

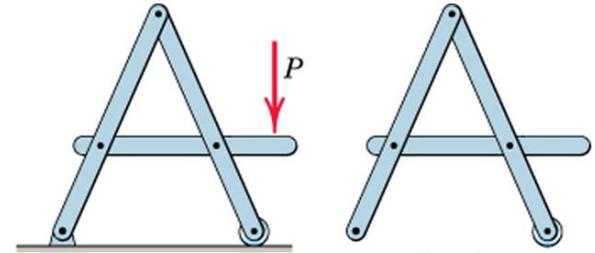
# Frames and Machines

## Interconnected Rigid Bodies with Multi-force Members

- **Rigid Non-collapsible**

- structure constitutes a rigid unit by itself  
when removed from its supports

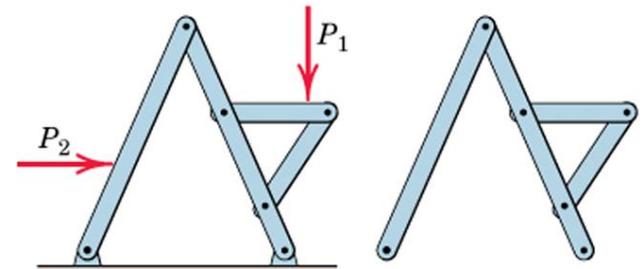
- first find all forces external to the structure treated as a single rigid body
- then dismember the structure & consider equilibrium of each part



- **Non-rigid Collapsible**

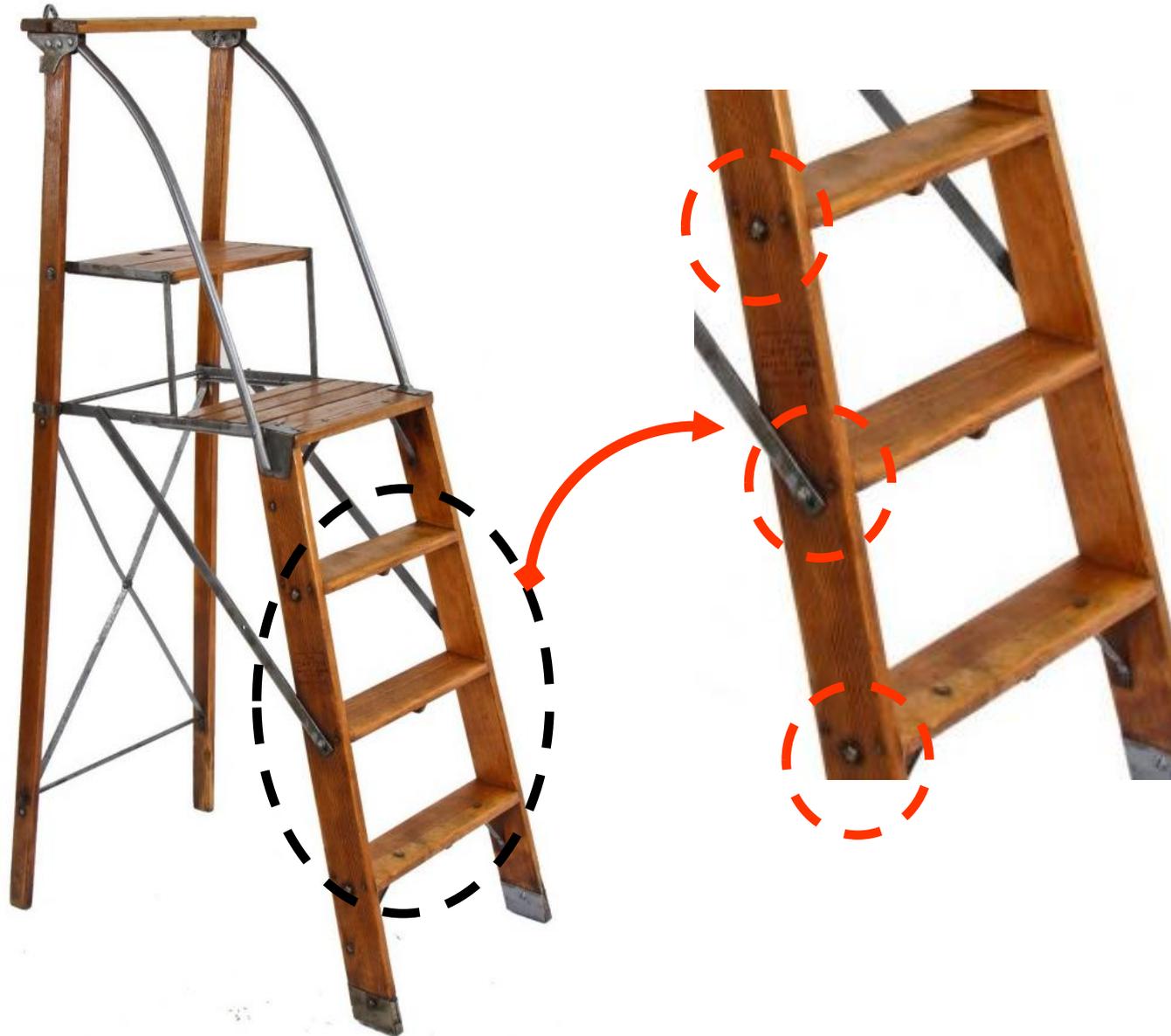
- structure is not a rigid unit by itself but depends on its external supports for rigidity

- calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analysed.

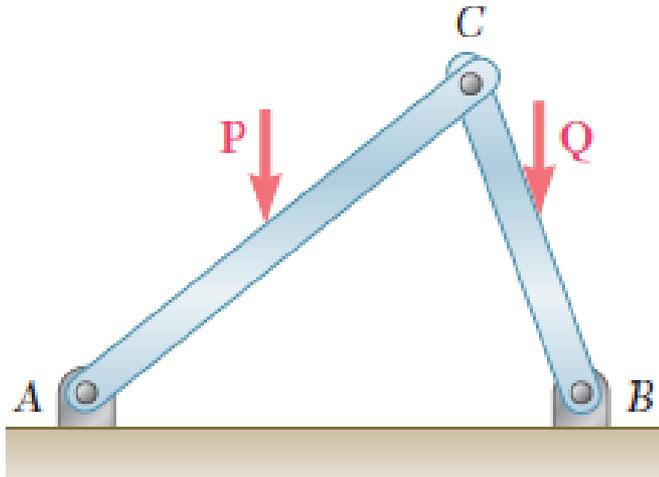


# Rigid Frame

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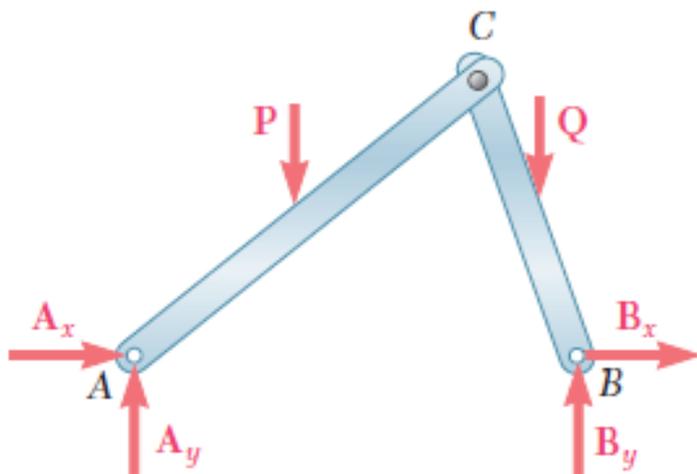


