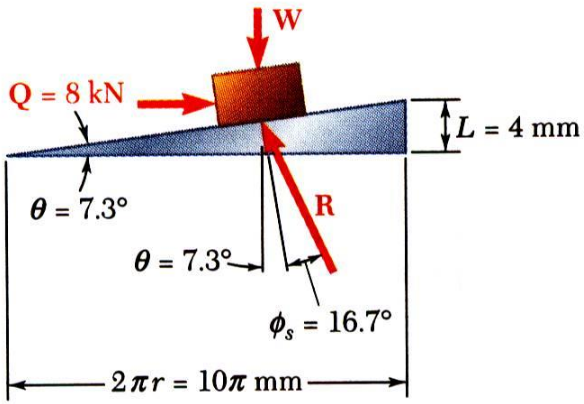
A clamp is used to hold two pieces of wood together as shown. The clamp has a double square thread of mean diameter equal to 10 mm with a pitch of 2 mm. The coefficient of friction between threads is $\mu_s = 0.30$.

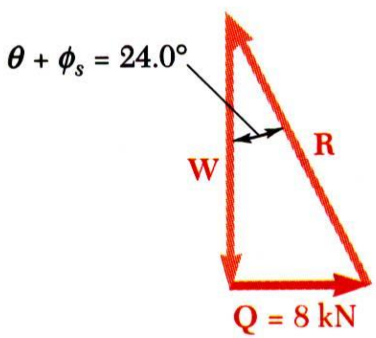
If a maximum torque of 40 N*m is applied in tightening the clamp, determine (a) the force exerted on the pieces of wood, and (b) the torque required to loosen the clamp.

<u>SOLUTION</u>

- Calculate lead angle and pitch angle.
- Using block and plane analogy with impending motion up the plane, calculate the clamping force with a force triangle.
- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

Sample Problem 8.5





SOLUTION

- Calculate lead angle and pitch angle. For the double threaded screw, the lead L is equal to twice the pitch.

$$\tan\theta = \frac{L}{2\pi r} = \frac{2(2\text{ mm})}{10\pi\text{ mm}} = 0.1273 \qquad \theta = 7.3°$$

$$\tan\phi_s = \mu_s = 0.30 \qquad \phi_s = 16.7°$$

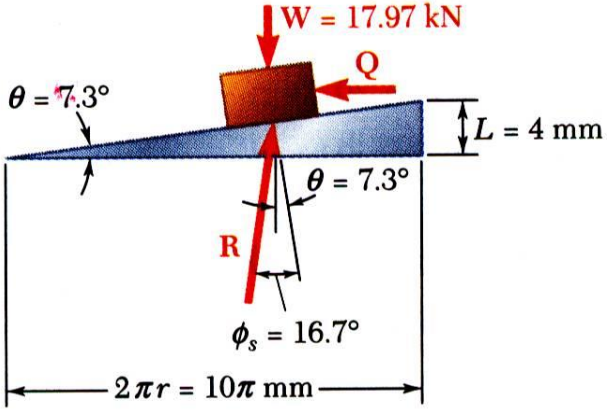
- Using block and plane analogy with impending motion up the plane, calculate clamping force with force triangle.

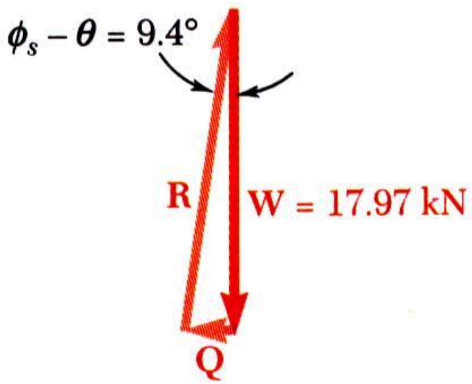
$$Qr = 40\text{ N}\cdot\text{m} \qquad Q = \frac{40\text{ N}\cdot\text{m}}{5\text{ mm}} = 8\text{ kN}$$

$$\tan(\theta+\phi_s) = \frac{Q}{W} \qquad W = \frac{8\text{ kN}}{\tan 24°}$$

$$\boxed{W = 17.97\text{ kN}}$$

Sample Problem 8.5





- With impending motion down the plane, calculate the force and torque required to loosen the clamp.

$$\tan(\phi_s - \theta) = \frac{Q}{W} \qquad Q = (17.97\,\text{kN})\tan 9.4^\circ$$

$$Q = 2.975\,\text{kN}$$

$$Torque = Q\,r = (2.975\,\text{kN})(5\,\text{mm})$$
$$= (2.975\times 10^3\,\text{N})(5\times 10^{-3}\,\text{m})$$

$$\boxed{Torque = 14.87\,\text{N}\cdot\text{m}}$$

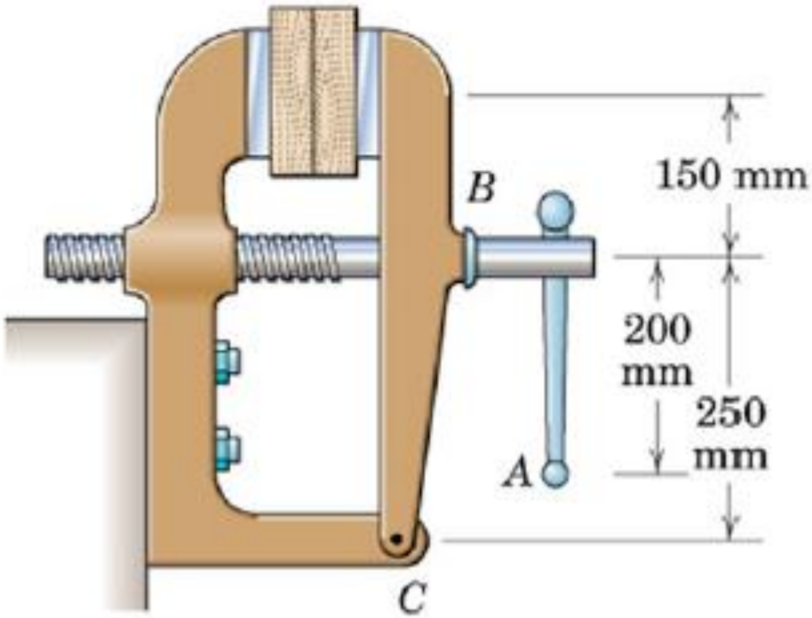
Example: Screw

Single threaded screw of the vise has a mean diameter of 25 mm and a lead of 5 mm. A 300 N pull applied normal to the handle at A produces a clamping force of 5 kN between the jaws of the vise. Determine:

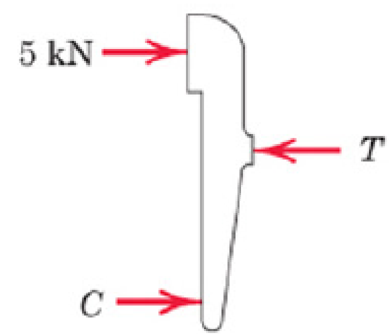
(a) Frictional moment M_B developed at B due to thrust of the screw against body of the jaw

(b) Force Q applied normal to the handle at A required to loosen the vise

μ_s in the threads = 0.20



Solution: Draw FBD of the jaw to find tension in the screw



$\sum M_C = 0$

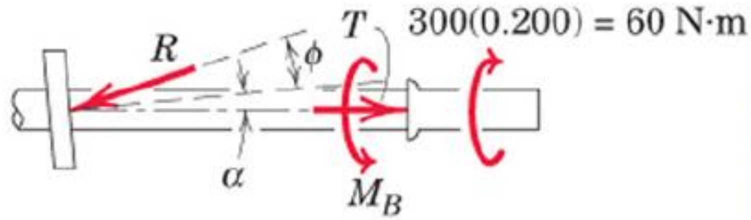
$\rightarrow T = 8$ kN

Find the helix angle α and the friction angle ϕ

$\alpha = tan^{-1}(L/2\pi r) = 3.64^{\circ}$

$\tan\phi = \mu \rightarrow \phi = 11.31^{\circ}$

(a) To tighten the vise
Draw FBD of the screw

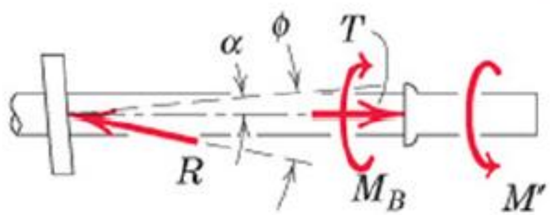


$M = T\,r\,\tan(\alpha + \phi)$

$60 - M_B = 8000(0.0125)\tan(3.64+11.31)$

$\boldsymbol{M_B = 33.3}$ **Nm**

(a) To loosen the vise (on the verge of being loosened)
Draw FBD of the screw: Net moment = applied moment M' minus M_B



