Engineering Mechanics: ME101

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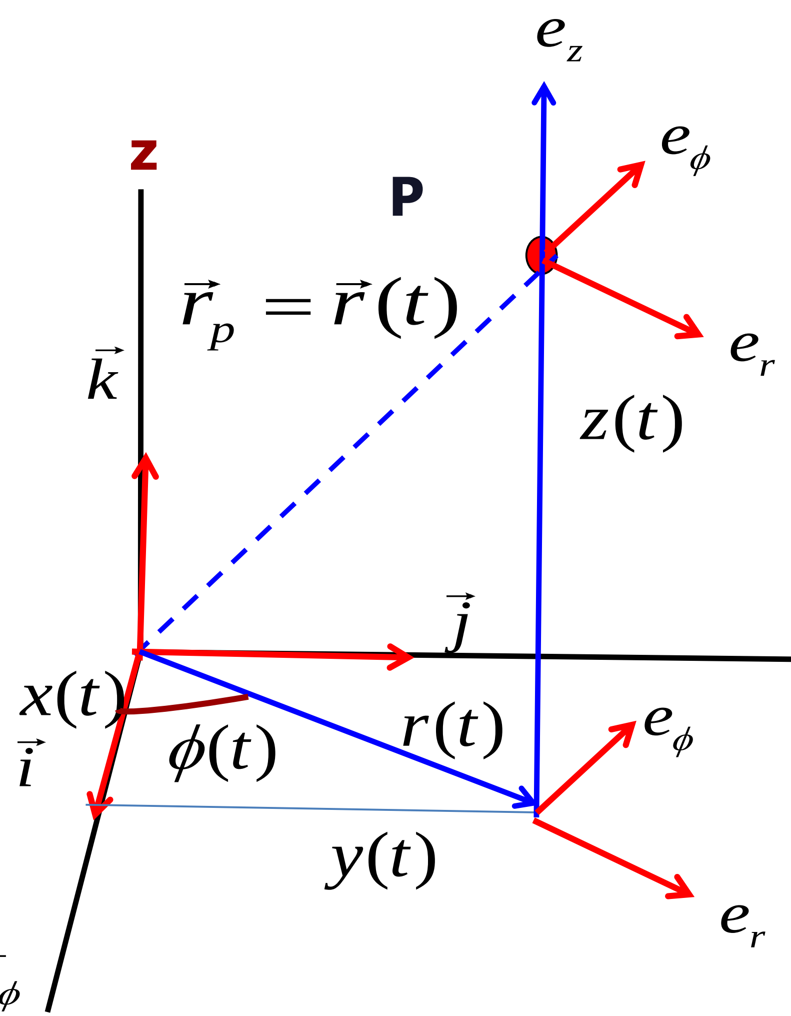
Kinematics of Points in Cylindrical coordinates

Let $r(t), \phi(t), z(t)$ be the cylindrical coordinates moving point P at time t

$\overline{e}_r$ **is the radial outward unit vector, in the direction of increasing r, tangent to the radial line r: (ϕ, z) constant**

$\overline{e}_\phi$ **Circumferential unit vector ,in the direction of increasing ϕ, tangent to the circum. Coordinate line ϕ : (r, z) constant**

$\overline{e}_z$ **Axial unit vector ,in the direction of increasing z, tangent to the axial. Coordinate line z: (r, ϕ) constant**

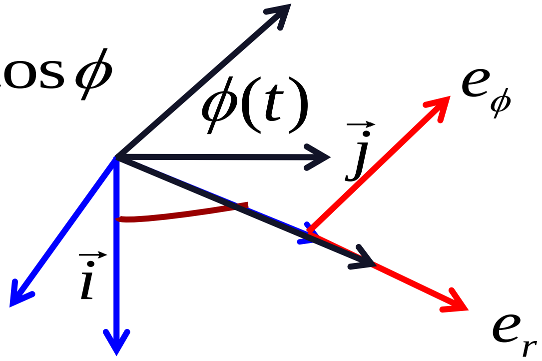


Right handed triad with

$$\overline{e}_r \times \overline{e}_\phi = \overline{e}_z, \quad \overline{e}_\phi \times \overline{e}_z = \overline{e}_r, \quad \overline{e}_z \times \overline{e}_r = \overline{e}_\phi$$

Kinematics of Points in Cylindrical polar coordinates

Relation and derivatives



$$\bar{e}_r(t) = \bar{i}\cos\phi + \bar{j}\sin\phi \qquad \bar{e}_\phi(t) = -\bar{i}\sin\phi + \bar{j}\cos\phi$$

$$\dot{\bar{e}}_r(t) = (-\bar{i}\sin\phi + \bar{j}\cos\phi)\dot{\phi} = \dot{\phi}\bar{e}_\phi$$

$$\dot{\bar{e}}_\phi(t) = (-\bar{i}\cos\phi - \bar{j}\sin\phi)\dot{\phi} = -\dot{\phi}\bar{e}_r$$

$$\dot{\bar{e}}_z(t) = 0 \qquad \bar{e}_z(t) = \bar{k}$$

Velocity and acceleration

$$\bar{r} = r(t)\bar{e}_r(t) + z(t)\bar{e}_z(t) \Rightarrow \bar{v} = \dot{r}\bar{e}_r + r\dot{\bar{e}}_r + \dot{z}\bar{e}_z + z\dot{\bar{e}}_z$$

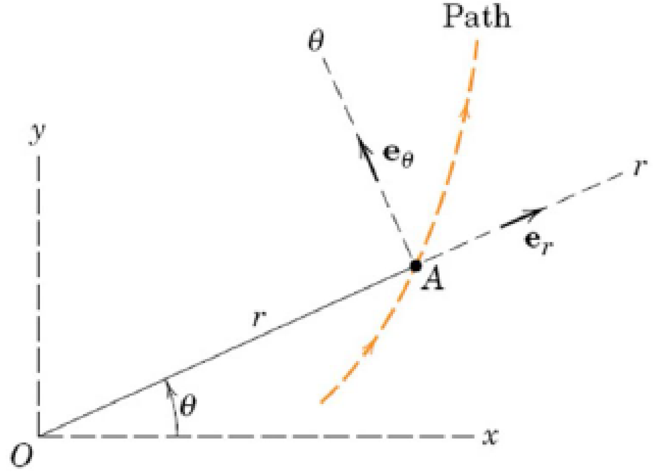
$$\bar{v} = \underbrace{\dot{r}}_{v_r}\bar{e}_r + \underbrace{r\dot{\phi}}_{v_\phi}\bar{e}_\phi + \underbrace{\dot{z}}_{v_z}\bar{e}_z \qquad v = (v_r^2 + v_\phi^2 + v_z^2)^{1/2}$$

$$\bar{a} = \dot{\bar{v}} = \ddot{r}\ \bar{e}_r + \dot{r}\ \dot{\bar{e}}_r + \dot{r}\dot{\phi}\bar{e}_\phi + r\ddot{\phi}\bar{e}_\phi + r\dot{\phi}\dot{\bar{e}}_\phi + \ddot{z}\ \bar{e}_z + z\dot{\bar{e}}_z$$

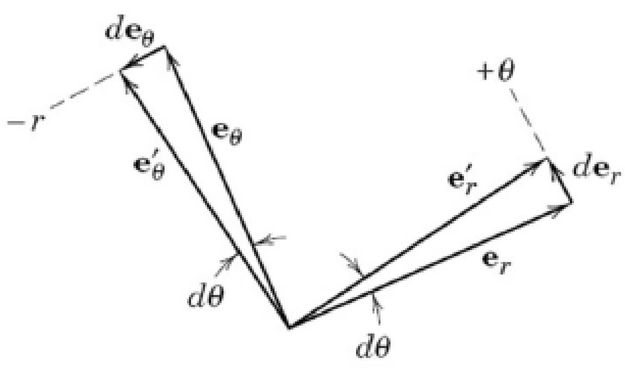
$$\bar{a} = \underbrace{(\ddot{r} - r\dot{\phi}^2)}_{a_r}\ \bar{e}_r + \underbrace{(2\dot{r}\dot{\phi} + r\ddot{\phi})}_{a_\phi}\ \bar{e}_\phi + \underbrace{\ddot{z}}_{a_z}\ \bar{e}_z$$

$$a = (a_r^2 + a_\phi^2 + a_z^2)^{1/2}$$

Time Derivative of the Unit Vectors in Polar (Cylindrical) Coordinates (2D)



(a)



(b)

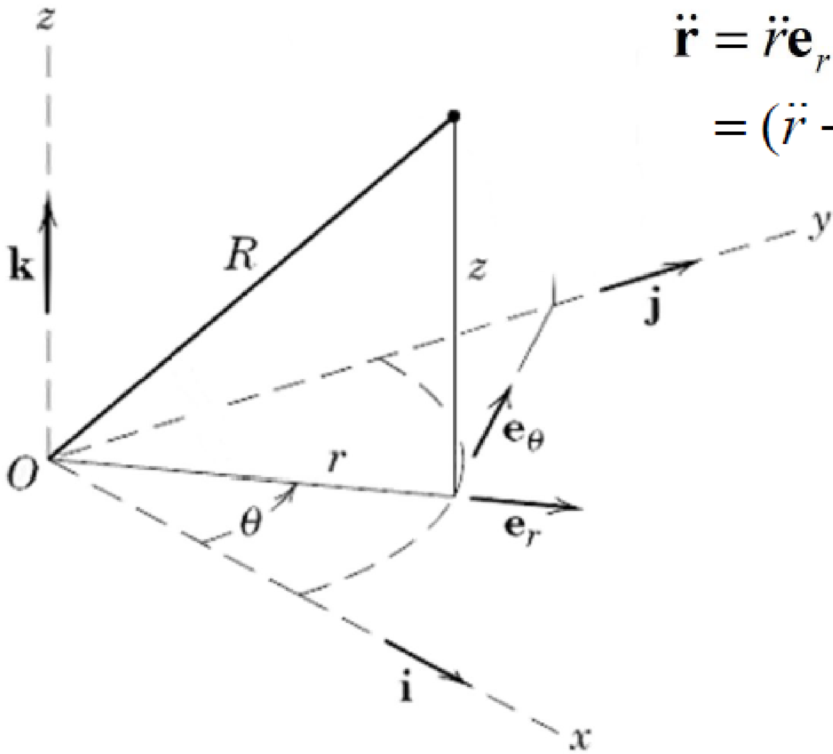
$$\frac{d\mathbf{e}_r}{d\theta} = \mathbf{e}_\theta$$

$$\frac{d\mathbf{e}_\theta}{d\theta} = -\mathbf{e}_r$$

$$\dot{\mathbf{e}}_r = \frac{d\mathbf{e}_r}{dt} = \frac{d\theta}{dt}\frac{d\mathbf{e}_r}{d\theta} = \dot{\theta}\mathbf{e}_\theta$$

$$\dot{\mathbf{e}}_\theta = \frac{d\mathbf{e}_\theta}{dt} = \frac{d\theta}{dt}\frac{d\mathbf{e}_\theta}{d\theta} = -\dot{\theta}\mathbf{e}_r$$

Cylindrical Coordinates (3D)



$$\mathbf{r} = r\mathbf{e}_r + z\mathbf{e}_z$$

$$\dot{\mathbf{r}} = \dot{r}\mathbf{e}_r + r\dot{\mathbf{e}}_r + \dot{z}\mathbf{e}_z + z\dot{\mathbf{e}}_z$$
$$= \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{e}_z$$

$$\ddot{\mathbf{r}} = \ddot{r}\mathbf{e}_r + \dot{r}\dot{\mathbf{e}}_r + \dot{r}\dot{\theta}\mathbf{e}_\theta + r\ddot{\theta}\mathbf{e}_\theta + r\dot{\theta}\dot{\mathbf{e}}_\theta + \ddot{z}\mathbf{e}_z$$
$$= (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{e}_z$$

$$v_r = \dot{r}$$
$$v_\theta = r\dot{\theta}$$
$$v_z = \dot{z}$$
$$v = \sqrt{{v_r}^2 + {v_\theta}^2 + {v_z}^2}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = \frac{1}{r}\frac{d}{dt}(r^2\dot{\theta})$$
$$a_z = \ddot{z}$$
$$a = \sqrt{{a_r}^2 + {a_\theta}^2 + {a_z}^2}$$

Kinematics of Points in spherical coordinates

Let $r(t), \theta(t), \phi(t)$ **be the spherical polar coordinates moving point P at time t**