# Example (1) on Frames

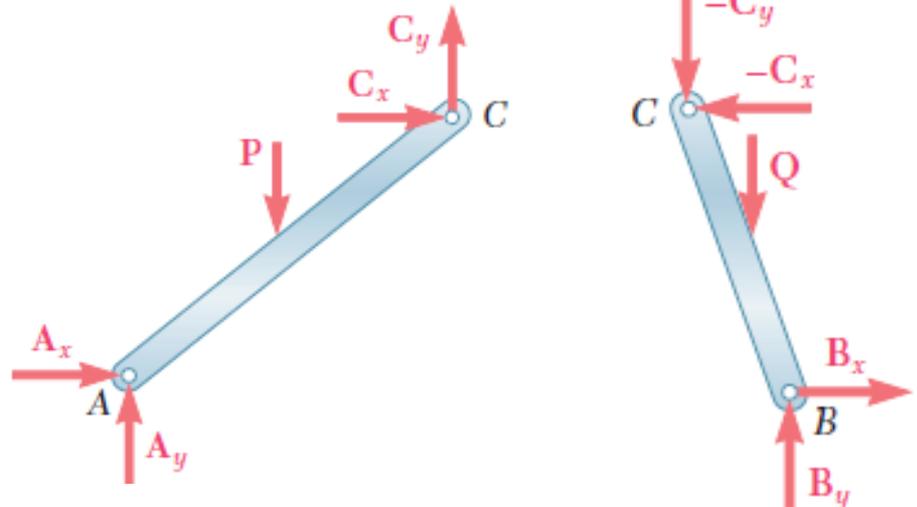


: Solve for the member forces

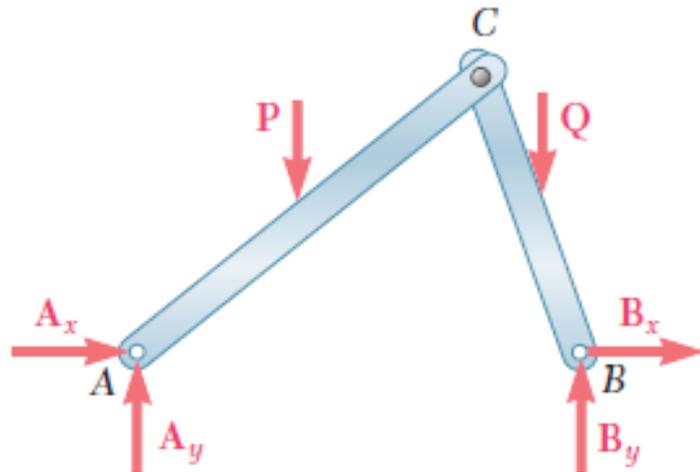
- Removal of support
- **Inherently unstable**
- **Collapsible frame**



Overall free body diagram



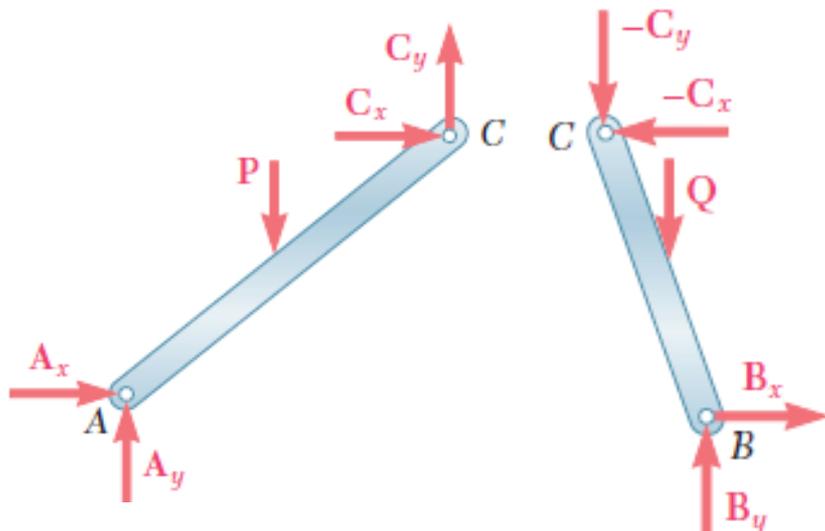
# Example (1) on Frames



Nos. of unknown support reactions = 4

Nos. of equilibrium equations = 3

∴ **All reactions cannot** be determined from the **overall free body diagram**



Nos. of unknown member forces = 6

Nos. of equilibrium equations = 6

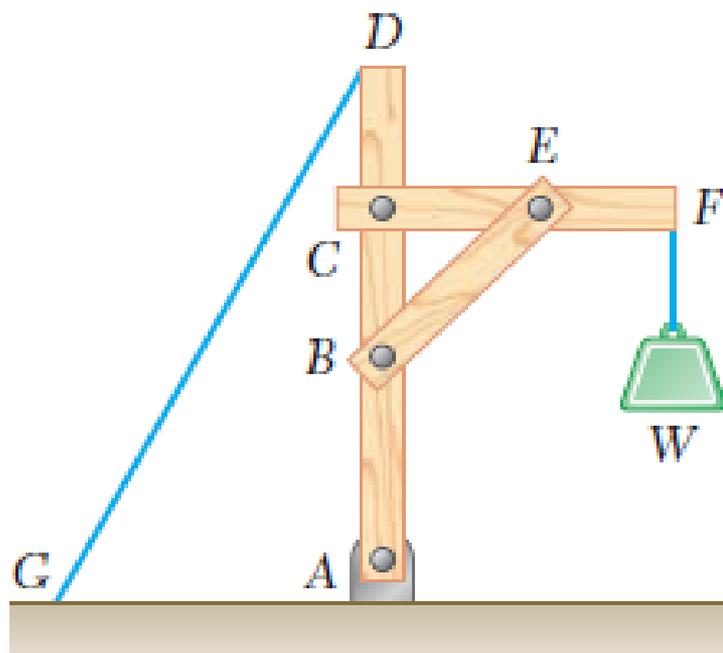
∴ **All member forces can** be determined from the **member free body diagrams**

# Frame Analysis

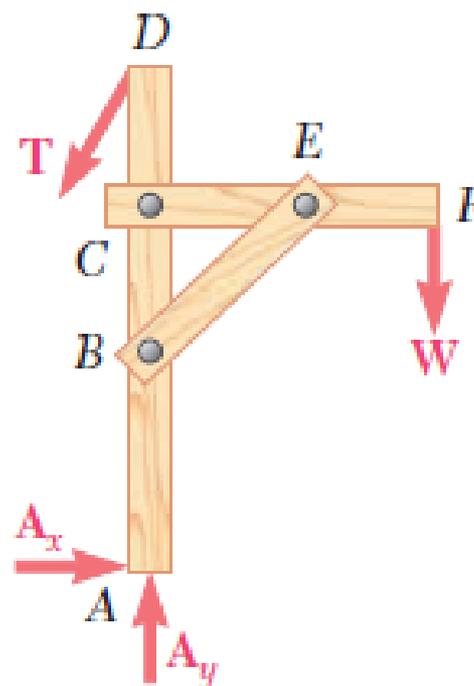
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- Collapsible frame
  - Overall free body diagram
    - **Necessary condition** to obtain **support reactions**
    - **Not sufficient condition**
  - Member free body diagram
    - Both **necessary** and **sufficient**