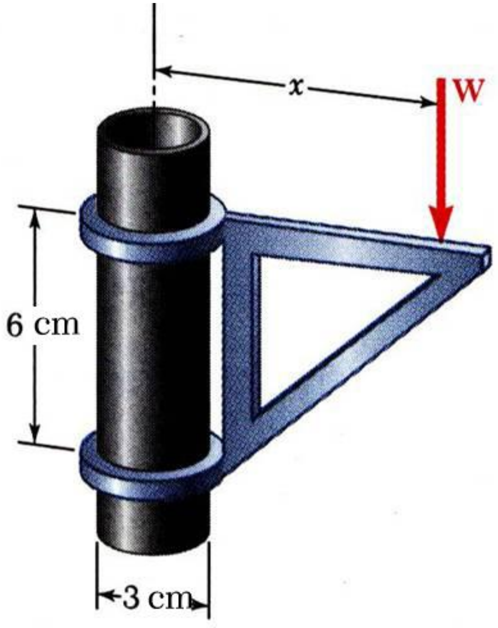
$M = T\,r\,\tan(\phi - \alpha)$

$M' - 33.3 = 8000(0.0125)\tan(11.31-3.64)$

$M' = 46.8$ Nm

$\boldsymbol{Q = M'/d = 46.8/0.2 = 234}$ **N**

Sample Problem 8.3

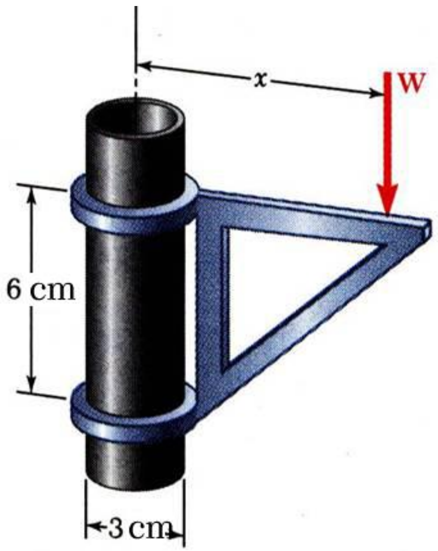


The moveable bracket shown may be placed at any height on the 3-cm diameter pipe. If the coefficient of friction between the pipe and bracket is 0.25, determine the minimum distance *x* at which the load can be supported. Neglect the weight of the bracket.

<u>SOLUTION</u>:

- When *W* is placed at minimum *x*, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.
- Apply conditions for static equilibrium to find minimum *x*.

Sample Problem 8.3



SOLUTION:

- When W is placed at minimum x, the bracket is about to slip and friction forces in upper and lower collars are at maximum value.

$$F_A = \mu_s N_A = 0.25 N_A$$

$$F_B = \mu_s N_B = 0.25 N_B$$

- Apply conditions for static equilibrium to find minimum x.

