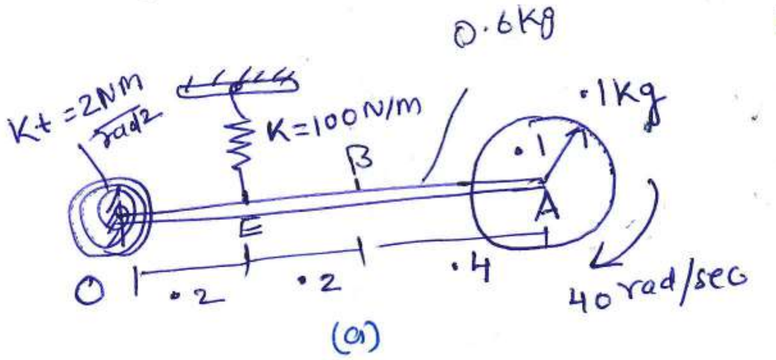
(1)

Example 3.16

A rod OA of mass 0.6 kg is hinged at O to a support and hinged to a thin disc of mass 0.1 kg at A. In a given position, the rod is held stationary, the disc rotates at 40 rad/s, the spring has a compression of 40 mm and the rotational spring is unstreched.



(a)

$M_{\cancel{I}} = \cancel{K\frac{\theta^2}{2}}$

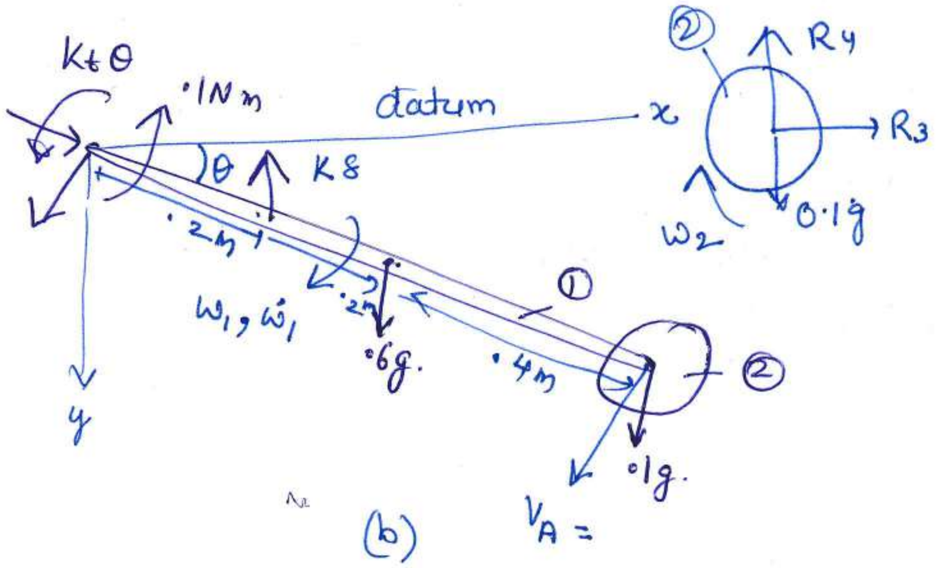
$\frac{Nm}{\cancel{rad}}^2$

The hinge at A is frictionless and there is constant frictional torque of 0.1 Nm on the rod from hinge at O.

(a) The rod is released. Find (a) the angular acceleration of the rod and the reaction from O just after release.

(b) The angular velocity of the rod, angular acceleration of the rod and the reaction from O when the rod has rotated by 30°

