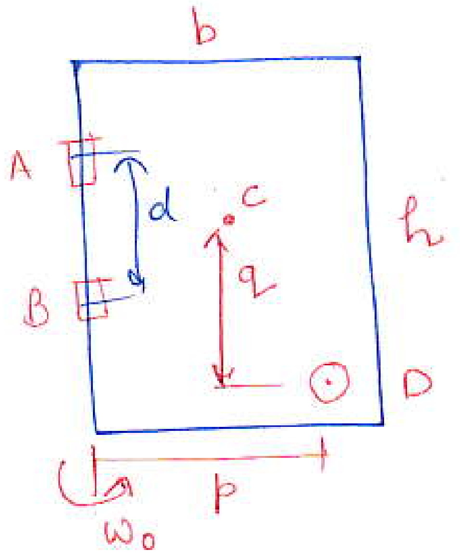
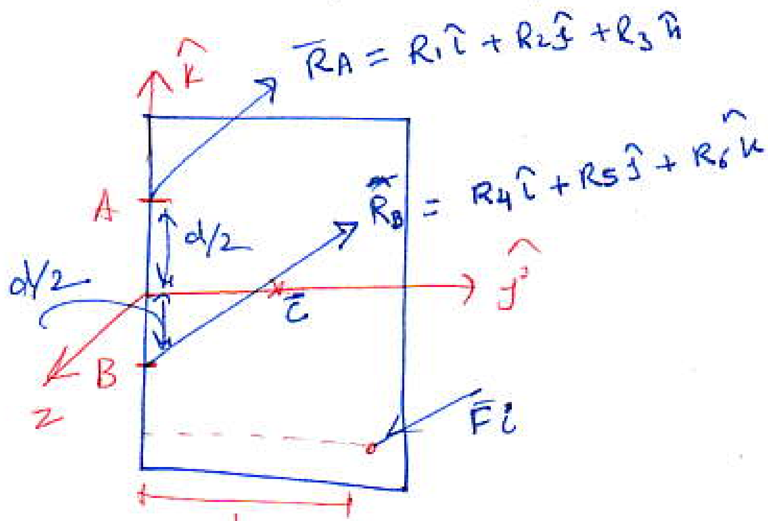
PC Dumir_ Example 3.5

Question: A uniform thin door of mass m, height h and width b is hinged at A and B about a vertical axis. It is rotating at angular velocity ω_0 when it bangs against a stop D at the ground level. The coefficient of restitution is 0.2. Find the impulsive force on the door from the hinges and the stop D during the instantaneous impact.



Solution: The FBD for impulsive forces is shown,



The line of impact is along $\hat{i}$. The angular velocity before impacy is $\bar{\omega} = \omega_0 \hat{k}$.

Let the angular velocity just after impact be $\bar{\omega}' = \omega' \hat{k}$ and the impulsive reactions ~~for~~ from the bearings be

$$\bar{R}_A = R_1 \hat{i} + R_2 \hat{j} + R_3 \hat{k}$$

$$\bar{R}_B = R_4 \hat{i} + R_5 \hat{j} + R_6 \hat{k}$$

The velocities of point E, D just before and just after impact are

$\bar{v}_E = -\omega_0 b \hat{i}$	$v_{Ex} = -\omega_0 b$
$\bar{v}_D = 0$	$v_{Dx} = 0$
$\bar{v}_E' = -\omega' b \hat{i}$	$v'_{Ex} = -\omega' b$
$\bar{v}_D' = 0$	$v'_{Dx} = 0$

coefficient of restitution

$$v'_{Ex} - v'_{Dx} = -e(v_{Ex} - v_{Dx})$$

$$\boxed{-\omega' b = -0.2(-\omega_0 b) = \omega' = -0.2\,\omega_0}$$

The fixed axis of rotation is not a principal axis of inertia at the fixed point A of the door.

$$I_{xz}^{A} = -m(0)(-d/2) = 0, \quad I_{yz}^{A} = m(b/2)(d/2) = \frac{mbd}{4}$$

$$\bar{H}_A = \left(I_{xz}^{A}\,\hat{i} + I_{yz}^{A}\,\hat{j} + I_{zz}^{A}\,\hat{k} \right) \omega_0$$

$$\Delta H_A = \left(I_{xz}^{A}\,\hat{i} + I_{yz}^{A}\,\hat{j} + I_{zz}^{A}\,\hat{k} \right) (\omega' - \omega_0)$$

$$= -1.2 \left(\frac{1}{4} m b d \hat{j} + \frac{1}{3} m b^2 \hat{k} \right) \omega_0$$

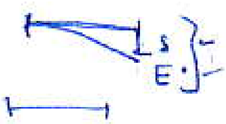
(A)

The angular impulse - moment of momentum relation for the fixed point A

$$I_{ang_A} = \Delta \bar{H}_A .$$

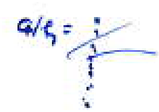
$$\left[-\left(\tfrac{1}{2} d + q \right) \hat{k} + p \hat{j} \right] \times \bar{F} \hat{i} - d \hat{k} \times \left(R_4 \hat{i} + R_5 \hat{j} + R_6 \hat{k} \right)$$

$$= -1.2 \left(\tfrac{1}{4} m b d \hat{j} + \tfrac{1}{3} m b^2 \hat{k} \right) \omega_0$$



$\hat{i}$: $d R_5 = 0 \Rightarrow R_5 = 0$

$\hat{k}$; $-p \hat{F} = -0.4 m b^2 \omega_0 \Rightarrow \hat{F} = 0.4 m b^2 \omega_0 / p$



$\hat{j}$: $-\frac{1}{2}(d+q)\hat{F} - d\bar{R}_4 = -0.3 m b d \omega_0$

$$\Rightarrow R_4 = m b \left[0.3 - 0.4 \left(0.5 + q/d \right) b/p \right] \omega_0$$

The velocity of centre of mass C just after the impact is $v_c = -0.5 \omega_0 b j$

$$\bar{v}_c = -0.5 \omega_0 b \hat{j} , \quad \bar{v}_c' = -0.5 \omega_0' b i = 0.1 \omega_0 \hat{i}$$

Using eq. (A), the impulse-momentum relation, $L = m \Delta \bar{V}_c$ yields

$$R_1\hat{i} + R_2\hat{j} + R_3\hat{k} + R_4\hat{i} + R_5\hat{j} + R_6\hat{k}$$

$$+ F\hat{i} = m(0.1 + 0.5)\, b\omega_0\hat{i}$$

$\Rightarrow$

$\hat{i}$: $R_1 + R_4 + F = 0.6\, m b\omega_0 \Rightarrow R_1 = mb(0.3 - 0.4(0.5 - \frac{g}{\ell})\frac{b}{\ell}\omega_0$

$\hat{j}$: $R_2 + R_5 = 0 \Rightarrow R_2 = 0$

$\hat{k}$: $R_3 + R_6 = 0 \Rightarrow R_3 = -\hat{R}_6$

R_3 & R_6 can not be evaluated individually.
If one hinge does not provides axial constraint, then $R_3 = R_6 = 0$