Right handed triad with

$$\bar{e}_r \times \bar{e}_\theta = \bar{e}_\phi, \quad \bar{e}_\theta \times \bar{e}_\phi = \bar{e}_r, \quad \bar{e}_\phi \times \bar{e}_r = \bar{e}_\theta$$

$$r = (x^2 + y^2 + z^2)^{1/2}, \quad \theta = \tan^{-1}[(x^2 + y^2)^{1/2}/(z)]$$

$$x = r\sin\theta\cos\phi, \quad y = r\sin\theta\sin\phi, \quad z = r\cos\theta$$

$$\bar{e}_r(t) = \sin\theta(\bar{i}\cos\phi + \bar{j}\sin\phi) + \bar{k}\cos\theta$$

$$\bar{e}_\phi(t) = -\bar{i}\sin\phi + \bar{j}\cos\phi$$

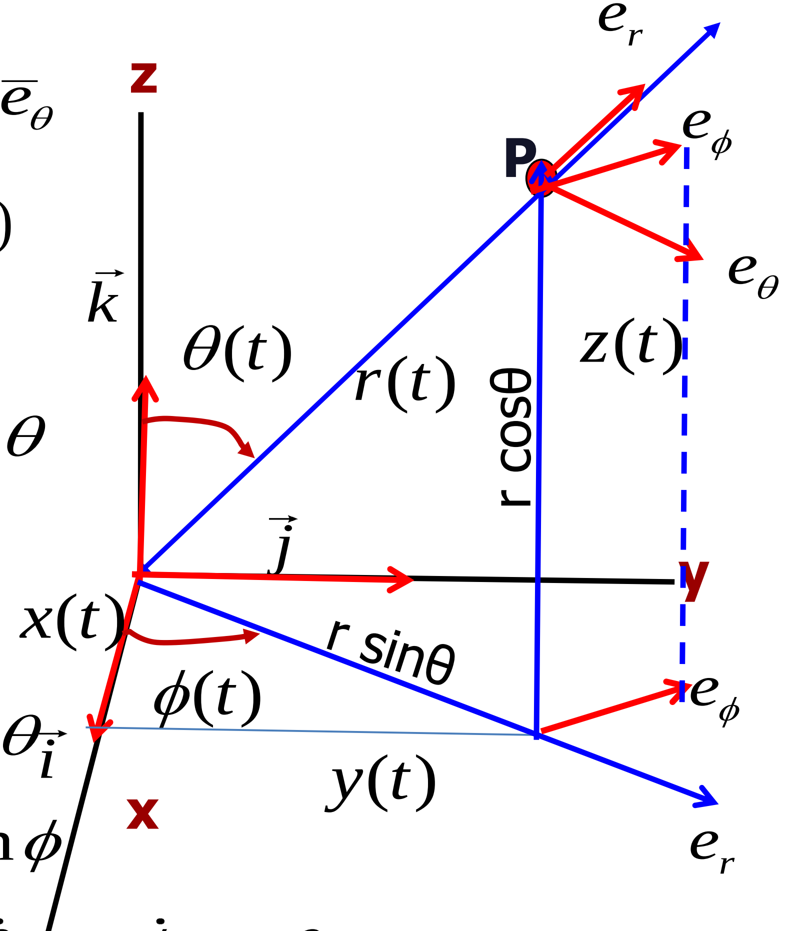
$$\bar{e}_\theta = \bar{e}_\phi \times \bar{e}_r$$

$$\bar{e}_\theta(t) = \cos\theta(\bar{i}\cos\phi + \bar{j}\sin\phi) - \bar{k}\sin\theta$$

$$\cos\theta\bar{e}_\theta(t) + \sin\theta\bar{e}_r(t) = \bar{i}\cos\phi + \bar{j}\sin\phi$$

$$\dot{\bar{e}}_r(t) = \dot{\theta}\bar{e}_\theta + \dot{\phi}\sin\theta\bar{e}_\phi \qquad \dot{\bar{e}}_\theta(t) = -\dot{\theta}\bar{e}_r + \dot{\phi}\cos\theta\bar{e}_\phi$$

$$\dot{\bar{e}}_\phi(t) = -\dot{\phi}(\sin\theta\bar{e}_r + \cos\theta\bar{e}_\theta)$$

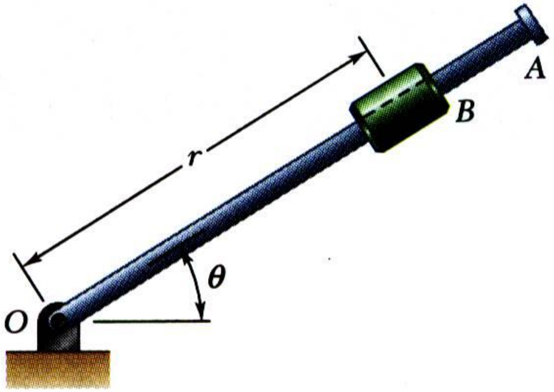


Kinematics of Points in spherical coordinates

$$\overline{v} = \dot{r}\overline{e}_r + r\dot{\theta}\overline{e}_\theta + r\dot{\phi}\sin\theta\overline{e}_\phi$$

$$\overline{a} = (\ddot{r} - r\dot{\theta}^2 - r\dot{\phi}^2\sin^2\theta)\ \overline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\phi}^2\sin\theta\cos\theta)\ \overline{e}_\theta$$
$$+[(r\ddot{\phi} + 2\dot{r}\dot{\phi})\sin\theta - 2r\dot{\theta}\dot{\phi}\cos\theta)]\ \overline{e}_\phi$$

Sample Problem 11.6



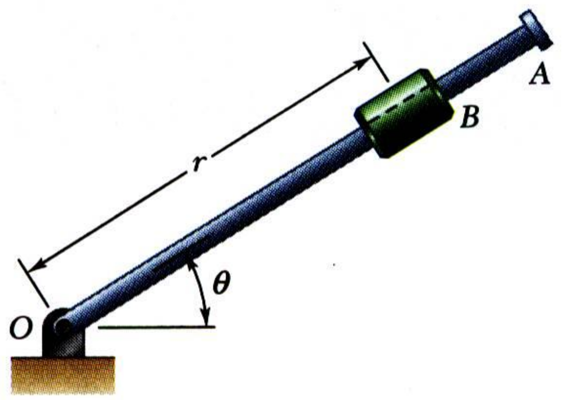
Rotation of the arm about O is defined by $\theta = 0.15t^2$ where θ is in radians and t in seconds. Collar B slides along the arm such that $r = 0.9 - 0.12t^2$ where r is in meters.

After the arm has rotated through 30^o, determine *(a)* the total velocity of the collar, *(b)* the total acceleration of the collar, and *(c)* the relative acceleration of the collar with respect to the arm.

SOLUTION:

- Evaluate time t for $\theta = 30^o$.
- Evaluate radial and angular positions, and first and second derivatives at time t.
- Calculate velocity and acceleration in cylindrical coordinates.
- Evaluate acceleration with respect to arm.

Sample Problem 11.6



SOLUTION:

- Evaluate time t for $\theta = 30^o$.

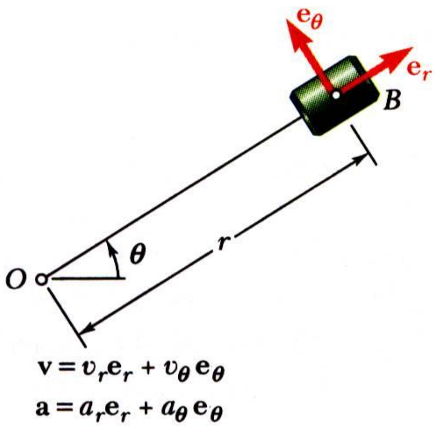
$$\theta = 0.15t^2$$
$$= 30° = 0.524\,\text{rad} \qquad t = 1.869\,\text{s}$$

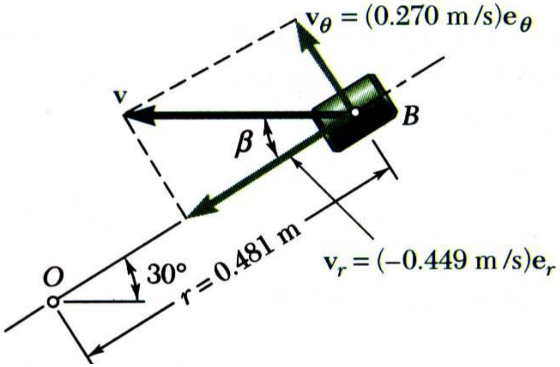
- Evaluate radial and angular positions, and first and second derivatives at time t.

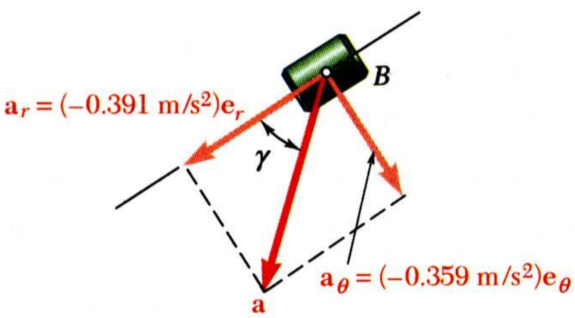
$$r = 0.9 - 0.12t^2 = 0.481\,\text{m}$$
$$\dot{r} = -0.24t = -0.449\,\text{m/s}$$
$$\ddot{r} = -0.24\,\text{m/s}^2$$

$$\theta = 0.15t^2 = 0.524\,\text{rad}$$
$$\dot{\theta} = 0.30t = 0.561\,\text{rad/s}$$
$$\ddot{\theta} = 0.30\,\text{rad/s}^2$$

Sample Problem 11.6







- Calculate velocity and acceleration.

$$v_r = \dot{r} = -0.449\,\mathrm{m/s}$$

$$v_\theta = r\dot{\theta} = (0.481\,\mathrm{m})(0.561\,\mathrm{rad/s}) = 0.270\,\mathrm{m/s}$$

$$v = \sqrt{v_r^2 + v_\theta^2} \qquad \beta = \tan^{-1}\frac{v_\theta}{v_r}$$

$$\boxed{v = 0.524\,\mathrm{m/s} \qquad \beta = 31.0^\circ}$$

$$a_r = \ddot{r} - r\dot{\theta}^2$$

$$= -0.240\,\mathrm{m/s^2} - (0.481\,\mathrm{m})(0.561\,\mathrm{rad/s})^2$$

$$= -0.391\,\mathrm{m/s^2}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

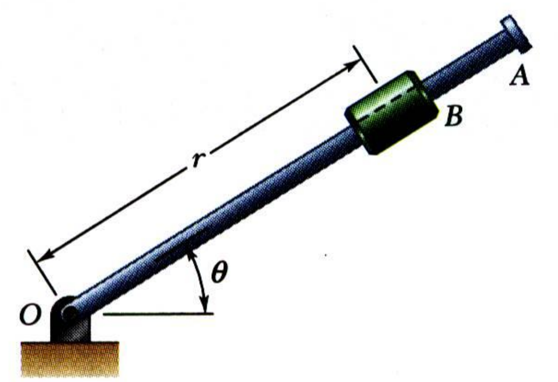
$$= (0.481\,\mathrm{m})\left(0.3\,\mathrm{rad/s^2}\right) + 2(-0.449\,\mathrm{m/s})(0.561\,\mathrm{rad/s})$$

$$= -0.359\,\mathrm{m/s^2}$$

$$a = \sqrt{a_r^2 + a_\theta^2} \qquad \gamma = \tan^{-1}\frac{a_\theta}{a_r}$$

$$\boxed{a = 0.531\,\mathrm{m/s} \qquad \gamma = 42.6^\circ}$$

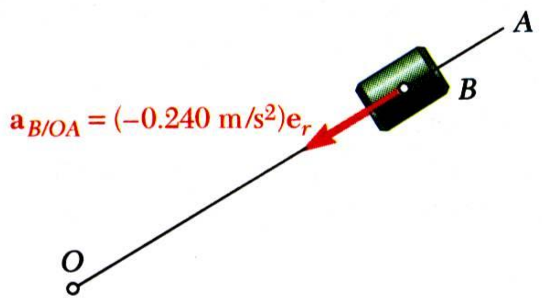
Sample Problem 11.6



- Evaluate acceleration with respect to arm.

