Example 15.8

An airplane moving at 200 ft/sec is undergoing a roll of 2 rad/min (Fig. 15.25). When the plane is horizontal, an antenna is moving out at a speed of 8 ft/sec relative to the plane and is at a position of 10 ft from the centerline of the plane. If we assume that the axis of roll corresponds to the centerline, what is the velocity of the antenna end relative to the ground when the plane is horizontal?

A *stationary reference XYZ on the ground* is shown in the diagram. A moving reference *xyz is fixed to the plane* with the *x* axis along the axis of roll and the *y* axis collinear with the antenna. We announce this formally as follows:

Fix *xyz* to plane.
Fix *XYZ* to ground.

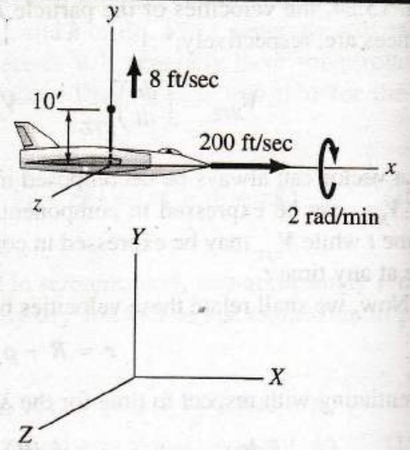


Figure 15.25. *xyz* fixed to plane; *XYZ* fixed to ground.

We then proceed in the following manner:

A. Motion of particle (antenna end) relative to *xyz*[10]

$$\boldsymbol{\rho} = 10\boldsymbol{j} \text{ ft}$$
$$\boldsymbol{V}_{xyz} = 8\boldsymbol{j} \text{ ft/sec}$$

B. Motion of *xyz* (moving reference) relative to *XYZ* (fixed reference)

$$\dot{\boldsymbol{R}} = 200\boldsymbol{i} \text{ ft/sec}$$
$$\boldsymbol{\omega} = -\tfrac{2}{60}\boldsymbol{i} = -\tfrac{1}{30}\boldsymbol{i} \text{ rad/sec}$$

We now employ Eq. 15.20 to get

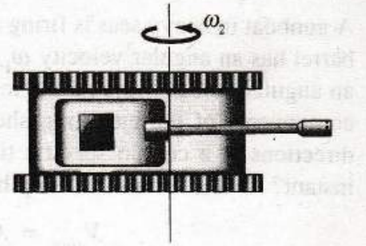
$$\boldsymbol{V}_{XYZ} = \boldsymbol{V}_{xyz} + \dot{\boldsymbol{R}} + \boldsymbol{\omega} \times \boldsymbol{\rho}$$
$$= 8\boldsymbol{j} + 200\boldsymbol{i} + \left(-\frac{\boldsymbol{i}}{30}\right) \times (10\boldsymbol{j})$$

$$\boxed{\boldsymbol{V}_{XYZ} = 200\boldsymbol{i} + 8\boldsymbol{j} - \tfrac{1}{3}\boldsymbol{k} \text{ ft/sec}}$$

[10]Note that since the corresponding axes of the references are parallel to each other at the instant of interest, the unit vectors ***i***, ***j***, and ***k*** apply to either reference at the instant of interest. We will arrange *xyz* and *XYZ* this way whenever possible.

Example 15.9

A tank is moving up an incline with a speed of 10 km/hr in Fig. 15.26. The turret is rotating at a speed ω_1 of 2 rad/sec relative to the tank, and the gun barrel is being lowered (rotating) at a speed ω_2 of .3 rad/sec relative to the turret. What is the velocity of point A of the gun barrel relative to the tank and relative to the ground? The gun barrel is 3 m in length. We proceed as follows (see Fig. 15.27).



Fix *xyz* to turret.
Fix *XYZ* to tank.