# Example (2) on Frames

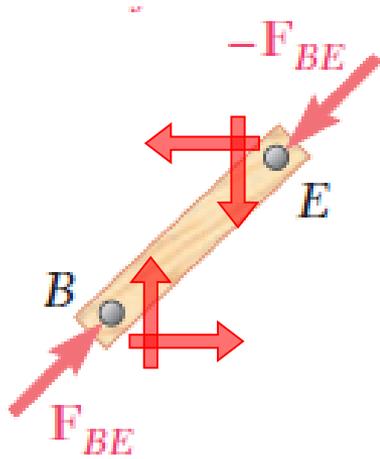


: Solve for the member forces

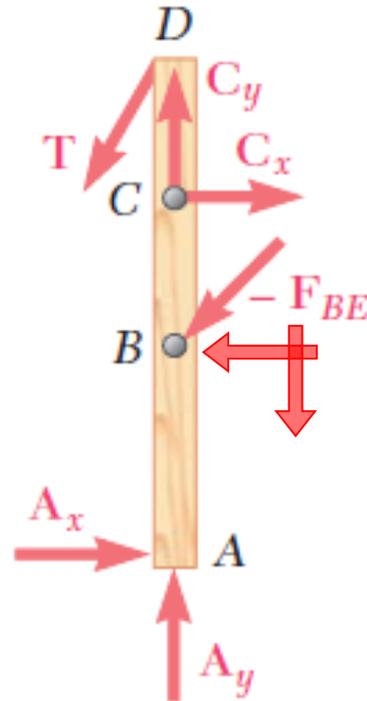


Overall free body diagram

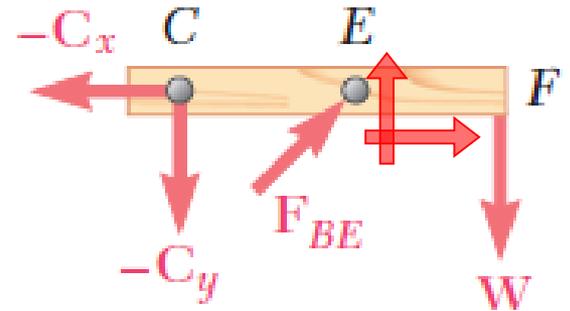
# Example (2) on Frames



BE :: **2- force** member



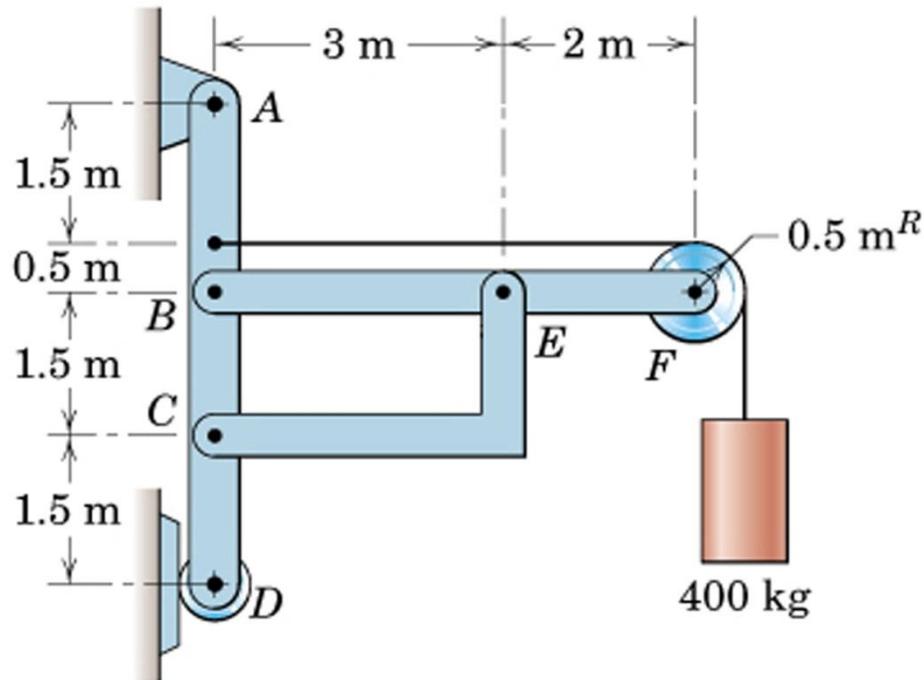
DCAB :: **Multi-force** member



CEF :: **Multi-force** member

# Example (3) on Frames

Compute the horizontal and vertical components of all forces acting on each of the members (neglect self weight)



# Example (3) on Frames

Example Solution:

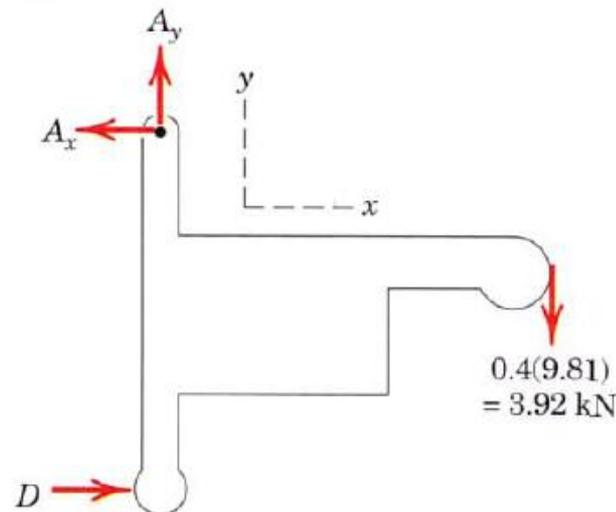
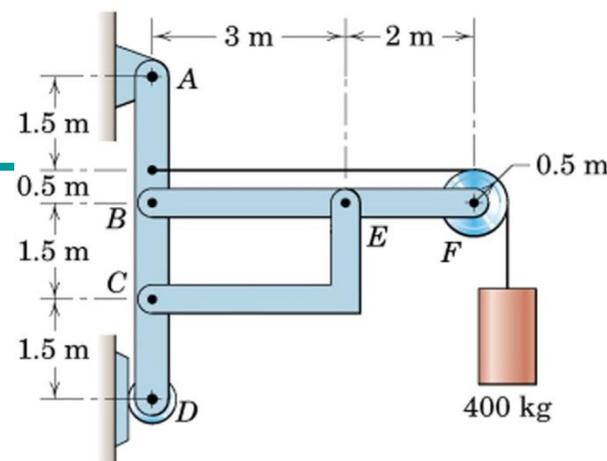
3 supporting members form a rigid non-collapsible assembly

Frame Statically Determinate Externally

Draw FBD of the entire frame

3 Equilibrium equations are available

Pay attention to sense of Reactions



$$[\Sigma M_A = 0] \quad 5.5(0.4)(9.81) - 5D = 0 \quad D = 4.32 \text{ kN}$$

$$[\Sigma F_x = 0] \quad A_x - 4.32 = 0 \quad A_x = 4.32 \text{ kN}$$

$$[\Sigma F_y = 0] \quad A_y - 3.92 = 0 \quad A_y = 3.92 \text{ kN}$$

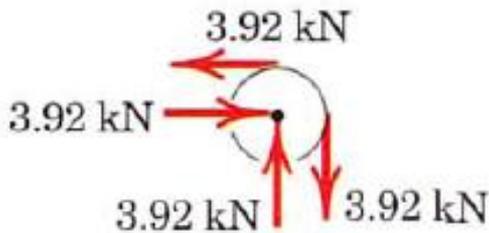
# Example (3) on Frames

Example Solution: Dismember the frame

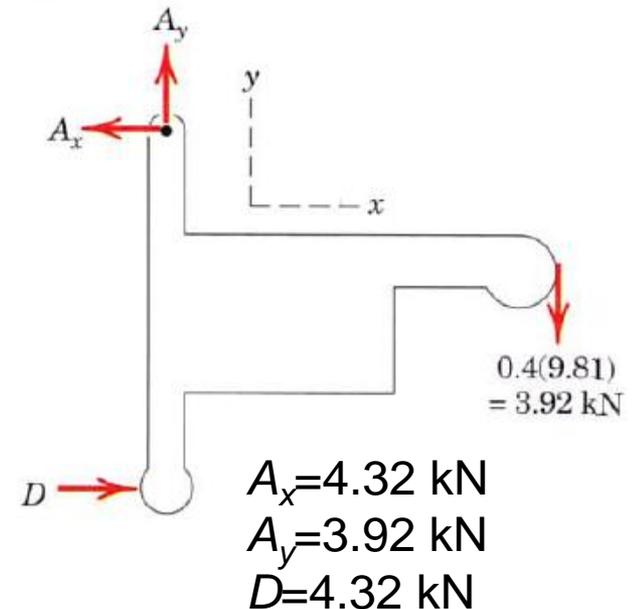
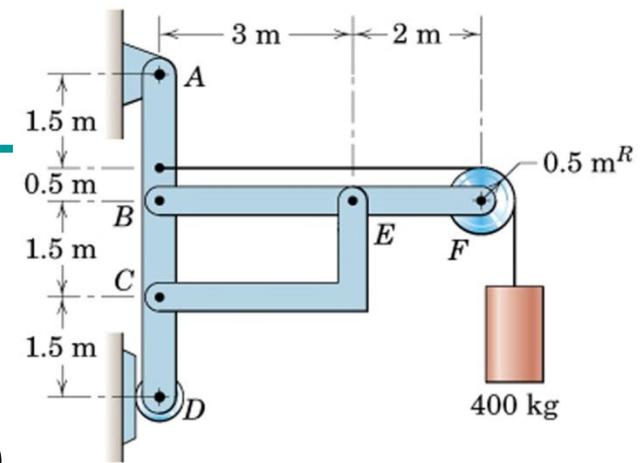
and draw separate FBDs of each member

- show loads and reactions on each member due to connecting members (interaction forces)