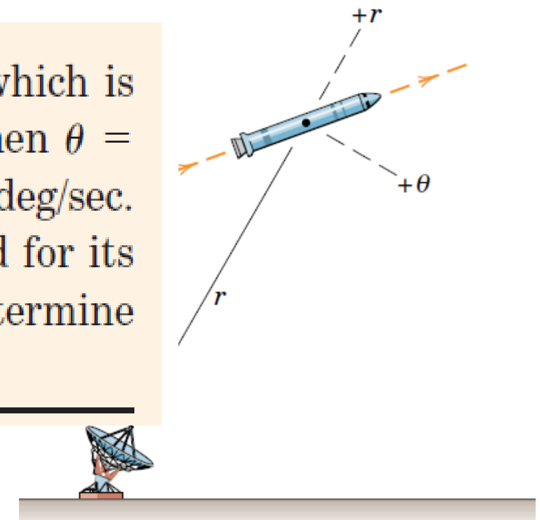
  Motion of collar with respect to arm is rectilinear and defined by coordinate *r*.

$$a_{B/OA} = \ddot{r} = -0.240\,\mathrm{m/s^2}$$



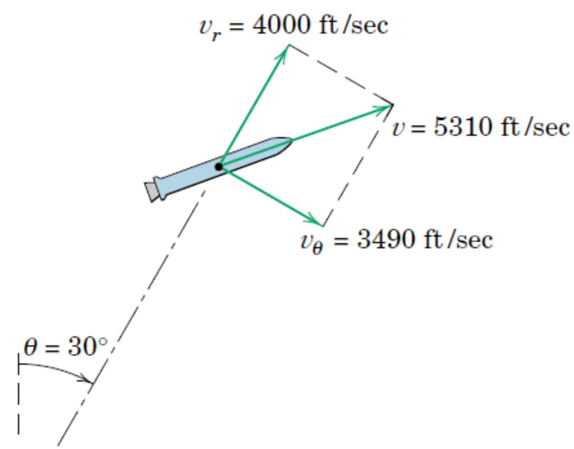
Sample Problem 2/10

A tracking radar lies in the vertical plane of the path of a rocket which is coasting in unpowered flight above the atmosphere. For the instant when $\theta = 30°$, the tracking data give $r = 25(10^4)$ ft, $\dot{r} = 4000$ ft/sec, and $\dot{\theta} = 0.80$ deg/sec. The acceleration of the rocket is due only to gravitational attraction and for its particular altitude is 31.4 ft/sec^2 vertically down. For these conditions determine the velocity v of the rocket and the values of $\ddot{r}$ and $\ddot{\theta}$.



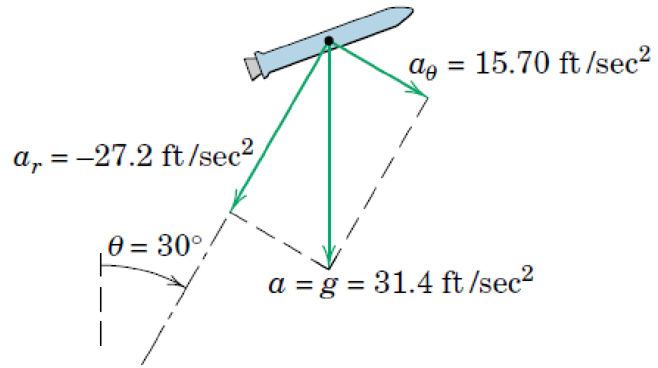
Solution steps

1. Obtain velocity component first then resultant velocity
2. Obtain acceleration components and using reltions, determine the



$$\theta = 30^0, r = 8\times10^4 m, \dot{r} = 1200 m/s,$$

$$\dot{\theta} = 0.80 \deg/s$$



Components of velocity

$$\bar{v} = \dot{r}\bar{e}_r + r\dot{\phi}\bar{e}_\phi$$

$$v_r = \dot{r} \Rightarrow 1200 m/s$$

$$v_\theta = r\dot{\theta} \Rightarrow 8\times10^4(0.80)(\frac{\pi}{180}) = 117 m/s$$

$$v = \sqrt{(v_r)^2 + (v_\theta)^2} \Rightarrow \sqrt{1200^2 + 1117^2} = 1639 m/s$$

Components of acceleration $\bar{a} = (\ddot{r} - r\dot{\phi}^2)\ \bar{e}_r + (2\dot{r}\dot{\phi} + r\ddot{\phi})\ \bar{e}_\phi$

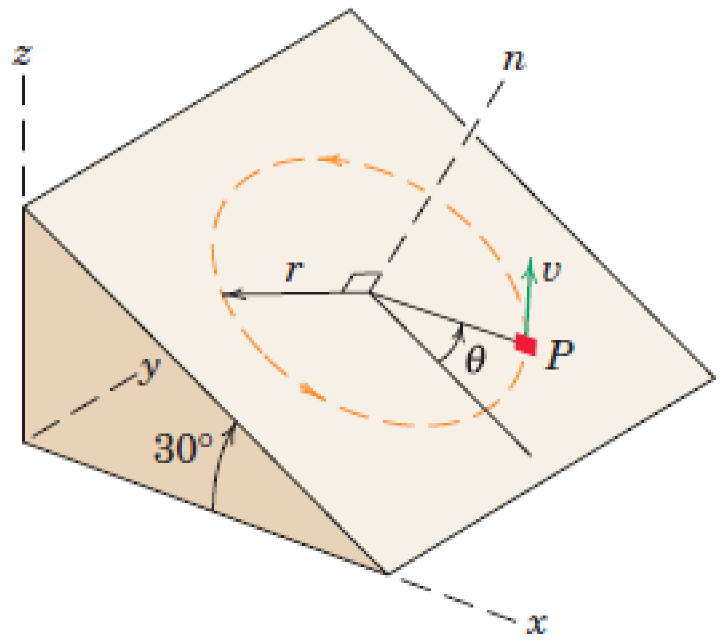
$$a_r = -g\cos\theta \Rightarrow -9.20\cos 30 = -7.97 m/s^2$$

$$a_\theta = g\sin\theta \Rightarrow 9.20\sin 30 = 4.60 m/s^2$$

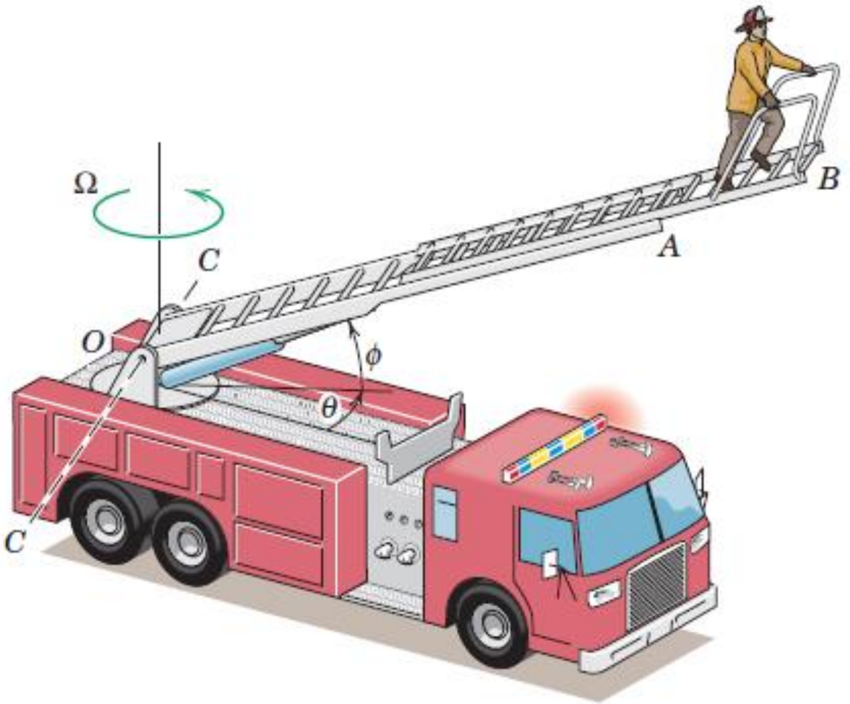
$$a_r = (\ddot{r} - r\dot{\theta}^2) = -7.97 \Rightarrow \ddot{r} = 7.63 m/s^2$$

$$a_\theta = (2\dot{r}\dot{\theta} + r\ddot{\theta}) = 4.60 \Rightarrow \ddot{\theta} = -3.61(10^{-4}) rad/s^2$$

2/175 The small block P travels with constant speed v in the circular path of radius r on the inclined surface. If $\theta = 0$ at time $t = 0$, determine the x-, y-, and z-components of velocity and acceleration as functions of time.

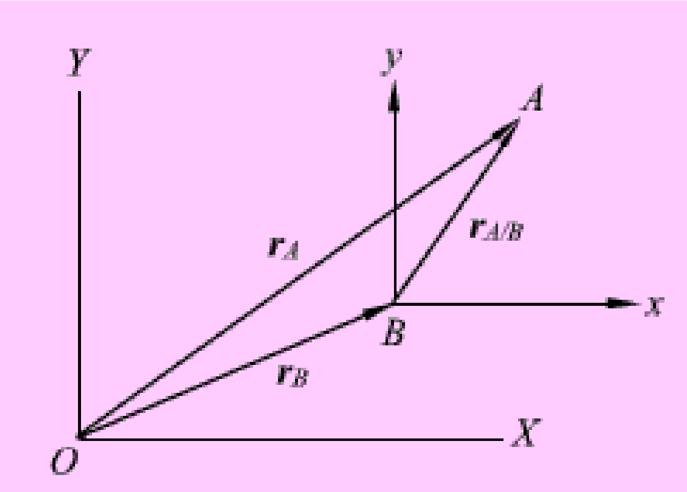


2/177 The base structure of the firetruck ladder rotates about a vertical axis through O with a constant angular velocity $\Omega = 10$ deg/s. At the same time, the ladder unit OB elevates at a constant rate $\dot{\phi} = 7$ deg/s, and section AB of the ladder extends from within section OA at the constant rate of 0.5 m/s. At the instant under consideration, $\phi = 30°$, $\overline{OA} = 9$ m, and $\overline{AB} = 6$ m. Determine the magnitudes of the velocity and acceleration of the end B of the ladder.



Relative Motion (Translating axes)

Supposing an axes system is moving with respect to other axes system, what is the relationship between velocity in two system?



X-Y is a fixed reference frame and *x-y* is a moving reference frame.

Now consider two particles A and B in a given plane. We will arbitrarily attach the origin of a set of translating axes x-y to particle B and observe the motion of A

$$r_{A/B} = x\bar{i} + y\bar{j}$$

Where A/B means A relative to B or A with respect to B

The absolute position of A can be written as .

$$r_A = r_B + r_{A/B}$$

The absolute velocity and acceleration of A can be written as .

$$\dot{r}_A = \dot{r}_B + \dot{r}_{A/B} \Rightarrow v_A = v_B + v_{A/B}$$

$$\ddot{r}_A = \ddot{r}_B + \ddot{r}_{A/B} \Rightarrow a_A = a_B + a_{A/B}$$

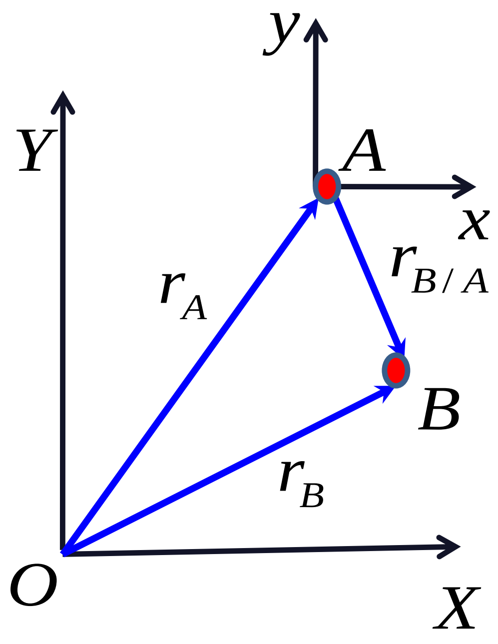
Note: rotation of moving frame is not allowed here

Relative Motion (Translating axes)

Selection of the moving point B for attachment of the reference coordinate system is arbitrary.