$$\sum F_x = 0: \quad N_B - N_A = 0 \qquad N_B = N_A$$

$$\sum F_y = 0: \quad F_A + F_B - W = 0$$

$$0.25 N_A + 0.25 N_B - W = 0$$

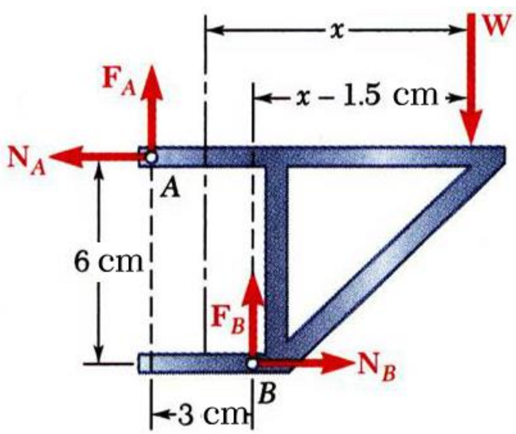
$$0.5 N_A = W \qquad N_A = N_B = 2W$$

$$\sum M_B = 0: N_A(6\text{ cm}) - F_A(3\text{ cm}) - W(x - 1.5\text{ cm}) = 0$$

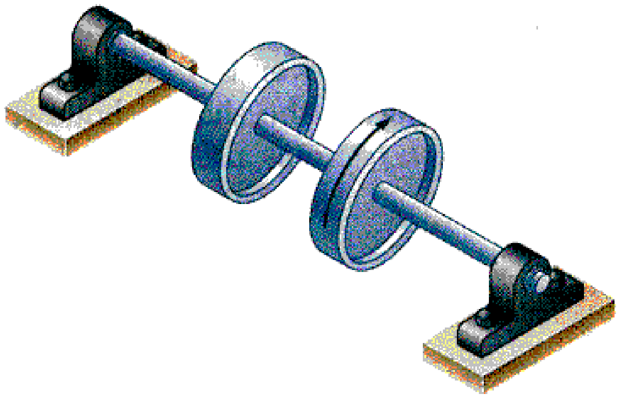
$$6N_A - 3(0.25 N_A) - W(x - 1.5) = 0$$

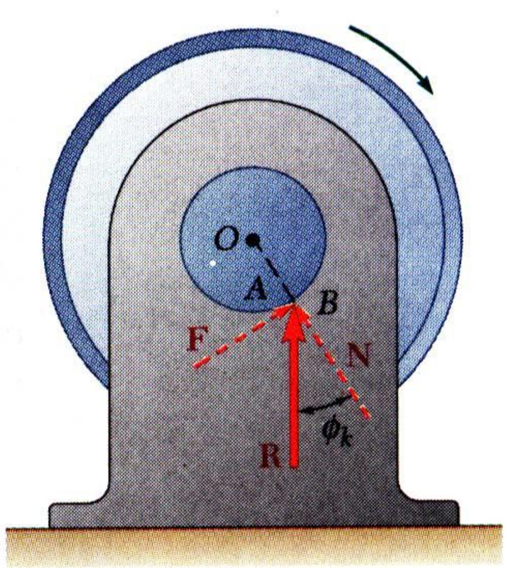
$$6(2W) - 0.75(2W) - W(x - 1.5) = 0$$

$$\boxed{x = 12\text{ cm}}$$



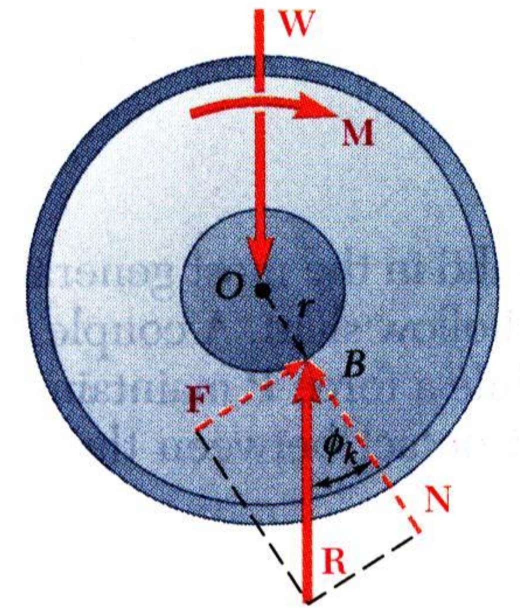
Journal Bearings. Axle Friction

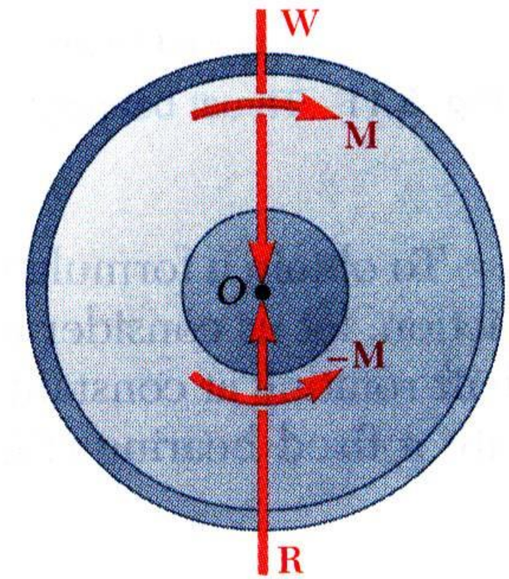


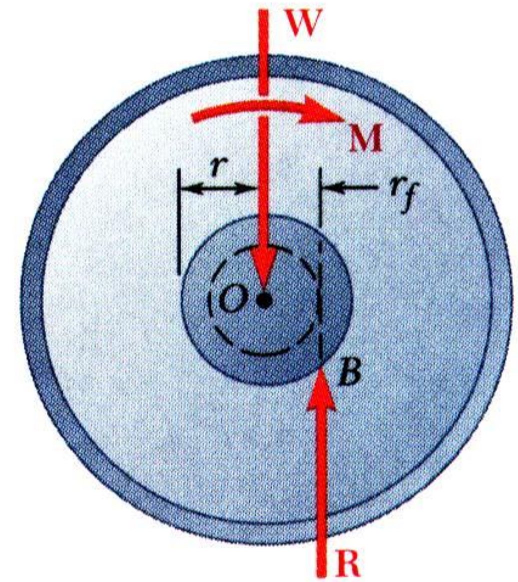


- Journal bearings provide lateral support to rotating shafts. Thrust bearings provide axial support
- Frictional resistance of fully lubricated bearings depends on clearances, speed and lubricant viscosity. Partially lubricated axles and bearings can be assumed to be in direct contact along a straight line.
- Forces acting on bearing are weight *W* of wheels and shaft, couple *M* to maintain motion, and reaction *R* of the bearing.
- Reaction is vertical and equal in magnitude to W.
- Reaction line of action does not pass through shaft center *O*; *R* is located to the right of *O*, resulting in a moment that is balanced by *M*.
- Physically, contact point is displaced as axle "climbs" in bearing.

Journal Bearings. Axle Friction







- Angle between *R* and normal to bearing surface is the angle of kinetic friction φ_k.

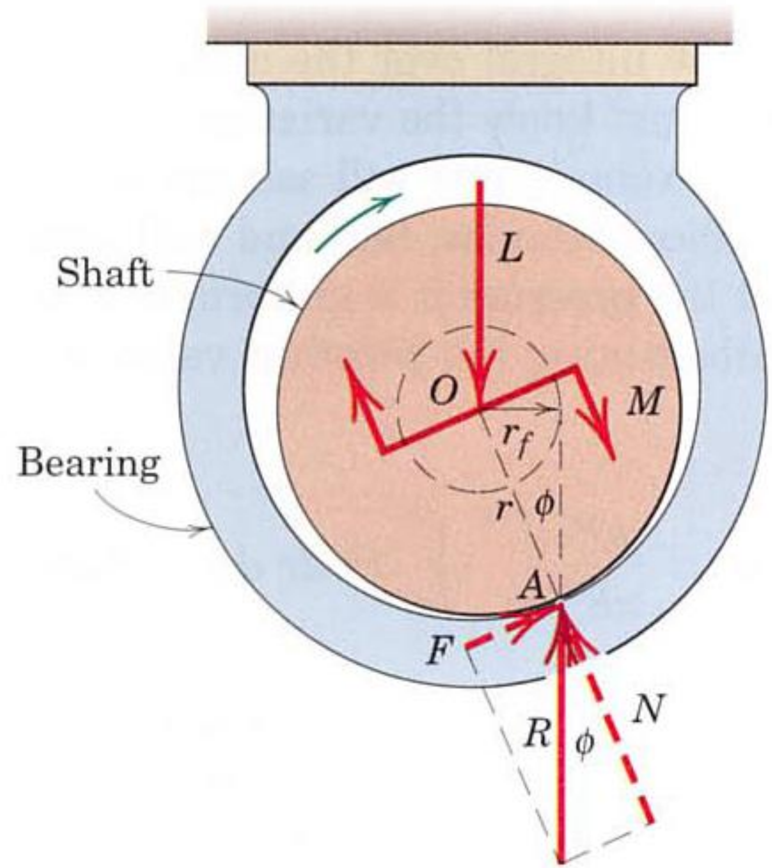
$$M = Rr\sin\phi_k$$
$$\approx Rr\mu_k$$

- May treat bearing reaction as force-couple system.

- For graphical solution, *R* must be tangent to *circle of friction*.

$$r_f = r\sin\phi_k$$
$$\approx r\mu_k$$

$$M = Lr_f = Lr \sin \phi$$



For a small coefficient of friction, the angle ϕ is small, and the sine and tangent may be interchanged with only small error. Since $\mu = \tan \phi$, a good approximation to the torque is

$$M = \mu Lr \qquad \textbf{(6/4a)}$$

Thrust Bearing-Disk friction

- Friction between circular surfaces under distributed normal pressure occurs in pivot bearings, clutch plates and disk brakes. To examine these applications,

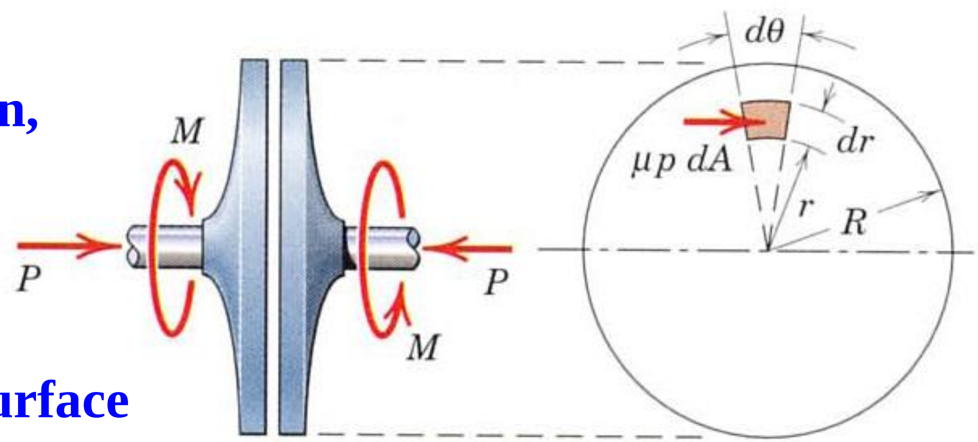
p is the normal pressure at any location

frictional force acting on an elemental area

$\mu p\, dA$, where μ is the friction coefficient and dA is the area $r\, dr\, d\theta$

- **Moment of the elemental friction,**

$$M = \int \mu p r\, dA$$



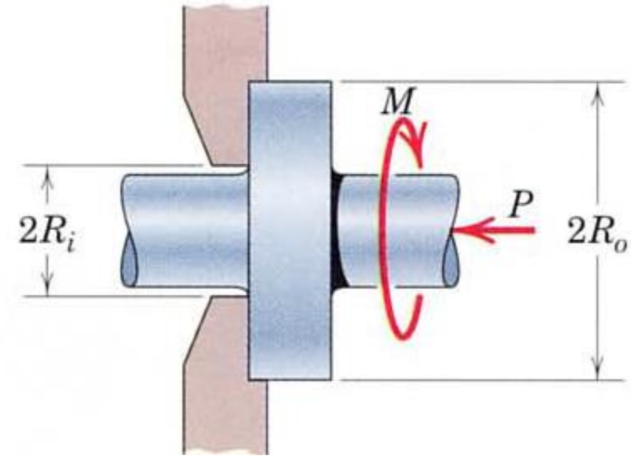
- **If p is uniform over the entire surface**

$$\pi R^2 p = P.$$

$$M = \frac{\mu P}{\pi R^2} \int_0^{2\pi} \int_0^R r^2\, dr\, d\theta = \tfrac{2}{3}\mu P R$$

