ME101: Engineering Mechanics (3 1 0 8)

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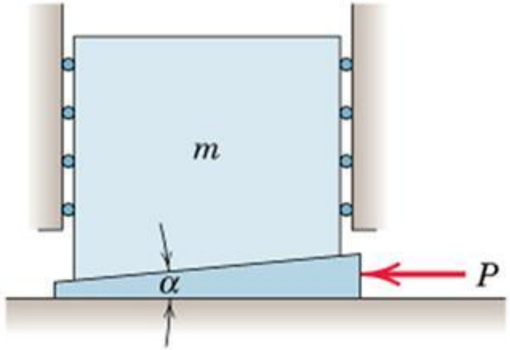
Applications of Friction in Machines

Wedges

- Simple machines used to raise heavy loads
- Force required to lift block is significantly less than block weight
- Friction prevents wedge from sliding out
- **Minimum force *P*** required to raise block???



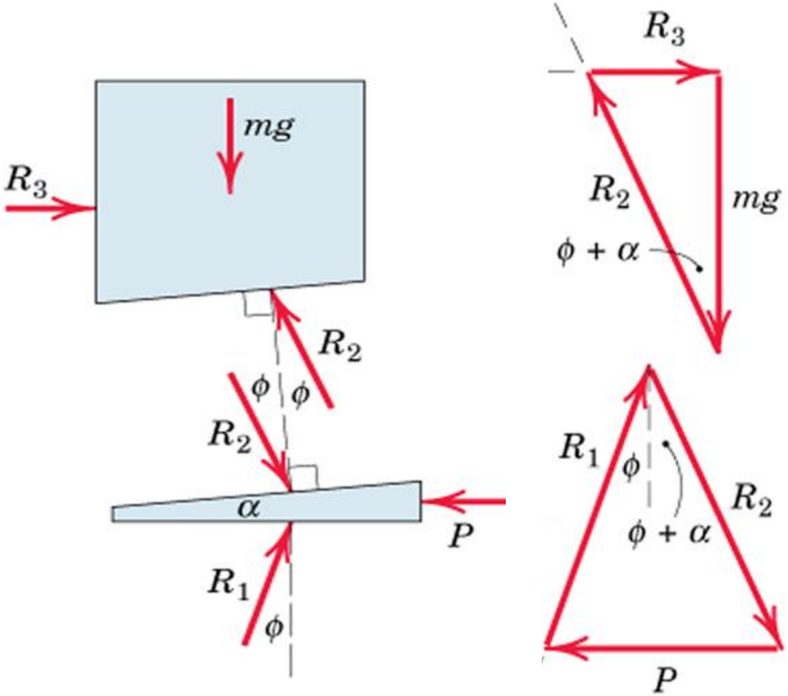
Coefficient of Friction for each pair of surfaces $\mu = \tan\phi$ (Static/Kinetic)

Free Body Diagrams
Reactions are inclined at an angle Φ from their respective normals and are in the direction opposite to the motion.

Force vectors acting on each body can also be shown.

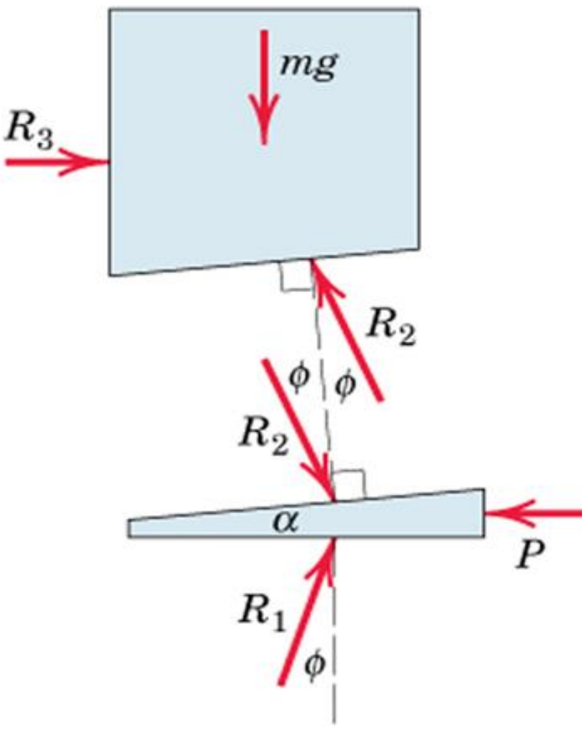
R_2 is first found from upper diagram since mg is known.

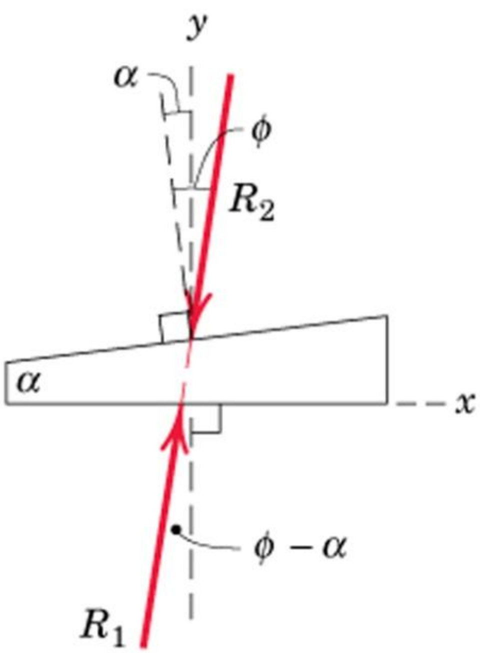
Then P can be found out from the lower diagra since R_2 is known.



Applications of Friction in Machines: Wedges

- Removal of force P
  - *Wedge remains in place*
  - *Equilibrium requirement :: Collinearity of R_1 and R_2*