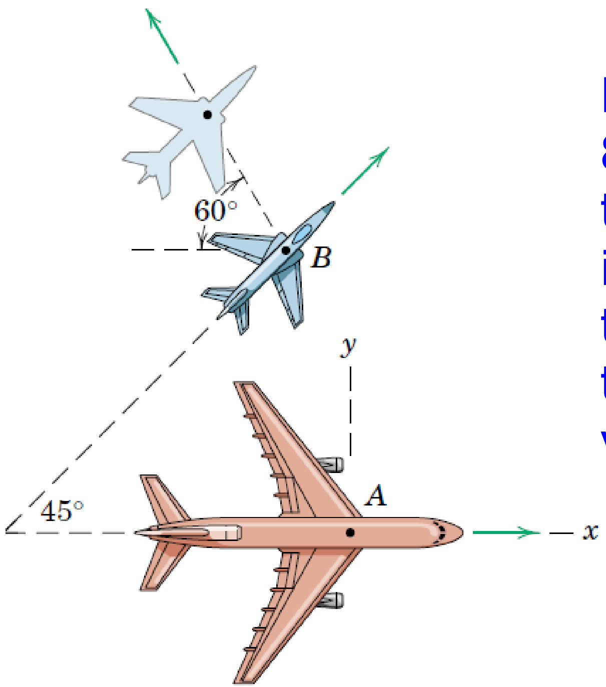
$$r_B = r_A + r_{B/A},\ v_B = v_A + v_{B/A},\ a_B = a_A + a_{B/A}$$

$$\Rightarrow r_{B/A} = -r_{A/B},\quad v_{B/A} = -v_{A/B},\quad a_{B/A} = -a_{A/B}$$



One example can be solved here

Sample Problem 2/13



Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45 northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60 angle as shown. Determine the true velocity of B.

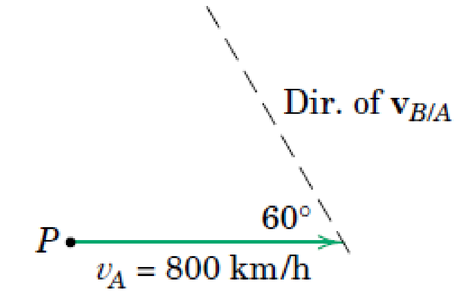
Assumption:

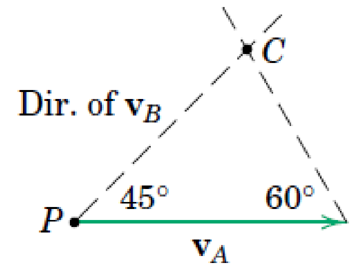
We treat each airplane as a particle.
We assume no side slip due to cross wind.

Solution. The moving reference axes x-y are attached to A, from which the relative observations are made. We write, therefore,

① $$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$

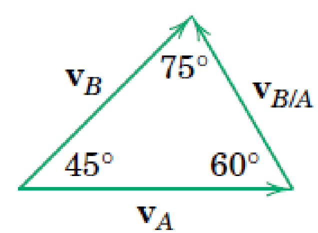
Next we identify the knowns and unknowns. The velocity $\mathbf{v}_A$ is given in both magnitude and direction. The 60° direction of $\mathbf{v}_{B/A}$, the velocity which B appears to ② have to the moving observers in A, is known, and the true velocity of B is in the 45° direction in which it is heading. The two remaining unknowns are the magni- ③ tudes of $\mathbf{v}_B$ and $\mathbf{v}_{B/A}$. We may solve the vector equation in any one of three ways.





(I) Graphical. We start the vector sum at some point P by drawing $\mathbf{v}_A$ to a convenient scale and then construct a line through the tip of $\mathbf{v}_A$ with the known direction of $\mathbf{v}_{B/A}$. The known direction of $\mathbf{v}_B$ is then drawn through P, and the intersection C yields the unique solution enabling us to complete the vector triangle and scale off the unknown magnitudes, which are found to be

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h} \qquad \textit{Ans.}$$



(II) Trigonometric. A sketch of the vector triangle is made to reveal the trigonometry, which gives

$$\frac{v_B}{\sin 60^\circ} = \frac{v_A}{\sin 75^\circ} \qquad v_B = 800\,\frac{\sin 60^\circ}{\sin 75^\circ} = 717 \text{ km/h} \qquad \textit{Ans.}$$

(III) Vector Algebra. Using unit vectors **i** and **j**, we express the velocities in vector form as

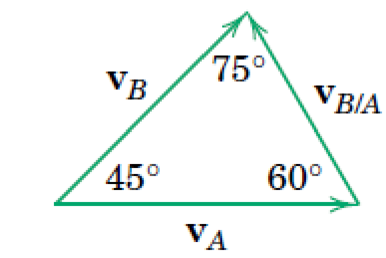
$$\mathbf{v}_A = 800\mathbf{i} \text{ km/h} \qquad \mathbf{v}_B = (v_B \cos 45^\circ)\mathbf{i} + (v_B \sin 45^\circ)\mathbf{j}$$

$$\mathbf{v}_{B/A} = (v_{B/A} \cos 60^\circ)(-\mathbf{i}) + (v_{B/A} \sin 60^\circ)\mathbf{j}$$

Substituting these relations into the relative-velocity equation and solving separately for the **i** and **j** terms give

$$(\mathbf{i}\text{-terms}) \qquad v_B \cos 45^\circ = 800 - v_{B/A} \cos 60^\circ$$

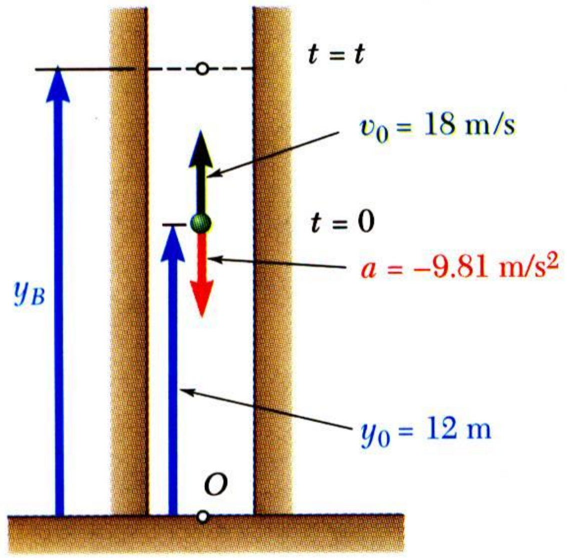
$$(\mathbf{j}\text{-terms}) \qquad v_B \sin 45^\circ = v_{B/A} \sin 60^\circ$$



Solving simultaneously yields the unknown velocity magnitudes

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h} \qquad \textit{Ans.}$$

Sample Problem 11.3



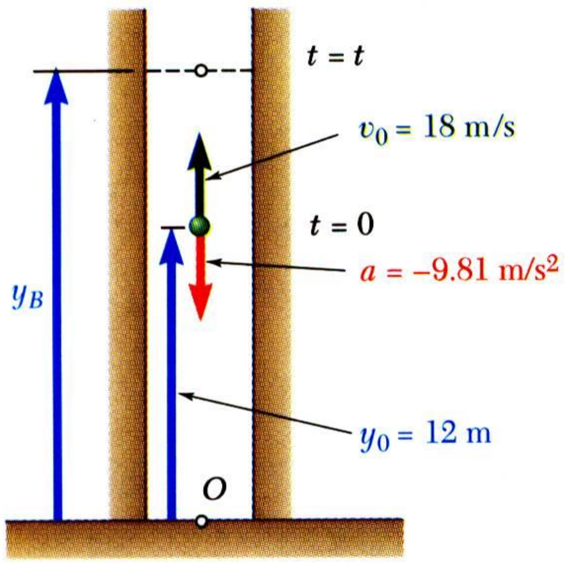
Ball thrown vertically from 12 m level in elevator shaft with initial velocity of 18 m/s. At same instant, open-platform elevator passes 5 m level moving upward at 2 m/s.

Determine *(a)* when and where ball hits elevator and *(b)* relative velocity of ball and elevator at contact.

SOLUTION:

- Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.
- Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.
- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.
- Substitute impact time into equation for position of elevator and relative velocity of ball with respect to elevator.

Sample Problem 11.3

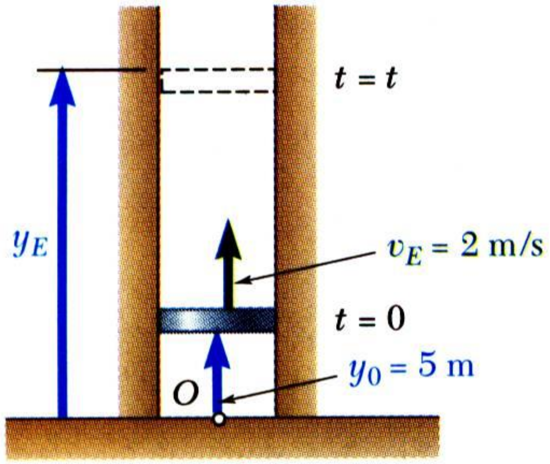


SOLUTION:

- Substitute initial position and velocity and constant acceleration of ball into general equations for uniformly accelerated rectilinear motion.

$$v_B = v_0 + at = 18\frac{\text{m}}{\text{s}} - \left(9.81\frac{\text{m}}{\text{s}^2}\right)t$$

$$y_B = y_0 + v_0 t + \tfrac{1}{2}at^2 = 12\,\text{m} + \left(18\frac{\text{m}}{\text{s}}\right)t - \left(4.905\frac{\text{m}}{\text{s}^2}\right)t^2$$

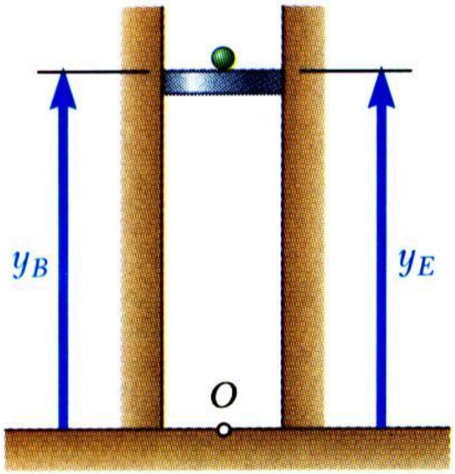


- Substitute initial position and constant velocity of elevator into equation for uniform rectilinear motion.

$$v_E = 2\frac{\text{m}}{\text{s}}$$

$$y_E = y_0 + v_E t = 5\,\text{m} + \left(2\frac{\text{m}}{\text{s}}\right)t$$

Sample Problem 11.3



- Write equation for relative position of ball with respect to elevator and solve for zero relative position, i.e., impact.

$$y_{B/E} = \left(12 + 18t - 4.905t^2\right) - \left(5 + 2t\right) = 0$$

$$t = -0.39\,\text{s} \quad (\text{meaningless})$$

$$t = 3.65\,\text{s}$$

- Substitute impact time into equations for position of elevator and relative velocity of ball with respect to elevator.

