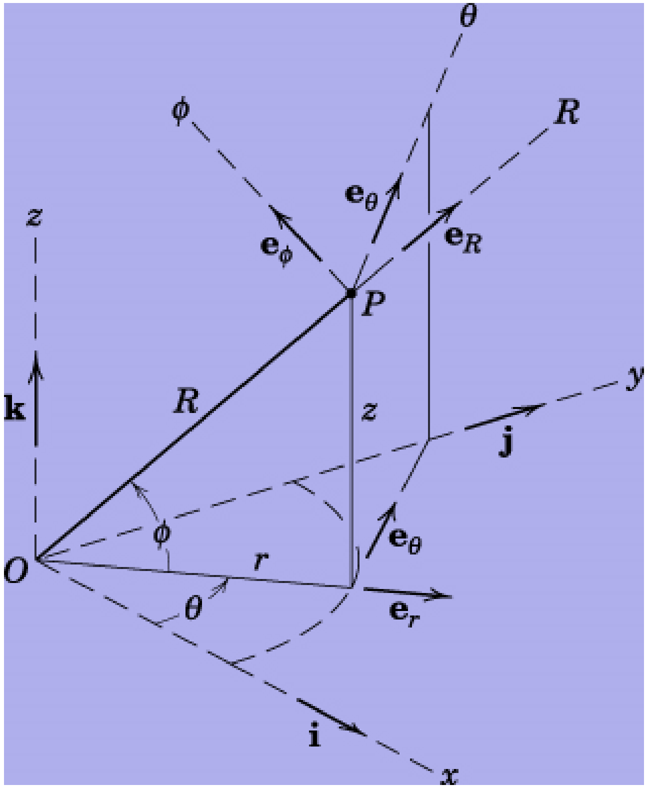
Kinematics of Particles

Space Curvilinear Motion

Three-dimensional motion of a particle along a space curve.

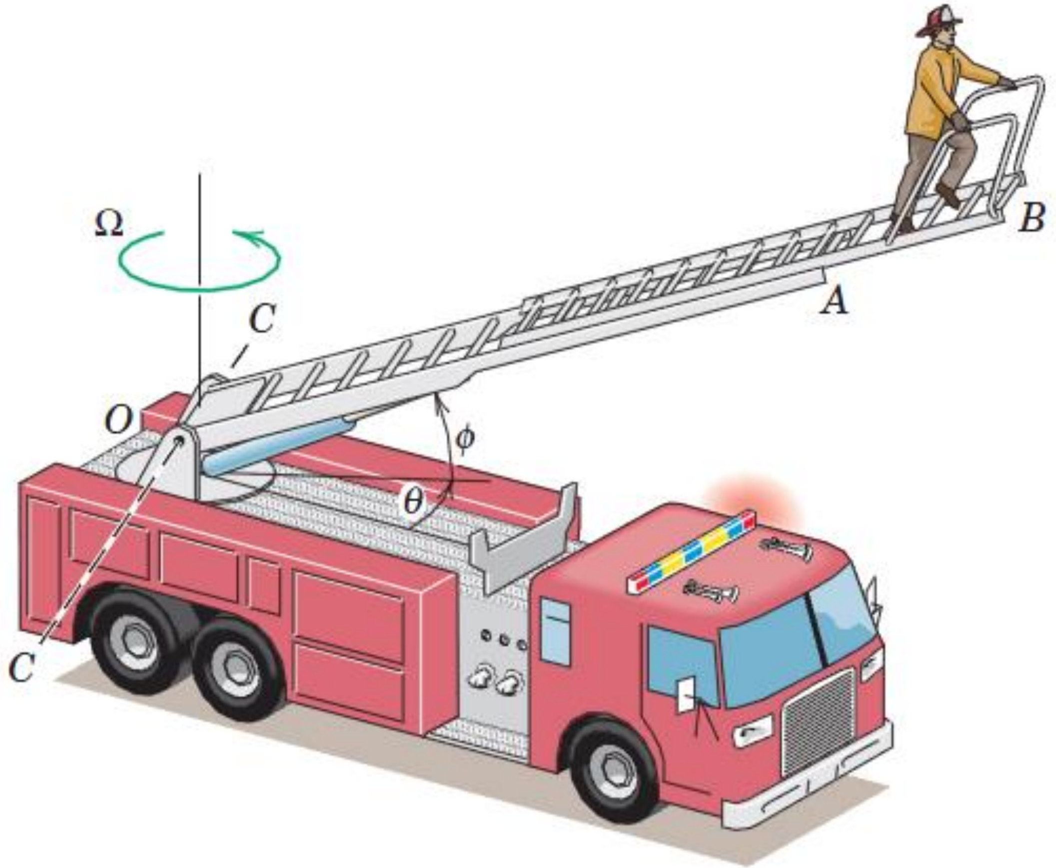
Three commonly used coordinate systems to describe this motion:

1. Rectangular Coordinate System (*x*-*y*-*z*)
2. Cylindrical Coordinate System (*r*-*θ*-*z*)
3. Spherical Coordinate System (*R*-*θ*-*Φ*)



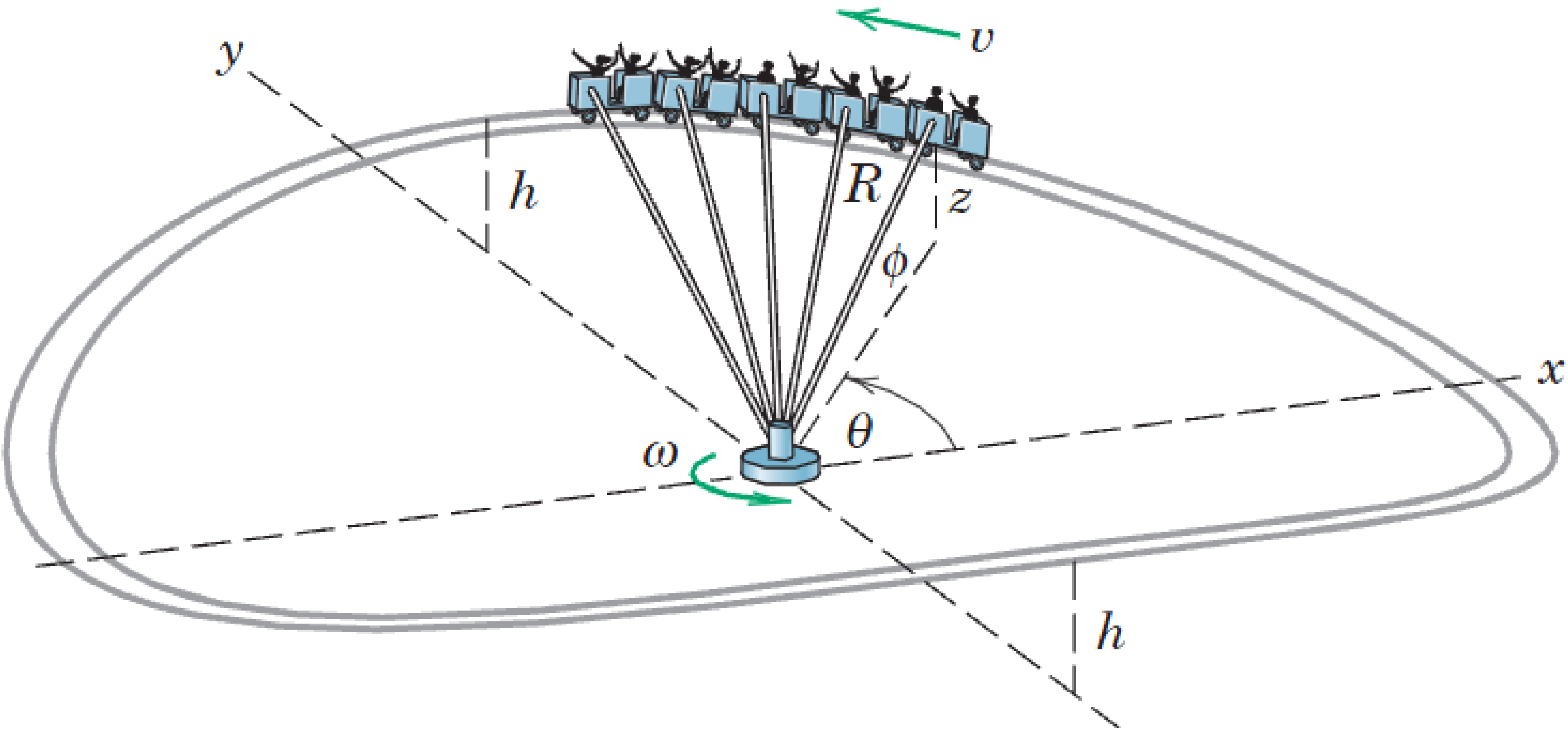
Kinematics of particles :: motion in space