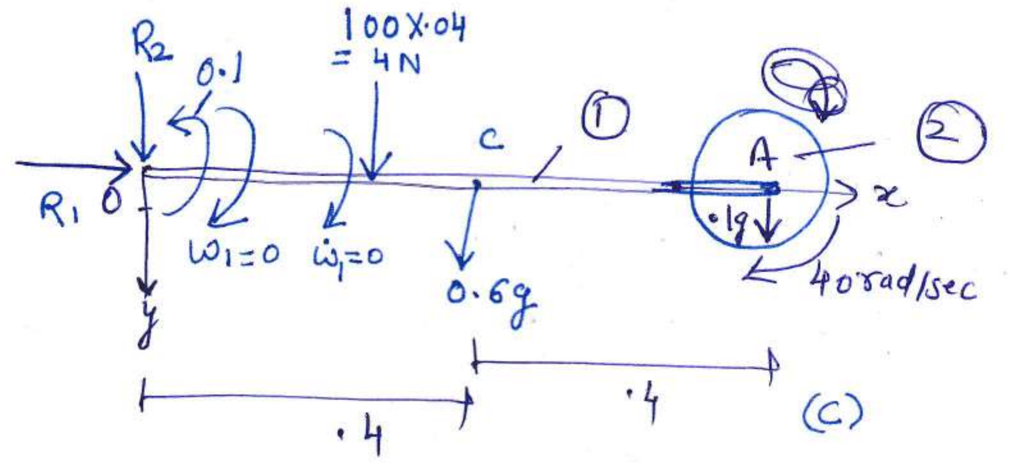
(c) The normal force, shear force and bending moment in the rod at B when rod has rotated by 30°.



(b)

Solution

FBD



For rod 1,

$$m_1 = 0.6 \text{ kg}$$

$$I_{zz}^{o} = \frac{m l^2}{3} = \frac{0.6 \times .8^2}{3} = .128 \text{ kg.m}^2$$

For disc $\quad m_2 = 0.1 \text{ kg}, \quad I_{zz}^{A} = \frac{m r^2}{2} = \frac{0.1\,(0.1)^2}{2} = 0.0005 \text{ kg.m}^2$

Let ω_1 = angular velocity of rod
ω_2 = angular velocity of disc

For the general position of the centre of mass A of disc 2

$$\bar{M}_A = 0, \quad I_{zz}^{A} \dot{\omega}_2 \Rightarrow 0, \quad \dot{\omega}_2 = 0 \Rightarrow \omega_2 = \text{Constant}$$

$$\omega_2 = 40 \text{ rad/s}$$

The extension of the spring is $\delta = 0.2 \sin\theta - 0.04\text{m}$

$$U_A = 0.8\, \omega_1\, \bar{e}_\phi$$

For rod + disc, H_{oz}, Kinetic Energy T, potential Energy V.
for general position θ.

③

$$H_{O_z} = I^{O}_{zz}\omega_1 + \left(I^{A}_{zz}\omega_2 + 0.8\, m_2 V_A\right)$$

$$= .128\omega_1 + 0.0005\times 40 + 0.8\times .1\times 0.8\omega_1 \Rightarrow .192\omega_1 + 0.02 \quad (1)$$

$$T = \frac{1}{2} I^{O}_{zz}\omega_1^2 + \left\{\frac{1}{2}\left[m_2 V_A^2\right] + \frac{1}{2} I^{A}_{zz}\omega_2^2\right]$$

$$= \frac{1}{2}\times .128\omega_1^2 + \cancel{\frac{1}{2}\left[(0.128)\omega_1^2\right.} + \frac{1}{2}\, .1\,(.8)\omega_1^2 + .0005\times 40^2$$

$$\Rightarrow 0.096\omega_1^2 + 0.4 \quad (2)$$

$$V = \frac{1}{2}K_t\theta^2 + \frac{1}{2}K(0.25\sin\theta - 0.04)^2 - m_1 g(0.4\sin\theta)$$

$$- m_2 g(0.8\sin\theta)$$

$$= \theta^2 + 50(0.25\sin\theta - 0.04)^2 - 0.32 g\sin\theta$$

(9) FBD of rod+disc is show en Fig.