Begin with FBD of Pulley



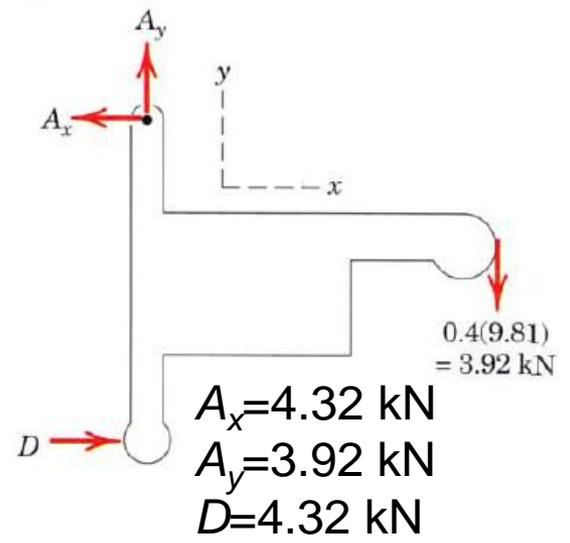
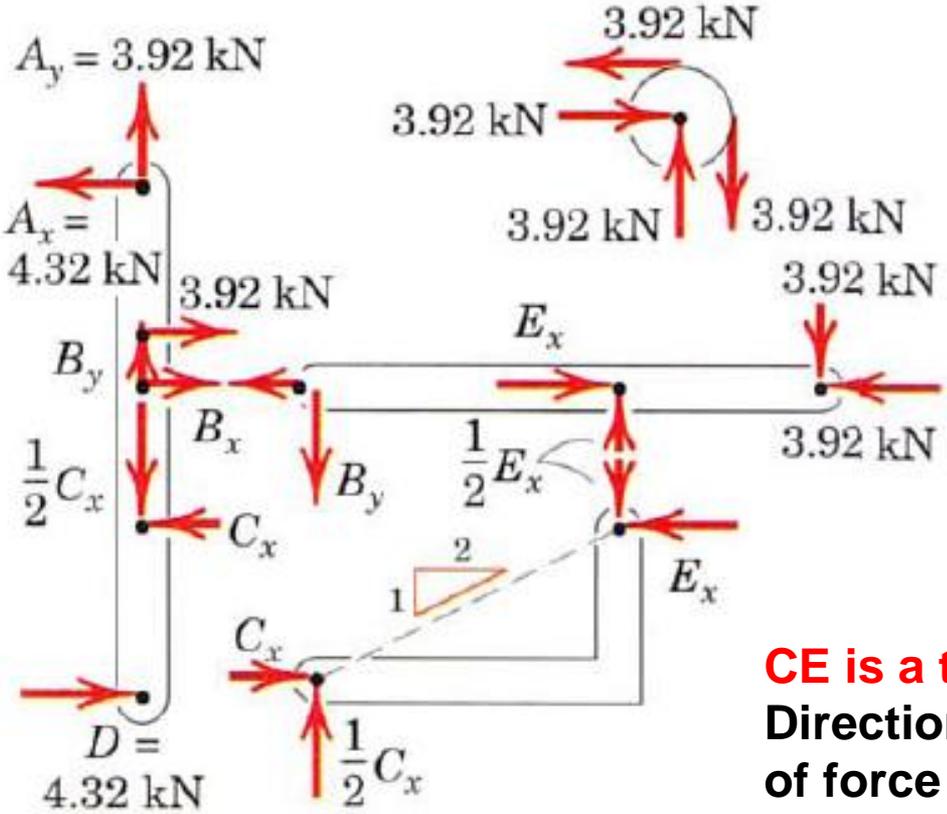
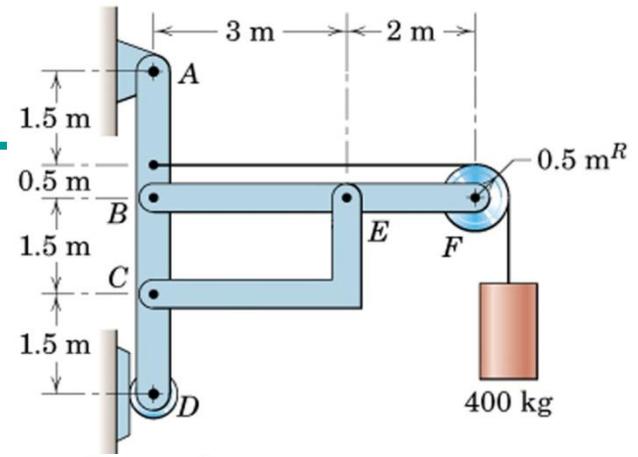
Then draw FBD of Members BF, CE, and AD



# Example (3) on Frames

Example Solution:

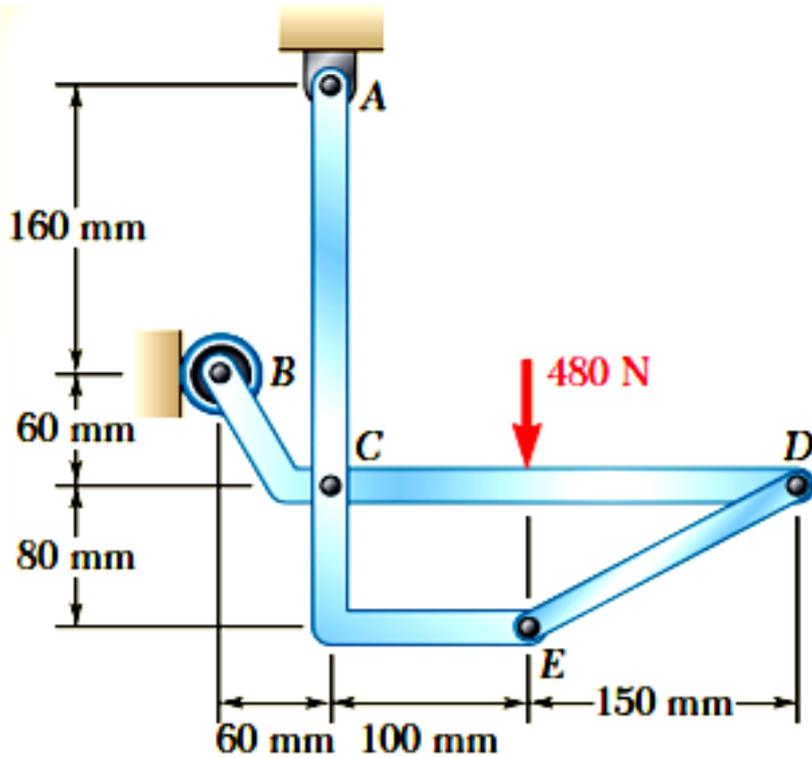
FBDs



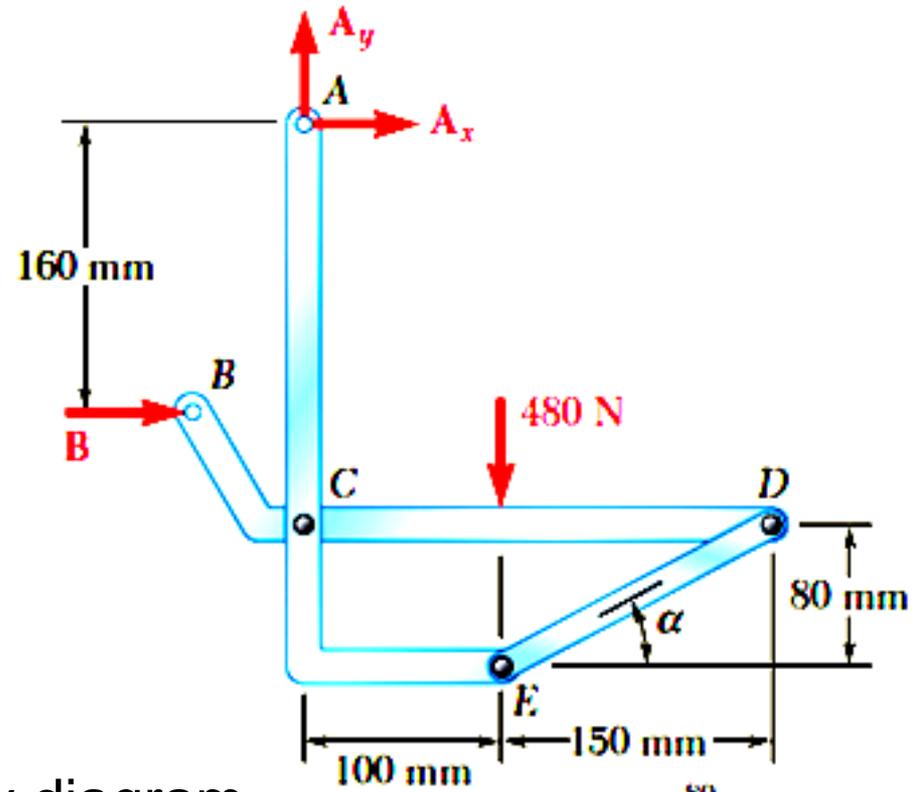
**CE is a two-force member**  
 Direction of the line joining the two points of force application determines the direction of the forces acting on a two-force member.  
**Shape of the member is not important.**