- Example 2



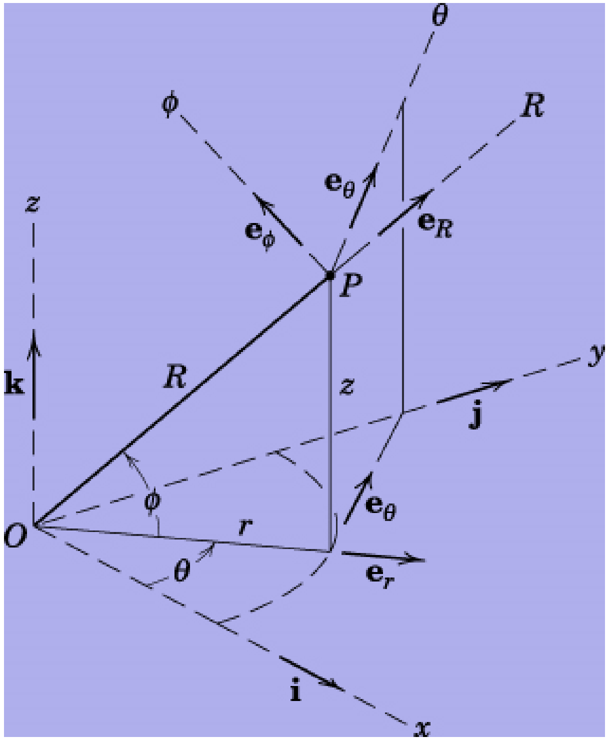
Kinematics of particles :: motion in space

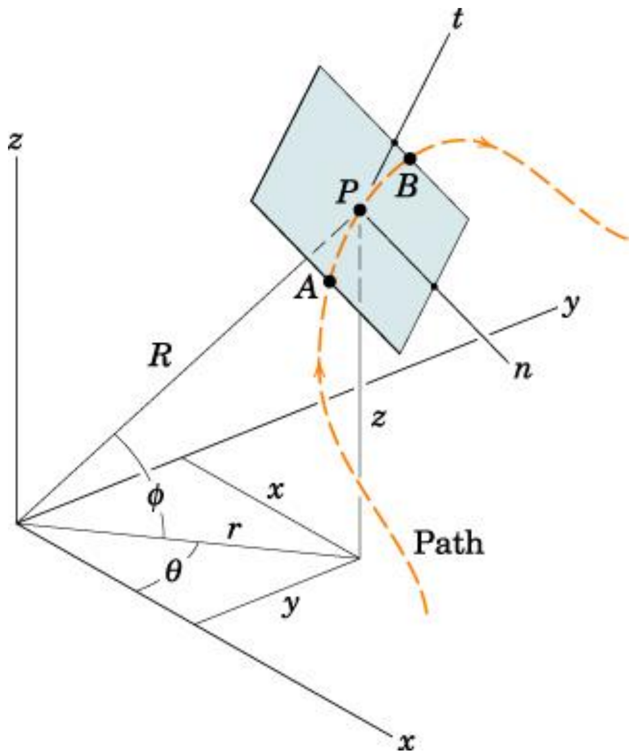
- Example 3



Kinematics of particles :: motion in space

***n*- and *t*-coordinates for plane curvilinear motion** can also be used for space curvilinear motion of a particle
:: Considering **a plane containing the curve and the *n*- and *t*-axes** **at a particular location (instance)**
•This plane will continuously shift its location and orientation in case of space curvilinear motion → difficult to use.

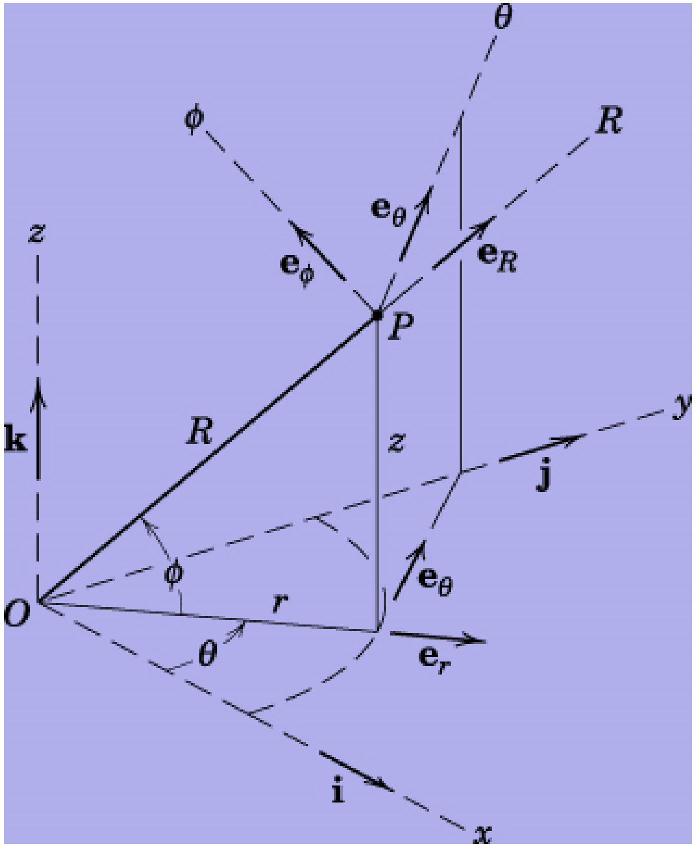




Kinematics of particles :: motion in space

Rectangular Coordinates (*x-y-z*)

•Simply extend the previously derived equations to include third dimension.



Plane Curvilinear Motion (2-D)

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j}$$

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{r}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

Space Curvilinear Motion (3-D)

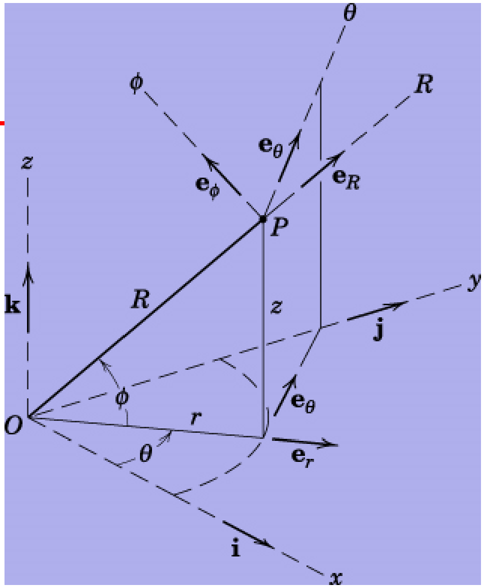
$$\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{v} = \dot{\mathbf{R}} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j} + \dot{z}\mathbf{k}$$

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{\mathbf{R}} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j} + \ddot{z}\mathbf{k}$$

In three dimensions, **R** is used in place of **r** for the position vector

Kinematics of Particles



Cylindrical Coordinates (*r*-*θ*-*z*)

•Extension of the Polar coordinate system.

•Addition of *z*-coordinate and its two time derivatives

Position vector **R** to the particle for cylindrical coordinates:

$\mathbf{R} = r\,\mathbf{e}_r + z\mathbf{k}$

Velocity:

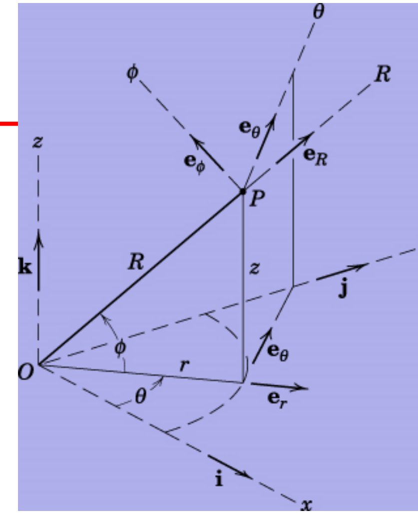
Polar	Cylindrical
$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta$	$\mathbf{v} = \dot{r}\mathbf{e}_r + r\dot{\theta}\mathbf{e}_\theta + \dot{z}\mathbf{k}$
$v_r = \dot{r}$	$v_r = \dot{r}$
$v_\theta = r\dot{\theta}$	$v_\theta = r\dot{\theta}$
	$v_z = \dot{z}$
$v = \sqrt{v_r^2 + v_\theta^2}$	$v = \sqrt{v_r^2 + v_\theta^2 + v_z^2}$

Acceleration:

Polar	Cylindrical
$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta$	$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{e}_\theta + \ddot{z}\mathbf{k}$
$a_r = \ddot{r} - r\dot{\theta}^2$	$a_r = \ddot{r} - r\dot{\theta}^2$
$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$	$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$
	$a_z = \ddot{z}$
$a = \sqrt{a_r^2 + a_\theta^2}$	$a = \sqrt{a_r^2 + a_\theta^2 + a_z^2}$

Unit vector **k** remains fixed in direction → has a zero time derivative

Kinematics of Particles



Spherical Coordinates (*R*-*θ*-*Φ*)

•Utilized when a radial distance and two angles are utilized to specify the position of a particle.
•The unit vector $\mathbf{e}_R$ is in the direction in which the particle *P* would move if *R* increases keeping *θ* and *Φ* constant.
•The unit vector $\mathbf{e}_\theta$ is in the direction in which the particle *P* would move if *θ* increases keeping *R* and *Φ* constant.
•The unit vector $\mathbf{e}_\Phi$ is in the direction in which the particle *P* would move if *Φ* increases keeping *R* and *θ* constant.