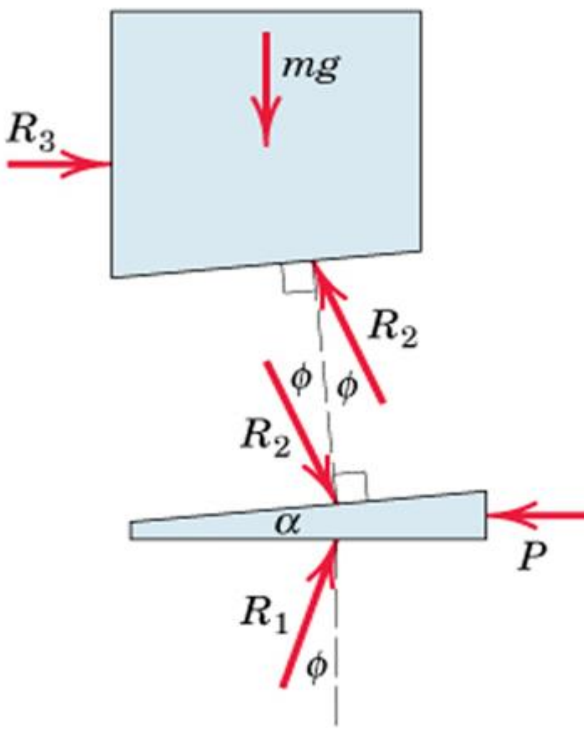


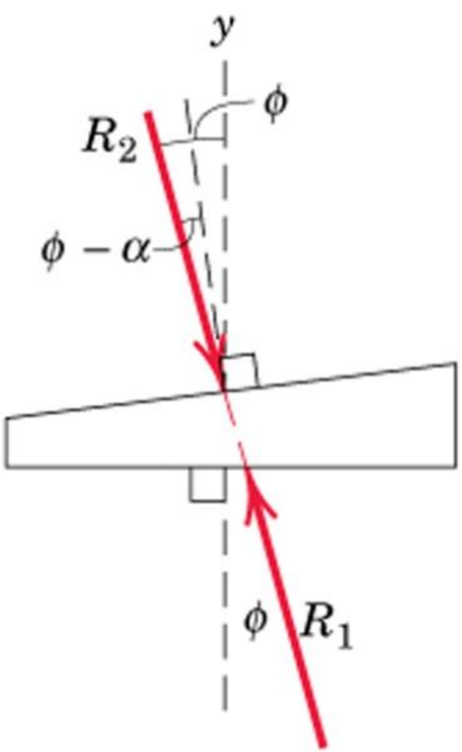
:: Impending slippage at upper surface

:: $\alpha < \phi$

Applications of Friction in Machines: Wedges

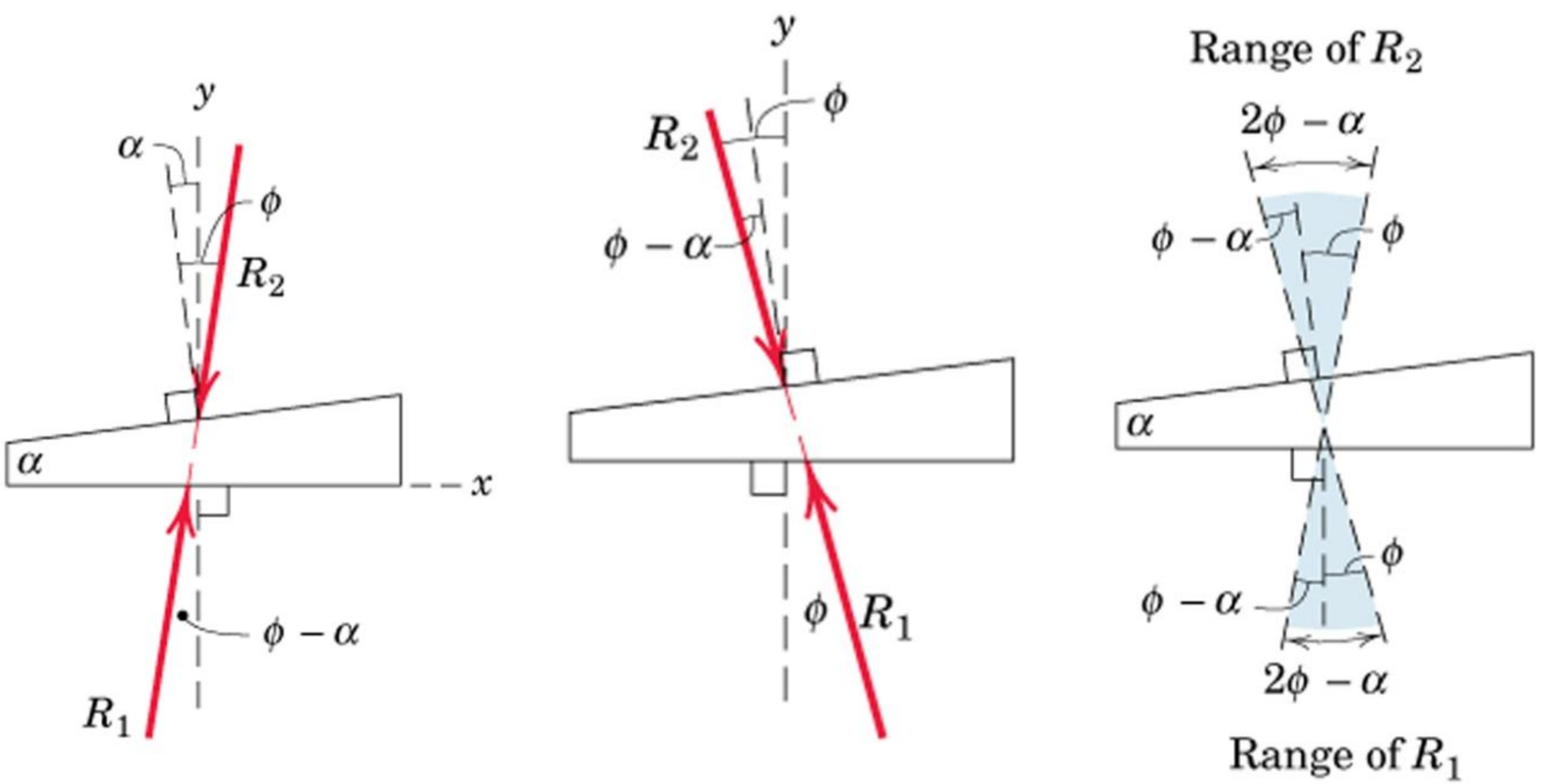
- Collinearity of reactions
  - *Impending slippage at lower surface* $\because \alpha < \phi$





Applications of Friction in Machines: Wedges

- **Simultaneous** occurring of **slippage** at both **upper** and **lower surfaces** (otherwise self-locking)
  - *Angular range for no slippage*
  - *Simultaneous slippage not possible for* $\alpha < 2\phi$



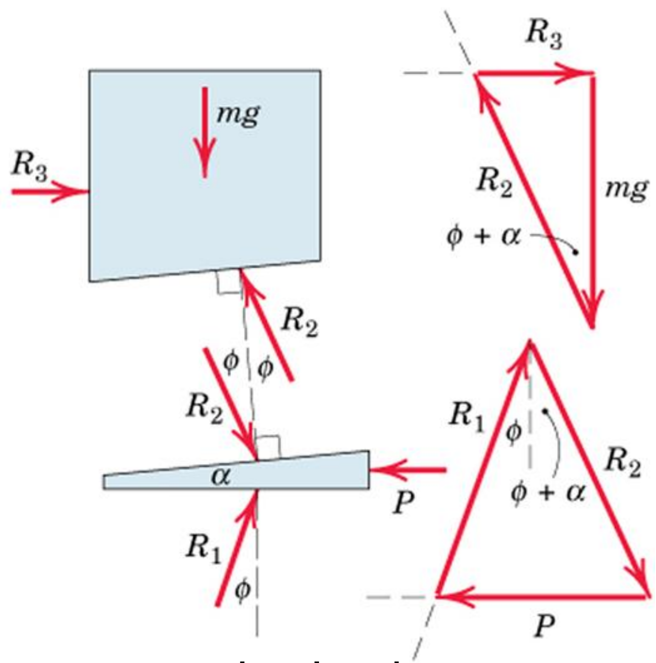
Applications of Friction in Machines: Wedges

Application of pull P on the wedge

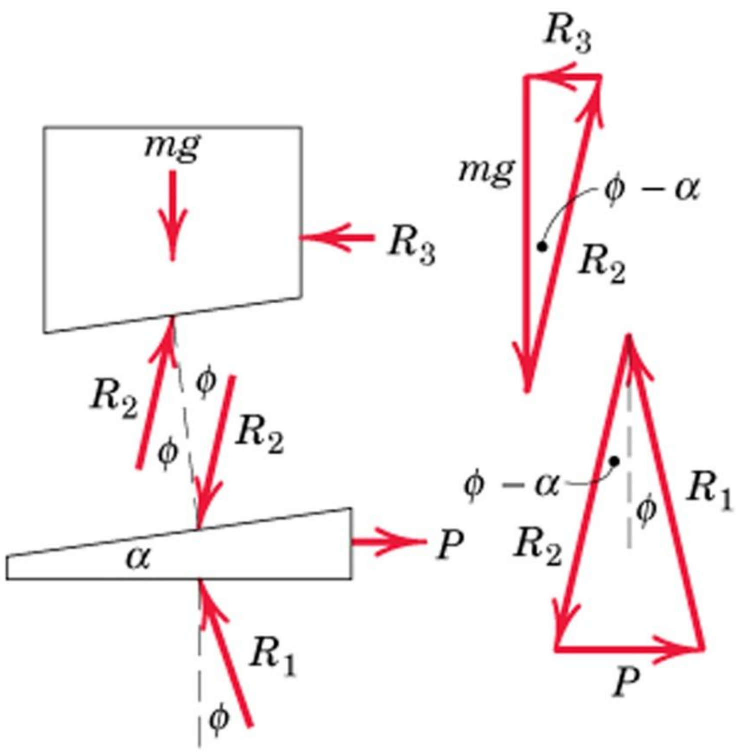
→**The reactions *R_1* and *R_2* must act on the opposite sides of their normal from those when the wedge was inserted**

→Solution by drawing FBDs and vector polygons

- → Graphical solution
- → Algebraic solutions from trigonometry



Forces to raise load



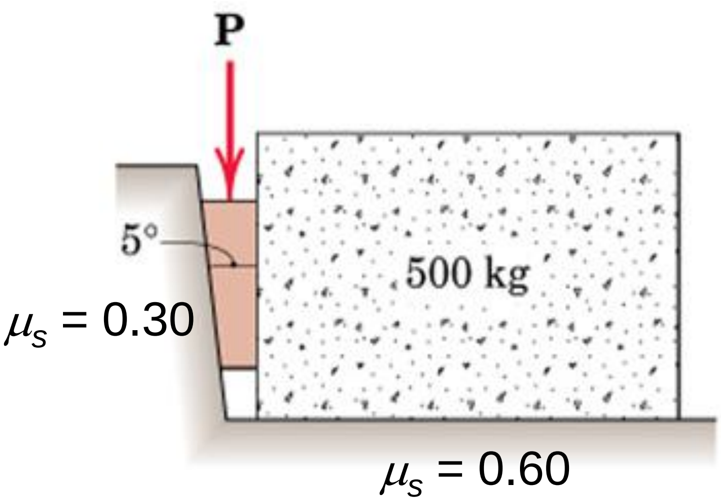
Forces to lower load

Example on Friction in Wedges

Find the least **P** required to move the block

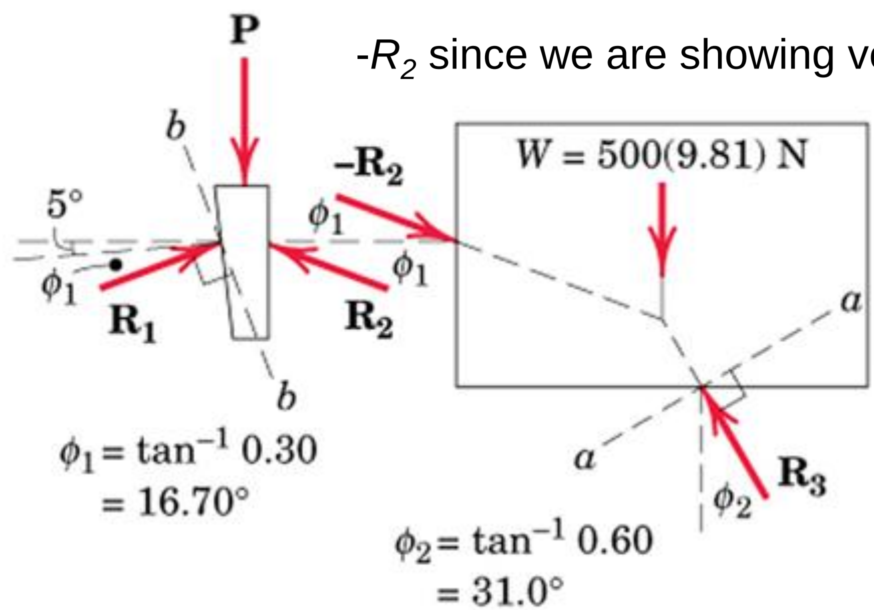
- Coefficient of static friction for both pairs of wedge = 0.3