Using equation (1). $\dot{H}_{Oz} = M_{Oz} = .192\dot{\omega}_1$

$$\cancel{\dot{H}_{Oz}} = .192\dot{\omega}_1 = \underbrace{-0.1}_{\text{Friction Torque}} + \underbrace{4\times 0.2}_{\text{spring}} + \underbrace{0.6g\times 0.4}_{\text{Rodwesh}} + \underbrace{0.1g(0.8)}_{\text{disk}}$$

$$\Rightarrow \dot{\omega}_1 = 20.00\ \text{rad/s}^2$$

Applying Newton's Law, R_1, R_2 can be calculated,

$$\bar{F} = m_1\bar{a}_c \,,$$

~~Initial there is not, a~~

For the present stath $\omega_1 = 0$, R_1, R_2 can be obtaned

$$\bar{a}_c = r\alpha + \omega\times(\omega\times r) = \dot{\omega}_1\hat{k}\times 0.4\hat{i} + \omega_1^2 r(-)\hat{i} \Rightarrow 8\hat{j}\ \text{m/s}^2$$

$$\bar{a}_A = -\omega_1^2(.8)\hat{i} + \dot{\omega}_1(.8)\hat{j} = 16\hat{j}\ \text{m/s}^2$$

$\boxed{\omega_1 = 0}$

$\boxed{\dot{\omega}_1 = c}$

$\cancel{\dot{\omega}_1 = \omega}$

$\omega = \frac{d\theta}{dt} = 0$

$\int \frac{d\theta}{dt} = \int 0$

$\theta \ne c(t)$

$\frac{d\theta}{dt} = 0$

$$\bar{F} = \sum m_i \bar{a}_{ci}$$

$$\Rightarrow R_1\hat{i} + R_2\hat{j} + (4 + .06g + .1g)\hat{j}$$

$$= .6 \times 8\hat{j} + .1 \times 16\hat{j}$$

$$R_1 = 0, \quad R_2 = -4.467\ N.$$

(b) Work-energy relation from the configuration 1 for $\theta = 0$ to configuration 2 for $\theta = 30^\circ = \pi/6$ yields ω_1.

Using eq (1), yield $\dot{\omega}_1$. R_{r_1}, R_4 are obtained using $\bar{F} = \sum m_i \bar{a}_{ic}$ and kinematic.

The pull on the spring for $\theta = 30^\circ$

$$100(0.2\sin 30 - 0.04) = 6N$$

$$(T_2 + V_2) - (T_1 + V_1) = W_{nc1-2}$$

Using eq.(2) and (3)

$$\left[0.096\omega_1^2 + 0.84) + \{(\pi/6)^2 + 50(0.2\sin 30^\circ - 0.04)^2 - 0.32g\sin 30\}\right] - \left[0.4 + 50(-0.04)^2\right] = \underbrace{-0.1\,\pi/6}_{\text{Work done by frictional Torque}}$$

$$\Rightarrow \omega_1 = 3.451\ \text{rad/sec.}$$

$$\dot{H}_{oz} = M_{oz}.$$

$$0.192\dot{\omega}_1 = -0.1 - 100(0.2\sin 30 - .04)(.2\cos 30) + 0.6g(0.4\cos 30) + 0.1g(0.8\cos 30) - 2(\pi/6)$$

$$\dot{\omega}_1 = 2.772\ \text{rad/s}^2$$

$$\bar{a}_C = -\omega_1^2(.4)\bar{e}_r + \dot{\omega}_1(0.4)\bar{e}_\phi$$
$$= -4.7638\,\bar{e}_r + 1.1088\,\bar{e}_\phi \;\; m/s^2$$

$$\bar{a}_A = -\omega_1^2(.8)\bar{e}_r + \dot{\omega}_1(0.8)\bar{e}_\phi$$
$$\Rightarrow -9.5275\,\bar{e}_r + 2.2176\,e_\phi \;\; m/s^2$$

