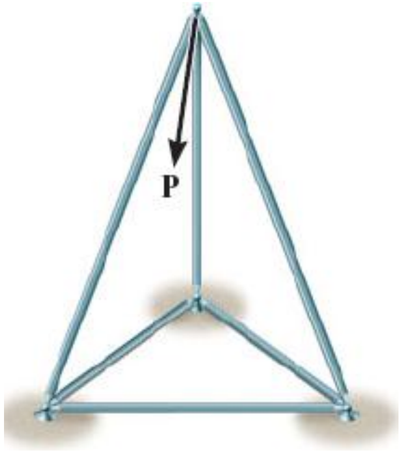
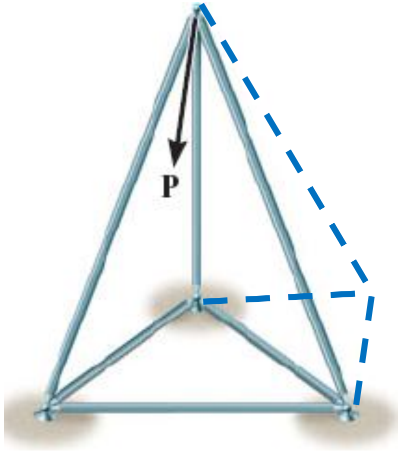
Structural Analysis: Space Truss

Space Truss



- 6 bars joined at their ends to form the edges of a tetrahedron as the basic non-collapsible unit
- 3 additional concurrent bars whose ends are attached to three joints on the existing structure are required to add a new rigid unit to extend the structure.

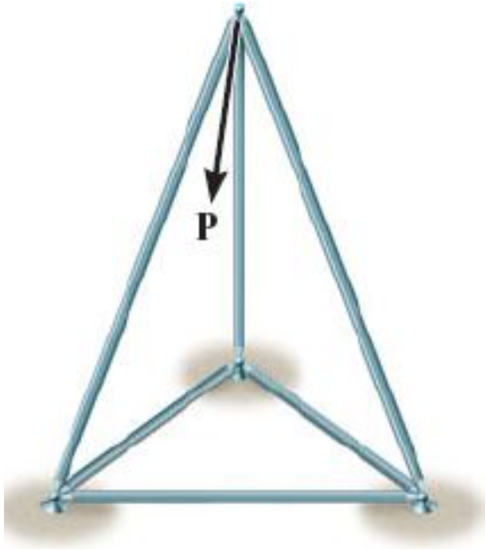


A space truss formed in this way is called a Simple Space Truss

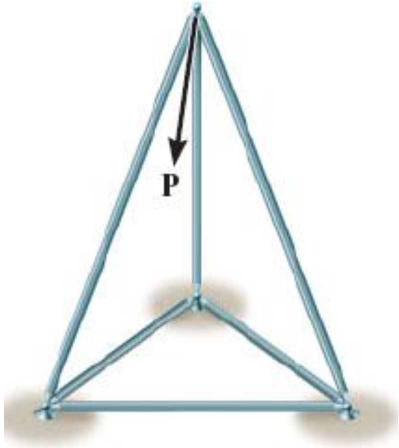
If center lines of joined members intersect at a point
→Two force members assumption is justified
→ Each member under Compression or Tension

Space Truss Analysis: Method of Joints

- Method of Joints
  - All the member forces are required
  - Scalar equation (***force***) at each joint
    - $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$
  - Solution of simultaneous equations