- Coefficient of static friction between block and horizontal surface = 0.6



Solution: Draw FBDs

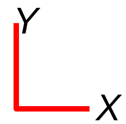
$-R_2$ since we are showing vectors

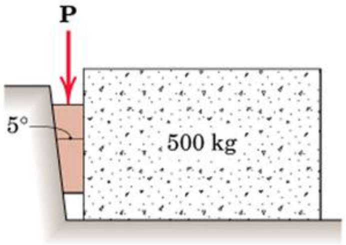


Example on Friction in Wedges

Solution: W = 500x9.81 = 4905 N

Three ways to solve





Method 1:

Equilibrium of FBD of the Block

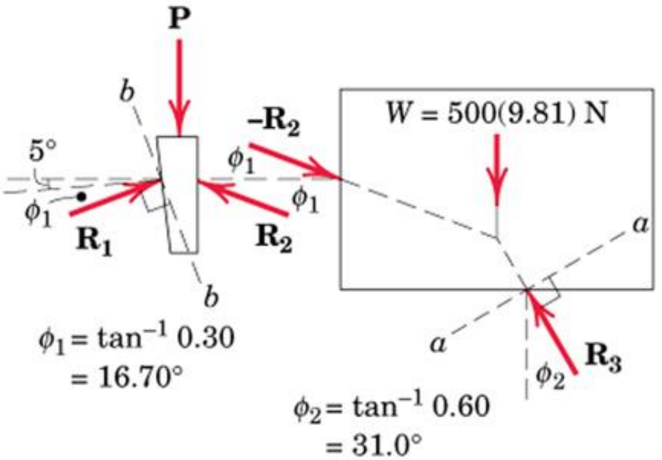
$\sum \mathbf{F}_X = \mathbf{0}$

$R_2 \cos \phi_1 = R_3 \sin \phi_2 \rightarrow R_2 = 0.538 R_3$

$\sum \mathbf{F}_Y = \mathbf{0}$

$4905 + R_2 \sin \phi_1 = R_3 \cos \phi_2 \rightarrow R_3 = 6970$ N

$\rightarrow R_2 = 3750$ N



Equilibrium of FBD of the Wedge

$\sum \mathbf{F}_X = \mathbf{0}$

$R_2 \cos \phi_1 = R_1 \cos(\phi_1 + 5) \rightarrow R_1 = 3871$ N

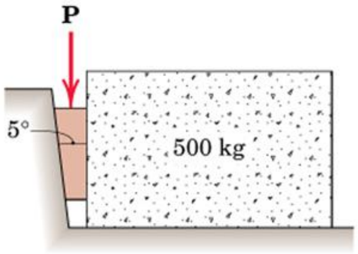
$\sum \mathbf{F}_Y = \mathbf{0}$

$R_1 \sin(\phi_1 + 5) + R_2 \sin \phi_1 = P$

$\rightarrow P = 2500$ N

Example on Friction in Wedges

Solution:

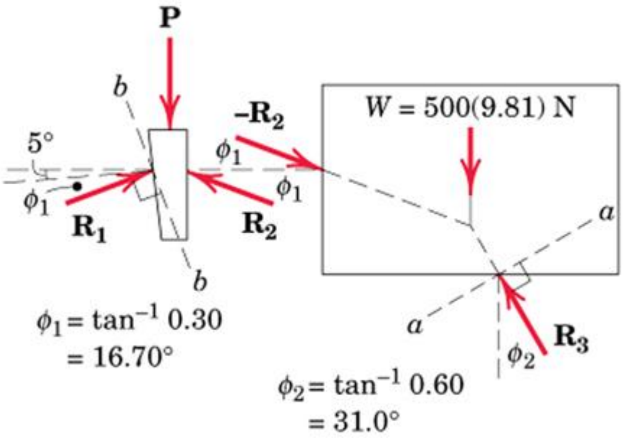


Method 2:

Using Equilibrium equations along reference axes *a-a* and *b-b*

→ No need to solve simultaneous equations

Angle between R_2 and *a-a* axis = 16.70+31.0 = 47.7^o



Equilibrium of Block:

$[\Sigma F_a = 0]$ $$500(9.81)\sin 31.0^\circ - R_2 \cos 47.7^\circ = 0$$
$$R_2 = 3750\text{ N}$$

Equilibrium of Wedge:

Angle between R_2 and *b-b* axis = 90-(2Φ_1+5) = 51.6^o

Angle between *P* and *b-b* axis = Φ_1+5 = 21.7^o

$[\Sigma F_b = 0]$ $$3750 \cos 51.6^\circ - P \cos 21.7^\circ = 0$$
$$P = 2500\text{ N}$$

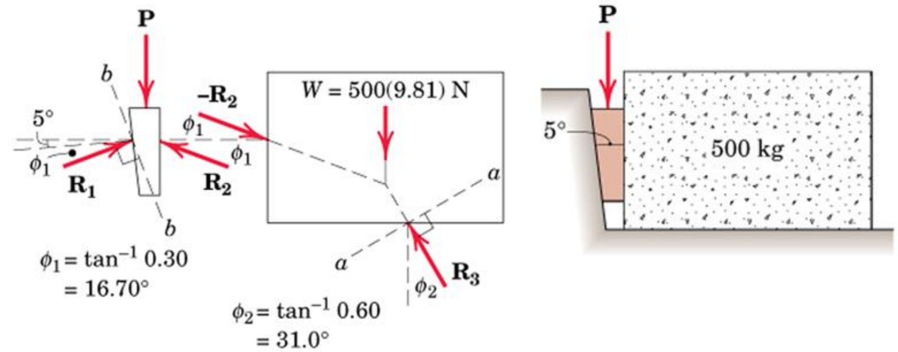
Example on Friction in Wedges

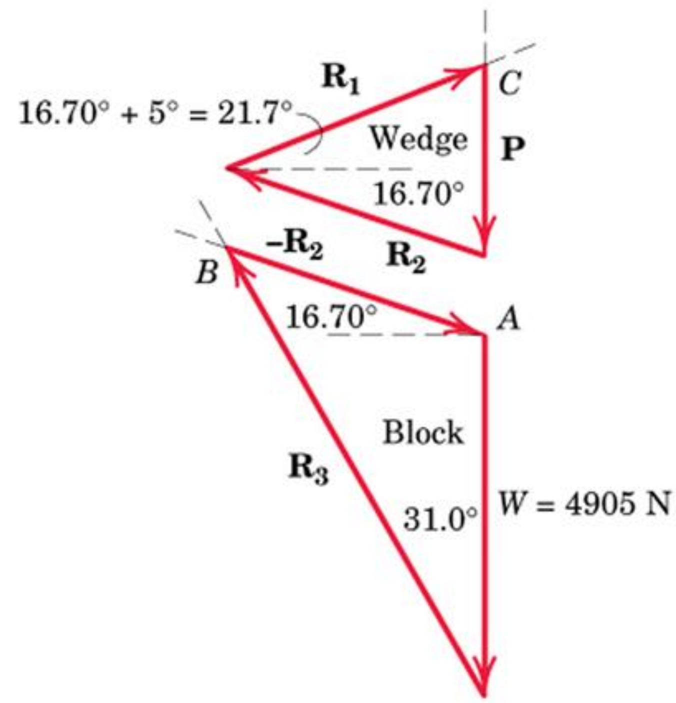
Solution:

Method 3:
Graphical solution using vector polygons

Starting with equilibrium of the block:
W is known, and directions of R_2 and R_3 are known
→ Magnitudes of R_2 and R_3 can be determined graphically