# Example (4) on Frames

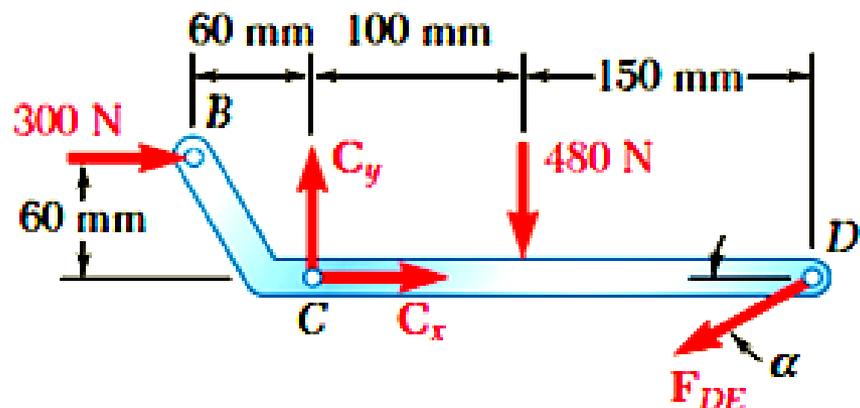


: Solve for the member forces

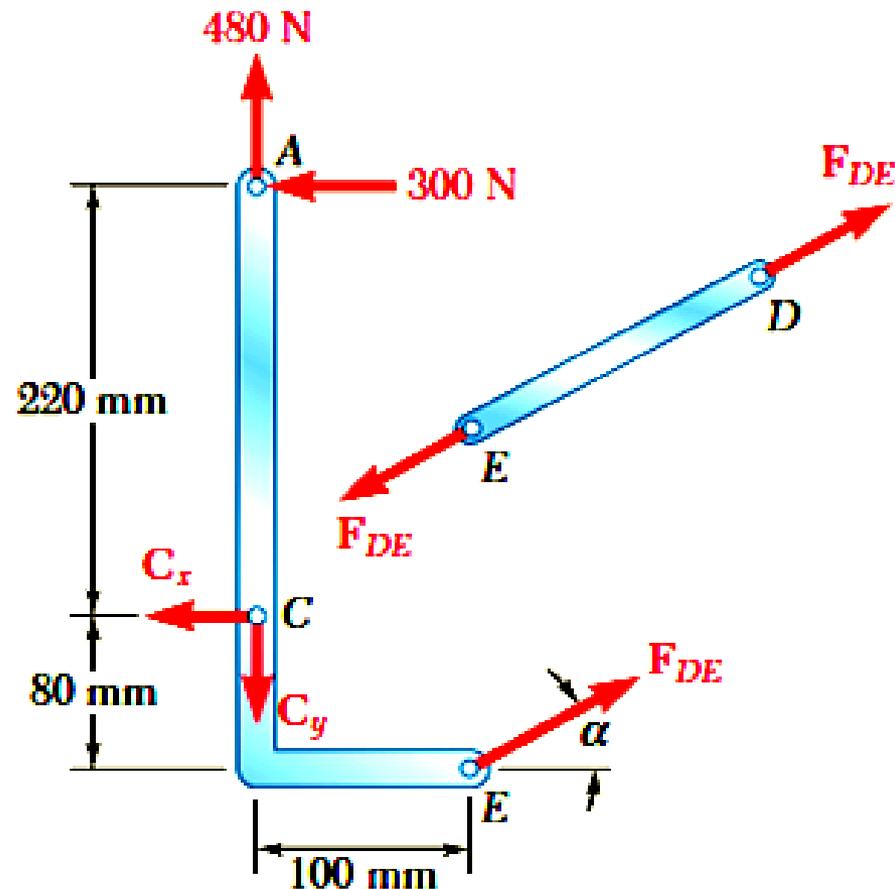


Overall free body diagram

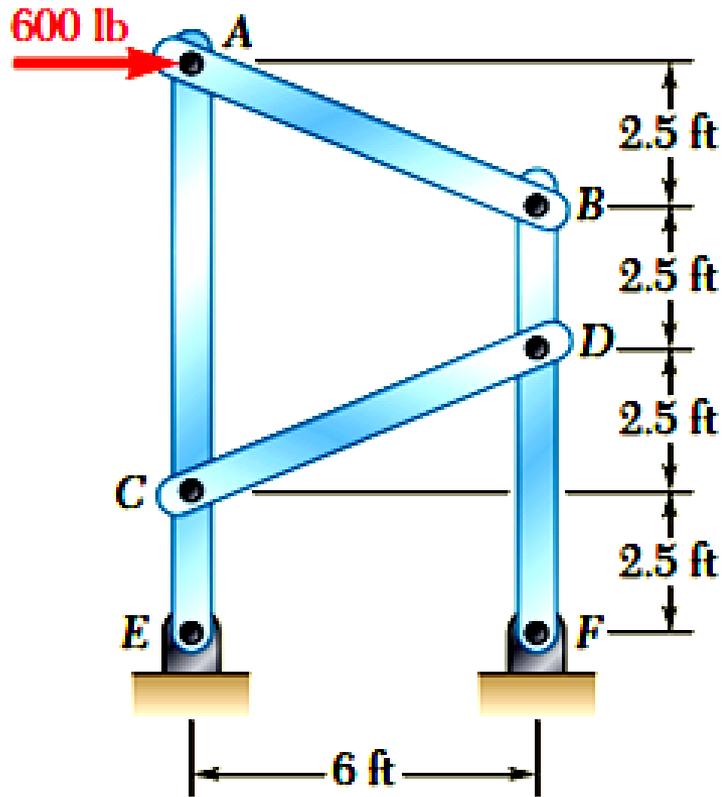
# Example (4) on Frames



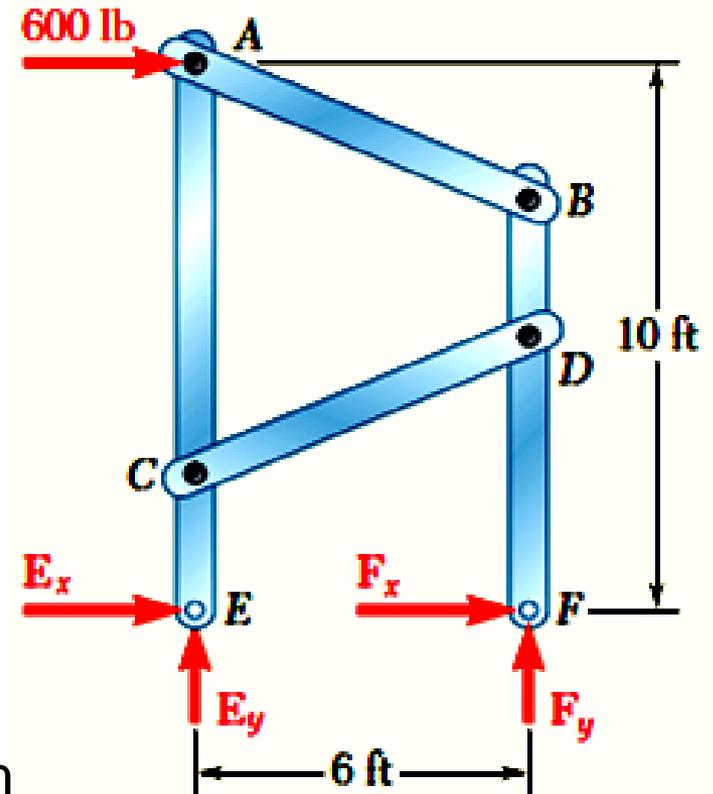
Member free body diagrams



# Example (5) on Frames

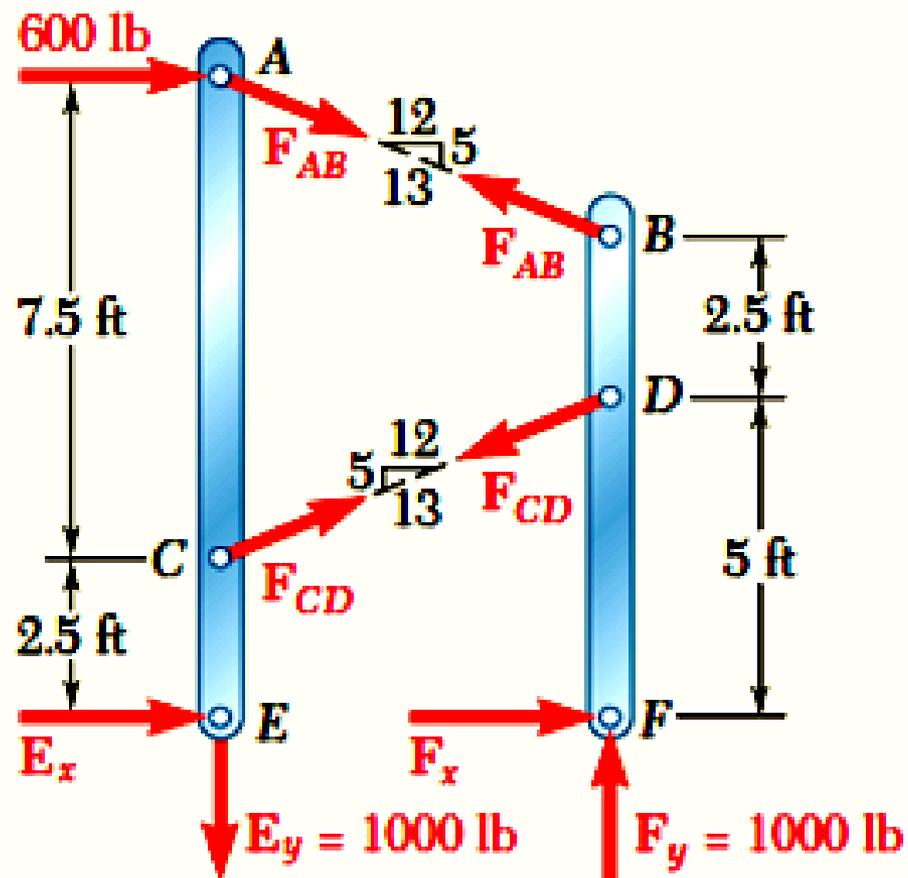
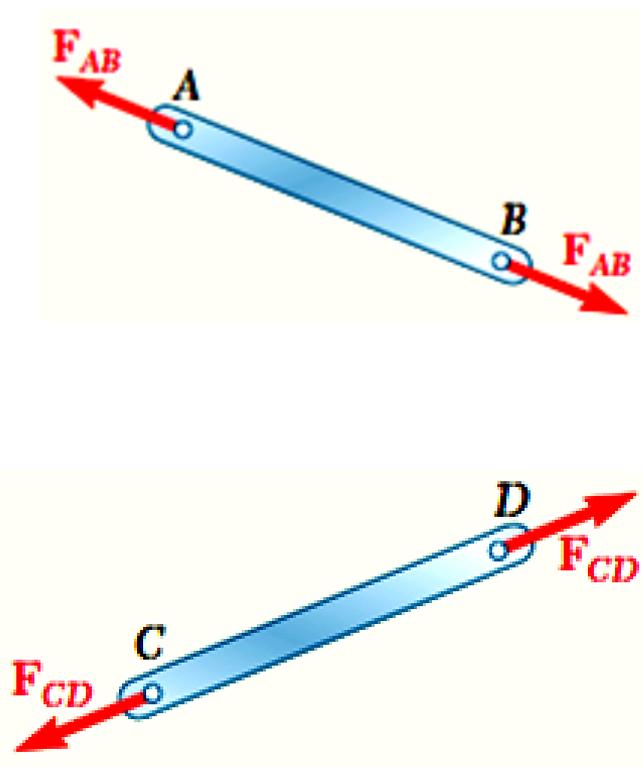


: Solve for the member forces



Overall free body diagram