If the friction disks are rings, as in the collar bearing shown in Fig

$$M = \frac{2}{3}\mu P \frac{R_o^{\ 3} - R_i^{\ 3}}{R_o^{\ 2} - R_i^{\ 2}}$$



After the initial wearing-in period is over, the surfaces retain their new relative shape and further wear is therefore constant over the surface. This wear depends on both the circumferential distance traveled and the pressure p. Since the distance traveled is proportional to r, the expression $rp = K$ may be written, where K is a constant. The value of K is determined from the equilibrium condition for the axial forces, which gives

$$P = \int p\, dA = K \int_0^{2\pi} \int_0^R dr\, d\theta = 2\pi KR$$

With $pr = K = P/(2\pi R)$, we may write the expression for M as

$$M = \int \mu pr\, dA = \frac{\mu P}{2\pi R} \int_0^{2\pi} \int_0^R r\, dr\, d\theta$$

which becomes

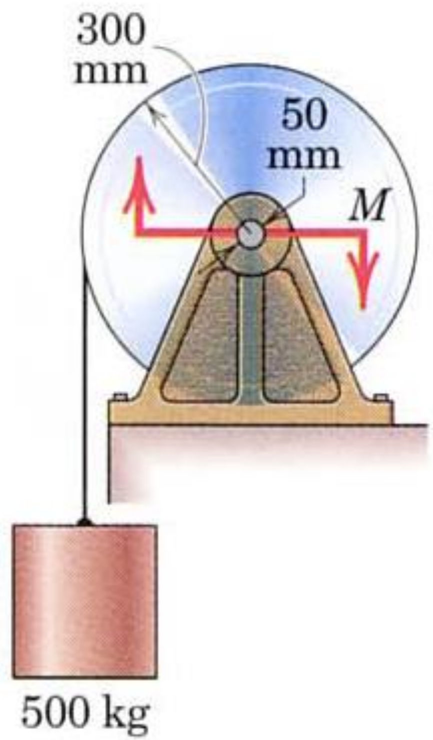
$$M = \tfrac{1}{2}\mu PR \qquad \textbf{(6/6)}$$

The frictional moment for worn-in plates is, therefore, only $(\frac{1}{2})/(\frac{2}{3})$, or $\frac{3}{4}$ as much as for new surfaces. If the friction disks are rings of inside radius R_i and outside radius R_o, substitution of these limits gives for the frictional torque for worn-in surfaces

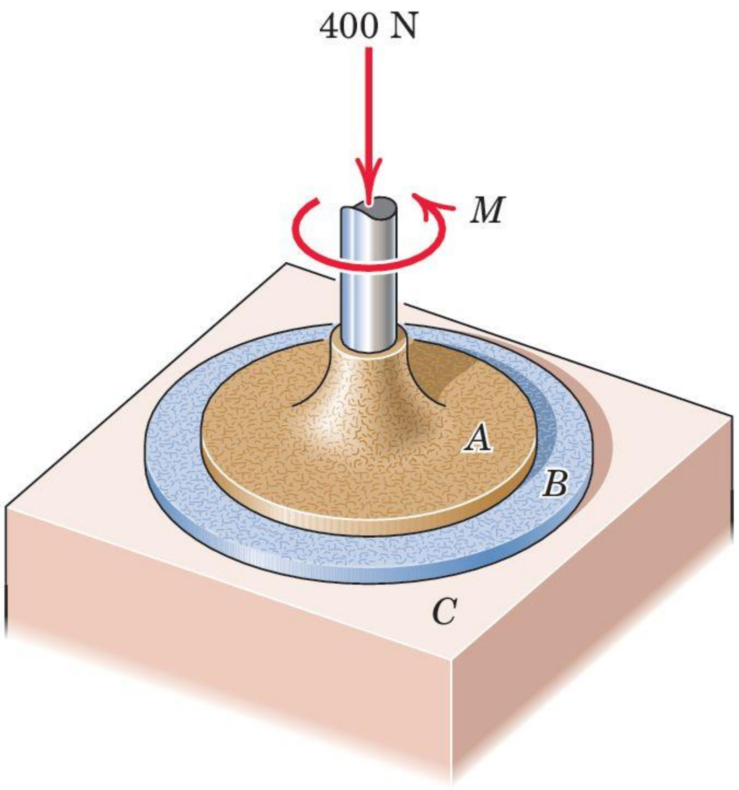
$$M = \tfrac{1}{2}\mu P(R_o + R_i) \qquad \textbf{(6/6}\boldsymbol{a}\textbf{)}$$

6/69 A torque M of 1510 N · m must be applied to the 50-mm-diameter shaft of the hoisting drum to raise the 500-kg load at constant speed. The drum and shaft together have a mass of 100 kg. Calculate the coefficient of friction μ for the bearing.

Ans. $\mu = 0.271$



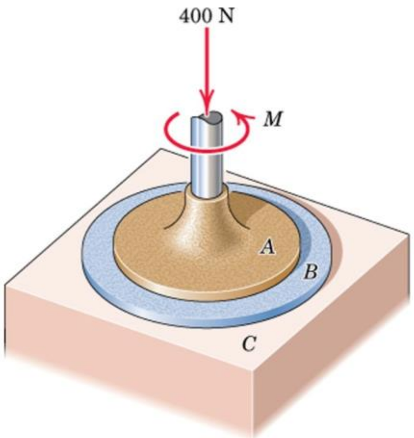
- **Diameter of A and B are 225 mm and 300mm and pressure under each disk is constant over the surface.**



If the coefficient of friction between A and B is 0.40, determine the couple M which will cause A to slip on B. Also, what is the minimum coefficient of friction μ between B and the supporting surface C which will prevent B from rotating?

Disk Friction

Circular disk *A* (225 mm dia) is placed on top of disk *B* (300 mm dia) and is subjected to a compressive force of 400 N. Pressure under each disk is constant over its surface. Coeff of friction betn *A* and *B* = 0.4.
Determine:
(a) the couple *M* which will cause *A* to slip on *B*.
(b) Min coeff of friction μ between *B* and supporting surface *C* which will prevent *B* from rotating.



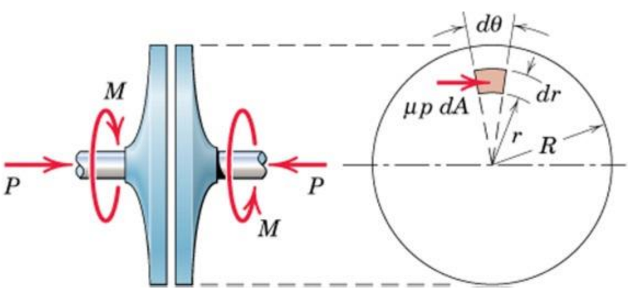
Solution

Solution:
(a) Impending slip between *A* and *B*:

μ=0.4, P=400 N, R=225/2 mm
M = 2/3 x 0.4 x 400 x 0.225/2 M = 12 Nm

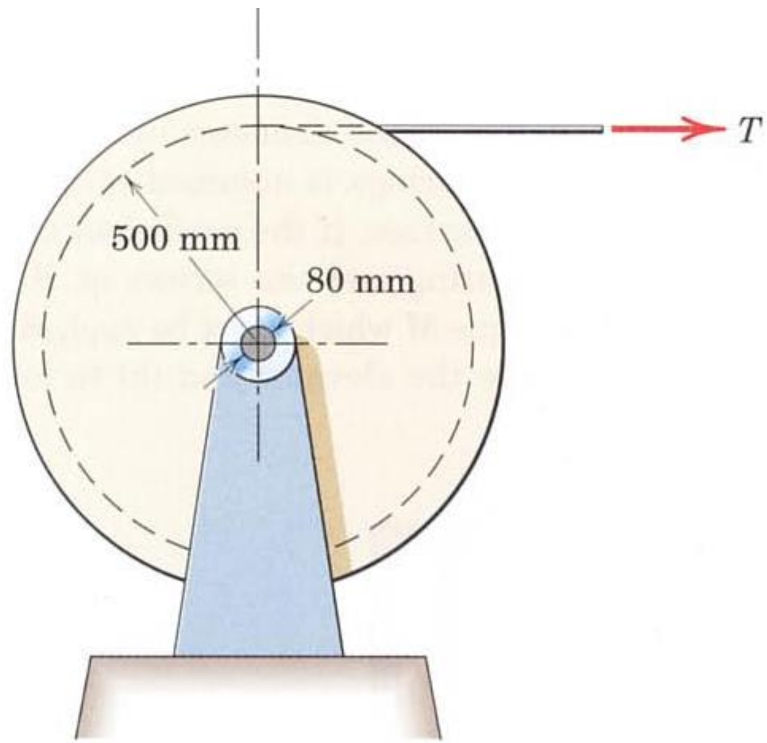
(b) Impending slip between *B* and *C* :
Slip between *A* and *B* M = 12 Nm