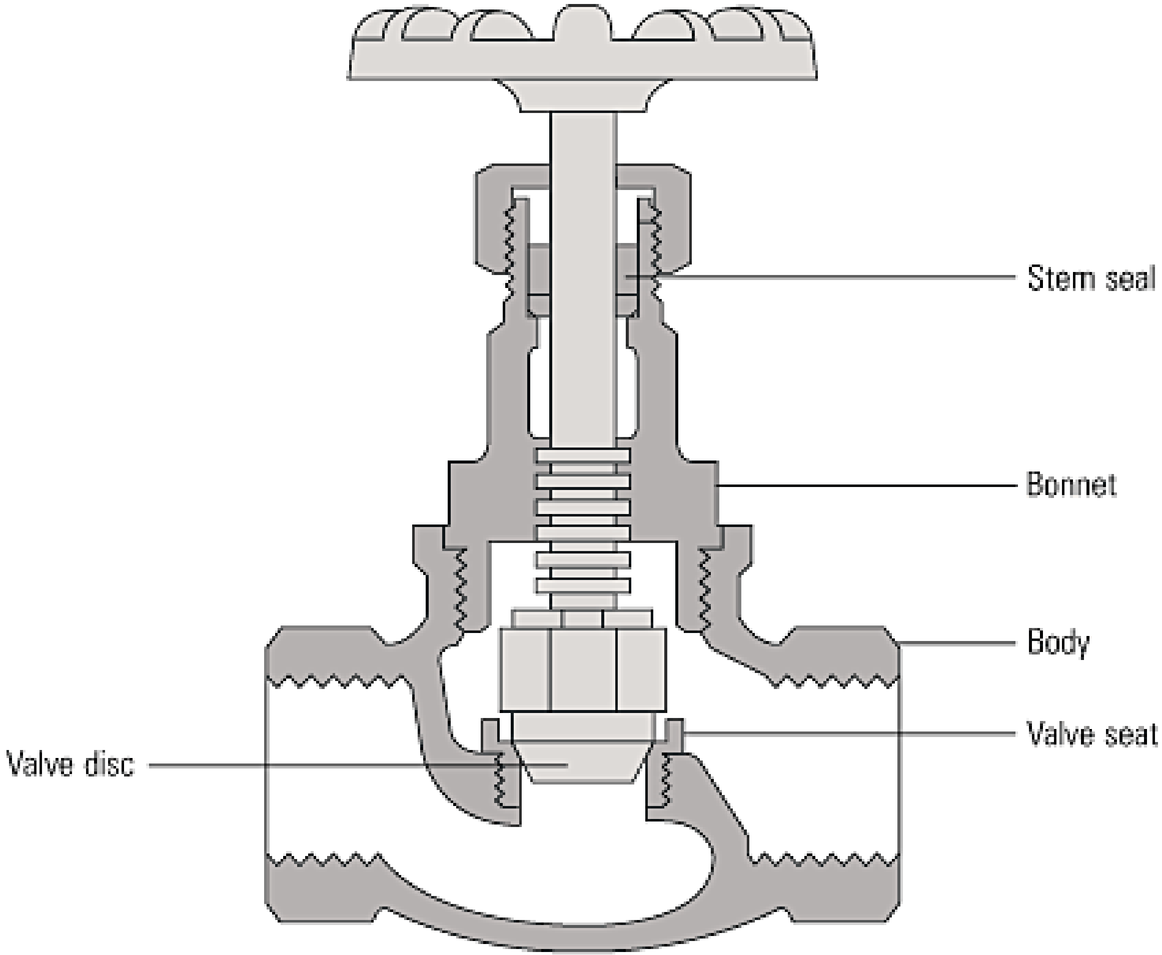
Similarly, construct vector polygon for the wedge from known magnitude of R_2, and known directions of R_2, R_1, *and P.*
→ Find out the magnitude of *P* graphically

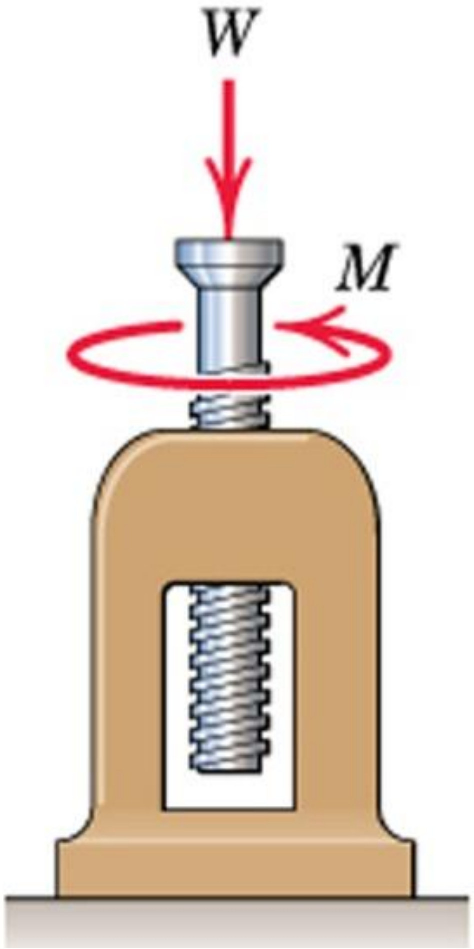




Applications of Friction in Machines: Screws

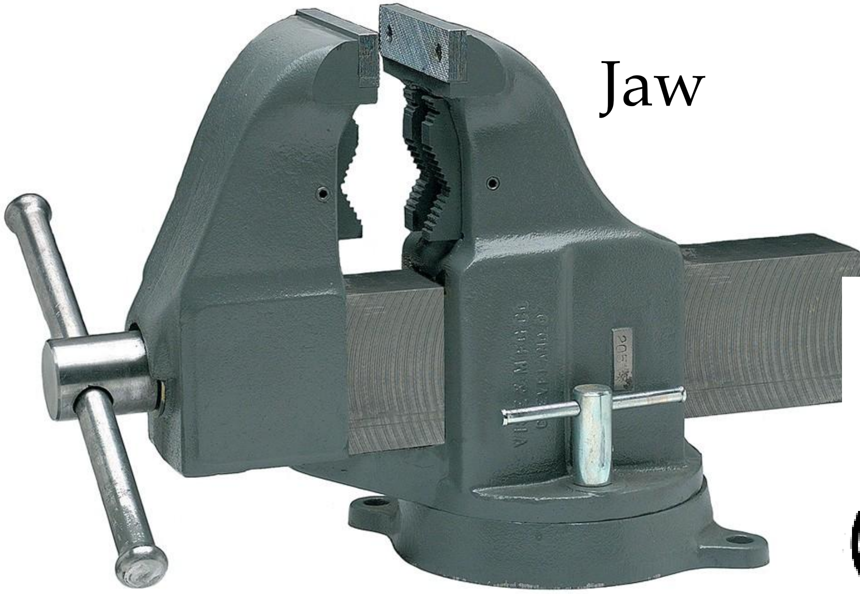
- Vertical direction of motion (*e.g.*, *isolation valve*)



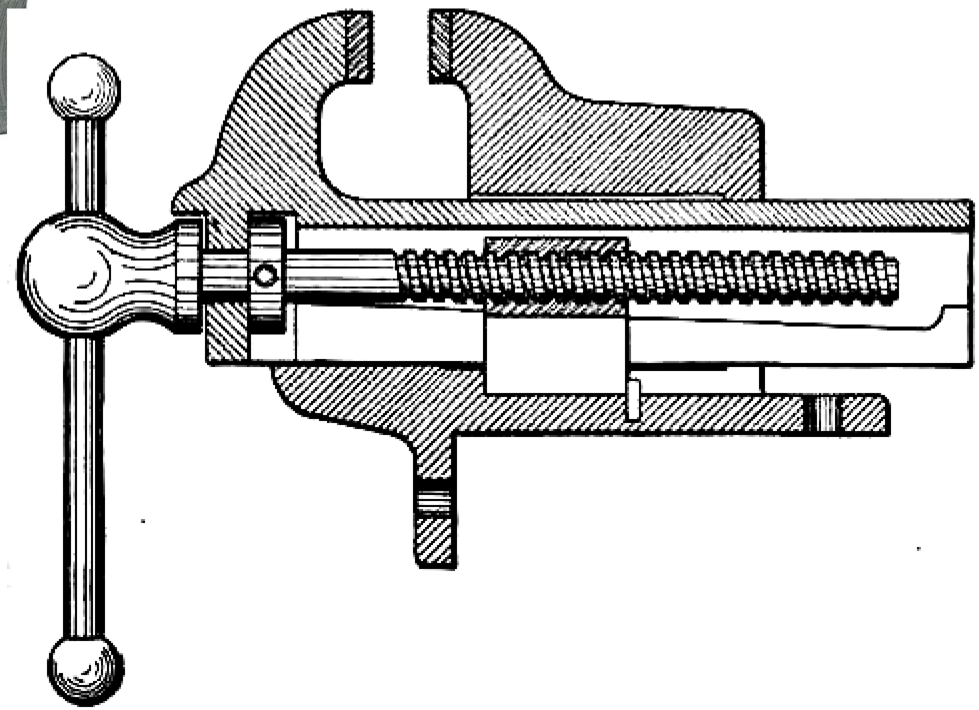


Applications of Friction in Machines: Screws

- Horizontal direction of motion (*e.g., vise*)



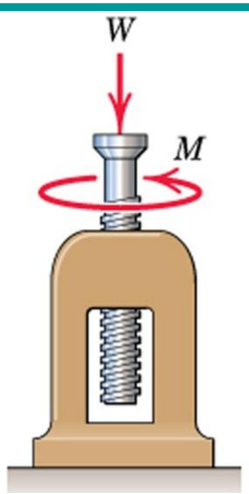
Jaw



Applications of Friction in Machines

Square Threaded Screws

- Used for fastening and for transmitting power or motion
- Square threads are more efficient
- Friction developed in the threads largely determines the action of the screw



FBD of the Screw: R exerted by the thread of the jack frame on a small portion of the screw thread is shown

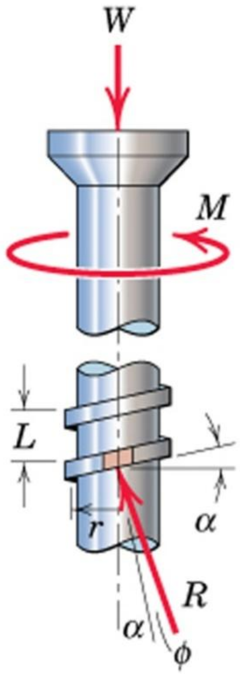
Lead = L = advancement per revolution

L = Pitch – for single threaded screw

L = 2xPitch – for double threaded screw (twice advancement per revolution)