Resulting expressions for **v** and **a**:

$$\mathbf{v} = v_R \mathbf{e}_R + v_\theta \mathbf{e}_\theta + v_\phi \mathbf{e}_\phi$$

$$v_R = \dot{R}$$

$$v_\theta = R\dot{\theta}\cos\phi$$

$$v_\phi = R\dot{\phi}$$

$$\mathbf{a} = a_R \mathbf{e}_R + a_\theta \mathbf{e}_\theta + a_\phi \mathbf{e}_\phi$$

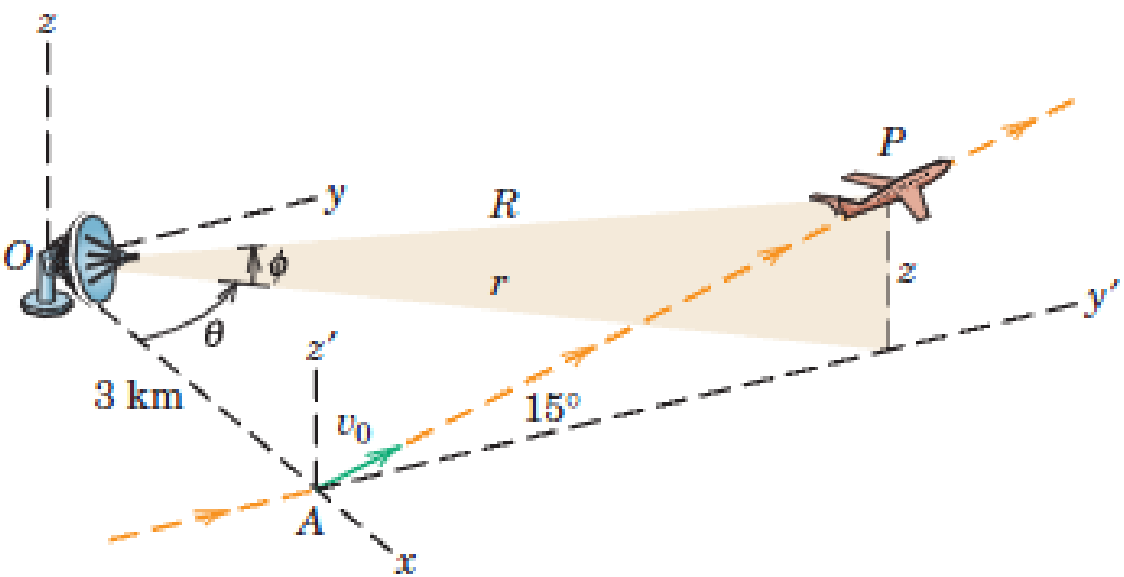
$$a_R = \ddot{R} - R\dot{\phi}^2 - R\dot{\theta}^2\cos^2\phi$$

$$a_\theta = \frac{\cos\phi}{R}\frac{d}{dt}\left(R^2\dot{\theta}\right) - 2R\dot{\theta}\dot{\phi}\sin\phi$$

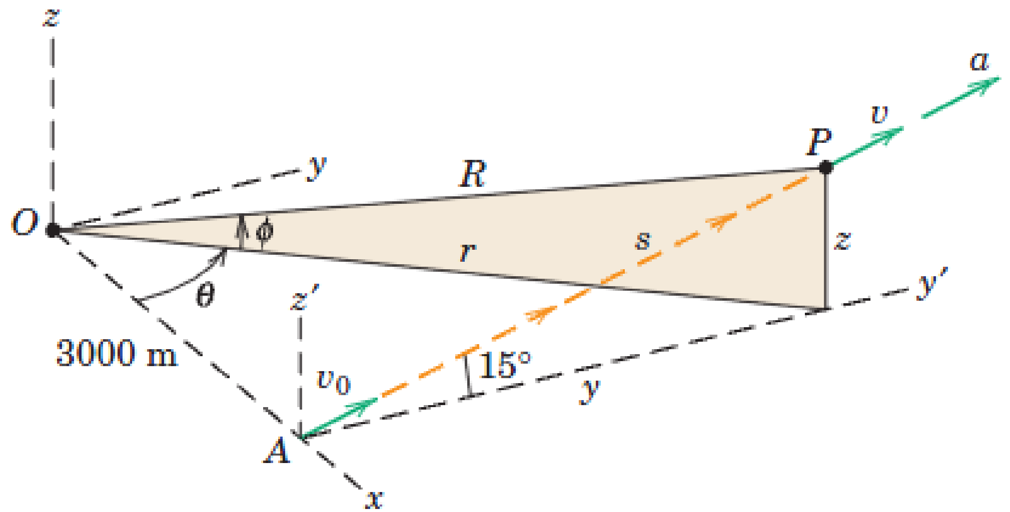
$$a_\phi = \frac{1}{R}\frac{d}{dt}\left(R^2\dot{\phi}\right) + R\dot{\theta}^2\sin\phi\cos\phi$$

Example (1) on cylindrical/spherical coordinate

An aircraft P takes off at A with a velocity v_0 of 250 km/h and climbs in the vertical y'-z' plane at the constant 15° angle with an acceleration along its flight path of 0.8 m/s^2. Flight progress is monitored by radar at point O. (a) Resolve the velocity of P into cylindrical-coordinate components 60 seconds after takeoff and find $\dot{r}$, $\dot{\theta}$, and $\dot{z}$ for that instant. (b) Resolve the velocity of the aircraft P into spherical-coordinate components 60 seconds after takeoff and find $\dot{R}$, $\dot{\theta}$, and $\dot{\phi}$ for that instant.



Example (1) on cylindrical/spherical coordinate



$$v_0 = \frac{250}{3.6} = 69.4 \text{ m/s}$$

and the speed after 60 seconds is

$$v = v_0 + at = 69.4 + 0.8(60) = 117.4 \text{ m/s}$$

The distance s traveled after takeoff is

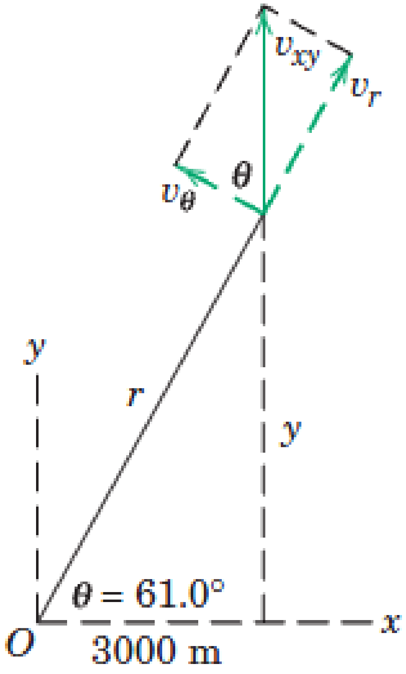
$$s = s_0 + v_0 t + \frac{1}{2} at^2 = 0 + 69.4(60) + \frac{1}{2}(0.8)(60)^2 = 5610 \text{ m}$$