μ=? P=400 N, R=300/2 mm
12 = 2/3 x μ x 400 x 0.300/2 μ = 0.3



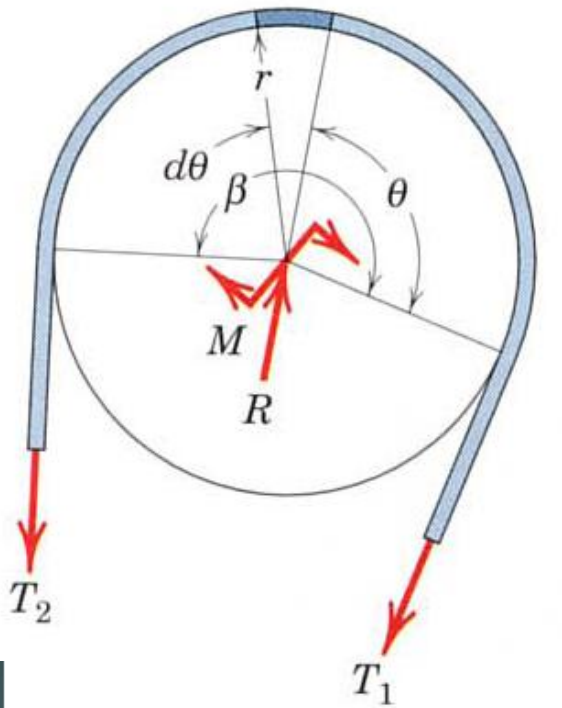
The telephone-cable reel has a mass of 250 kg and is mounted on an 80-mm-diameter shaft. If the coefficient of friction between the shaft and its bearing is 0.30, calculate the horizontal tension T required to turn the reel.

Ans. $T = 56.4$ N

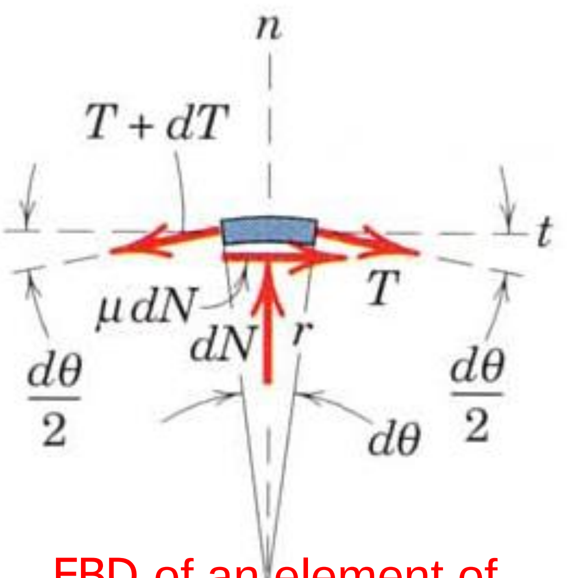


Belt Friction

Impending slippage of flexible cables, belts, ropes over sheaves, wheels, drums
It is necessary to estimate the frictional forces developed between the belt and its contacting surface.



Consider a drum subjected to two belt tensions (*T1* and *T2*)
M is the torque necessary to prevent rotation of the drum
R is the bearing reaction
r is the radius of the drum
β is the total contact angle between belt and surface (in radians)
T2 > *T1* since *M* is clockwise



FBD of an element of the belt of length *r dθ*

Frictional force for impending motion = μ *dN*

Equilibrium in the t-direction:

$$T\cos\frac{d\theta}{2} + \mu\, dN = (T + dT)\cos\frac{d\theta}{2}$$

$$\mu\, dN = dT$$

(As cosine of a differential quantity is unity in the limit)

$$dN = (T + dT)\sin\frac{d\theta}{2} + T\sin\frac{d\theta}{2}$$

Equilibrium in the *n*-direction:

or

$$dN = T\, d\theta$$

(As sine of a differential in the limit equals the angle, and product of two differentials can be neglected)

Combining the two equilibrium relations gives

$$\frac{dT}{T} = \mu\, d\theta$$

Integrating between corresponding limits yields

$$\int_{T_1}^{T_2}\frac{dT}{T} = \int_0^{\beta}\mu\, d\theta$$

or $\ln\dfrac{T_2}{T_1} = \mu\beta$ or $\boxed{T_2 = T_1 e^{\mu\beta}}$

Wheel Friction or Rolling Resistance

Resistance of a wheel to roll over a surface is caused by deformation between two materials of contact.

- This resistance is not due to tangential frictional forces
- Entirely different phenomenon from that of dry friction

Steel is very stiff
Low Rolling Resistance

Significant Rolling Resistance between rubber tyre and tar road

Large Rolling Resistance due to wet field

Actually materials are not rigid and deformation occurs at the point of contact. Reaction of surface on the cylinder consists of a distribution of normal pressure.

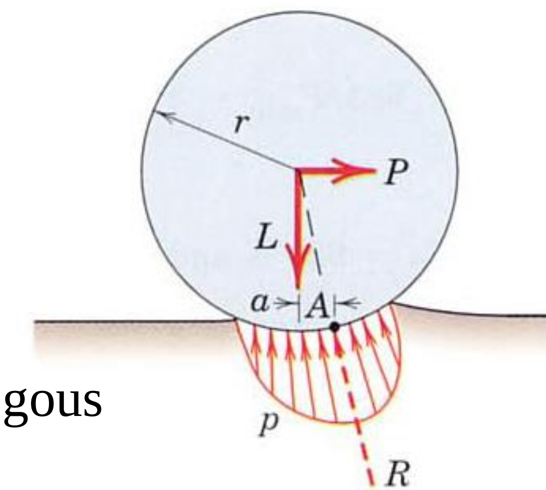
Consider a wheel under action of a load *L* on axle and a force *P* applied at its center to produce rolling

- Deformation of wheel and supporting surface as shown in Fig.
- Resultant *R* of the distribution of normal pressure must pass through wheel center for the wheel to be in equilibrium (i.e., rolling at a constant speed)
- *R* acts at point *A* on right of wheel center for rightwards motion

Force *P* required to maintain rolling at constant speed can be apprx. estimated as:

$$\sum M_A = 0 \Rightarrow L\,a = P\,r\cancelto{1}{\cos\theta}$$

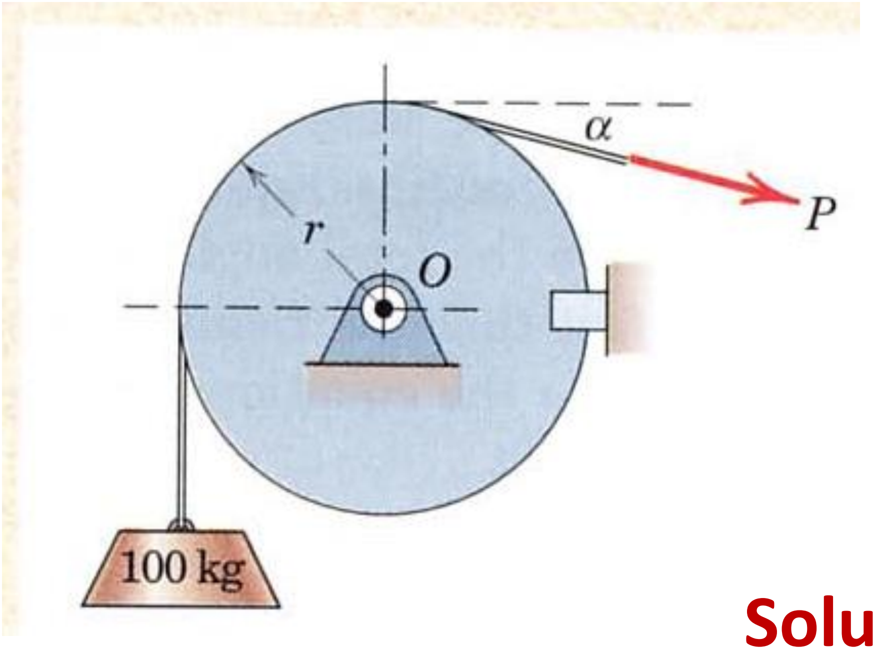
$$\Rightarrow \; P = \frac{a}{r}L = \mu_r L$$