Pitch = axial distance between adjacent threads on a helix or screw

Mean Radius = r ; α = Helix Angle



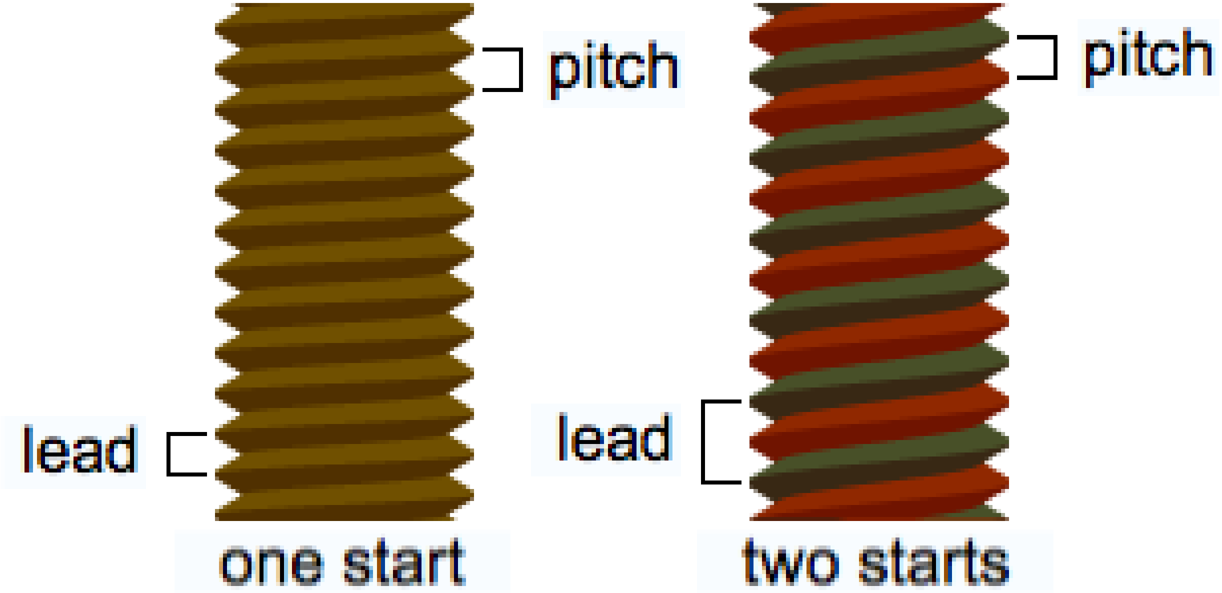
Similar reactions exist on all segments of the screw threads

Analysis similar to block on inclined plane since friction force does not depend on area of contact.

- Thread of base can be "unwrapped" and shown as straight line. Slope is $2\pi r$ horizontally and lead L vertically.

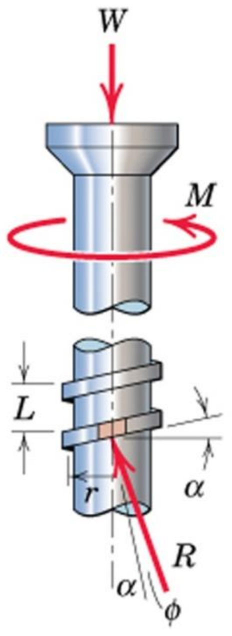
Applications of Friction in Machines

- Thread of a screw



Single thread **Double thread**

Applications of Friction in Machines: Screws



If M is just sufficient to turn the screw → Motion Impending

Angle of friction = ϕ (made by R with the axis normal to the thread)

→ $\tan\phi = \mu$

Moment of R @ vertical axis of screw = $R\sin(\alpha+\phi)r$

→ Total moment due to all reactions on the thread = $\sum R\sin(\alpha+\phi)r$

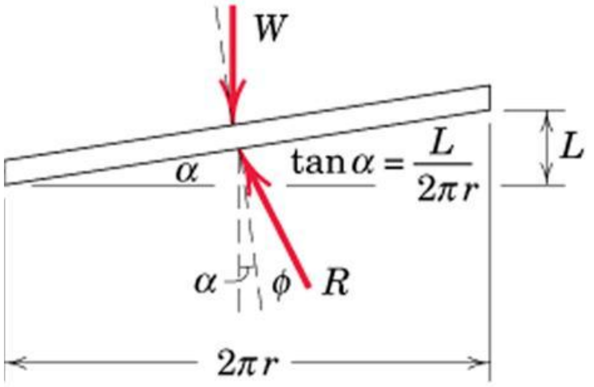
→ Moment Equilibrium Equation for the screw:

$$\rightarrow M = [r\sin(\alpha+\phi)]\sum R$$

Equilibrium of forces in the axial direction: $W = \sum R\cos(\alpha+\phi)$

$$\rightarrow W = [\cos(\alpha+\phi)]\sum R$$

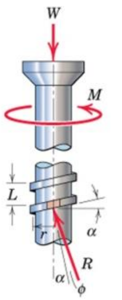
Finally → $\boldsymbol{M = W\,r\tan(\alpha+\phi)}$



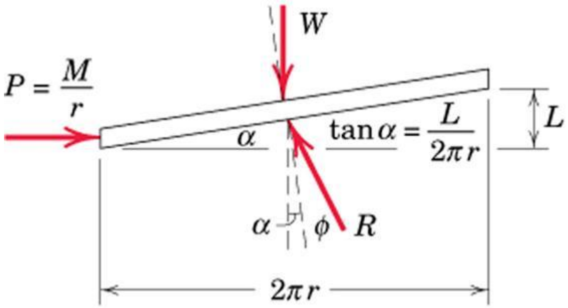
Helix angle α can be determined by unwrapping the thread of the screw for one complete turn

$$\boldsymbol{\alpha = \tan^{-1}(L/2\pi r)}$$

Applications of Friction in Machines: Screws



Alternatively, action of the entire screw can be simulated using unwrapped thread of the screw



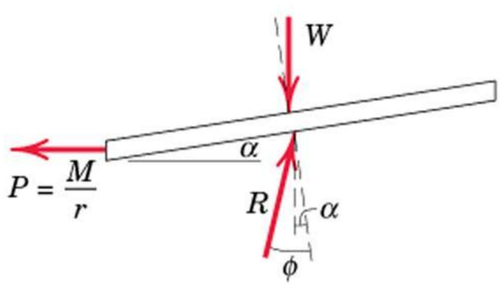
To Raise Load

Equivalent force required to push the movable thread up the fixed incline is:
P = *M*/*r*

From Equilibrium:

$M = W\, r \tan(\alpha + \phi)$

If M is removed: the screw will remain in place and be self-locking provided $\alpha<\phi$ and will be on the verge of unwinding if $\alpha=\phi$



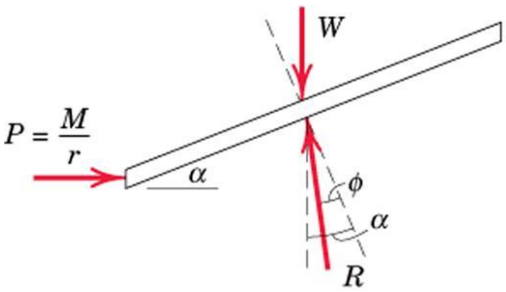
To Lower Load ($\alpha<\phi$)

To lower the load by unwinding the screw, We must reverse the direction of *M* as long as $\alpha<\phi$

From Equilibrium:

$M = W\, r \tan(\phi - \alpha)$

→ This is the moment required to unwind the screw



To Lower Load ($\alpha>\phi$)

If $\alpha > \phi$, the screw will unwind by itself. Moment required to prevent unwinding:

From Equilibrium:

$M = W\, r \tan(\alpha - \phi)$

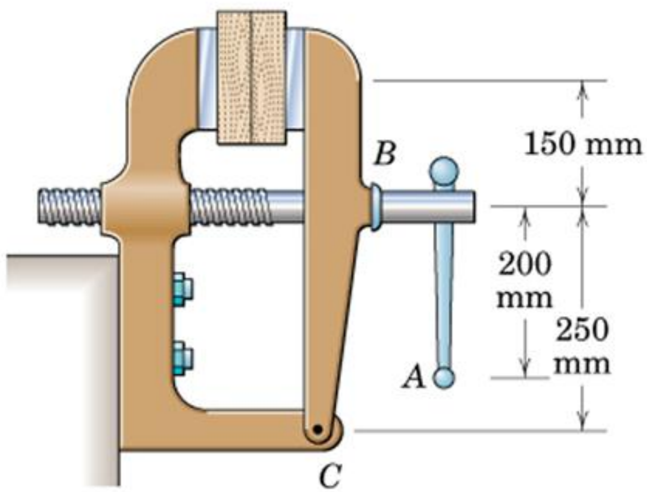
Example of Screw Action: Vise

Single threaded screw of the vise has a mean diameter of 25 mm and a lead of 5 mm. A 300 N pull applied normal to the handle at *A* produces a clamping force of 5 kN between the jaws of the vise. Determine:

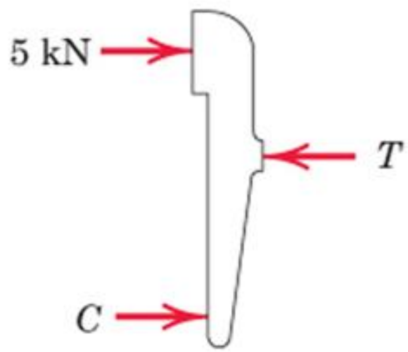
(a)Frictional moment M_B developed at *B* due to thrust of the screw against body of the jaw

(b)Force *Q* applied normal to the handle at *A* required to loosen the vise

μ_s in the threads = 0.20



Solution: Draw FBD of the jaw to find tension in the screw



$\sum M_C = 0$

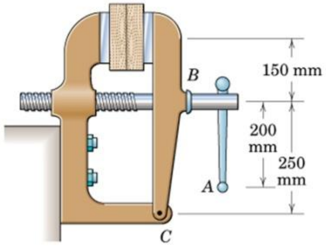
$\rightarrow$ T = 8 kN

Find the helix angle α and the friction angle ϕ

$\alpha = tan^{-1}(L/2\pi r) = 3.64^o$

$\tan \phi = \mu \rightarrow \phi = 11.31^o$

Example of Screw Action: Vise

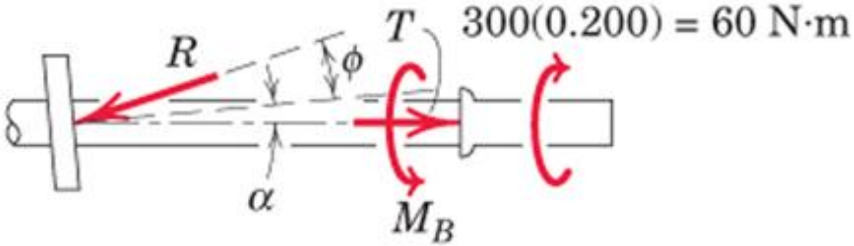


Example: Screw

Solution:

(a) To tighten the vise

Draw FBD of the screw



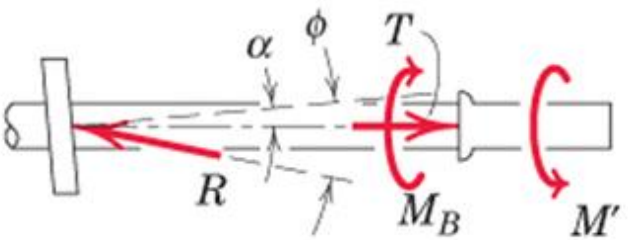
$M = T\, r \tan(\alpha + \phi)$

$60\text{-}M_B = 8000(0.0125)\tan(3.64° + 11.31°)$

$\boldsymbol{M_B = 33.3\ \mathrm{Nm}}$

(a) To loosen the vise (on the verge of being loosened)

Draw FBD of the screw: Net moment = (Applied moment) - M_B



$M = T\, r \tan(\phi - \alpha)$

$M' - 33.3 = 8000(0.0125)\tan(11.31° - 3.64°)$

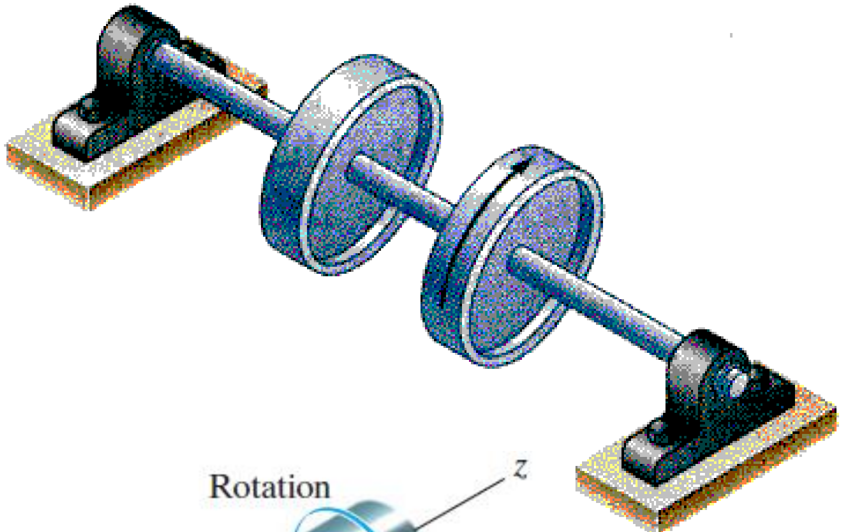
$M' = 46.8$ Nm

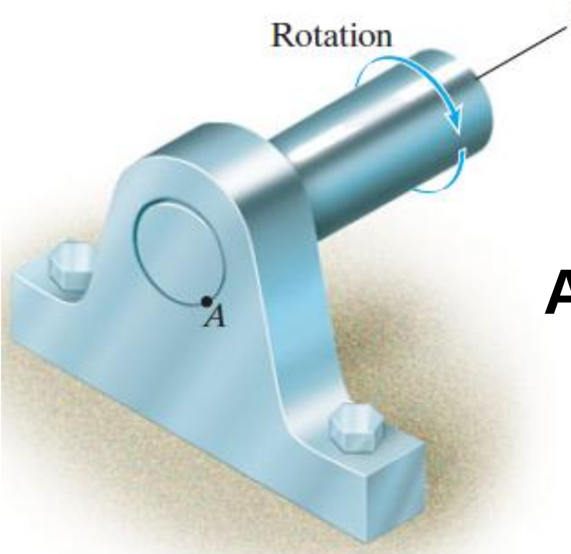
$\boldsymbol{Q = M'/d = 46.8/0.2 = 234\ \mathrm{N}}$

(Applied moment: M')

Friction in Machines :: Journal Bearing

- Rotating shaft

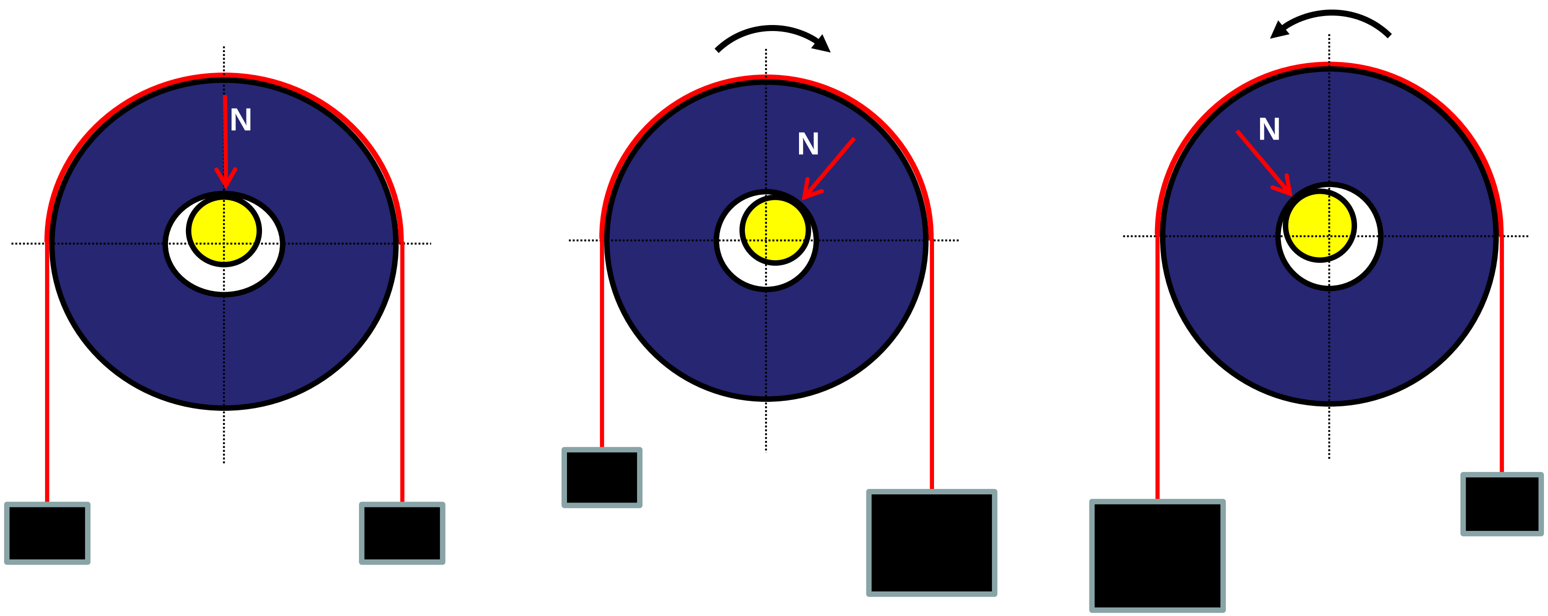




Axle Friction

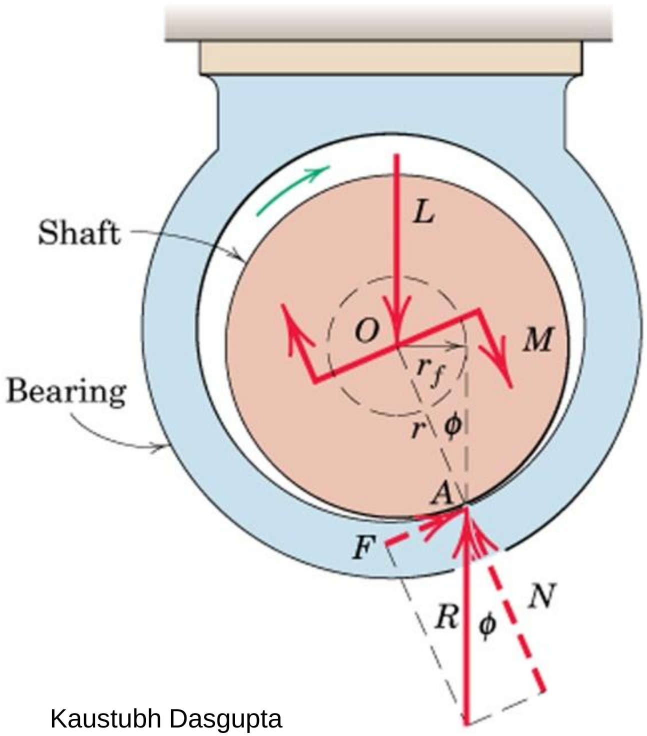
Friction in Machines :: Journal Bearing

- Normal reaction on bearing
  - Point of application
  - Friction tends to oppose the motion
  - Friction angle ϕ for the resultant force