The y-coordinate and associated angle θ are

$$y = 5610 \cos 15^\circ = 5420 \text{ m}$$

$$\theta = \tan^{-1} \frac{5420}{3000} = 61.0^\circ$$

Example (1) on cylindrical/spherical coordinate



$$r = \sqrt{3000^2 + 5420^2} = 6190 \text{ m}$$

$$v_{xy} = v \cos 15° = 117.4 \cos 15° = 113.4 \text{ m/s}$$

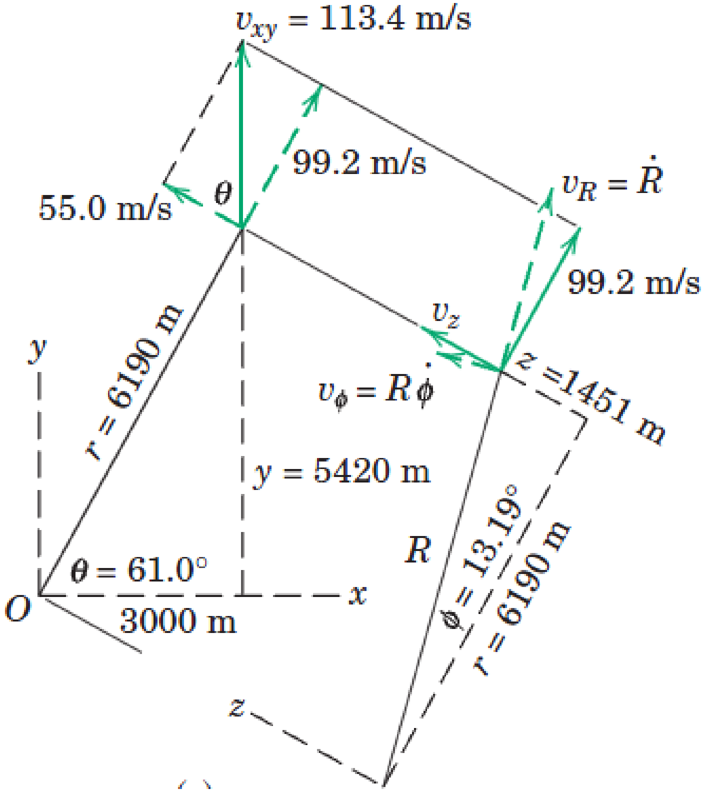
$$v_r = \dot{r} = v_{xy} \sin\theta = 113.4 \sin 61.0° = 99.2 \text{ m/s} \qquad \textit{Ans.}$$

$$v_\theta = r\dot{\theta} = v_{xy} \cos\theta = 113.4 \cos 61.0° = 55.0 \text{ m/s}$$

So $\dot{\theta} = \frac{55.0}{6190} = 8.88(10^{-3}) \text{ rad/s}$ *Ans.*

Finally $\dot{z} = v_z = v \sin 15° = 117.4 \sin 15° = 30.4 \text{ m/s}$ *Ans.*

Example (1) on cylindrical/spherical coordinate



$$z = y \tan 15^\circ = 5420 \tan 15^\circ = 1451 \text{ m}$$

$$\phi = \tan^{-1}\frac{z}{r} = \tan^{-1}\frac{1451}{6190} = 13.19^\circ$$

$$R = \sqrt{r^2 + z^2} = \sqrt{6190^2 + 1451^2} = 6360 \text{ m}$$

$$v_R = \dot{R} = 99.2 \cos 13.19^\circ + 30.4 \sin 13.19^\circ = 103.6 \text{ m/s} \qquad Ans.$$

$$\dot{\theta} = 8.88(10^{-3}) \text{ rad/s, as in part } (a) \qquad Ans.$$

$$v_\phi = R\dot{\phi} = 30.4 \cos 13.19^\circ - 99.2 \sin 13.19^\circ = 6.95 \text{ m/s}$$

$$\dot{\phi} = \frac{6.95}{6360} = 1.093(10^{-3}) \text{ rad/s} \qquad Ans.$$

Kinematics of Particles

Relative Motion (Translating Axes)

- Till now particle motion described using fixed reference axes
  → Absolute Displacements, Velocities, and Accelerations
- Relative motion analysis is extremely important for some cases
  → measurements made wrt a moving reference system

Motion of a moving coordinate system is specified wrt a fixed coordinate system (whose absolute motion is negligible for the problem at hand).

Current Discussion:

- Moving reference systems that translate but do not rotate
- Relative motion analysis for plane motion

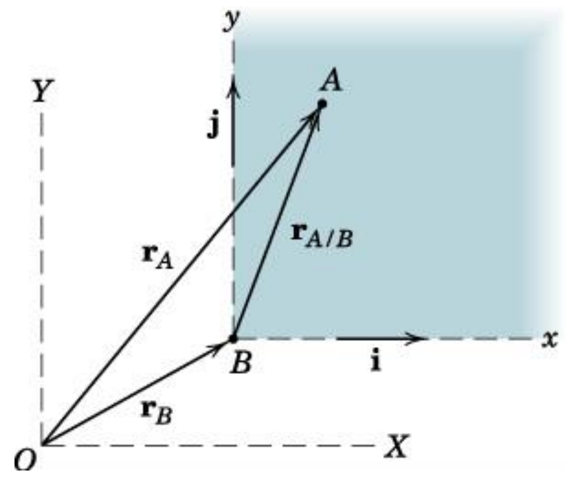
Relative Motion Analysis is critical even if aircrafts are not rotating

Kinematics of Particles

Relative Motion (Translating Axes)

Vector Representation

Two particles *A* and *B* have separate curvilinear motions in a given plane or in parallel planes.



- Attaching the origin of translating (non-rotating) axes *x*-*y* to B.
- Observing the motion of *A* from moving position on *B*.
- Position vector of *A* measured relative to the frame *x*-*y* is $\mathbf{r}_{A/B} = x\mathbf{i} + y\mathbf{j}$. Here *x* and *y* are the coordinates of *A* measured in the x-y frame. (*A/B* → *A* relative to *B*)
- Absolute position of B is defined by vector $\mathbf{r}_B$ measured from the origin of the fixed axes *X*-*Y*.
- Absolute position of A → $\mathbf{r}_A = \mathbf{r}_B + \mathbf{r}_{A/B}$
- Differentiating wrt time →

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

Velocity of *A* wrt *B*:

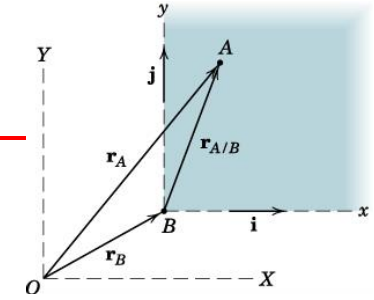
$$\dot{\mathbf{r}}_{A/B} = \mathbf{v}_{A/B} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

Acceleration of *A* wrt *B*:

$$\ddot{\mathbf{r}}_{A/B} = \dot{\mathbf{v}}_{A/B} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

Unit vector **i** and **j** have constant direction → zero derivatives

Kinematics of Particles



Relative Motion (Translating Axes)

Vector Representation

$$\dot{\mathbf{r}}_A = \dot{\mathbf{r}}_B + \dot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{v}_A = \mathbf{v}_B + \mathbf{v}_{A/B}$$

$$\ddot{\mathbf{r}}_A = \ddot{\mathbf{r}}_B + \ddot{\mathbf{r}}_{A/B} \quad \text{or} \quad \mathbf{a}_A = \mathbf{a}_B + \mathbf{a}_{A/B}$$

Velocity of *A* wrt *B*:

$$\dot{\mathbf{r}}_{A/B} = \mathbf{v}_{A/B} = \dot{x}\mathbf{i} + \dot{y}\mathbf{j}$$

Acceleration of *A* wrt *B*:

$$\ddot{\mathbf{r}}_{A/B} = \dot{\mathbf{v}}_{A/B} = \ddot{x}\mathbf{i} + \ddot{y}\mathbf{j}$$

→ Absolute Velocity or Acceleration of *A* = Absolute Velocity or Acceleration of *B* + Velocity or Acceleration of *A* relative to *B*.

- The relative motion terms can be expressed in any convenient coordinate system (rectangular, normal-tangential, or polar)
- Already derived formulations can be used.
  → The appropriate fixed systems of the previous discussions becomes the moving system in this case.