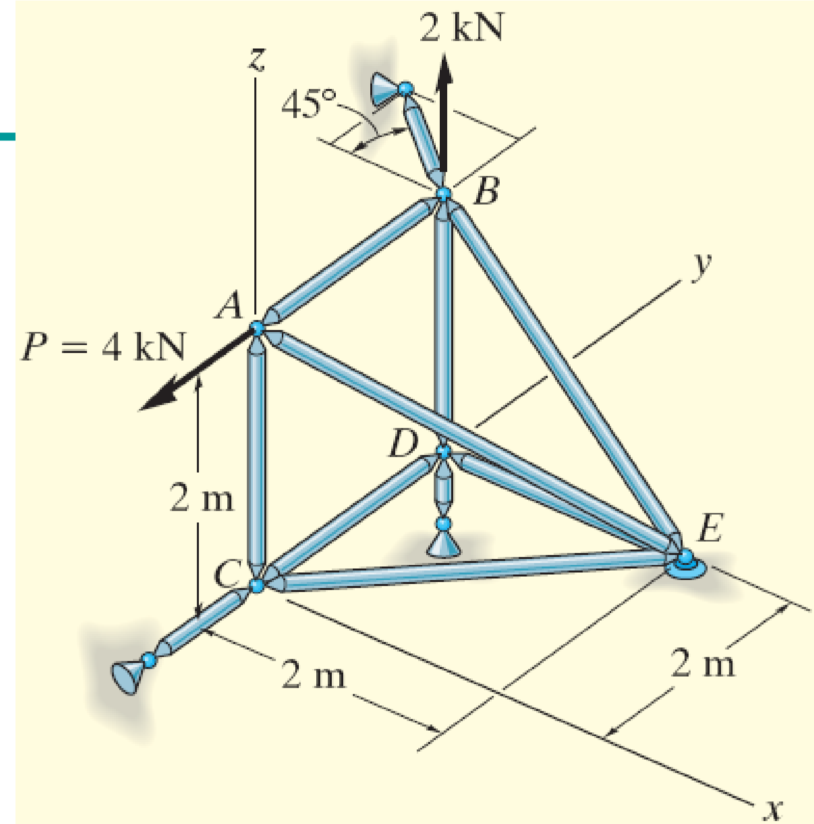
Space Truss Analysis: Method of Sections



- Method of Sections
  - A few member forces are required
  - Vector equations (***force*** and ***moment***)
    - $\Sigma\mathbf{F} = \mathbf{0}$, $\Sigma\mathbf{M} = \mathbf{0}$
  - Scalar equations
    - 6 nos.:: F_x, F_y, F_z and M_x, M_y, M_z
  - **Section should not** pass through more than **6 members**
    - More number of unknown forces

Space Truss: Example

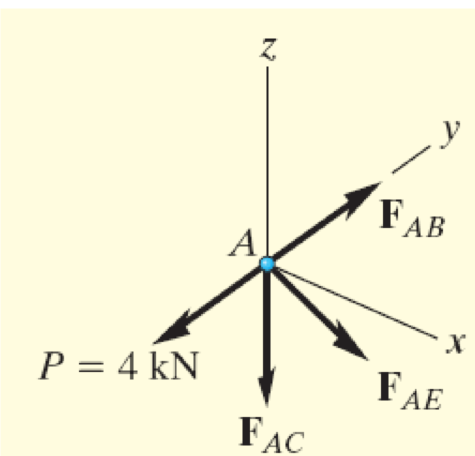
Determine the forces acting in members of the space truss.



Solution:

Start at joint A: Draw free body diagram

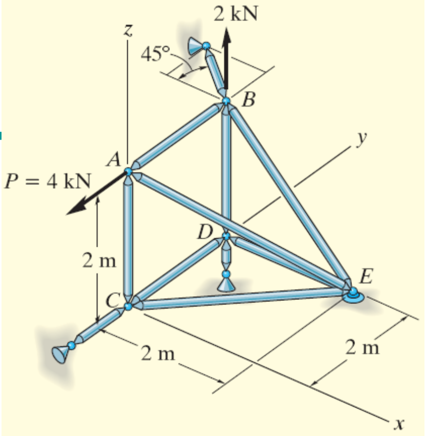
Express each force in vector notation



$$\mathbf{P} = \{-4\mathbf{j}\}\ \text{kN}, \qquad \mathbf{F}_{AB} = F_{AB}\mathbf{j}, \quad \mathbf{F}_{AC} = -F_{AC}\mathbf{k},$$

$$\mathbf{F}_{AE} = F_{AE}\left(\frac{\mathbf{r}_{AE}}{r_{AE}}\right) = F_{AE}(0.577\mathbf{i} + 0.577\mathbf{j} - 0.577\mathbf{k})$$