# Example (5) on Frames



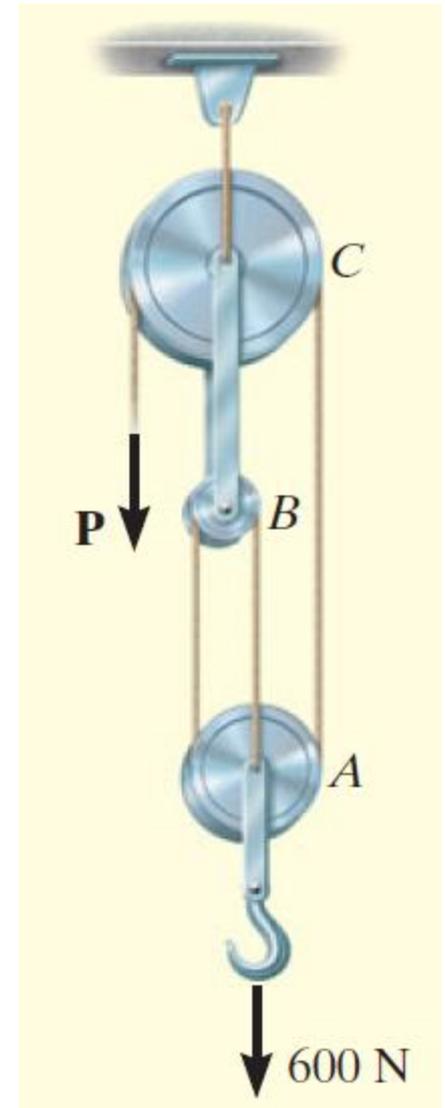
# Example (6) on Frames

## Example:

Find the tension in the cables and the force  $P$  required to support the 600 N force using the frictionless pulley system (neglect self weight)

## Solution:

Draw the FBD



# Example (6) on Frames

Example Solution:  
Draw FBD and apply  
equilibrium equations

*Pulley A*

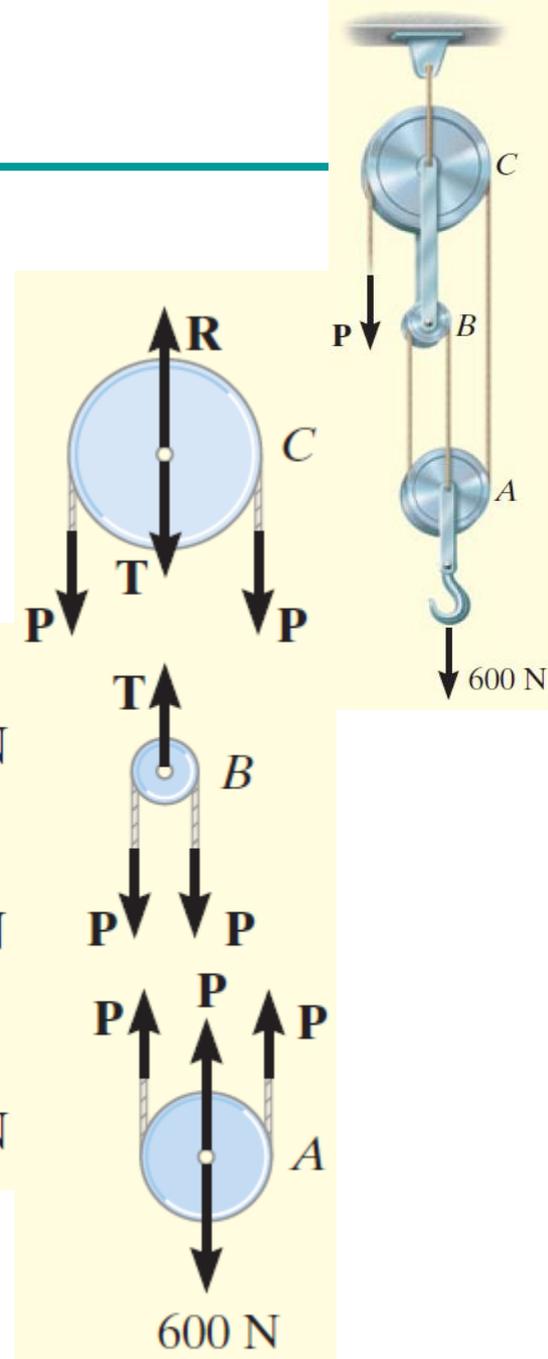
$$+\uparrow \Sigma F_y = 0; \quad 3P - 600 \text{ N} = 0 \quad P = 200 \text{ N}$$