Kinematics of Particles

Relative Motion (Translating Axes)

Selection of Translating Axes

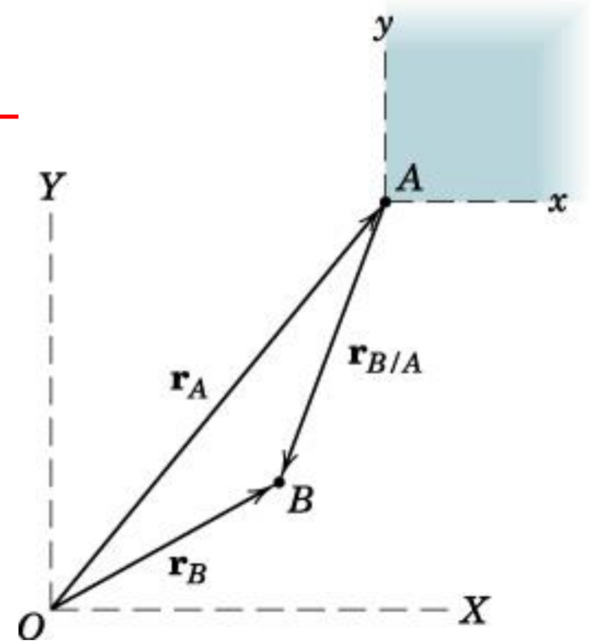
Instead of *B*, if *A* is used for the attachment of the moving system: →

$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A}$

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$

→ $\mathbf{r}_{B/A} = -\mathbf{r}_{A/B}$; $\mathbf{v}_{B/A} = -\mathbf{v}_{A/B}$; $\mathbf{a}_{B/A} = -\mathbf{a}_{A/B}$



Relative Motion Analysis:

- Acceleration of a particle in translating axes (*x*-*y*) will be the same as that observed in a fixed system (*X*-*Y*) if the moving system has a constant velocity

→ A set of axes which have a constant absolute velocity may be used in place of a fixed system for the determination of accelerations

→ Interesting applications of Newton's Second law of motion in Kinetics

A translating reference system that has no acceleration → Inertial System

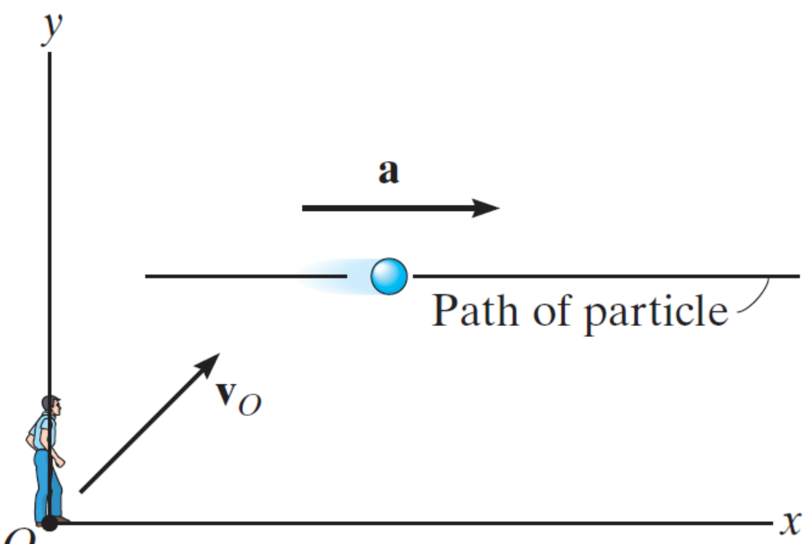
Kinematics of Particles

Relative Motion (Translating Axes)

Inertial Reference Frame Or Newtonian Reference Frame

- When applying the eqn of motion (Newton's Second Law of Motion), it is important that the acceleration of the particle be measured wrt a reference frame that is either fixed or translates with a constant velocity.
- The reference frame should not rotate and should not accelerate.
- In this way, the observer will not accelerate and measurements of particle's acceleration will be the same from any reference of this type.

→ Inertial or Newtonian Reference Frame

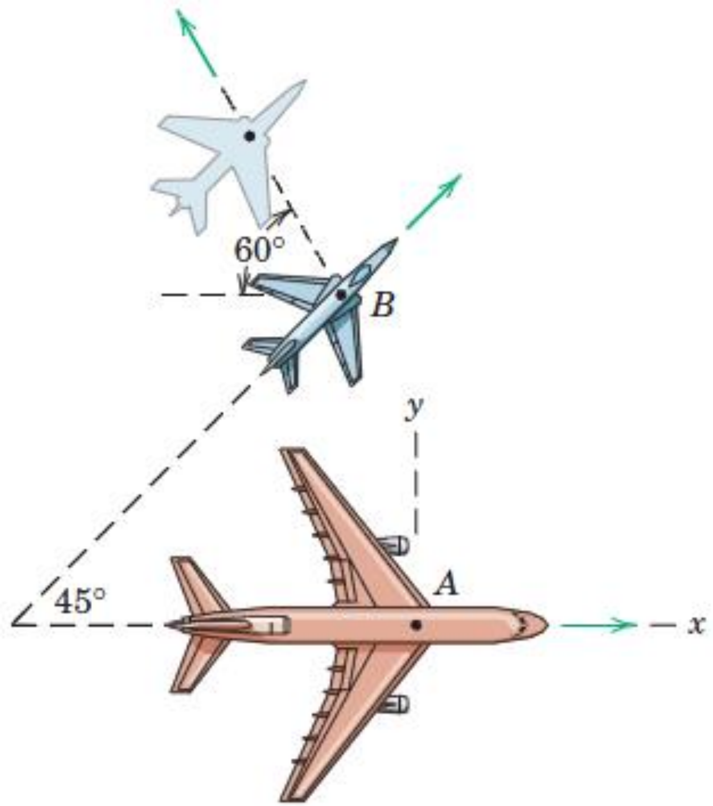


Study of motion of rockets and satellites: inertial reference frame may be considered to be fixed to the stars.

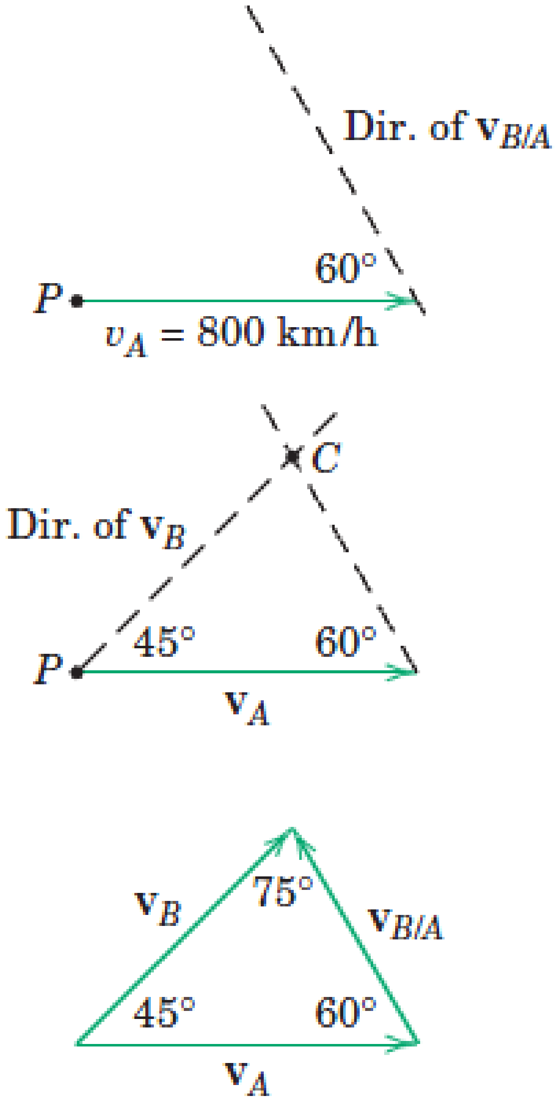
Motion of bodies near the surface of the earth: inertial reference frame may be considered to be fixed to the earth. Though the earth rotates @ its own axis and revolves around the sun, the accelerations created by these motions of the earth are relatively small and can be neglected.

Example on relative motion

Passengers in the jet transport A flying east at a speed of 800 km/h observe a second jet plane B that passes under the transport in horizontal flight. Although the nose of B is pointed in the 45° northeast direction, plane B appears to the passengers in A to be moving away from the transport at the 60° angle as shown. Determine the true velocity of B.