$$F = \Sigma M_i \bar{a}_{ci}$$

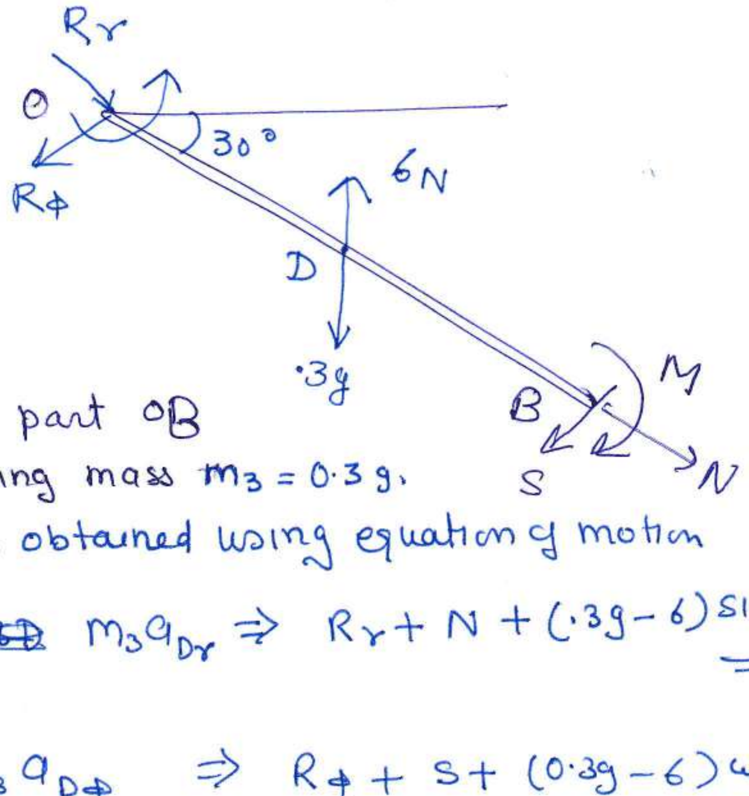
$\bar{e}_r$: $R_r + (0.6g + 0.1g - 6)\sin 30^\circ = .6 \times -4.7638 + 0.1(-9.527)$

$$\Rightarrow R_r = -4.245\ N$$

e_ϕ : $R_\phi + (0.6g + 0.1g - 6)\cos 30 = 0.6(1.1088) + 0.1 \times 2.2176$

$$\Rightarrow R_\phi = 0.1362\ N$$

(c)



The FBD of part OB of the rod having mass $m_3 = 0.3\ g.$

N & M are obtained using equation of motion

$F_r = \cancel{m_3 a_D}\ m_3 a_{Dr} \Rightarrow R_r + N + (.3g - 6)\sin 30 = -m_3\omega_1^2\, 0.2$

$$\Rightarrow N = 5.059\,N$$

$F_\phi = m_3 a_{D\phi} \Rightarrow R_\phi + S + (0.3g - 6)\cos 30^\circ = m_3\dot{\omega}_1(0.2)$

$$S = 2.814$$

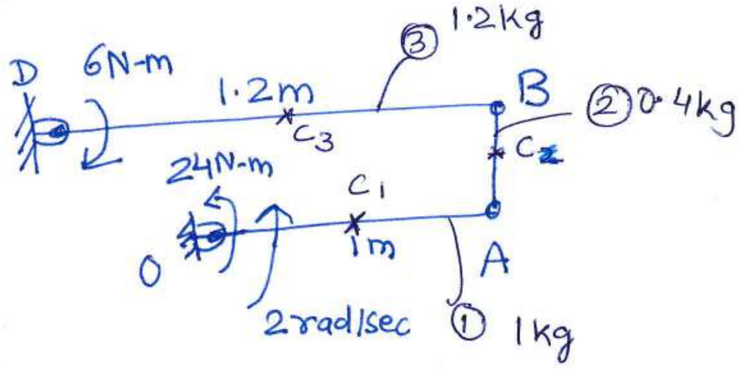
$M_{oz} = I_{zz}^{O}\dot{\omega}_1 \Rightarrow M - 0.1 - 2\times\pi/6 + (0.3g - 6)(.2\cos 30)$

$$+ S(\times 0.4) = 0.3(0.4)^2/3\ \dot{\omega}_1$$

$M = 0.5954$ N-m

Example No 3.17 PC Dumir

A 4- bar Linkage moves in a vertical plane under input torque of 24 N-m on the driver link 1 and a torque of 6 N-m on the driven link 3 (see Fig). Find the support reactions at given instant



Solution:

First step: We first express $\omega_2, \omega_3, \mathring{\omega}_2, \mathring{\omega}_3$ interms of ω_1 and $\mathring{\omega}_1$ using kinematics of the connected linkages. Point O, A on body 1;, A, B, C on body 2 and D, B on body 3 have the same plane of motion.