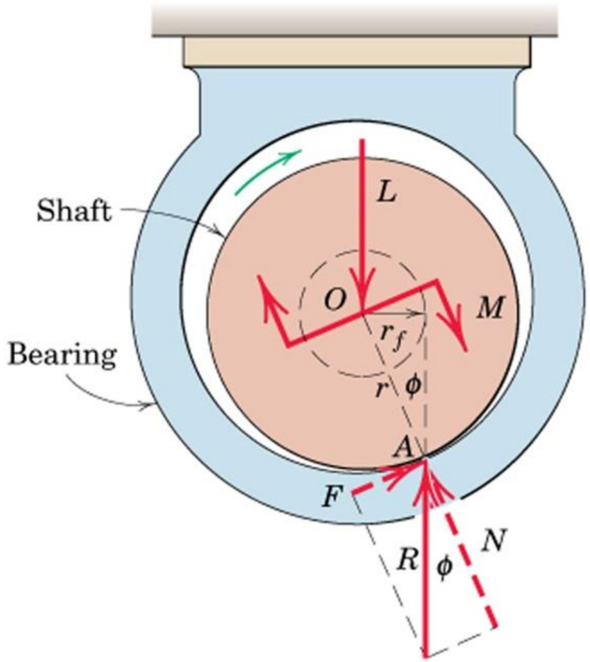


Friction in Machines :: Journal Bearing

- Lateral/Vertical load on shaft is $\boldsymbol{L}$
- Partially lubricated bearing
  – *Direct contact along a line*
- Fully lubricated bearing
  – *Clearance, speed, lubricant viscosity*



Friction in Machines :: Journal Bearing



-*R* will be tangent to a small circle of radius r_f called the *friction circle*

$\sum M_A = 0 \rightarrow M = Lr_f = Lr \sin\phi$

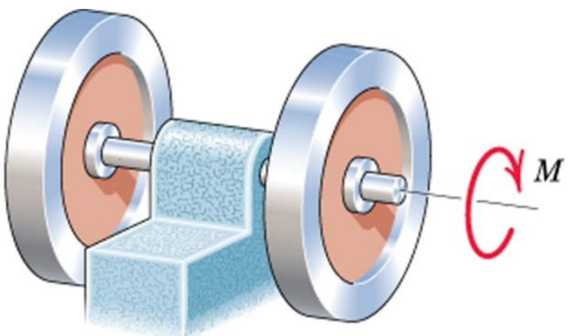
For a small coefficient of friction, ϕ is small → $\sin\phi \approx \tan\phi$

→$M = \mu Lr$ (since $\mu = \tan\phi$) → Use equilibrium equations to solve a problem

→ Moment that must be applied to the shaft to overcome friction for a dry or partially lubricated journal bearing

Example (1) on Journal Bearing

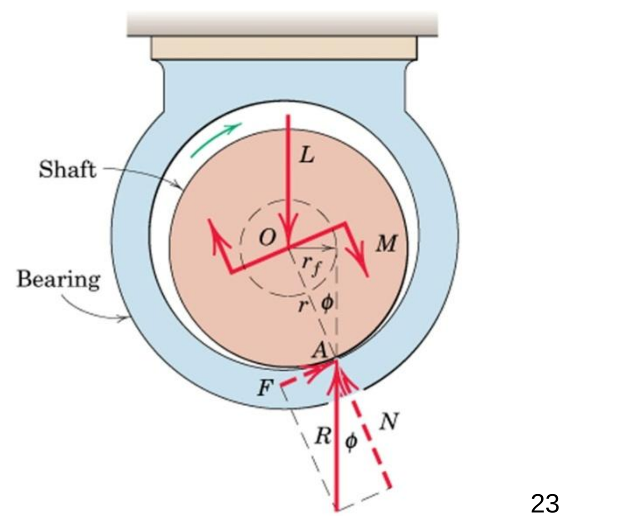
Two flywheels (each of mass 40 kg and diameter 40 mm) are mounted on a shaft, which is supported by a journal bearing. M = 3 Nm couple is reqd on the shaft to maintain rotation of the flywheels and shaft at a constant low speed.
Determine: (a) coeff of friction in the bearing, and (b) radius r_f of the friction circle.



Solution: Draw the FBD of the shaft and the bearing

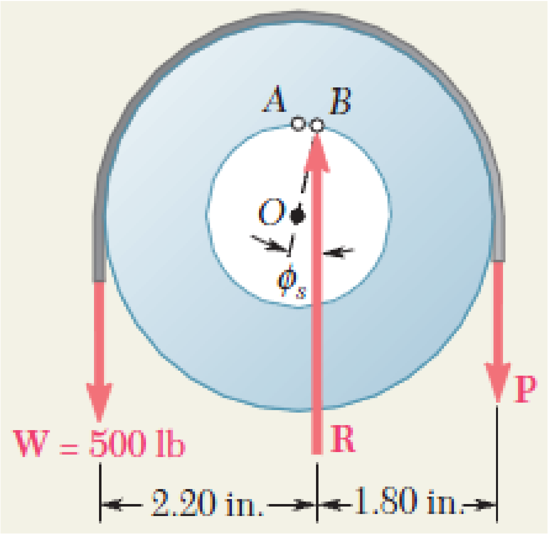
(a) Moment equilibrium at O
$M = Rr_f = Rr\sin\phi$
M = 3 Nm, R = 2x40x9.81 = 784.8 N, r = 0.020 m
→ $\sin\phi = 0.1911$ → $\phi = 11.02^{o}$

(b) $r_f = r\sin\phi$ = 3.82 mm



Example (2) on Journal Bearing

A pulley of diameter 4 in. can rotate about a fixed shaft of diameter 2 in. The coefficient of static friction between the pulley and shaft is 0.20. Determine (*a*) the smallest vertical force **P** required to start raising a 500-lb load, (*b*) the smallest vertical force **P** required to hold the load, (*c*) the smallest horizontal force **P** required to start raising the same load.



(a) For equal tension on both sides, contact point is A; for slight rotation of the pulley, under increased P, the contact point shifts to B.

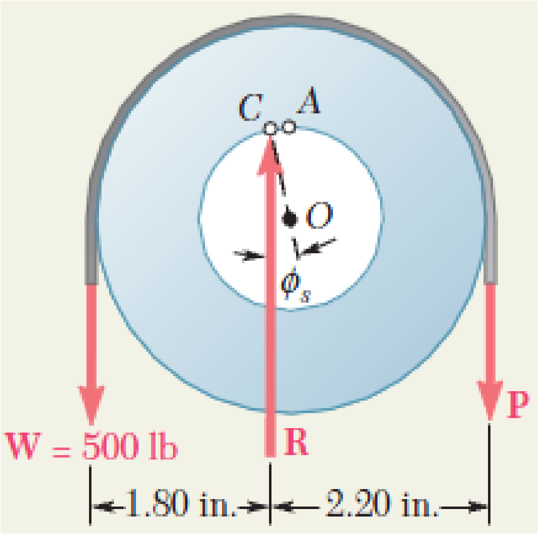
Friction circle radius,

$$r_f = r \sin \phi_s \approx r\mu_s \qquad r_f \approx (1 \text{ in.})0.20 = 0.20 \text{ in.}$$

Summing moments about B, we write

$$+\circlearrowleft \Sigma M_B = 0: \qquad (2.20 \text{ in.})(500 \text{ lb}) - (1.80 \text{ in.})P = 0$$
$$P = 611 \text{ lb}$$