- μ_r is the ratio of resisting force to the normal force analogous to μ_s *or* μ_k and called *Coefficient of Rolling Resistance*
- No slippage or impending slippage in interpretation of μ_r

Sample Problem

A flexible cable which supports the 100-kg load is passed over a fixed circular drum and subjected to a force P to maintain equilibrium. The coefficient of static friction μ between the cable and the fixed drum is 0.30. (a) For $\alpha = 0$, determine the maximum and minimum values which P may have in order not to raise or lower the load. (b) For $P = 500$ N, determine the minimum value which the angle α may have before the load begins to slip.



Solution on next page

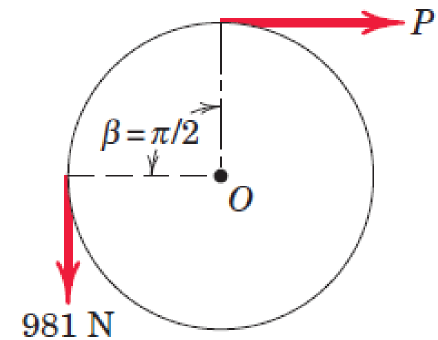
Impending slippage of the cable over the fixed drum is given by: $T_2 = T_1 e^{\mu\beta}$

(a) With $\alpha = 0$ the angle of contact is $\beta = \pi/2$ rad. For impending upward m tion of the load, $T_2 = P_{max}$, $T_1 = 981$ N, and we have

$$P_{max}/981 = e^{0.30(\pi/2)} \qquad P_{max} = 981(1.602) = 1572 \text{ N} \qquad \textit{An}$$

For impending downward motion of the load, $T_2 = 981$ N and $T_1 = P_{min}$. Thus,

$$981/P_{min} = e^{0.30(\pi/2)} \qquad P_{min} = 981/1.602 = 612 \text{ N} \qquad \textit{Ans.}$$



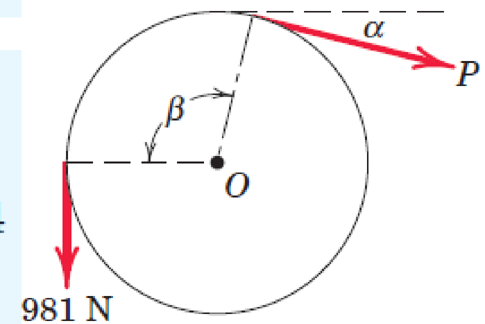
(a) $\alpha = 0$

(b) With $T_2 = 981$ N and $T_1 = P = 500$ N, Eq. 6/7 gives us

$$981/500 = e^{0.30\beta} \qquad 0.30\beta = \ln(981/500) = 0.674$$

$$\beta = 2.25 \text{ rad} \qquad \text{or} \qquad \beta = 2.25\left(\frac{360}{2\pi}\right) = 128.7°$$