$\bar{\omega}_1 = 2\hat{k}$ rad/sec, $\bar{\omega}_2 = \omega_2\hat{k}$, $\bar{\omega}_3 = \omega_3\hat{k}$, $\bar{V}_o = \bar{V}_D = 0$

$AB = 0.4\hat{j}$ m, $\bar{DB} = 1.2\hat{i}$,

$\dot{\bar{\omega}}_1 = \dot{\omega}_1\hat{k}$, $\dot{\bar{\omega}}_2 = \dot{\omega}_2\hat{k}$, $\dot{\bar{\omega}}_3 = \omega_3\hat{k}$, $\bar{a}_o = \bar{a}_D = 0$

$AC = 0.2\hat{j}$, $\bar{OA} = 1\hat{i}$ m

$\boxed{\bar{V}_A = \omega_1(OA)\hat{j} = 2\times 1\hat{j} = 2\hat{j},}$ $\bar{V}_B = V_A + \omega_2\hat{k}\times\bar{AB} = \omega_3(DB)\hat{j}$

Now $2\hat{j} + \omega_2\hat{k}\times 0.4\hat{j} = \omega_3(1.2)\hat{j} \Rightarrow 2\hat{j} - 0.4\omega_2\hat{i} = 1.2\omega_3\hat{j}$

Equation coefficient of $\hat{i}$ & $\hat{j}$ $-0.4\omega_2 = 0$, $2 = 1.2\omega_3$

$\Rightarrow \omega_2 = 0$, $\omega_3 = 1.667$ rad/s.

$$\bar{a}_A = -\omega_1^2 (OA)\hat{i} + \dot{\omega}_1 (OA)\hat{j} \Rightarrow -4\hat{i} + \dot{\omega}_1\hat{j}$$

$$\bar{a}_B = a_A - \omega_2^2 \overline{AB} + \dot{\omega}_2 \hat{k}\times\overline{AB} = -\omega_3^2 (DB)\hat{i} + \dot{\omega}_3 (DB)\hat{j}$$

$$\Rightarrow \quad -4\hat{i} + \dot{\omega}_1\hat{j} + \dot{\omega}_2\hat{k}\times 0.4\hat{j} = -1.667^2(1.2)\hat{i} + \dot{\omega}_3(1.2)\hat{j}$$

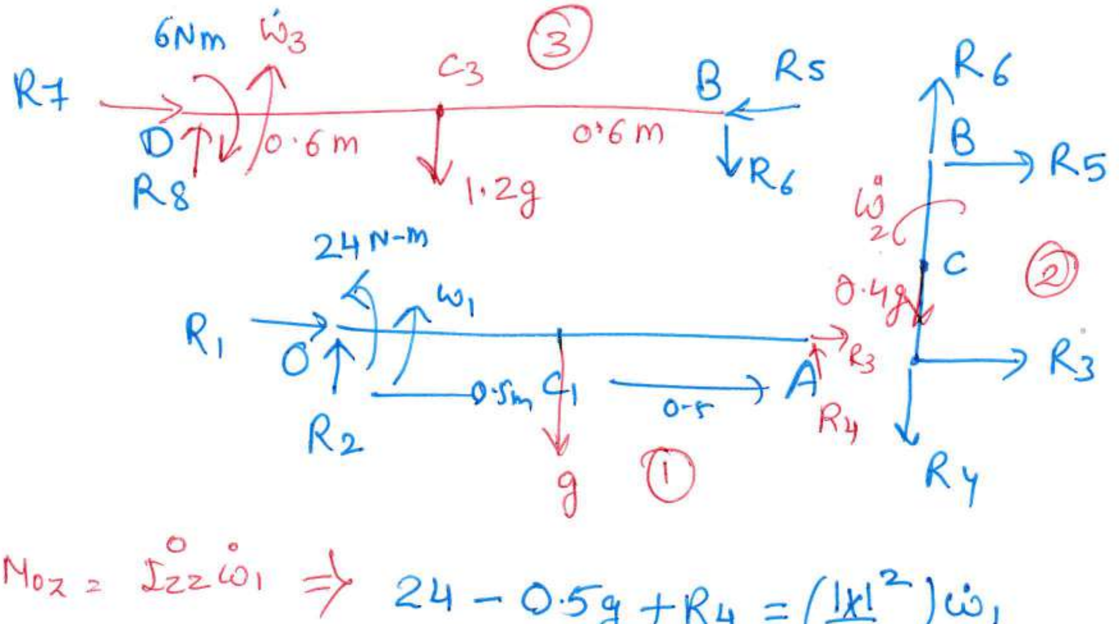
Equating i & j component.