$$y_E = 5 + 2(3.65)$$

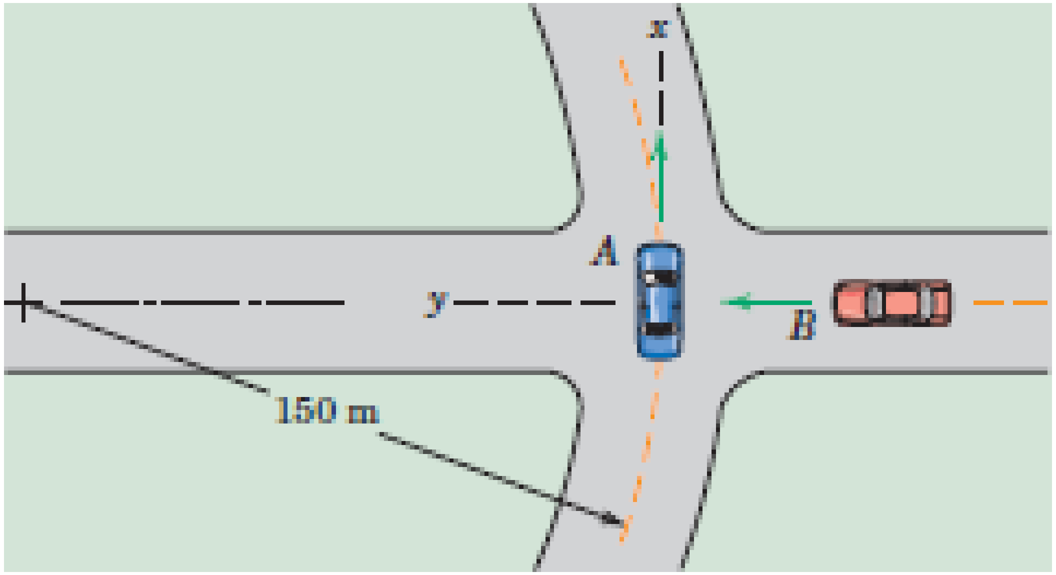
$$y_E = 12.3\,\text{m}$$

$$v_{B/E} = \left(18 - 9.81t\right) - 2$$

$$= 16 - 9.81(3.65)$$

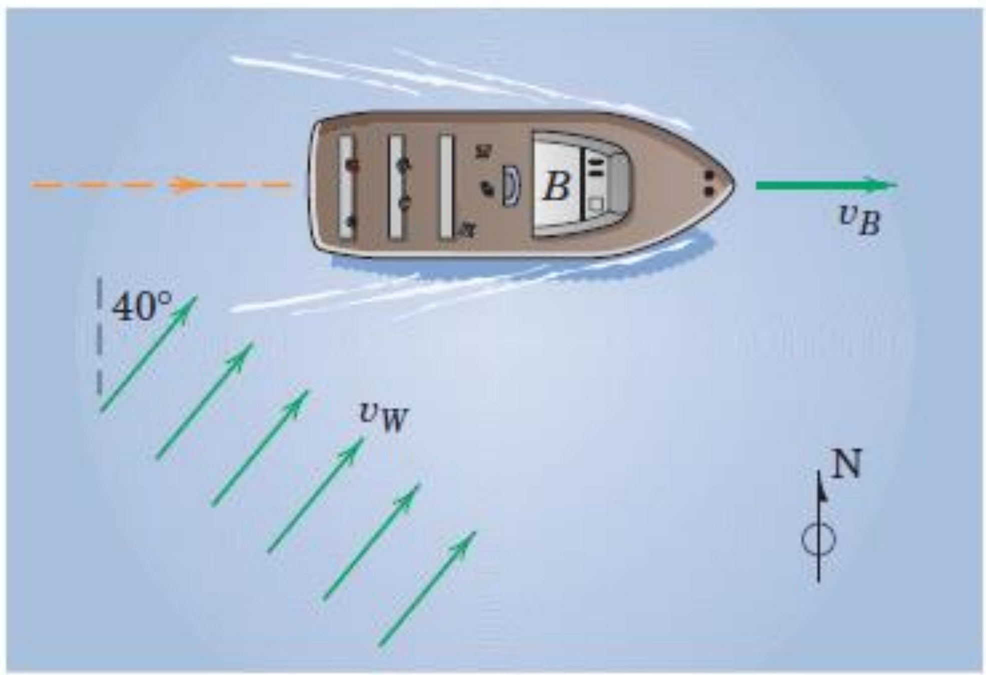
$$v_{B/E} = -19.81\,\frac{\text{m}}{\text{s}}$$

2/183 Car A rounds a curve of 150-m radius at a constant speed of 54 km/h. At the instant represented, car B is moving at 81 km/h but is slowing down at the rate of 3 m/s^2. Determine the velocity and acceleration of car A as observed from car B.

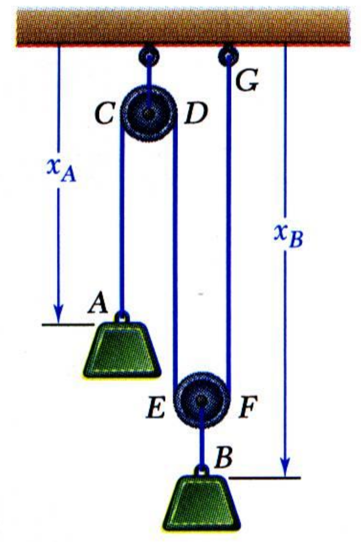


Problem 2/183

2/191 A ferry is moving due east and encounters a southwest wind of speed $v_W = 10$ m/s as shown. The experienced ferry captain wishes to minimize the effects of the wind on the passengers who are on the outdoor decks. At what speed v_B should he proceed?



Constrained Motion of Connected Particles



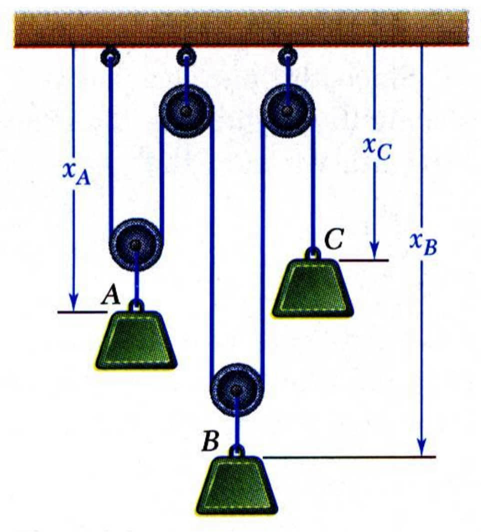
- Position of a particle may *depend* on position of one or more other particles.

- Position of block *B* depends on position of block *A*. Since rope is of constant length, it follows that sum of lengths of segments must be constant.

$$x_A + 2x_B = \text{constant} \quad \text{(one degree of freedom)}$$

- Positions of three blocks are dependent.

$$2x_A + 2x_B + x_C = \text{constant (two degrees of freedom)}$$



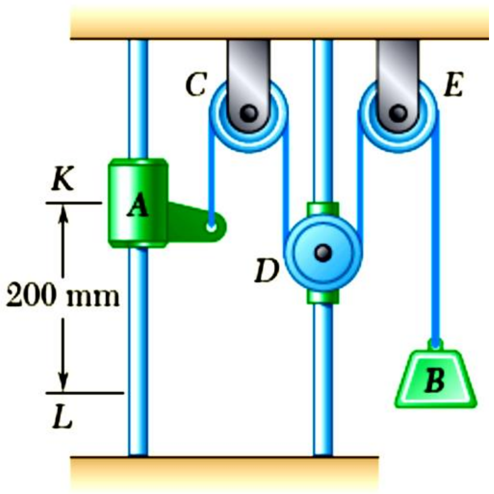
- For linearly related positions, similar relations hold between velocities and accelerations.

$$2\frac{dx_A}{dt} + 2\frac{dx_B}{dt} + \frac{dx_C}{dt} = 0 \quad \text{or} \quad 2v_A + 2v_B + v_C = 0$$

$$2\frac{dv_A}{dt} + 2\frac{dv_B}{dt} + \frac{dv_C}{dt} = 0 \quad \text{or} \quad 2a_A + 2a_B + a_C = 0$$

One example can be solved here

Sample Problem 11.4

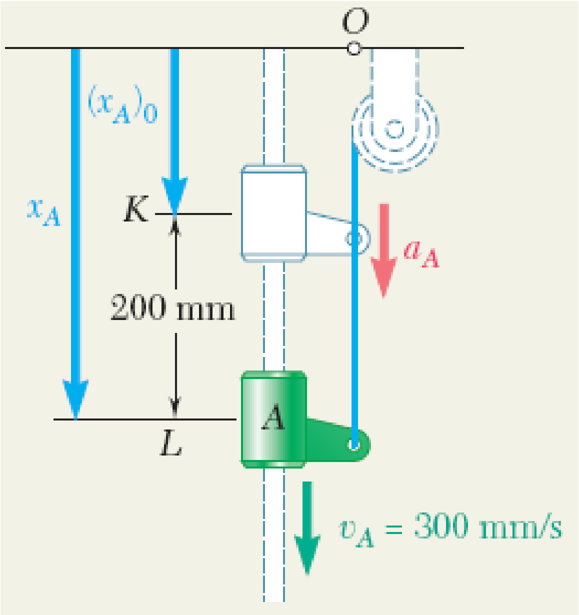


Pulley *D* is attached to a collar which is pulled down at 75mm/s. At $t = 0$, collar *A* starts moving down from *K* with constant acceleration and zero initial velocity. Knowing that velocity of collar *A* is 300 mm/s as it passes *L*, determine the change in elevation, velocity, and acceleration of block *B* when block *A* is at *L*.

SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.
- Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.
- Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.
- Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*.
- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.

Sample Problem 11.4



SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.

- Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.

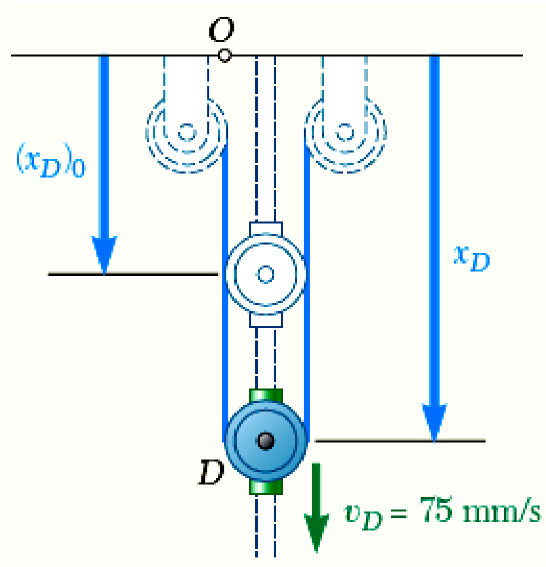
$$v_A^2 = (v_A)_0^2 + 2a_A\left[x_A - (x_A)_0\right]$$

$$\left(300\,\frac{\text{mm}}{\text{s}}\right)^2 = 2a_A(200\,\text{mm}) \qquad a_A = 225\,\frac{\text{mm}}{\text{s}^2}$$

$$v_A = (v_A)_0 + a_A t$$

$$300\,\frac{\text{mm}}{\text{s}} = 225\,\frac{\text{mm}}{\text{s}^2}\,t \qquad t = 1.333\,\text{s}$$

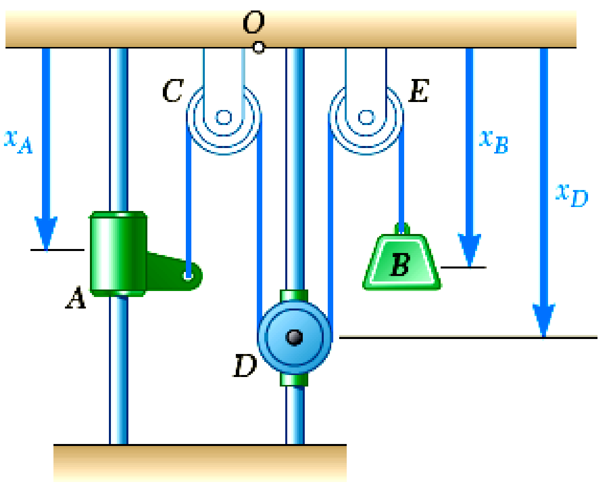
Sample Problem 11.4



- Pulley *D* has uniform rectilinear motion. Calculate change of position at time *t*.

$$x_D = (x_D)_0 + v_D t$$

$$x_D - (x_D)_0 = \left(75\frac{\text{mm}}{\text{s}}\right)(1.333\,\text{s}) = 100\text{ mm}$$



- Block *B* motion is dependent on motions of collar *A* and pulley *D*. Write motion relationship and solve for change of block *B* position at time *t*.