A. Motion of particle relative to *xyz*

$$\boldsymbol{\rho} = 3(\cos 30°\boldsymbol{j} + \sin 30°\boldsymbol{k}) = 2.60\boldsymbol{j} + 1.50\boldsymbol{k} \text{ m}$$

Since $\boldsymbol{\rho}$ is fixed in the gun barrel, which has an angular velocity $\boldsymbol{\omega}_2$ relative to *xyz*, we have

$$\begin{aligned} \boldsymbol{V}_{xyz} &= \left(\frac{d\boldsymbol{\rho}}{dt}\right)_{xyz} = \boldsymbol{\omega}_2 \times \boldsymbol{\rho} = (-3\boldsymbol{i}) \times (2.60\boldsymbol{j} + 1.5\boldsymbol{k}) \\ &= -.780\boldsymbol{k} + .45\boldsymbol{j} \text{ m/sec} \end{aligned}$$

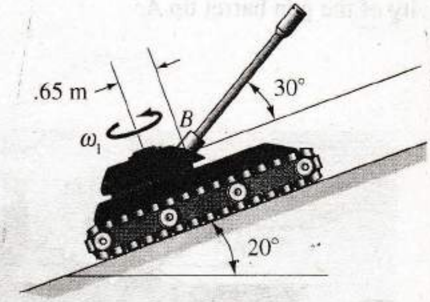


Figure 15.26. Tank with turret and gun barrel in motion.

B. Motion of *xyz* relative to *XYZ*

$$\boldsymbol{R} = .65\boldsymbol{j}$$

Since $\boldsymbol{R}$ is fixed in the turret, which is rotating with angular speed $\boldsymbol{\omega}_1$ relative to *XYZ*, we have

$$\begin{aligned} \dot{\boldsymbol{R}} &= \boldsymbol{\omega}_1 \times \boldsymbol{R} = 2\boldsymbol{k} \times .65\boldsymbol{j} = -1.3\boldsymbol{i} \text{ m/sec} \\ \boldsymbol{\omega} &= \boldsymbol{\omega}_1 = 2\boldsymbol{k} \text{ rad/sec} \end{aligned}$$

We can now substitute into the basic equation relating $\boldsymbol{V}_{xyz}$ to $\boldsymbol{V}_{XYZ}$. That is,

$$\begin{aligned} \boldsymbol{V}_{XYZ} &= \boldsymbol{V}_{xyz} + \dot{\boldsymbol{R}} + \boldsymbol{\omega} \times \boldsymbol{\rho} \\ &= (-.780\boldsymbol{k} + .45\boldsymbol{j}) - 1.3\boldsymbol{i} + (2\boldsymbol{k}) \times (2.60\boldsymbol{j} + 1.50\boldsymbol{k}) \end{aligned}$$

$$\boxed{\boldsymbol{V}_{XYZ} = -6.5\boldsymbol{i} + .45\boldsymbol{j} - .780\boldsymbol{k} \text{ m/sec}}$$

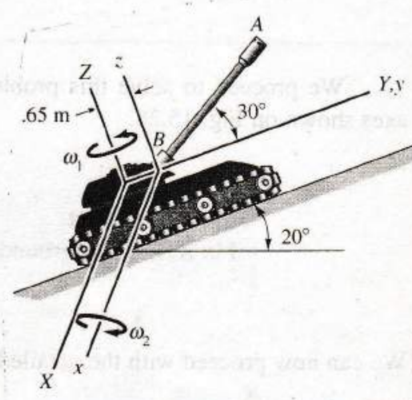


Figure 15.27. *xyz* fixed to turret; *XYZ* fixed to tank.

This result is the desired velocity of A relative to the tank. Since the tank is moving with a speed of (10)(1,000)/(3,600) = 2.78 m/sec relative to the ground, we can say that A has a velocity relative to the ground given as

$$\boldsymbol{V}_{\text{ground}} = \boldsymbol{V}_{XYZ} + 2.78\boldsymbol{j}$$

$$\boxed{\boldsymbol{V}_{\text{ground}} = -6.5\boldsymbol{i} + 3.23\boldsymbol{j} - .780\boldsymbol{k} \text{ m/sec}}$$

Example 15.10

A gunboat in heavy seas is firing its main battery (see Fig. 15.28). The gun barrel has an angular velocity ω_1 relative to the turret, while the turret has an angular velocity ω_2 relative to the ship. If we wish to have the velocity components of the emerging shell to be zero in the stationary X and Z directions at a certain specific time t, what should ω_1 and ω_2 be at this instant? At this instant, the ship has a translational velocity given as

$$V_{ship} = .02\boldsymbol{i} + .016\boldsymbol{k} \text{ m/s}$$

Take the inclination of the barrel to be $\theta = 30°$. Determine also the velocity of the gun barrel tip A.