Example on relative motion



Graphical method

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h}$$

Trigonometric method

$$\frac{v_B}{\sin 60^\circ} = \frac{v_A}{\sin 75^\circ} \qquad v_B = 800\,\frac{\sin 60^\circ}{\sin 75^\circ} = 717 \text{ km/h}$$

Vector algebra

$$\mathbf{v}_A = 800\mathbf{i} \text{ km/h} \qquad \mathbf{v}_B = (v_B \cos 45^\circ)\mathbf{i} + (v_B \sin 45^\circ)\mathbf{j}$$

$$\mathbf{v}_{B/A} = (v_{B/A} \cos 60^\circ)(-\mathbf{i}) + (v_{B/A} \sin 60^\circ)\mathbf{j}$$

Substituting these relations into the relative-velocity equation and solving separately for the **i** and **j** terms give

$$(\mathbf{i}\text{-terms}) \qquad v_B \cos 45^\circ = 800 - v_{B/A} \cos 60^\circ$$

$$(\mathbf{j}\text{-terms}) \qquad v_B \sin 45^\circ = v_{B/A} \sin 60^\circ$$

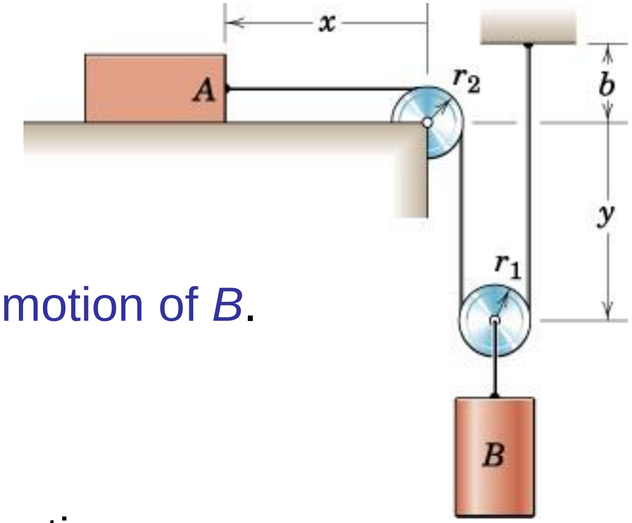
Solving simultaneously yields the unknown velocity magnitudes

$$v_{B/A} = 586 \text{ km/h} \qquad \text{and} \qquad v_B = 717 \text{ km/h} \qquad \textit{Ans.}$$

Kinematics of Particles

Constrained Motion of Connected Particles

- Inter-related motion of particles



One Degree of Freedom System

Establishing the position coordinates *x* and *y* measured from a convenient fixed datum.

→ We know that horz motion of *A* is twice the vertical motion of *B*.

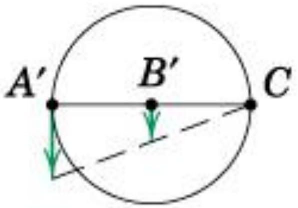
Total length of the cable:

$$L = x + \frac{\pi r_2}{2} + 2y + \pi r_1 + b$$

L, r_2, r_1 and b are constant. First and second time derivatives:

$$0 = \dot{x} + 2\dot{y} \quad \text{or} \quad 0 = v_A + 2v_B$$

$$0 = \ddot{x} + 2\ddot{y} \quad \text{or} \quad 0 = a_A + 2a_B$$



Lower Pulley

→ Signs of velocity and acceleration of *A* and *B* must be opposite

→ v_A is positive to the left. v_B is positive to the down

→ Equations do not depend on lengths or pulley radii

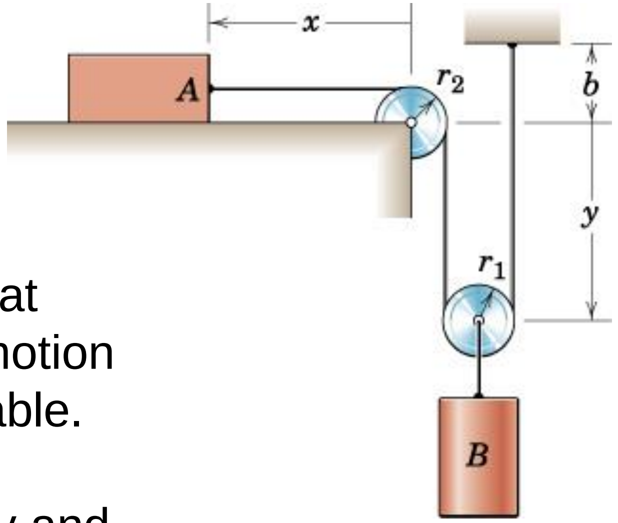
Alternatively, the velocity and acceleration magnitudes can be determined by inspection of lower pulley.

SDOF: since only one variable (*x* or *y*) is needed to specify the positions of all parts of the system

Kinematics of Particles

Constrained Motion of Connected Particles

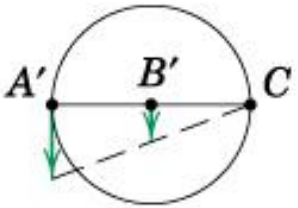
One Degree of Freedom System



Applying an infinitesimal motion of *A'* in lower pulley.

- *A*' and *A* will have same motion magnitudes
- *B*' and *B* will have same motion magnitudes
- From the triangle shown in lower figure, it is clear that *B*' moves half as far as *A*' because point *C* has no motion momentarily since it is on the fixed portion on the cable.

- Using these observations, we can obtain the velocity and acceleration magnitude relationships by inspection.
- The pulley is actually a wheel which rolls on the fixed cable.



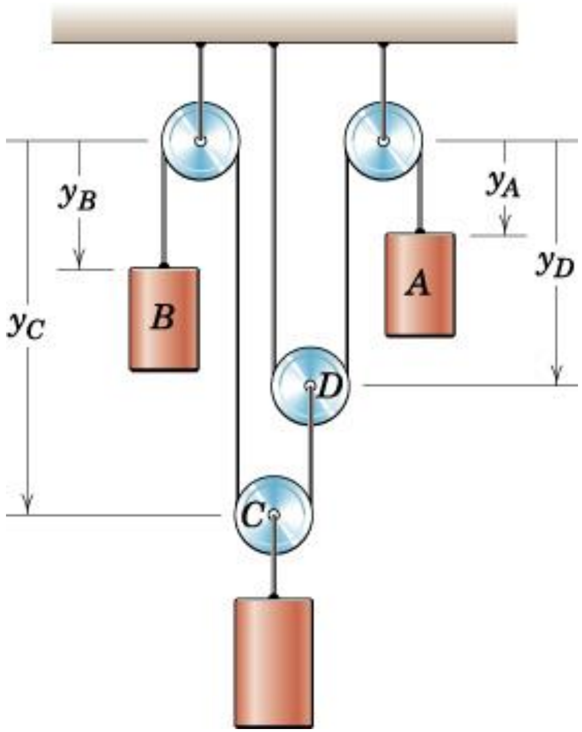
Lower Pulley

Kinematics of Particles

Constrained Motion of Connected Particles

Two Degrees of Freedom System

Two separate coordinates are required to specify the position of lower cylinder and pulley *C* → y_A and y_B.



Lengths of the cables attached to cylinders *A* and *B*:

$$L_A = y_A + 2y_D + \text{constant}$$

$$L_B = y_B + y_C + (y_C - y_D) + \text{constant}$$

Their time derivatives:

$$0 = \dot{y}_A + 2\dot{y}_D \quad \text{and} \quad 0 = \dot{y}_B + 2\dot{y}_C - \dot{y}_D$$

$$0 = \ddot{y}_A + 2\ddot{y}_D \quad \text{and} \quad 0 = \ddot{y}_B + 2\ddot{y}_C - \ddot{y}_D$$

Eliminating the terms in $\dot{y}_D$ and $\ddot{y}_D$

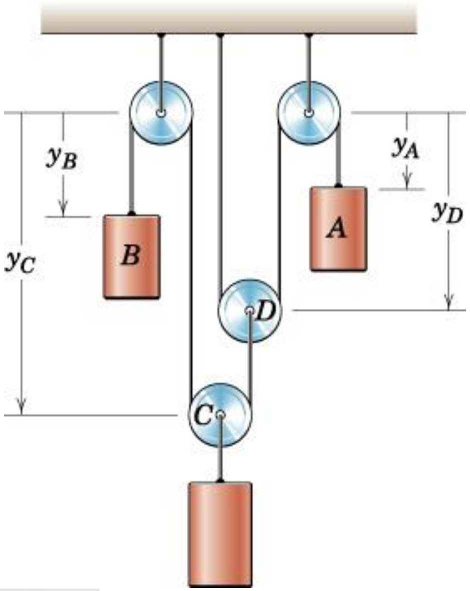
$$\dot{y}_A + 2\dot{y}_B + 4\dot{y}_C = 0 \quad \text{or} \quad v_A + 2v_B + 4v_C = 0$$

$$\ddot{y}_A + 2\ddot{y}_B + 4\ddot{y}_C = 0 \quad \text{or} \quad a_A + 2a_B + 4a_C = 0$$

→ It is impossible for the signs of all three terms to be positive simultaneously.
→ If *A* and *B* have downward (+ve) velocity, *C* will have an upward (-ve) velocity.