Total length of cable remains constant,

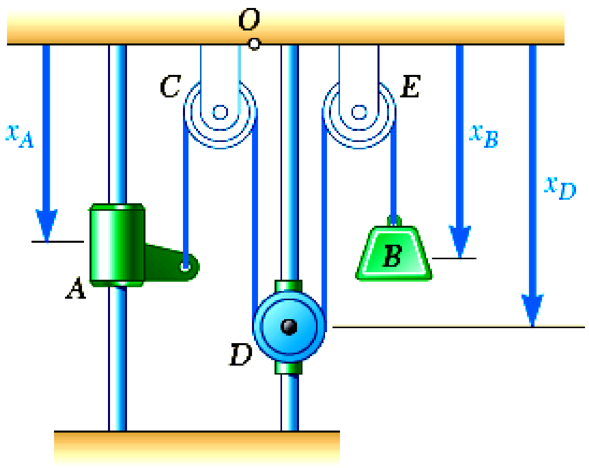
$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0$$

$$\left[x_A - (x_A)_0\right] + 2\left[x_D - (x_D)_0\right] + \left[x_B - (x_B)_0\right] = 0$$

$$(200\text{mm}) + 2(100\text{ mm}) + \left[x_B - (x_B)_0\right] = 0$$

$$\boxed{x_B - (x_B)_0 = -400\text{ mm}}$$

Sample Problem 11.4



- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.

$$x_A + 2x_D + x_B = \text{constant}$$

$$v_A + 2v_D + v_B = 0$$

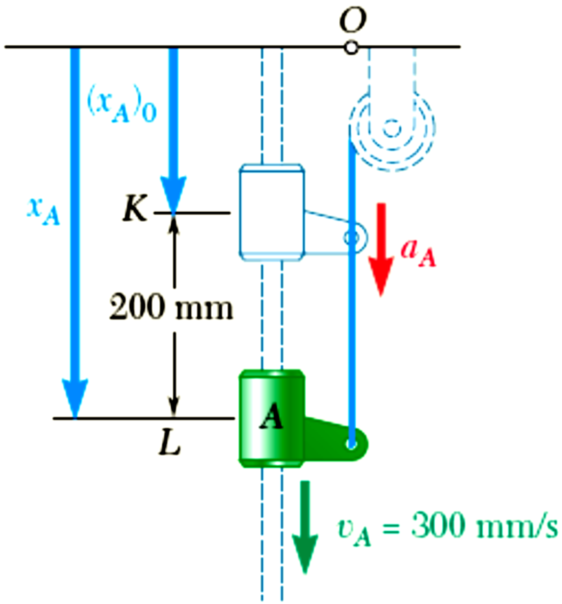
$$\left(300\frac{\text{mm}}{\text{s}}\right) + 2\left(75\frac{\text{mm}}{\text{s}}\right) + v_B = 0$$

$$\boxed{v_B = 450\frac{\text{mm}}{\text{s}}}$$

$$a_A + 2a_D + a_B = 0$$

$$\left(225\frac{\text{mm}}{\text{s}^2}\right) + v_B = 0$$

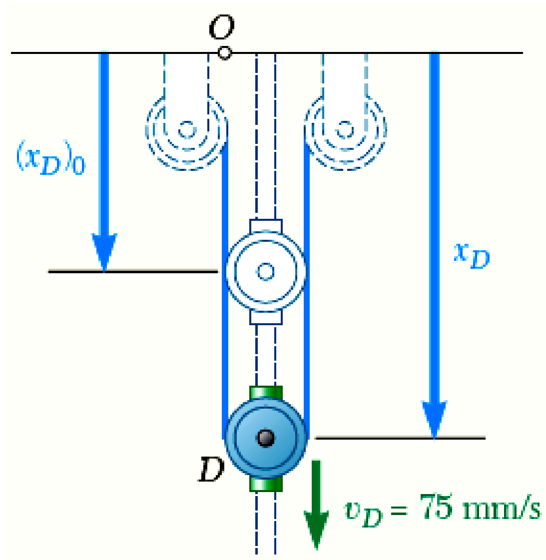
$$\boxed{a_B = -225\frac{\text{mm}}{\text{s}^2}}$$

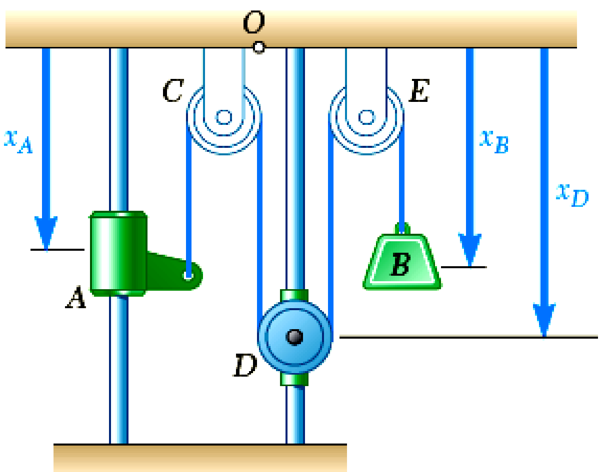


SOLUTION:

- Define origin at upper horizontal surface with positive displacement downward.

- Collar *A* has uniformly accelerated rectilinear motion. Solve for acceleration and time *t* to reach *L*.





- Pulley D has uniform rectilinear motion. Calculate change of position at time t.

$$x_D = (x_D)_0 + v_D t$$

$$x_D - (x_D)_0 = \left(75 \frac{\text{mm}}{\text{s}}\right)(1.333\,\text{s}) = 100\ \text{mm}$$

- Block B motion is dependent on motions of collar A and pulley D. Write motion relationship and solve for change of block B position at time t.

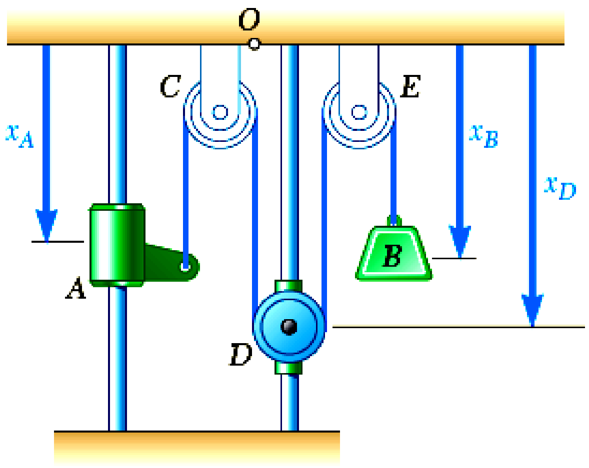
Total length of cable remains constant,

$$x_A + 2x_D + x_B = (x_A)_0 + 2(x_D)_0 + (x_B)_0$$

$$\left[x_A - (x_A)_0\right] + 2\left[x_D - (x_D)_0\right] + \left[x_B - (x_B)_0\right] = 0$$

$$(200\text{mm}) + 2(100\ \text{mm}) + \left[x_B - (x_B)_0\right] = 0$$

$$x_B - (x_B)_0 = -400\ \text{mm}$$



- Differentiate motion relation twice to develop equations for velocity and acceleration of block *B*.

$$x_A + 2x_D + x_B = \text{constant}$$

$$v_A + 2v_D + v_B = 0$$

$$\left(300\frac{\text{mm}}{\text{s}}\right) + 2\left(75\frac{\text{mm}}{\text{s}}\right) + v_B = 0$$

$$\boxed{v_B = 450\frac{\text{mm}}{\text{s}}}$$

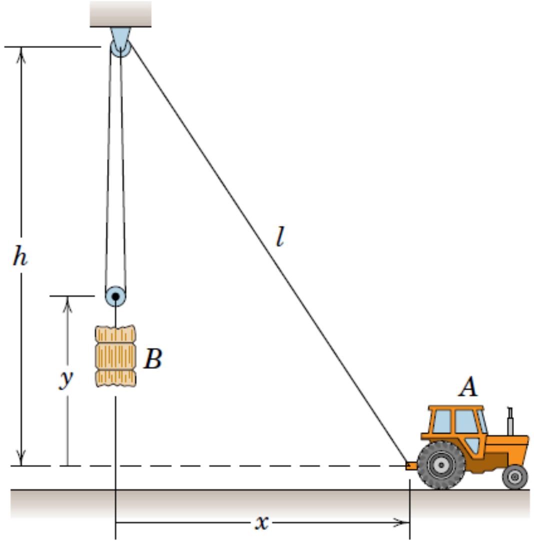
$$a_A + 2a_D + a_B = 0$$

$$\left(225\frac{\text{mm}}{\text{s}^2}\right) + v_B = 0$$

$$\boxed{a_B = -225\frac{\text{mm}}{\text{s}^2}}$$

Sample Problem 2/16

The tractor A is used to hoist the bale B with the pulley arrangement shown. If A has a forward velocity v_A, determine an expression for the upward velocity v_B of the bale in terms of x.



Solution. We designate the position of the tractor by the coordinate x and the position of the bale by the coordinate y, both measured from a fixed reference. The total constant length of the cable is

$$L = 2(h - y) + l = 2(h - y) + \sqrt{h^2 + x^2}$$

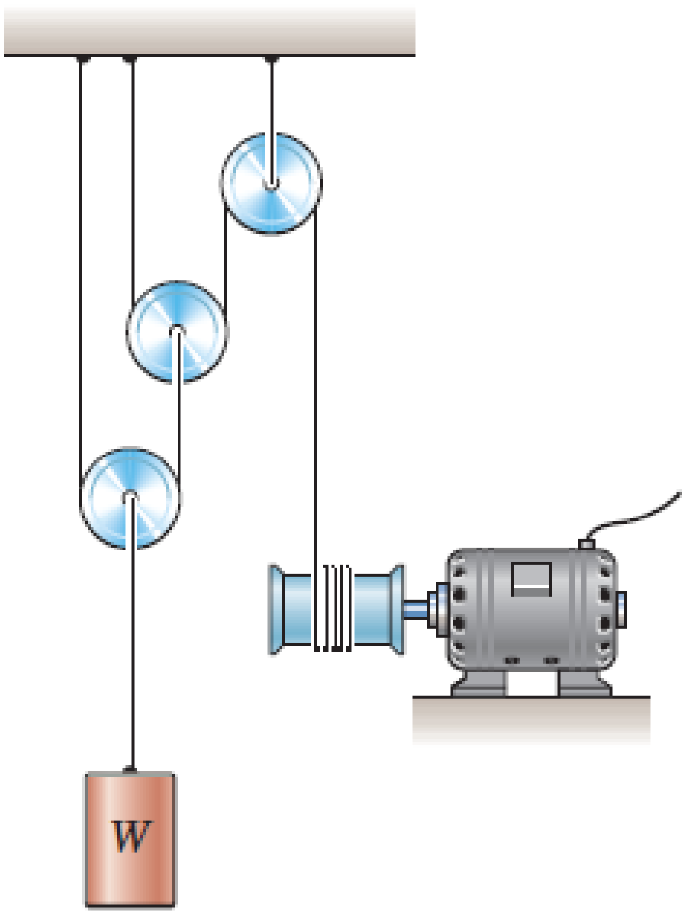
Differentiation with time yields

$$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{h^2 + x^2}}$$

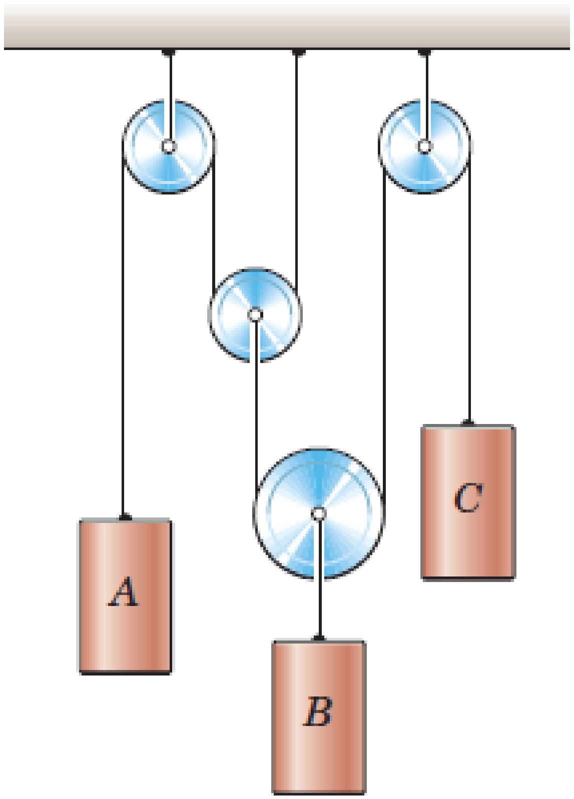
Substituting $v_A = \dot{x}$ and $v_B = \dot{y}$ gives

$$v_B = \frac{1}{2}\frac{xv_A}{\sqrt{h^2 + x^2}}$$

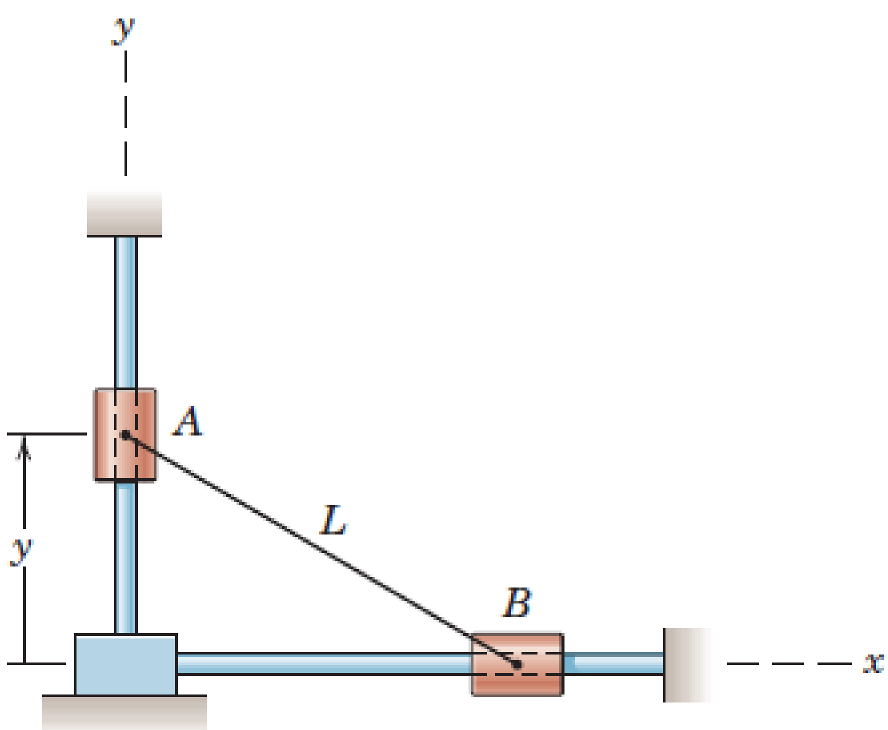
2/211 Determine the vertical rise h of the load W during 5 seconds if the hoisting drum wraps cable around it at the constant rate of 320 mm/s.