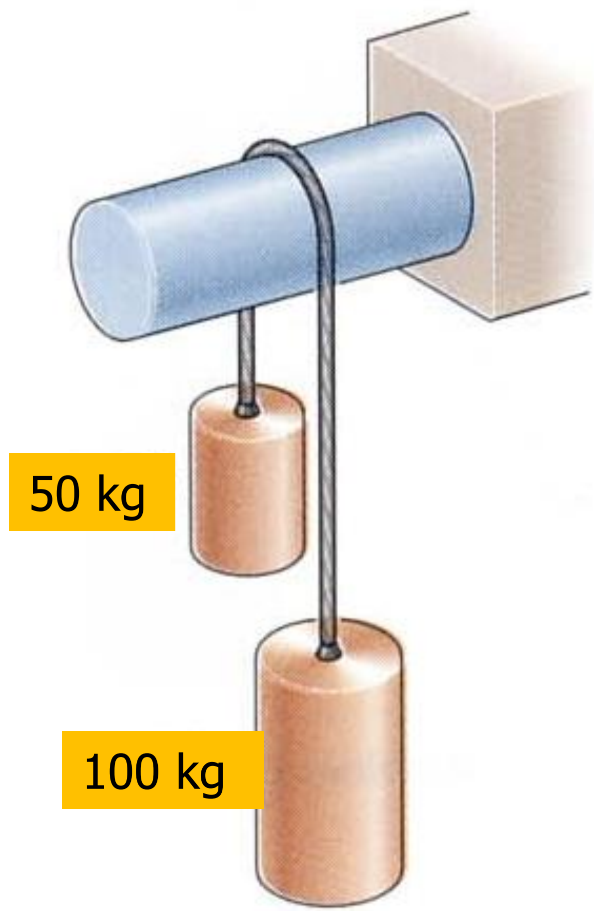
$$\alpha = 128.7° - 90° = 38.7°$$



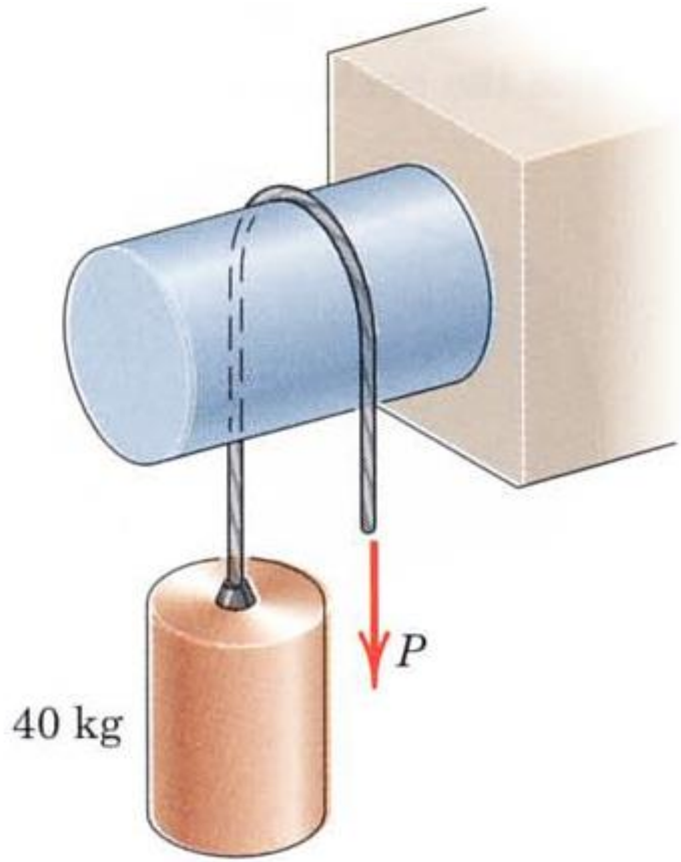
(b) $P = 500$ N

FBD

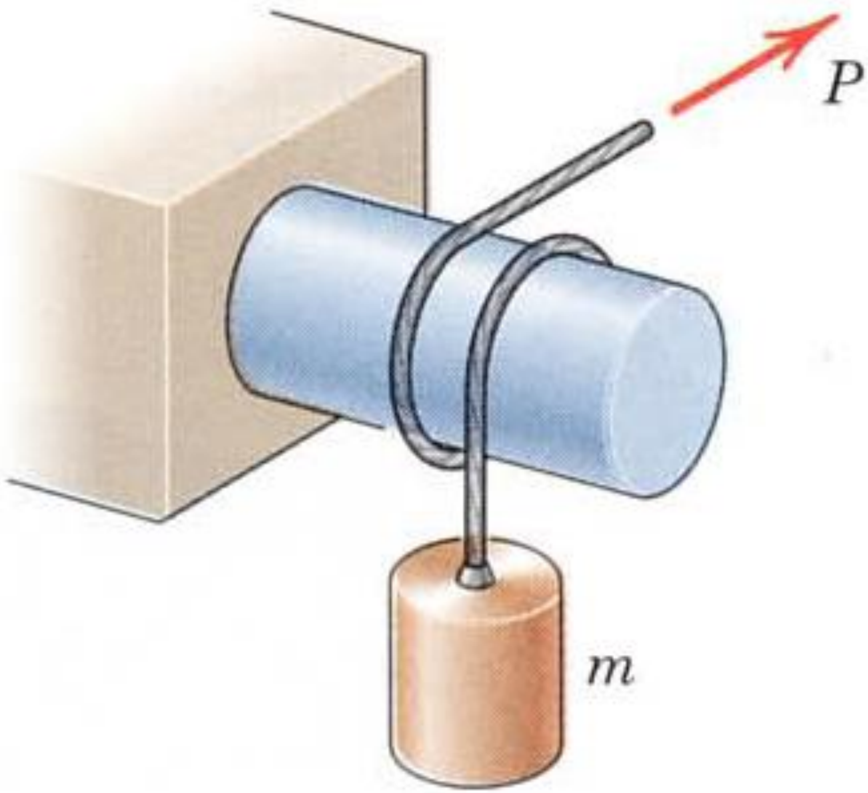
What is the minimum coefficient of friction μ between the rope and the fixed shaft which will prevent the unbalanced cylinders from moving?



Determine the force P required to (a) raise and (b) lower the 40-kg cylinder at a slow steady speed. The coefficient of friction between the cord and its supporting surface is 0.30.



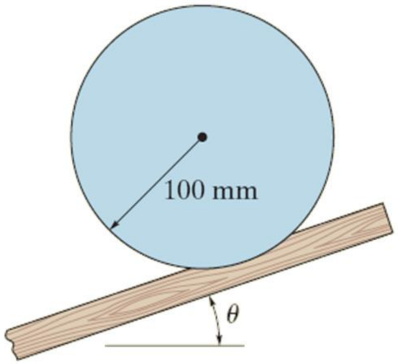
6/90 A force $P = mg/6$ is required to lower the cylinder at a constant slow speed with the cord making $1\frac{1}{4}$ turns around the fixed shaft. Calculate the coefficient of friction μ between the cord and the shaft.



Problem 6/90

Rolling Resistance

A 10 kg steel wheel (radius = 100 mm) rests on an inclined plane made of wood. At θ =1.2º, the wheel begins to roll-down the incline with constant velocity. Determine the coefficient of rolling resistance.



Solution

When the wheel has impending motion, the normal reaction *N* acts at point *A* defined by the dimension *a*. Draw the FBD for the wheel: *r* = 100 mm, 10 kg = 98.1 N

Using simplified equation directly: $P = \frac{a}{r} L = \mu_r L$

Here P = 98.1(sin1.2) = 2.05 N

L = 98.1(cos1.2) = 98.08 N

Coeff of Rolling Resistance $\mu_r = 0.0209$

Alternatively, $\sum M_A = 0$

98.1(*sin*1.2)(*r* appx) = 98.1(*cos*1.2)*a*

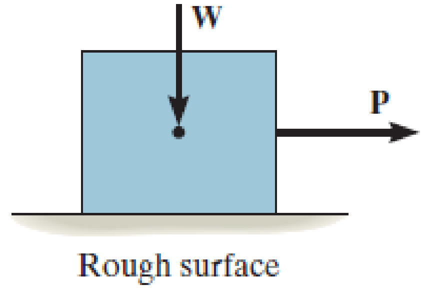
(since *r*cos1.2 = *r*x0.9998 *r*)

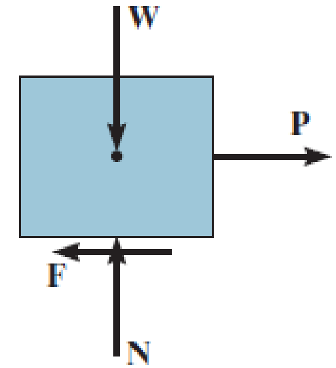
$a/r = \mu r = 0.0209$

Chapter review

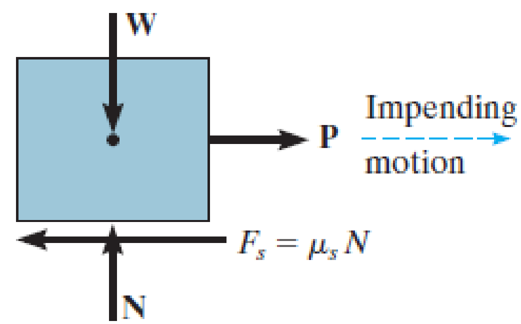
Dry Friction

Frictional forces exist between two rough surfaces of contact. These forces act on a body so as to oppose its motion or tendency of motion.

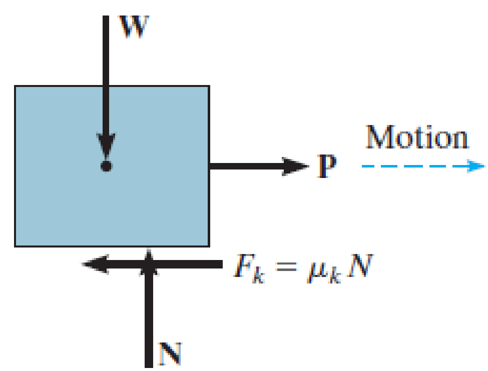




A static frictional force approaches a maximum value of $F_s = \mu_s N$, where μ_s is the *coefficient of static friction*. In this case, motion between the contacting surfaces is *impending*.

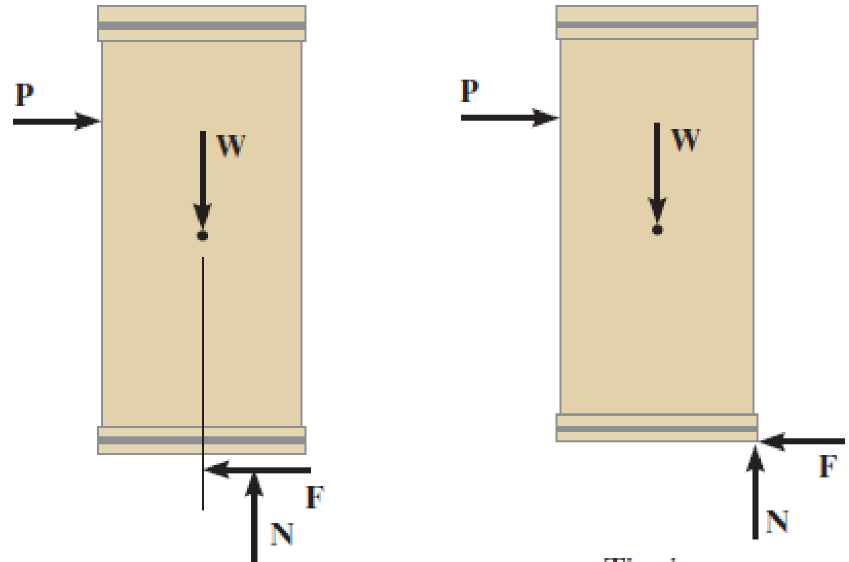


If slipping occurs, then the friction force remains essentially constant and equal to $F_k = \mu_k N$. Here μ_k is the *coefficient of kinetic friction*.



The solution of a problem involving friction requires first drawing the free-body diagram of the body. If the unknowns cannot be determined strictly from the equations of equilibrium, and the possibility of slipping occurs, then the friction equation should be applied at the appropriate points of contact in order to complete the solution.