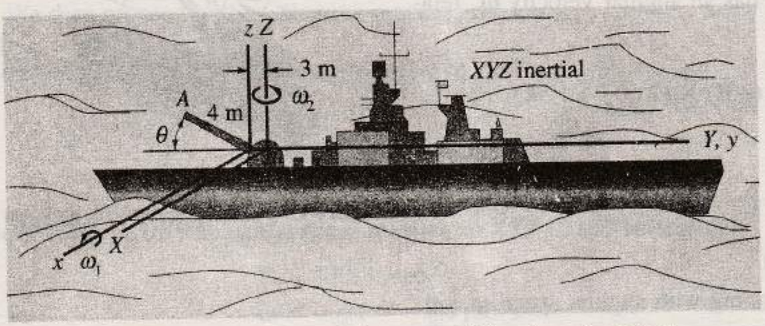


Figure 15.28. A gunboat in heavy seas firing its main battery.

We proceed to solve this problem by the following positioning of axes shown on Fig. 15.28.

Fix *xyz* to turret.
Fix *XYZ* to the ground (inertial reference).

We can now proceed with the detailed analysis of the problem.

A. Motion of *A* relative to *xyz*

$$\boldsymbol{\rho} = -(4)(.866)\boldsymbol{j} + (4)(.5)\boldsymbol{k} = -3.464\boldsymbol{j} + 2\boldsymbol{k} \text{ m}$$

$$V_{xyz} = \boldsymbol{\omega}_1 \times \boldsymbol{\rho} = \omega_1\boldsymbol{i} \times (-3.464\boldsymbol{j} + 2\boldsymbol{k}) = -3.464\omega_1\boldsymbol{k} - 2\omega_1\boldsymbol{j} \text{ m/s}$$

Example 15.10 (Continued)

B. Motion of *xyz* relative to *XYZ*

$$\boldsymbol{R} = -3\boldsymbol{j}\text{ m}$$
$$\dot{\boldsymbol{R}} = \omega_2 \boldsymbol{k} \times (-3\boldsymbol{j}) + (.02\boldsymbol{i} + .016\boldsymbol{k}) = (3\omega_2 + .02)\boldsymbol{i} + .016\boldsymbol{k}\text{ m/s}$$
$$\boldsymbol{\omega} = \omega_2 \boldsymbol{k}\text{ rad/s}$$

We can now proceed with the calculations.

$$\begin{aligned}
\boldsymbol{V}_{XYZ} &= \boldsymbol{V}_{xyz} + \dot{\boldsymbol{R}} + \boldsymbol{\omega} \times \boldsymbol{\rho} \\
&= (-3.464\omega_1 \boldsymbol{k} - 2\omega_1 \boldsymbol{j}) + (3\omega_2 + .02)\boldsymbol{i} + .016\boldsymbol{k} + (\omega_2 \boldsymbol{k}) \times (-3.464\boldsymbol{j} + 2\boldsymbol{k} \\
&= -3.464\omega_1 \boldsymbol{k} - 2\omega_1 \boldsymbol{i} + 3\omega_2 \boldsymbol{i} + .02\boldsymbol{i} + .016\boldsymbol{k} + 3.464\omega_2 \boldsymbol{i}
\end{aligned}$$
$$\therefore \boldsymbol{V}_{XYZ} = (3\omega_2 + 3.464\omega_2 + .02)\boldsymbol{i} + (-2\omega_1)\boldsymbol{j} + (-3.464\omega_1 + .016)\boldsymbol{k}\text{ m/s}$$

Let $(\boldsymbol{V}_{XYZ})_X = \mathbf{0}$

$$\therefore 6.464\omega_2 = -.02 \qquad \omega_2 = -.003094\text{ rad/s}$$

Let $(\boldsymbol{V}_{XYZ})_Z = \mathbf{0}$

$$\therefore -3.464\omega_1 = -.016 \qquad \omega_1 = .004619\text{ rad/s}$$

Finally, we can give $\boldsymbol{V}_{XYZ}$ as $\qquad \boldsymbol{V}_{XYZ} = -2\omega_1 \boldsymbol{j} = -.009238\boldsymbol{j}\text{ m/s}$

(P)

15.72. A particle rotates at a constant angular speed of 10 rad/sec on a platform, while the platform rotates with a constant angular speed of 50 rad/sec about axis A–A. What is the velocity of the particle P at the instant the platform is in the XY plane and the radius vector to the particle forms an angle of 30° with the Y axis as shown?