2/215 The pulley system of the previous problem is modified as shown with the addition of a fourth pulley and a third cylinder C. Determine the relationship which governs the velocities of the three cylinders, and state the number of degrees of freedom. Express all velocities as positive down.



2/221 Collars A and B slide along the fixed right-angle rods and are connected by a cord of length L. Determine the acceleration a_x of collar B as a function of y if collar A is given a constant upward velocity v_A.



Relative Motion (rotating axes)

Let us consider axes system xy which rotates with respect to *XY*.

Angular Velocity ω= $\dot{\theta}$

$$\frac{d\bar{i}}{dt} = \bar{j}\,\omega \qquad \frac{d\bar{j}}{dt} = -\bar{i}\,\omega$$

Angular Velocity $\boldsymbol{\omega} = \omega\hat{k} = \dot{\theta}\hat{k}$

Relative Velocity

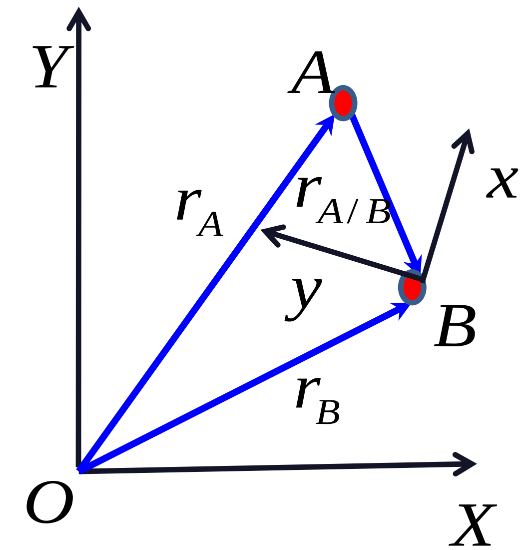
$$\dot{\boldsymbol{r}}_A = \dot{\boldsymbol{r}}_B + \frac{d}{dt}(x\hat{i} + y\hat{j})$$

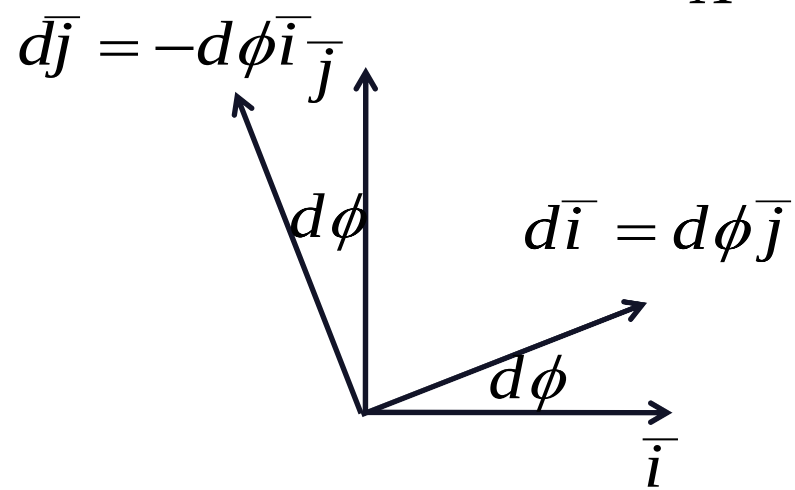
$$= \dot{\boldsymbol{r}}_B + (x\dot{\hat{i}} + y\dot{\hat{j}}) + (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= \dot{\boldsymbol{r}}_B + \boldsymbol{\omega}\times x\hat{i} + \boldsymbol{\omega}\times y\hat{j} + (\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= \boldsymbol{V}_B + \boldsymbol{\omega}\times\boldsymbol{r} + \boldsymbol{V}_{rel}$$

$$V_A = V_B + \omega\times r + V_{rel}$$





Relative Motion (rotating axes)

The relative acceleration may be obtained by differentiating the Relative velocity

$$a_A = a_B + \dot{\omega} \times r + \dot{r} \times \omega + \dot{V}_{rel}$$

Now, using previous relation

$$\dot{r} = \omega \times r + V_{rel}$$

$$\dot{V}_{rel} = \frac{d}{dt}(\dot{x}\hat{i} + \dot{y}\hat{j})$$

$$= (\dot{x}\dot{\hat{i}} + \dot{y}\dot{\hat{j}}) + (\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$= \boldsymbol{\omega} \times (\dot{x}\hat{i} + \dot{y}\hat{j}) + (\ddot{x}\hat{i} + \ddot{y}\hat{j})$$

$$= \boldsymbol{\omega} \times V_{rel} + \boldsymbol{a}_{rel}$$

Thus Finally

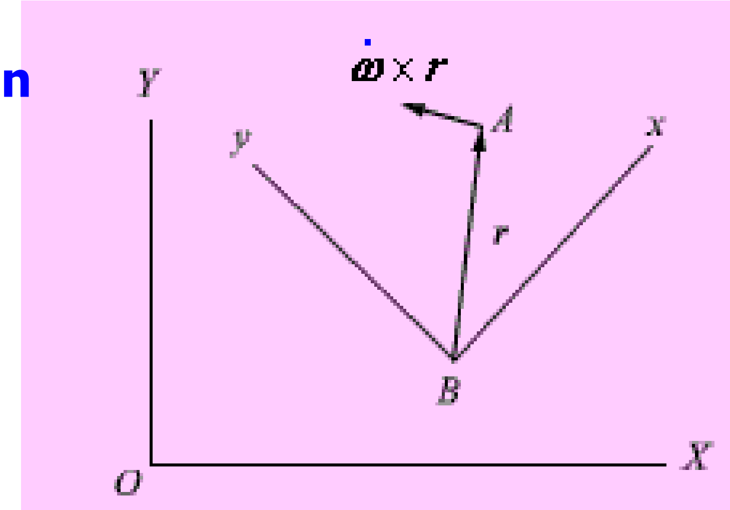
$$\boldsymbol{a}_A = \boldsymbol{a}_B + \dot{\boldsymbol{\omega}} \times \boldsymbol{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \boldsymbol{r}) + 2\boldsymbol{\omega} \times V_{rel} + \boldsymbol{a}_{rel}$$

Relative Motion (rotating axes)

$\dot{\omega} \times r$ **is the tangential acceleration, since it is perpendicular to unit vector** $\bar{k}$ and r

$\omega \times (\omega \times r)$ **is normal acceleration, since it is directed towards *B***

$2\omega \times V_{rel}$ **is the Coriolis acceleration**

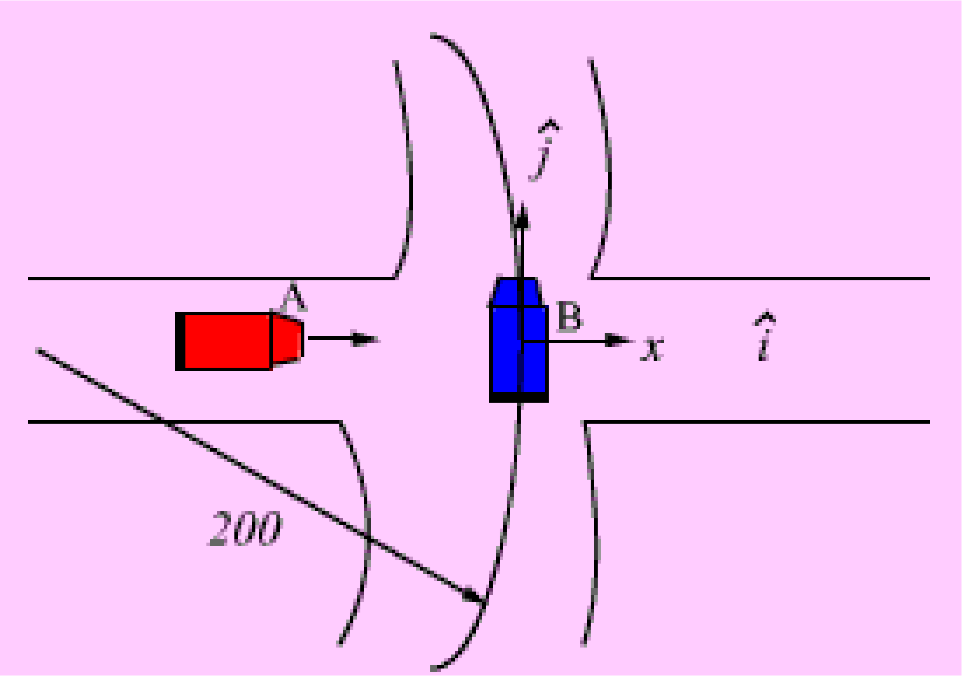


Relative Motion (rotating axes)

Example

Car B is rounding the curve with constant speed of 15 m/sec and car A is approaching car B in the intersection with a constant speed of 20 m/sec. The distance separating the cars is 40 meters at the instant depicted.

At the instant



$$V_A = 20\hat{i} \qquad V_B = 15\hat{j}$$

$$V_A = V_B + \omega \times r + V_{rel}$$

$$20\hat{i} = 15\hat{j} + \frac{15}{200} \times (-40)\hat{j} + V_{rel}$$

$$V_{rel} = 20\hat{i} - 12\hat{j}$$

This is the velocity of car A as seen by the driver in car B

Relative Motion (rotating axes)