*Pulley B*

$$+\uparrow \Sigma F_y = 0; \quad T - 2P = 0 \quad T = 400 \text{ N}$$

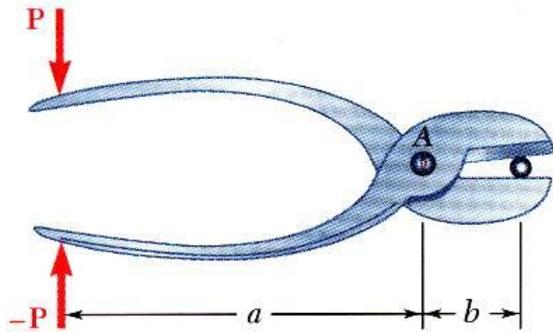
*Pulley C*

$$+\uparrow \Sigma F_y = 0; \quad R - 2P - T = 0 \quad R = 800 \text{ N}$$

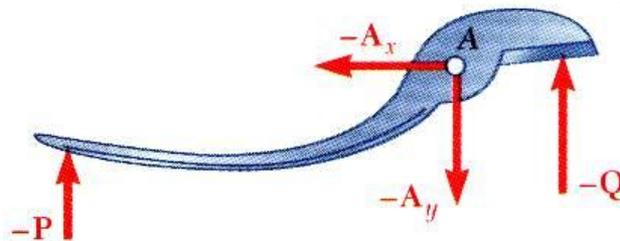
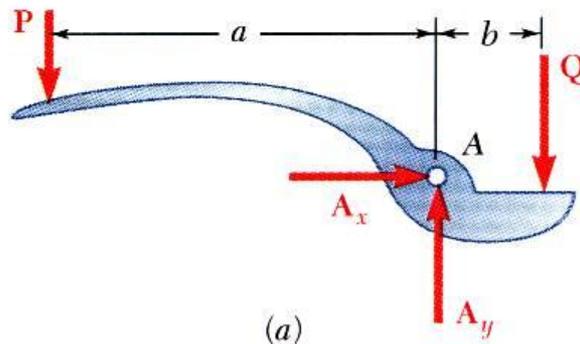
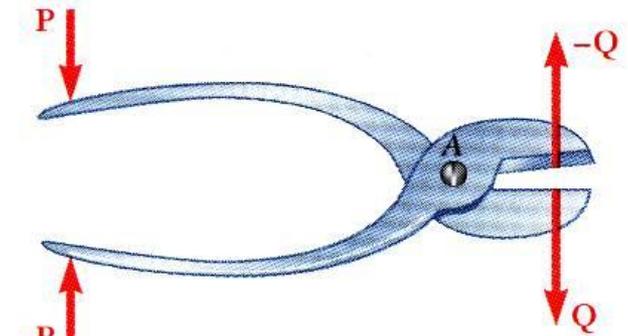


# Example (6) on Frames

Example: Pliers: Given the magnitude of  $P$ , determine the magnitude of  $Q$



FBD of Whole Pliers



FBD of individual parts

Taking moment about pin A

$$Q = Pa/b$$

Also pin reaction  $A_x = 0$

$$A_y = P(1 + a/b) \text{ OR } A_y = P + Q$$

# Frames and Machines

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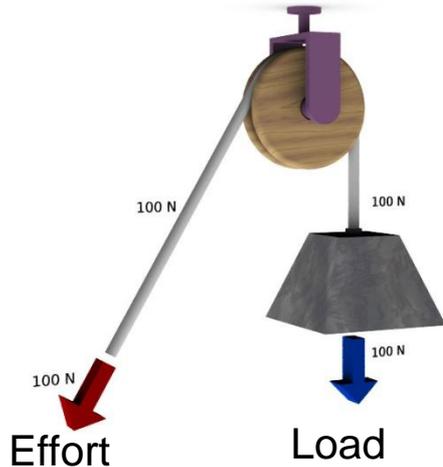
## Definitions

- **Effort:** Force required to overcome the resistance to get the work done by the machine.
- **Mechanical Advantage:** Ratio of load lifted ( $W$ ) to effort applied ( $P$ ).  
Mechanical Advantage =  $W/P$
- **Velocity Ratio:** Ratio of the distance moved by the effort ( $D$ ) to the distance moved by the load ( $d$ ) in the same interval of time.  
Velocity Ratio =  $D/d$
- **Input:** Work done by the effort  $\rightarrow$  Input =  $PD$
- **Output:** Useful work got out of the machine, i.e. the work done by the load  $\rightarrow$  Output =  $Wd$
- **Efficiency:** Ratio of output to the input.

Efficiency of an ideal machine is 1. In that case,  $Wd = PD \rightarrow W/P = D/d$ .

For an ideal machine, mechanical advantage is equal to velocity ratio.

# Frames and Machines: Pulley System



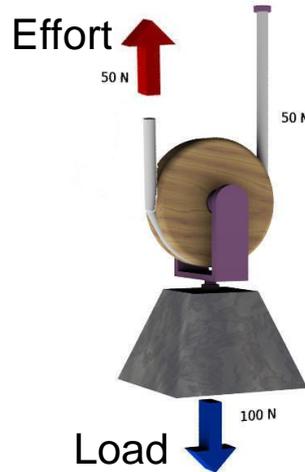
**Fixed Pulley**

Effort = Load  
 → Mechanical Advantage = 1

Distance moved by effort is equal to the distance moved by the load.

→ Velocity Ratio = 1

[www.Petervaldivia.com](http://www.Petervaldivia.com)



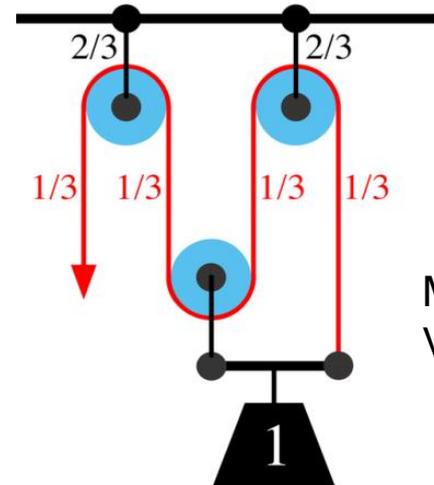
**Movable Pulley**

Effort = Load/2  
 → Mechanical Advantage = 2

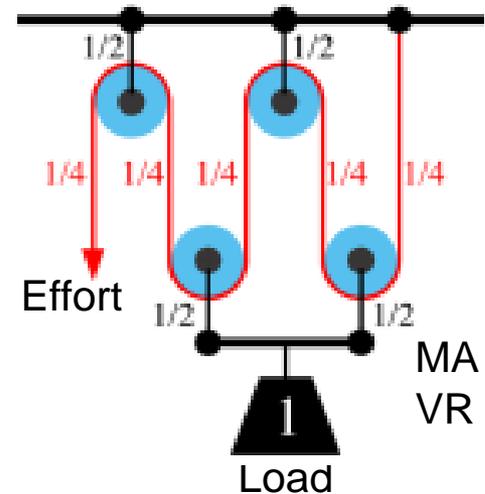
Distance moved by effort is twice the distance moved by the load (both rope should also accommodate the same displacement by which the load is moved).

→ Velocity Ratio = 2

**Compound Pulley**



MA = 3  
 VR = 3



MA = 4  
 VR = 4

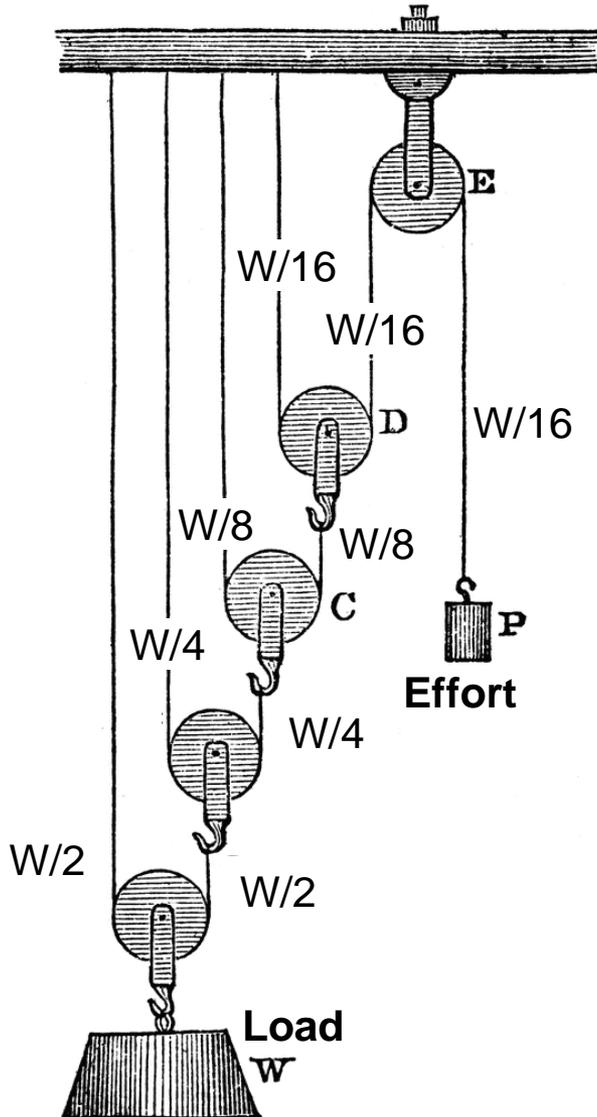
# Frames and Machines: Pulley System

## Compound Pulley

Effort required is  $1/16^{\text{th}}$  of the load.

Mechanical Advantage = 16.  
(neglecting frictional forces).

Velocity ratio is 16, which means in order to raise a load to 1 unit height; effort has to be moved by a distance of 16 units.