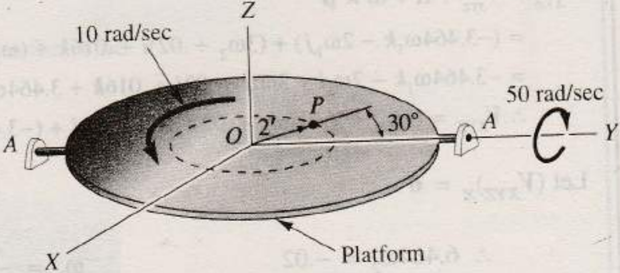


Figure P.15.72.

(P)

15.76. A tank is moving over rough terrain while firing its main gun at a fixed target. The barrel and turret of the gun partly compensate for the motion of the tank proper by giving the barrel an angular velocity $\boldsymbol{\omega}_1$ relative to the turret and, simultaneously, by giving the turret an angular velocity $\boldsymbol{\omega}_2$ relative to the tank proper such that any instant the velocity of end A of the barrel has zero velocity in the X and Z directions relative to the ground reference. What should these angular velocities be for the following translational motion of the tank:

$$V_{\text{TANK}} = 10\boldsymbol{i} + 4\boldsymbol{k} \text{ m/s}$$

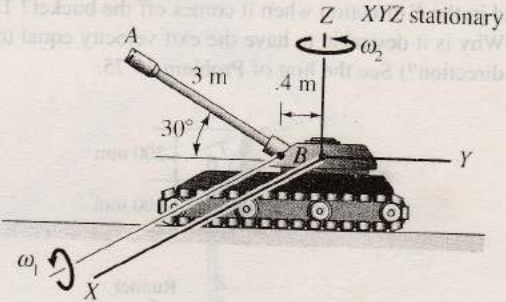


Figure P.15.76.

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15.80. A crane moves to the right at a speed of 5 km/hr. The boom OB, which is 15 m long, is being raised at an angular speed ω_2 relative to the cab of .4 rad/sec, while the cab is rotating at an angular speed ω_1 of .2 rad/sec relative to the base. What is the velocity of pin B relative to the ground at the instant when OB is at an angle of 35° with the ground? The axis of rotation O of the boom is 1 m from the axis of rotation A–A of the cab, as shown in the diagram.

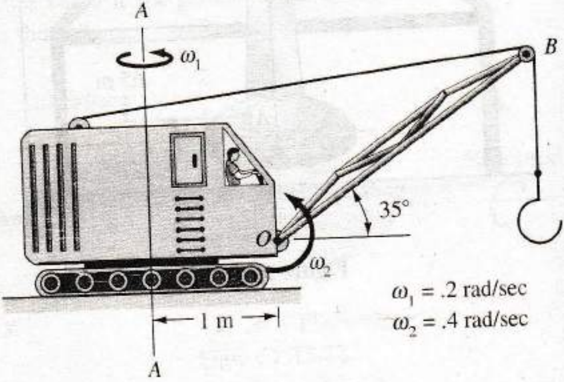


Figure P.15.80.

Figure P.15.83.

15.84. Find the velocity of gear tooth A relative to the ground reference XYZ. Note that ω_1 and ω_2 are both relative to the ground. Bevel gear A is free to rotate in the collar at C. Take $\omega_1 = 2$ rad/sec and $\omega_2 = 4$ rad/sec.

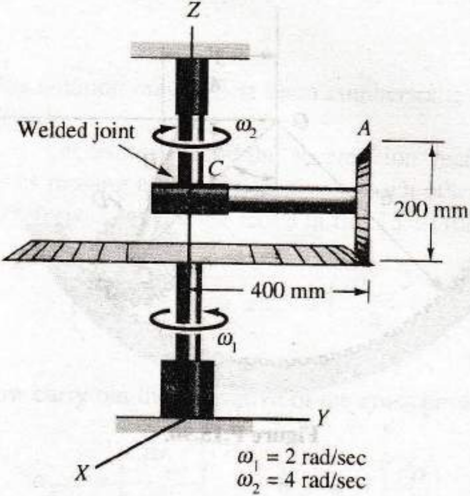


Figure P.15.84.

15.92. A wheel rotates with an angular speed ω_2 of 5 rad/sec relative to a platform, which rotates with a speed ω_1 of 10 rad/sec relative to the ground as shown. A collar moves down the spoke of the wheel, and, when the spoke is vertical, the collar has a speed of 20 ft/sec, an acceleration of 10 ft/sec^2 along the spoke, and is positioned 1 ft from the shaft centerline of the wheel. Compute the velocity and acceleration of the collar relative to the ground at this instant. Fix xyz to platform and use cylindrical coordinates.