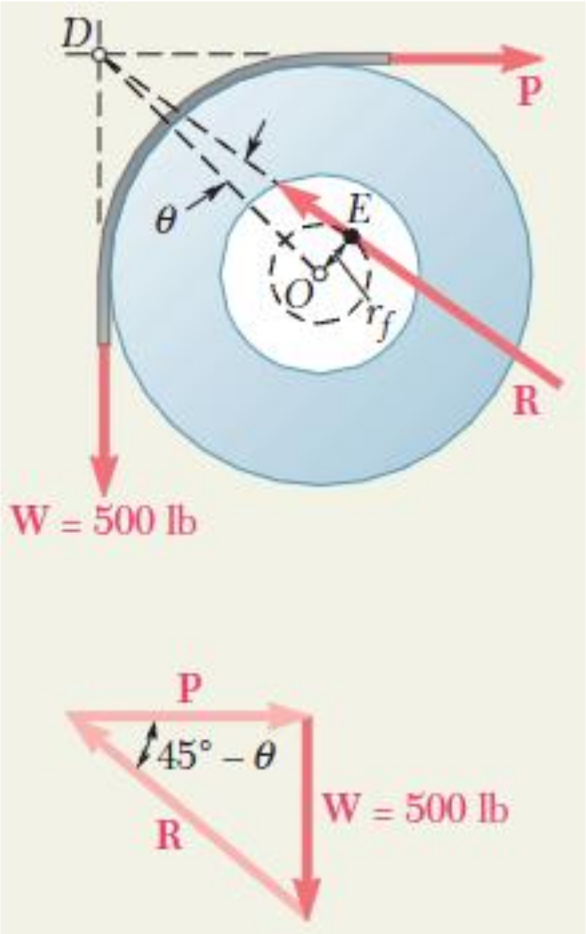
Example (2) on Journal Bearing



(b) With reduction of $\boldsymbol{P}$, contact point shifts to C.
Free body diagram of pulley with moment @ C,

$$+\circlearrowleft \Sigma M_C = 0: \qquad (1.80 \text{ in.})(500 \text{ lb}) - (2.20 \text{ in.})P = 0$$
$$P = 409 \text{ lb}$$

Example (2) on Journal Bearing



(c) ***P***, ***W*** and ***R*** must be concurrent.

R is also the tangent to the friction circle

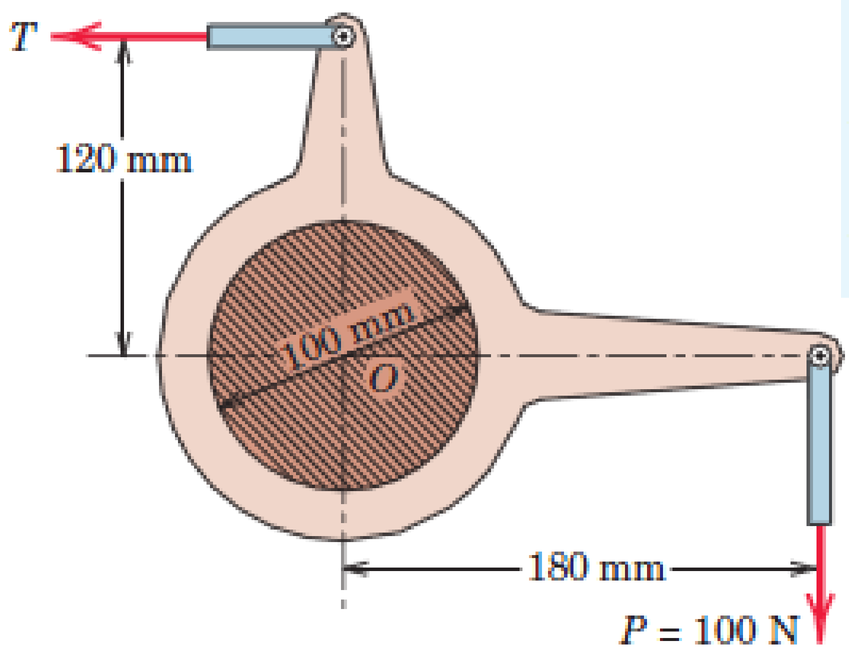
$$\sin\theta = \frac{OE}{OD} = \frac{0.20 \text{ in.}}{(2 \text{ in.})\sqrt{2}} = 0.0707 \qquad \theta = 4.1°$$

From the force triangle, we obtain

$$P = W \cot(45° - \theta) = (500 \text{ lb}) \cot 40.9°$$
$$= 577 \text{ lb}$$

Example (3) on Journal Bearing

The bell crank fits over a 100-mm-diameter shaft which is fixed and cannot rotate. The horizontal force T is applied to maintain equilibrium of the crank under the action of the vertical force $P = 100$ N. Determine the maximum and minimum values which T may have without causing the crank to rotate in either direction. The coefficient of static friction μ between the shaft and the bearing surface of the crank is 0.20.



Friction angle $\phi = \tan^{-1}\mu = \tan^{-1} 0.20 = 11.31°$

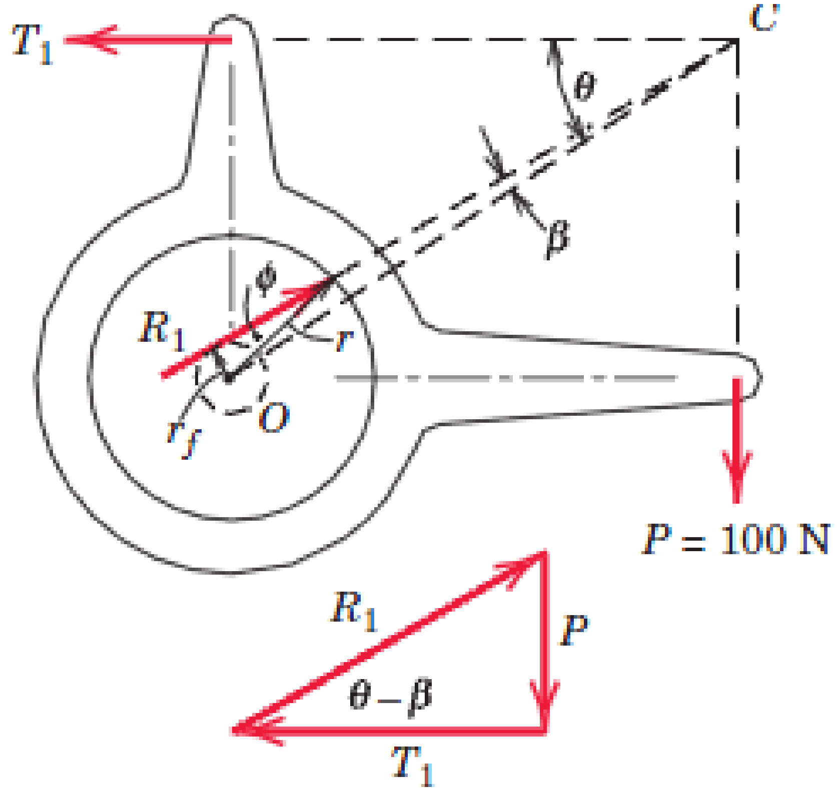
Radius of friction circle $r_f = r \sin\phi = 50 \sin 11.31° = 9.81$ mm

Angle $\theta = \tan^{-1}\dfrac{120}{180} = 33.7°$

Angle $\beta = \sin^{-1}\dfrac{r_f}{\overline{OC}} = \sin^{-1}\dfrac{9.81}{\sqrt{(120)^2 + (180)^2}} = 2.60°$

Example (3) on Journal Bearing

- Impending motion

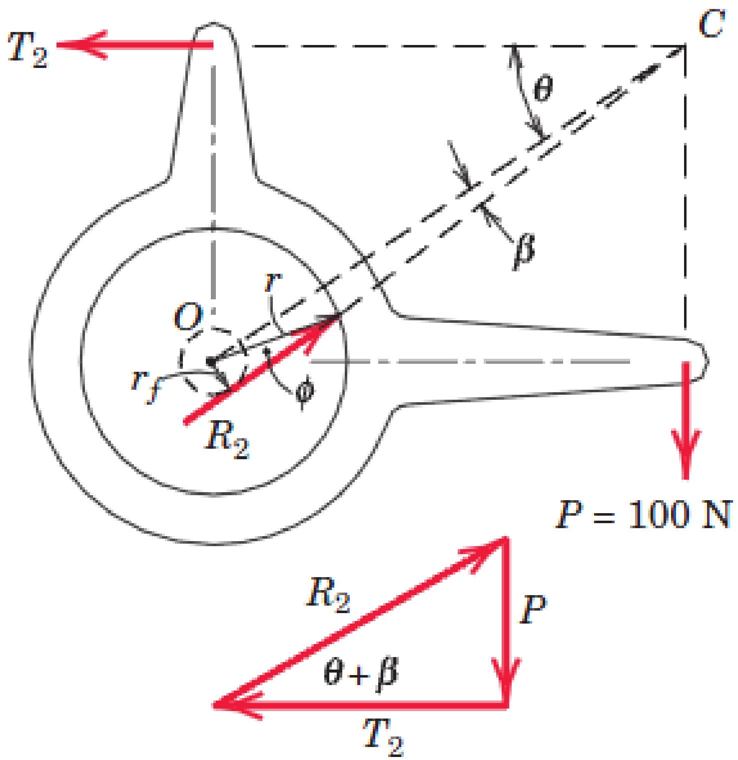


(a) Impending counterclockwise motion. The equilibrium triangle of forces is drawn and gives

$$T_1 = P \cot(\theta - \beta) = 100 \cot(33.7° - 2.60°)$$

$$T_1 = T_{\max} = 165.8 \text{ N} \qquad \textit{Ans.}$$

Example (3) on Journal Bearing



(b) Impending clockwise motion. The equilibrium triangle of forces for this case gives

$$T_2 = P \cot(\theta + \beta) = 100 \cot(33.7° + 2.60°)$$

$$T_2 = T_{min} = 136.2 \text{ N} \qquad \textit{Ans.}$$