Kinematics of Particles

Constrained Motion of Connected Particles



Two Degrees of Freedom System

Same results can be obtained by observing the motions of the two pulleys at *C* and *D*.

- Apply increment dy_A (keeping y_B fixed)
  → *D* moves up an amount $dy_A/2$
  → this causes an upward movement $dy_A/4$ of *C*
- Similarly, for an increment dy_B (keeping y_A fixed)
  → *C* moves up an amount $dy_B/2$

- A combination of the two movements gives an upward movement:
  → $-v_c = v_A/4 + v_B/2$ as obtained earlier.

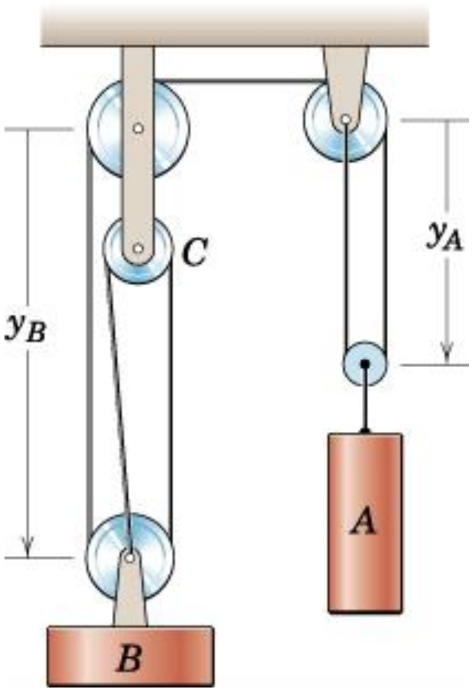
$$-dy_C = \frac{dy_A}{4} + \frac{dy_B}{2}$$

Kinematics of Particles

Constrained Motion of Connected Particles

Example

Determine the velocity of B if the cylinder A has a downward velocity of 0.3 m/s.
Use two different methods.



Solution

Method I: Centers of pulleys at A and B are located by the coordinates y_A and y_B measured from fixed positions.

Total constant length of the cable in the system:
$L = 3\, y_B + 2\, y_A$ + constants
Differentiating wrt time: $0 = 3\dot{y}_B + 2\dot{y}_A$

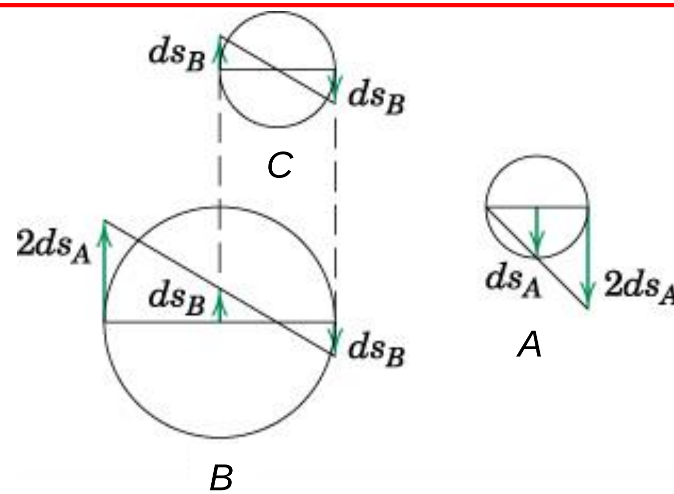
Substituting $v_A = \dot{y}_A = 0.3$ m/s and $v_B = \dot{y}_B$

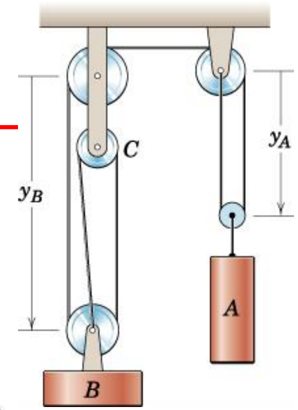
→ $v_B = -\,0.2$ m/s

Kinematics of Particles

Constrained Motion of Connected Particles

Example





Solution

Method II: Graphical method:

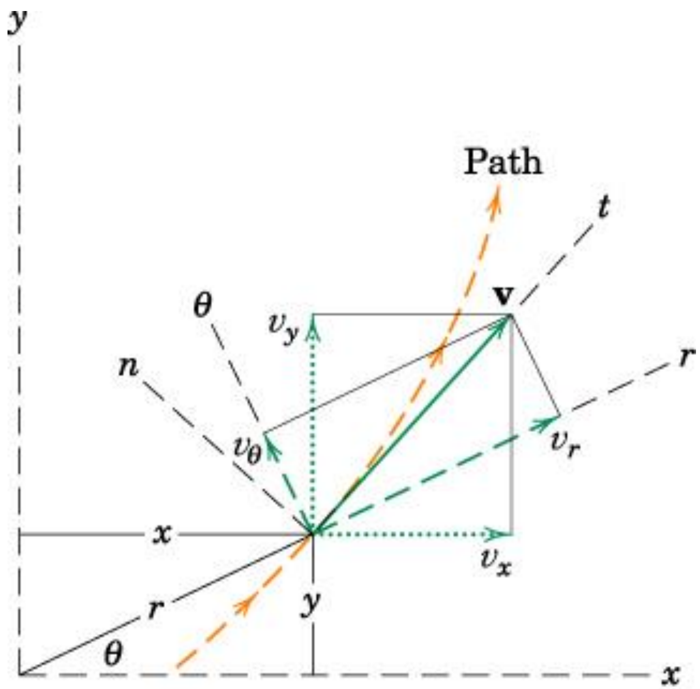
Enlarged views of the pulleys at *A*, *B*, and *C* are shown.

- Apply a differential movement ds_A at center of pulley *A*
  → no motion at left end of its horz diameter since it is attached to the fixed part of the cable → right end will move by $2ds_A$
  → this movement will be transmitted to the left end of horz diameter of pulley *B* but in the upward direction.
- Pulley *C* has a fixed center → disp on each side are equal and opposite (ds_B)
  → Right end of pulley *B* will also have a downward displacement equal to the upward displacement of the center of the pulley *B* (both ds_B)
  → $2ds_A = 3\ ds_B$ → $ds_B = 2/3\ ds_A$

  Dividing by dt → $|v_B| = 2/3\ v_A = 0.2$ m/s (upwards)

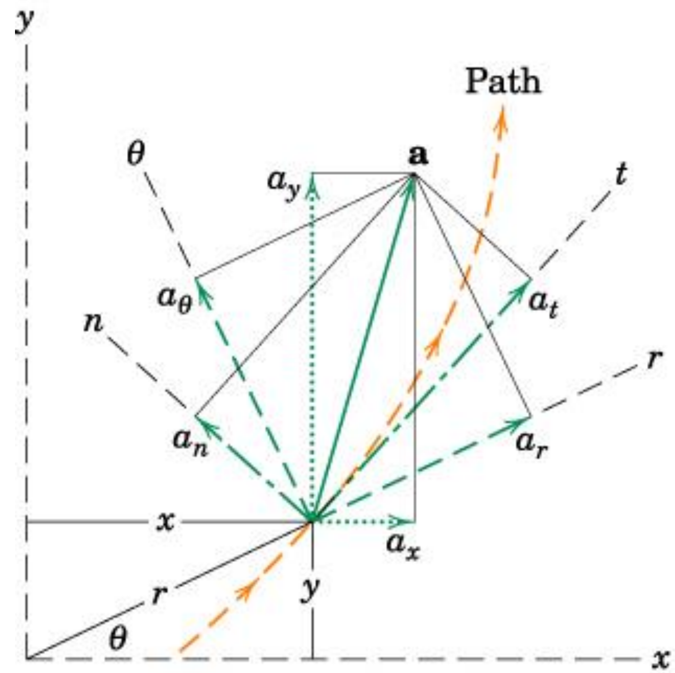
Kinematics of Particles

Summary



$$v_x = \dot{x} \qquad v_y = \dot{y}$$
$$v_n = 0 \qquad v_t = v$$
$$v_r = \dot{r} \qquad v_\theta = r\dot{\theta}$$

(*a*) Velocity components



$$a_x = \ddot{x} \qquad a_y = \ddot{y}$$
$$a_n = v^2/\rho \qquad a_t = \dot{v}$$
$$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

(*b*) Acceleration components

Kinetics of Particles

Kinetics:

Study of the relations between unbalanced forces and the resulting changes in motion.