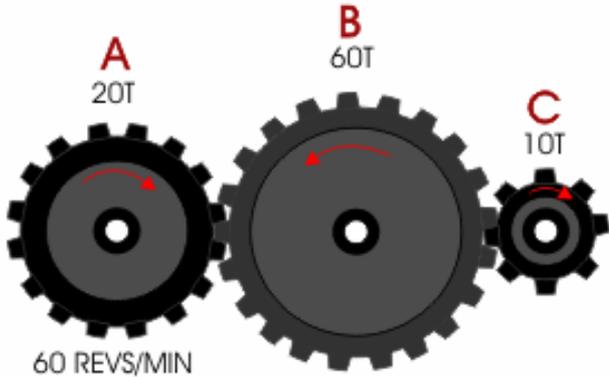


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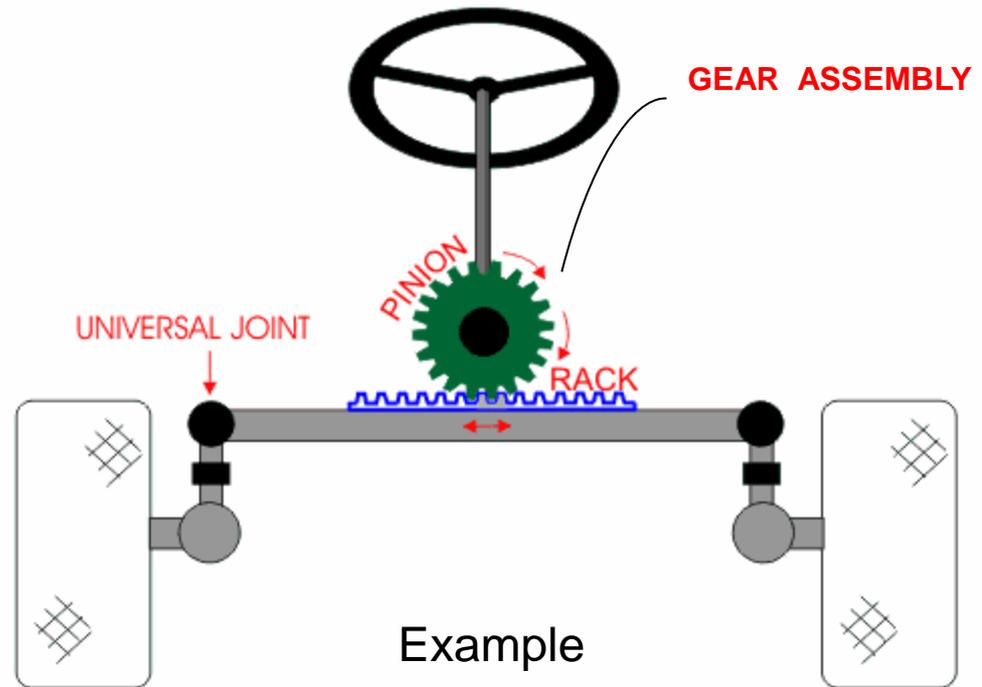
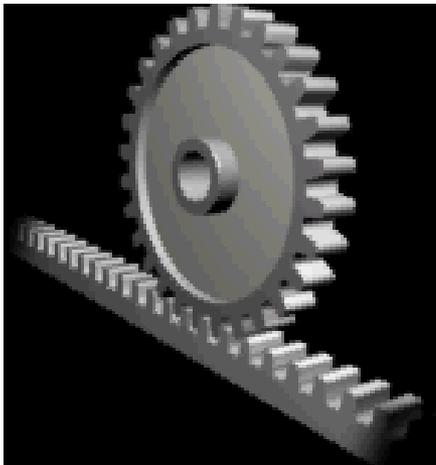
# Frames and Machines

## Gears

$$\begin{aligned} & (\text{Speed gear A} * \text{Number of teeth Gear A}) = \\ & (\text{Speed gear B} * \text{Number of teeth gear B}) \end{aligned}$$



## Rack & Pinion

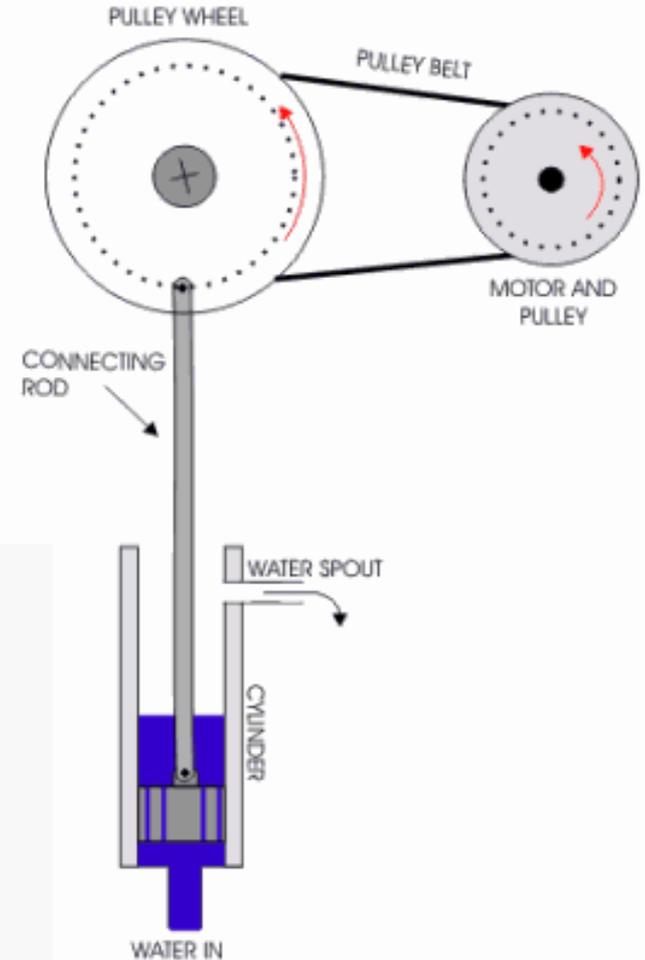


# Frames and Machines

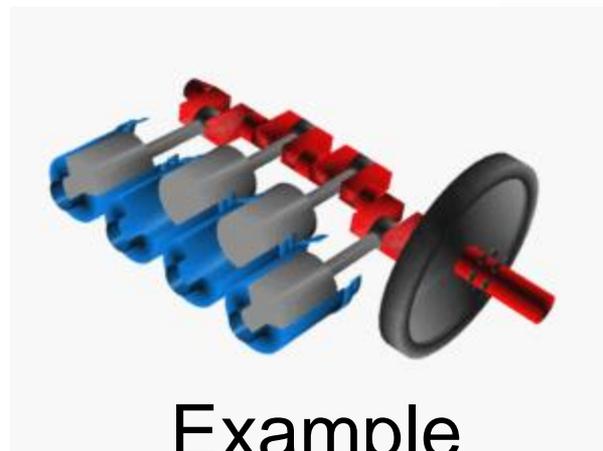
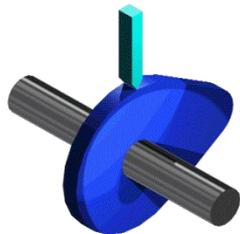
## Gear and Belt



## Crank Connecting Rod



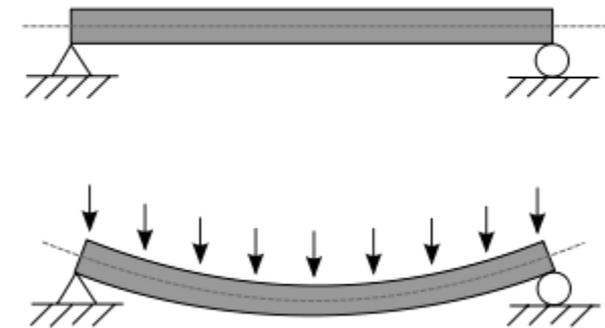
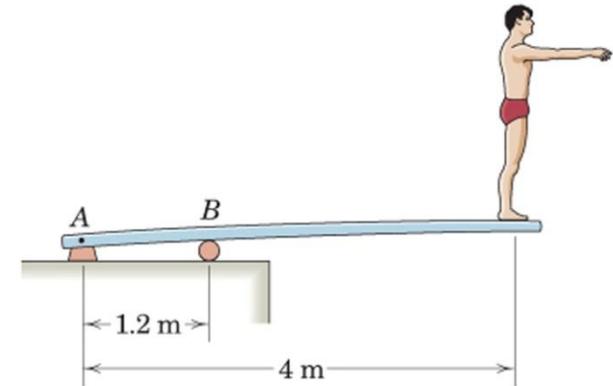
## Cam and Follower System



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Example

# Beams



Beams are structural members that offer resistance to bending due to applied load