Space Truss: Example



For equilibrium,

$$\Sigma \mathbf{F} = \mathbf{0}; \qquad \mathbf{P} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AE} = \mathbf{0}$$

$$-4\mathbf{j} + F_{AB}\mathbf{j} - F_{AC}\mathbf{k} + 0.577F_{AE}\mathbf{i} + 0.577F_{AE}\mathbf{j} - 0.577F_{AE}\mathbf{k} = \mathbf{0}$$

Rearranging the terms and equating the coefficients of **i**, **j**, and **k** unit vector to zero will give:

$$\Sigma F_x = 0; \qquad 0.577F_{AE} = 0$$

$$\Sigma F_y = 0; \qquad -4 + F_{AB} + 0.577F_{AE} = 0$$

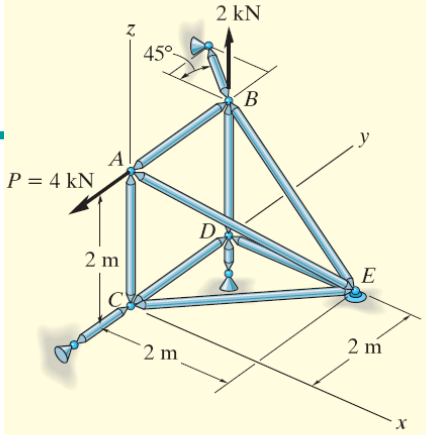
$$\Sigma F_z = 0; \qquad -F_{AC} - 0.577F_{AE} = 0$$

$$F_{AC} = F_{AE} = 0$$

$$F_{AB} = 4\text{ kN}$$

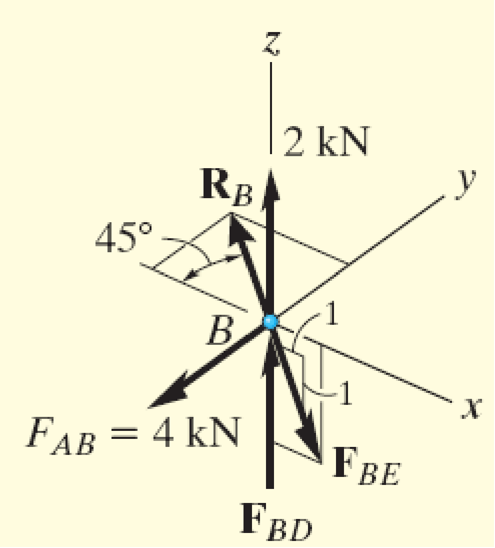
Next Joint B may be analysed.

Space Truss: Example



Joint B: Draw the Free Body Diagram

Scalar equations of equilibrium may be used at joint B



$$\Sigma F_x = 0; \quad -R_B \cos 45^\circ + 0.707 F_{BE} = 0$$

$$\Sigma F_y = 0; \quad -4 + R_B \sin 45^\circ = 0$$

$$\Sigma F_z = 0; \quad 2 + F_{BD} - 0.707 F_{BE} = 0$$

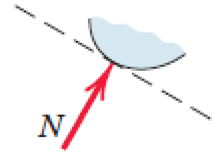
$$R_B = F_{BE} = 5.66 \text{ kN} \quad (\text{T}), \qquad F_{BD} = 2 \text{ kN}$$

Using Scalar equations of equilibrium at joints D and C will give:

$$F_{DE} = F_{DC} = F_{CE} = 0$$

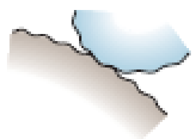
Recapitulation :: Support Reaction

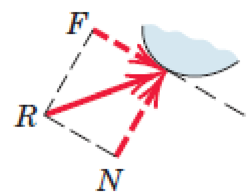
Smooth surfaces



Contact force is compressive and is normal to the surface.