$$\boxed{\begin{aligned} -4 - 0.4\dot{\omega}_2 &= -3.333 \Rightarrow \dot{\omega}_2 = -1.667 \text{ rad/sec}^2 \\ \dot{\omega}_1 &= 1.2\dot{\omega}_3 \Rightarrow \dot{\omega}_3 \Rightarrow .8333\,\omega_1 \end{aligned}}$$

$$\bar{a}_C = \bar{a}_A - \omega_2^2\overline{AC} + \dot{\omega}_2\hat{k}\times\overline{AC} = -4\hat{i} + \dot{\omega}_1\hat{j} + \dot{\omega}_2\hat{k}\times 0.2\hat{j}$$

$$\boxed{\Rightarrow -3.6667\hat{i} + \dot{\omega}_1\hat{j}}$$

FBD



For link 1

$$M_{Oz} = I^O_{zz}\dot{\omega}_1 \Rightarrow 24 - 0.5g + R_4 = \left(\frac{1\times1^2}{3}\right)\dot{\omega}_1$$

$$\Rightarrow R_4 = -19.095 + 0.333\dot{\omega}_1$$

For line 3.

$$M_{Dz} = I^D_{zz}\dot{\omega}_3 = -6 - 1.2g(0.6) - 1.2R_6 = \frac{1.2(1.2)^2}{3}\dot{\omega}_3$$

$$\Rightarrow R_6 = -10.866 - 0.4\dot{\omega}_1$$

For link 2

$$F_y = m_2 a_{cy} = R_6 - R_4 - 0.4g = 0.4\dot{\omega}_1$$

Now substitute the value of R_6 & R_4 in the above equation.

$$\dot{\omega}_1 = 3.781 \text{ rad/s}^2$$

For link 2 $M_{c_2} = I^c_{zz}\dot{\omega}_2 = -0.2(R_5+R_3) = \left(\frac{0.4(0.4)^2}{12}\right)(-1.667)$

$$R_5 + R_3 = 0.0444 \quad \text{—— (a)}$$

$$F_x = m_2 a_{cx} = R_5 - R_3 = 0.4(-3.667) = -1.4667 \quad (b)$$

From (a) & (b)

$R_5 = -0.711\text{N}$, $R_3 = .755\text{ N}$

Link 1

$$F_x = m_1 a_{c_1x} \Rightarrow R_1 + R_3 = 1(-\omega_1^2(0.5)) = R_1 = -2.756\text{ N}$$

$$F_y = m_1 a_{c_1y} \Rightarrow R_2 + R_4 - g = 1[\dot{\omega}_1(0.5)]$$

$$\Rightarrow R_2 = 29.53\text{ N}$$

Line 3

$$F_x = m_3 a_{c_3x} \Rightarrow R_7 - R_5 = 1.2(-\omega_3^2(0.5))$$

$$R_7 = -2.711\text{ N}$$

$$F_y = m_3 a_{c_3y} \Rightarrow R_8 - R_6 - 1.2g = 1.2(\dot{\omega}_3(0.4))$$

$$\Rightarrow R_8 = 1.641\text{ N.}$$