Newton's Second Law of Motion : The acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this force.
→ **A particle will accelerate when it is subjected to unbalanced forces**

Three approaches to solution of Kinetics problems:

1. Force-Mass-Acceleration method (direct application of Newton's Second Law)
2. Use of Work and Energy principles
3. Impulse and Momentum methods

Limitations of this chapter:

- Motion of bodies that can be treated as particles (motion of the mass centre of the body)
- Forces are concurrent through the mass center (action of non-concurrent forces on the motion of bodies will be discussed in chapter on Kinetics of rigid bodies).

Kinetics of Particles

Force-Mass-Acceleration method

Newton's Second Law of Motion

Subject a mass particle to a force $\mathbf{F}_1$ and measure accln of the particle $\mathbf{a}_1$. Similarly, $\mathbf{F}_2$ and $\mathbf{a}_2$… → The ratio of magnitudes of force and resulting acceleration will remain constant.

$$\frac{F_1}{a_1} = \frac{F_2}{a_2} = \cdots = \frac{F}{a} = C$$

Constant C is a measure of some invariable property of the particle → Inertia of the particle

Inertia = Resistance to rate of change of velocity

Mass m is used as a quantitative measure of Inertia.

$C = km$ where k is a constant introduced to account for the units used.

→ $F = kma$

F = magnitude of the resultant force acting on the particle of mass m

a = magnitude of the resulting acceleration of the particle.

Accln is always in the direction of the applied force → Vector Relation: $\mathbf{F} = km\mathbf{a}$

In **Kinetic System of units**, k is taken as unity → $\mathbf{F} = m\mathbf{a}$

→ units of force, mass and acceleration are not independent

→ Absolute System since the force units depend on the absolute value of mass.

Values of *g* at Sea level and 45^0 latitude**:**

For measurements relative to rotating earth: Relative g → 9.80665 ~ 9.81 m/s^2

For measurements relative to non-rotating earth: Absolute g → 9.8236 m/s^2

Kinetics of Particles

Force-Mass-Acceleration method

Equation of Motion

Particle of mass *m* subjected to the action of concurrent forces $\mathbf{F}_1$, $\mathbf{F}_2$,… whose vector sum is $\sum\mathbf{F}$:

→ Equation of motion: $\sum\mathbf{F} = m\mathbf{a}$

→ Force-Mass-Acceleration equation

Equation of Motion gives the instantaneous value of the acceleration corresponding to the instantaneous value of the forces.

- The equation of motion can be used in scalar component form in any coordinate system.
- For a 3 DOF problem, all three scalar components of equation of motion will be required to be integrated to obtain the space coordinates as a function of time.
- All forces, both applied or reactive, which act on the particle must be accounted for while using the equation of motion.

Free Body Diagrams:

In Statics: Resultant of all forces acting on the body = 0

In Dynamics: Resultant of all forces acting on the body = *ma* → Motion of body

Kinetics of Particles: Force-Mass-Acceleration method

Rectilinear Motion

Motion of a particle along a straight line

For motion along *x*-direction, accelerations along *y*- and *z*-direction will be zero

→ $\sum F_x = ma_x$

$\sum F_y = 0$

$\sum F_z = 0$

For a general case:

→ $\sum F_x = ma_x$

$\sum F_y = ma_y$

$\sum F_z = ma_z$

The acceleration and resultant force are given by:

$$\mathbf{a} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

$$a = \sqrt{a_x^{\,2} + a_y^{\,2} + a_z^{\,2}}$$

$$\Sigma\mathbf{F} = \Sigma F_x\mathbf{i} + \Sigma F_y\mathbf{j} + \Sigma F_z\mathbf{k}$$

$$|\Sigma\mathbf{F}| = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2 + (\Sigma F_z)^2}$$

Kinetics of Particles: Force-Mass-Acceleration method

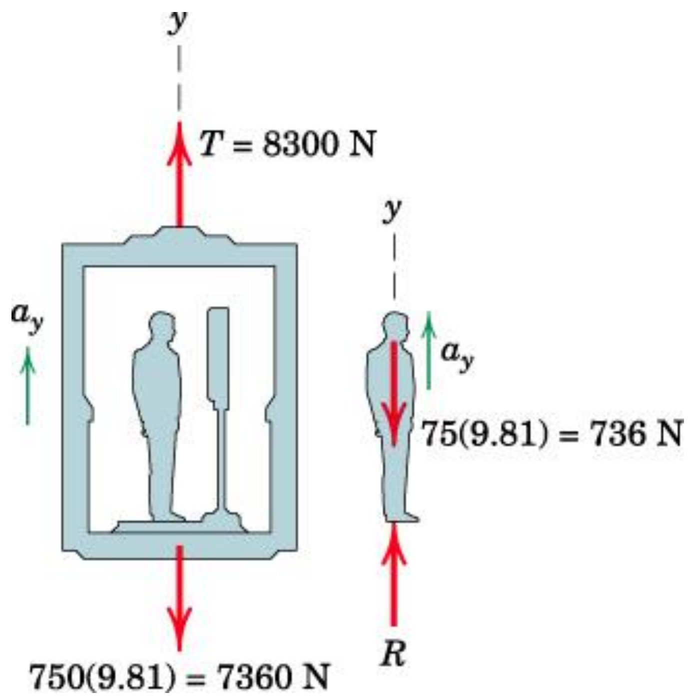
Rectilinear Motion

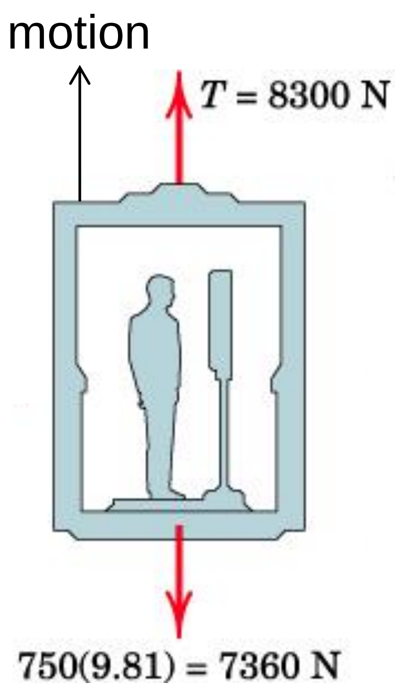
Example

A 75 kg man stands on a spring scale in an elevator. During the first 3 seconds of motion from rest, the tension *T* in the hoisting cable is 8300 N. Find the reading *R* of the scale in Newton during this interval and the upward velocity v of the elevator at the end of the 3 seconds. Total mass of elevator, man, and scale is 750 kg.

Solution

Draw the FBD of the elevator and the man alone





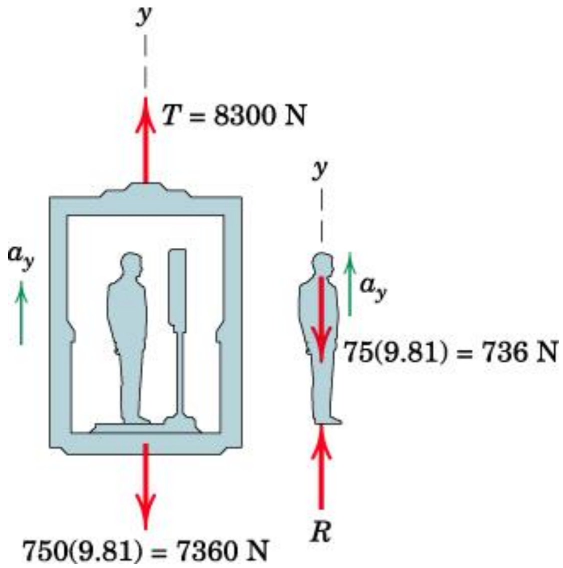
Kinetics of Particles: Force-Mass-Acceleration method

Rectilinear Motion

Example

Solution

During first 3 seconds, the forces acting on the elevator are constant. Therefore, the acceleration a_y will also remain constant during this time.
Force registered by the scale and the velocity of the elevator depend on the acceleration a_y



From FBD of the elevator, scale, and man taken together:
$\sum F_y = ma_y \rightarrow 8300\text{-}7360 = 750a_y \rightarrow a_y = 1.257 \text{ m/s}^2$

From FBD of the man alone:
$\sum F_y = ma_y \rightarrow R\text{-}736 = 75a_y \rightarrow R = 830 \text{ N}$

Velocity reached at the end of the 3 sec:

$\Delta v = \int a\, dt \rightarrow v - 0 = \int_0^3 1.257\, dt$

$v = 3.77 \text{ m/s}$