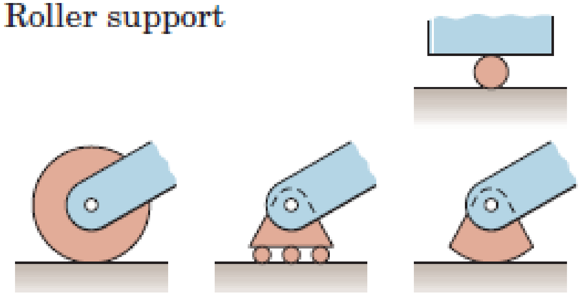
Rough surfaces

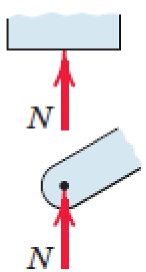




Rough surfaces are capable of supporting a tangential component F (frictional force) as well as a normal component N of the resultant contact force R.

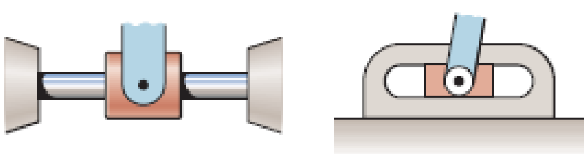
Roller support

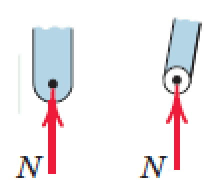




Roller, rocker, or ball support transmits a compressive force normal to the supporting surface.

Freely sliding guide

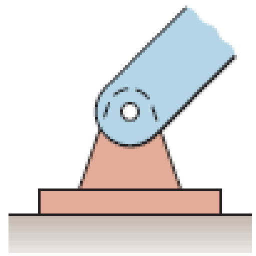




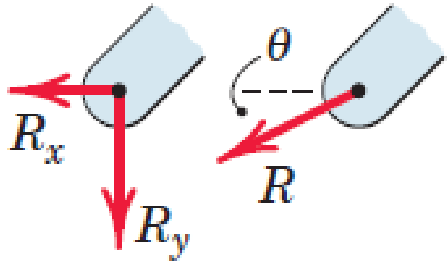
Collar or slider free to move along smooth guides; can support force normal to guide only.