It may also be possible for slender objects, like crates, to tip over, and this situation should also be investigated.



Impending slipping
$F = \mu_s N$

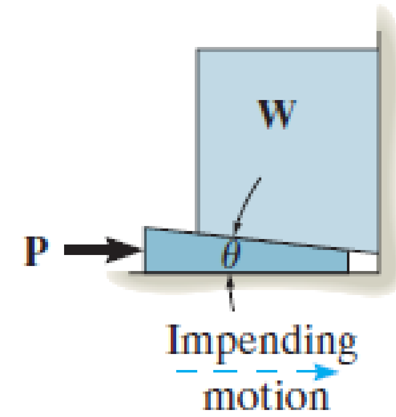
Tipping

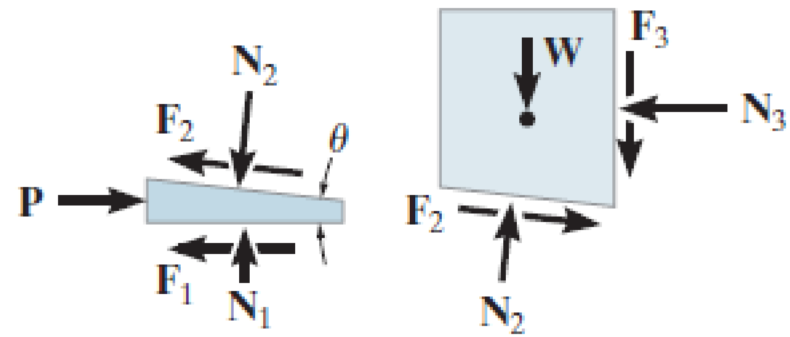
Wedges

Wedges are inclined planes used to increase the application of a force. The two force equilibrium equations are used to relate the forces acting on the wedge.

An applied force **P** must push on the wedge to move it to the right.

If the coefficients of friction between the surfaces are large enough, then **P** can be removed, and the wedge will be self-locking and remain in place.





$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$

Screws

Square-threaded screws are used to move heavy loads. They represent an inclined plane, wrapped around a cylinder.

The moment needed to turn a screw depends upon the coefficient of friction and the screw's lead angle θ.

If the coefficient of friction between the surfaces is large enough, then the screw will support the load without tending to turn, i.e., it will be self-locking.

$$M = rW\tan(\theta + \phi_s)$$

Upward Impending Screw Motio

$$M' = rW\tan(\theta - \phi_s)$$

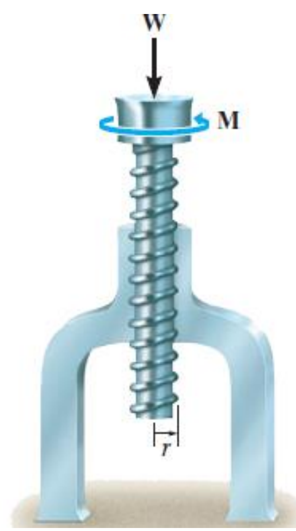
Downward Impending Screw Motion

$$\theta > \phi_s$$

$$M'' = rW\tan(\phi_s - \theta)$$

Downward Screw Motion

$$\phi_s > \theta$$

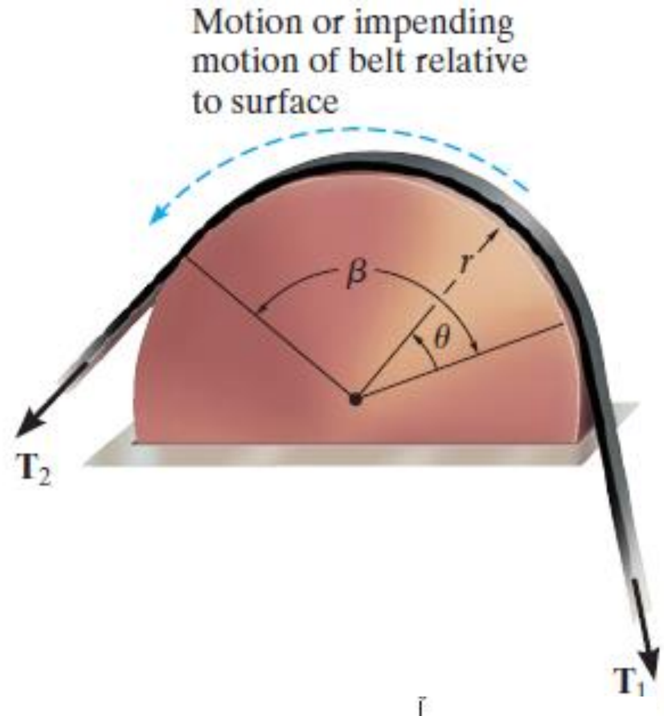


Flat Belts

The force needed to move a flat belt over a rough curved surface depends only on the angle of belt contact, β, and the coefficient of friction.

$$T_2 = T_1 e^{\mu\beta}$$

$$T_2 > T_1$$

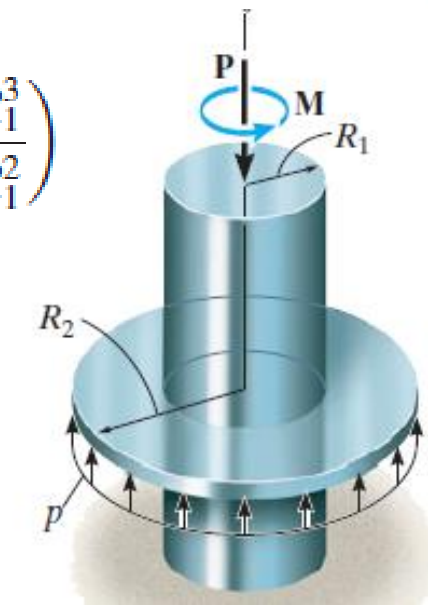


Collar Bearings and Disks

The frictional analysis of a collar bearing or disk requires looking at a differential element of the contact area. The normal force acting on this element is determined from force equilibrium along the shaft, and the moment needed to turn the shaft at a constant rate is determined from moment equilibrium about the shaft's axis.

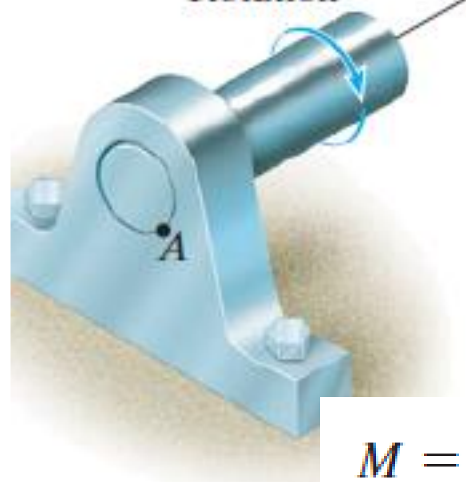
If the pressure on the surface of a collar bearing is uniform, then integration gives the result shown.

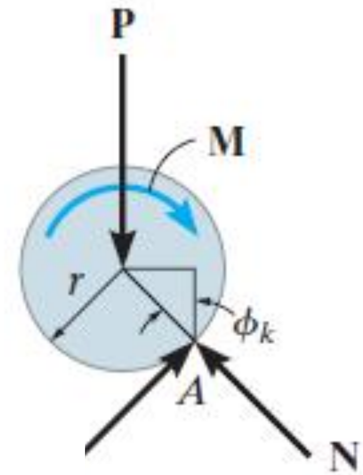
$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$



Journal Bearings

When a moment is applied to a shaft in a nonlubricated or partially lubricated journal bearing, the shaft will tend to roll up the side of the bearing until slipping occurs. This defines the radius of a friction circle, and from it the moment needed to turn the shaft can be determined.





$$M = Rr \sin \phi_k$$

Rolling Resistance

The resistance of a wheel to rolling over a surface is caused by localized *deformation* of the two materials in contact. This causes the resultant normal force acting on the rolling body to be inclined so that it provides a component that acts in the opposite direction of the applied force **P** causing the motion. This effect is characterized using the *coefficient of rolling resistance*, *a*, which is determined from experiment.

$$P \approx \frac{Wa}{r}$$