Suppose the axes system is attached to A

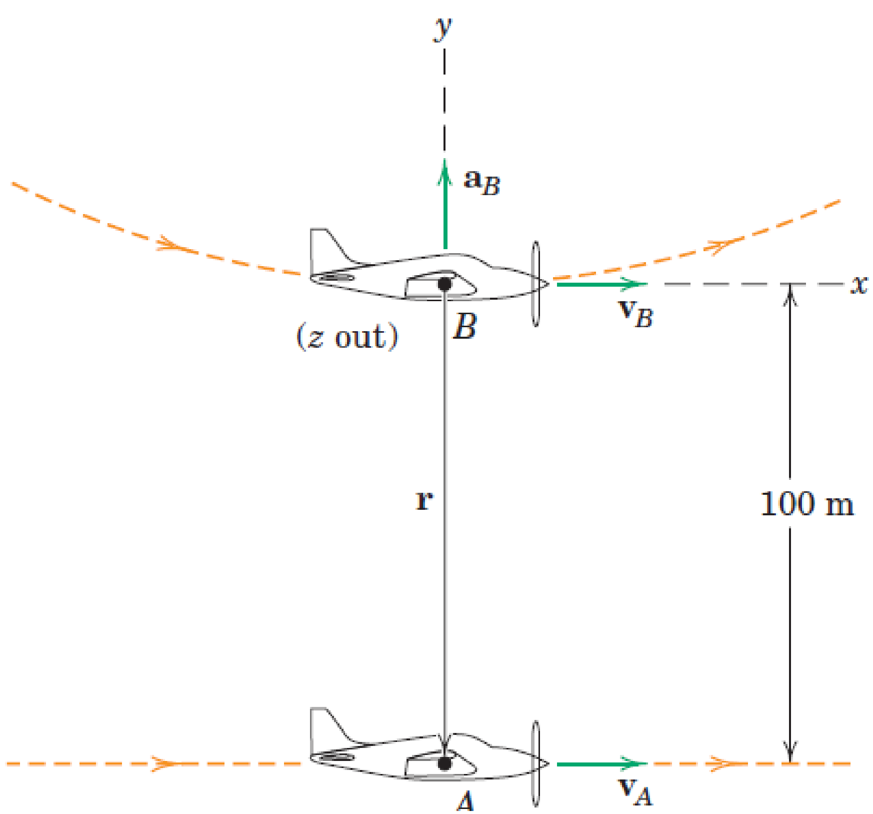
$$V_A = V_B + V_{rel}$$

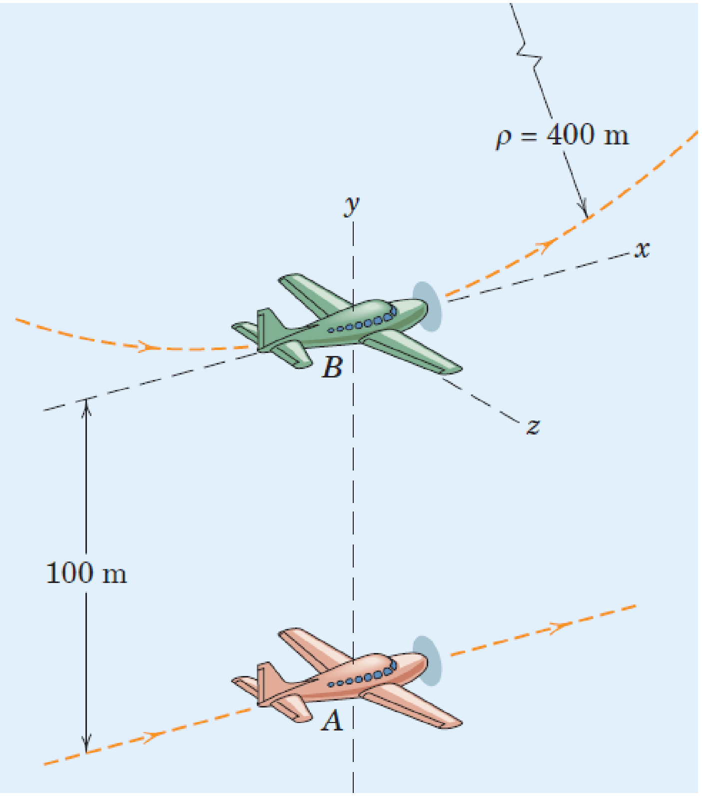
$$V_{rel} = 15\hat{j} - 20\hat{i}$$

Thus, we see that observations of both drivers are not the negative of each other

Sample Problem 5/19

Aircraft B has a constant speed of 150 m/s as it passes the bottom of a circular loop of 400-m radius. Aircraft A flying horizontally in the plane of the loop passes 100 m directly below B at a constant speed of 100 m/s. (a) Determine the instantaneous velocity and acceleration which A appears to have to the pilot of B, who is fixed to his rotating aircraft. (b) Compare your results for part (a) with the case of erroneously treating the pilot of aircraft B as nonrotating.





$$\mathbf{v}_A = 100\mathbf{i} \text{ m/s}$$

$$\mathbf{v}_B = 150\mathbf{i} \text{ m/s}$$

$$\boldsymbol{\omega} = \frac{v_B}{\rho}\mathbf{k} = \frac{150}{400}\mathbf{k} = 0.375\mathbf{k} \text{ rad/s}$$

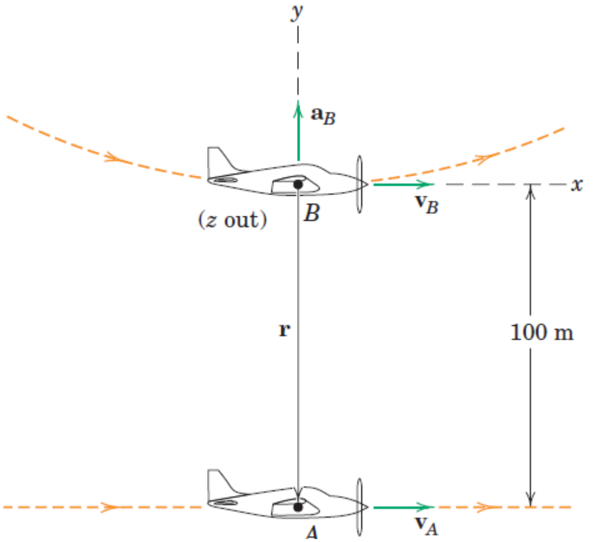
$$\mathbf{r} = \mathbf{r}_{A/B} = -100\mathbf{j} \text{ m}$$

Eq. 5/12:

$$\mathbf{v}_A = \mathbf{v}_B + \boldsymbol{\omega} \times \mathbf{r} + \mathbf{v}_{\text{rel}}$$

$$100\mathbf{i} = 150\mathbf{i} + 0.375\mathbf{k} \times (-100\mathbf{j}) + \mathbf{v}_{\text{rel}}$$

Solving for $\mathbf{v}_{\text{rel}}$ gives $$\mathbf{v}_{\text{rel}} = -87.5\mathbf{i} \text{ m/s}$$



The terms in Eq. 5/14, in addition to those listed above, are

$$\mathbf{a}_A = \mathbf{0}$$

$$\mathbf{a}_B = \frac{v_B^2}{\rho}\mathbf{j} = \frac{150^2}{400}\mathbf{j} = 56.2\mathbf{j} \text{ m/s}^2$$

$$\dot{\boldsymbol{\omega}} = \mathbf{0}$$

Eq. 5/14:

$$\mathbf{a}_A = \mathbf{a}_B + \dot{\boldsymbol{\omega}} \times \mathbf{r} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}) + 2\boldsymbol{\omega} \times \mathbf{v}_{\text{rel}} + \mathbf{a}_{\text{rel}}$$

$$\mathbf{0} = 56.2\mathbf{j} + \mathbf{0} \times (-100\mathbf{j}) + 0.375\mathbf{k} \times [0.375\mathbf{k} \times (-100\mathbf{j})]$$

$$+ 2[0.375\mathbf{k} \times (-87.5\mathbf{i})] + \mathbf{a}_{\text{rel}}$$

Solving for $\mathbf{a}_{\text{rel}}$ gives $$\mathbf{a}_{\text{rel}} = -4.69\mathbf{k} \text{ m/s}^2$$

(b) For motion relative to translating frames, we use Eqs. 2/20 and 2/21 of Chapter 2:

$$\mathbf{v}_{A/B} = \mathbf{v}_A - \mathbf{v}_B = 100\mathbf{i} - 150\mathbf{i} = -50\mathbf{i} \text{ m/s}$$

$$\mathbf{a}_{A/B} = \mathbf{a}_A - \mathbf{a}_B = \mathbf{0} - 56.2\mathbf{j} = -56.2\mathbf{j} \text{ m/s}^2$$

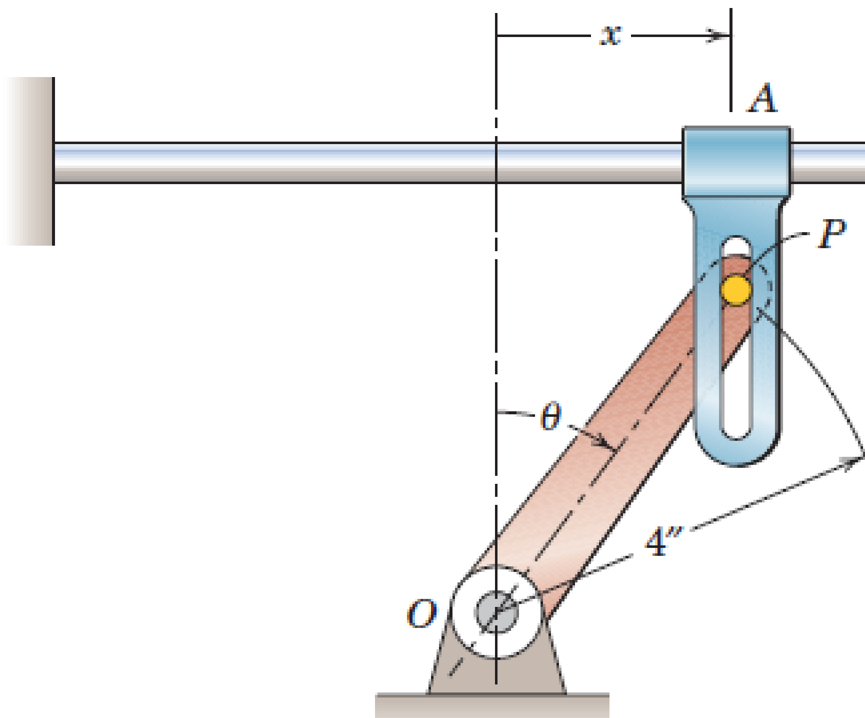
Again, we see that $\mathbf{v}_{rel} \neq \mathbf{v}_{A/B}$ and $\mathbf{a}_{rel} \neq \mathbf{a}_{A/B}$. The rotation of pilot B makes a difference in what he observes!

The scalar result $\omega = \dfrac{v_B}{\rho}$ can be obtained by considering a complete circular motion of aircraft B, during which it rotates 2π radians in a time $t = \dfrac{2\pi\rho}{v_B}$:

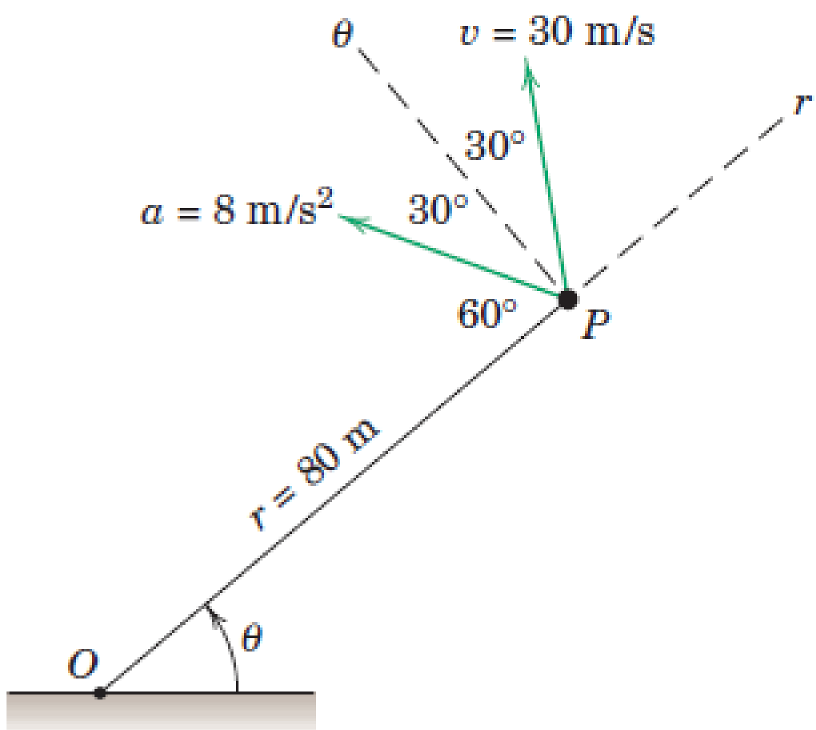
$$\omega = \frac{2\pi}{2\pi\rho/v_B} = \frac{v_B}{\rho}$$

Because the speed of aircraft B is constant, there is no tangential acceleration and thus the angular acceleration $\boldsymbol{\alpha} = \dot{\boldsymbol{\omega}}$ of this aircraft is zero.

2/233 Rotation of the arm PO is controlled by the horizontal motion of the vertical slotted link. If $\dot{x} = 4$ ft/sec and $\ddot{x} = 30$ ft/sec^2 when $x = 2$ in., determine $\dot{\theta}$ and $\ddot{\theta}$ for this instant.



2/243 At the instant depicted, assume that the particle P, which moves on a curved path, is 80 m from the pole O and has the velocity v and acceleration a as indicated. Determine the instantaneous values of $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, $\ddot{\theta}$, the n- and t-components of acceleration, and the radius of curvature ρ.



2/245 The rod of the fixed hydraulic cylinder is moving to the left with a constant speed $v_A = 25$ mm/s. Determine the corresponding velocity of slider B when $s_A = 425$ mm. The length of the cord is 1600 mm, and the effects of the radius of the small pulley at A may be neglected.