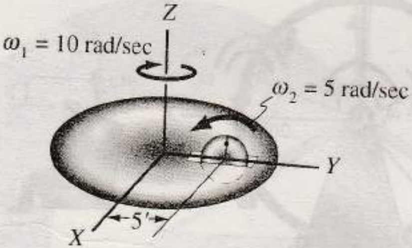
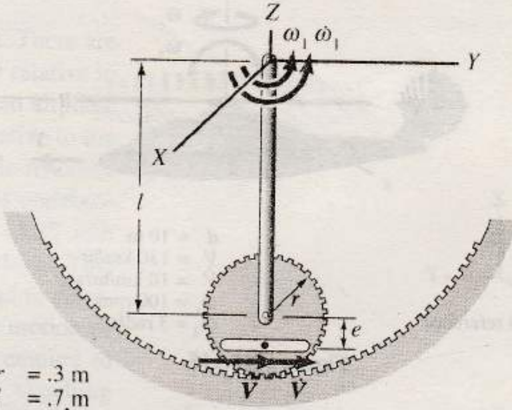


Figure P.15.92.

8

15.113. A particle moves in a slot of a gear with speed $V = 2$ m/s and a rate of change of speed $\dot{V} = 1.2$ m/s² both relative to the gear. Find the acceleration vector for the particle at the configuration shown relative to the ground reference XYZ.



$r = .3$ m
$l = .7$ m
$V = 2$ m/s relative to gear
$\dot{V} = 1.2$ m/s² relative to gear
$\omega_1 = .08$ rad/s
$\dot{\omega}_1 = .02$ rad/s²
$e = .15$ m

Figure P.15.113.

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15.114. A submarine is undergoing an evasive maneuver. At the instant of interest, it has a speed $V_s = 10$ m/s and an acceleration $a_s = 15$ m/s² at its center of mass. It also has an angular velocity about its center of mass C of $\omega_1 = .5$ rad/s and an angular acceleration $\dot{\omega}_1 = .02$ rad/s². Inside is a part of an inertial guidance system that consists of a wheel spinning with speed $\omega_2 = 20$ rad/s about a vertical axis of the ship. Along a spoke shown at the instant of interest, a particle is moving toward the center with r, $\dot{r}$ and $\ddot{r}$ as given in the diagram. What is the acceleration of the particle relative to inertial reference XYZ? Just write out the formulations for $\boldsymbol{a}_{XYZ}$ but do not carry out the cross products.

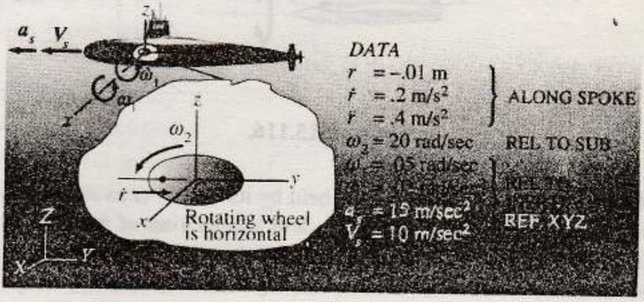
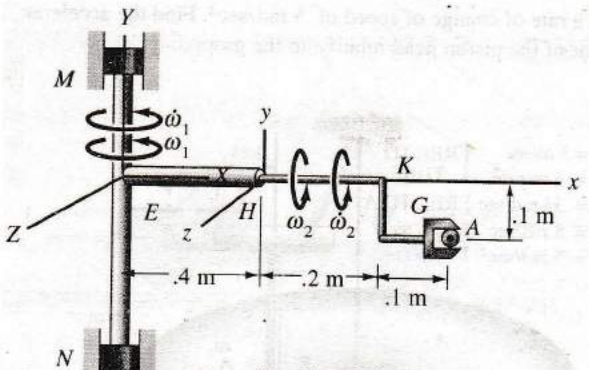


Figure P.15.114.

15.117. A robot moves a body held by its "jaws" G as shown in the diagram. What is the velocity and acceleration of point A at the instant shown relative to the ground? Arm EH is welded to the vertical shaft MN. Arm HKG is one rigid member which rotates about EH. How do you want to fix xyz?



$\omega_1 = .2$ rad/sec, $\dot{\omega}_1 = .1$ rad/sec² } Relative to ground

$\omega_2 = .4$ rad/sec, $\dot{\omega}_2 = .3$ rad/sec² } Relative to arm EH

Figure P.15.117.