Recapitulation :: Support Reaction

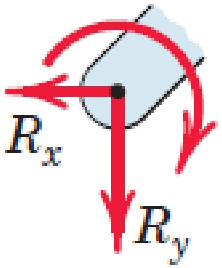
Pin connection



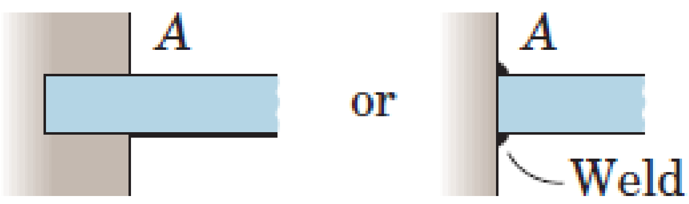
Pin free to turn

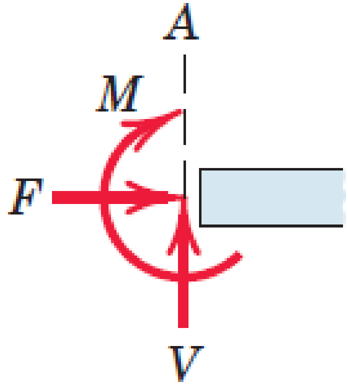


Pin not free to turn



Built-in or fixed support

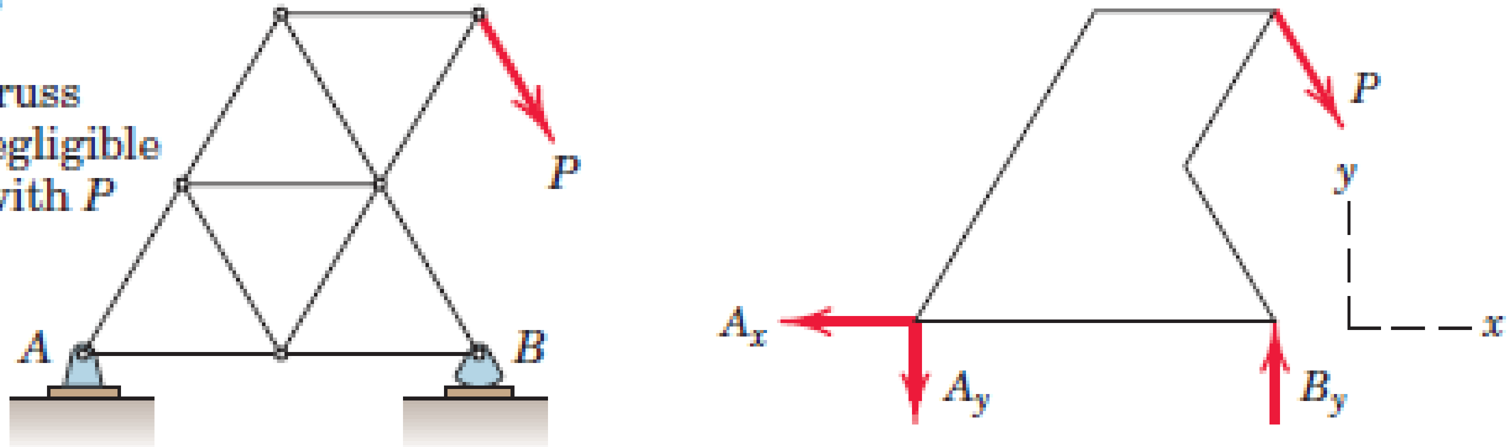




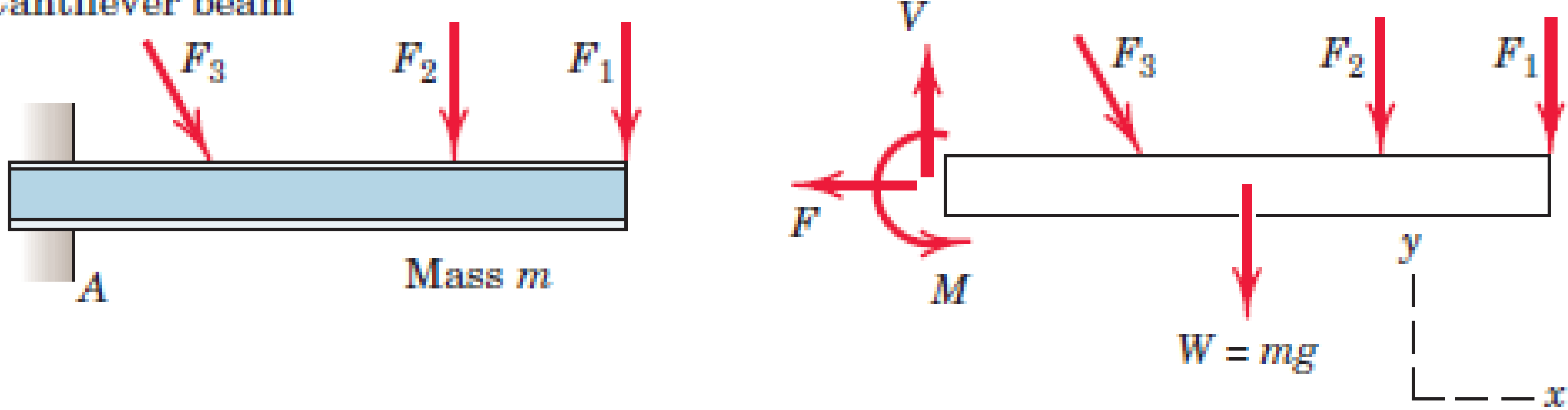
Recapitulation :: Free Body Diagram

1. Plane truss

Weight of truss assumed negligible compared with P

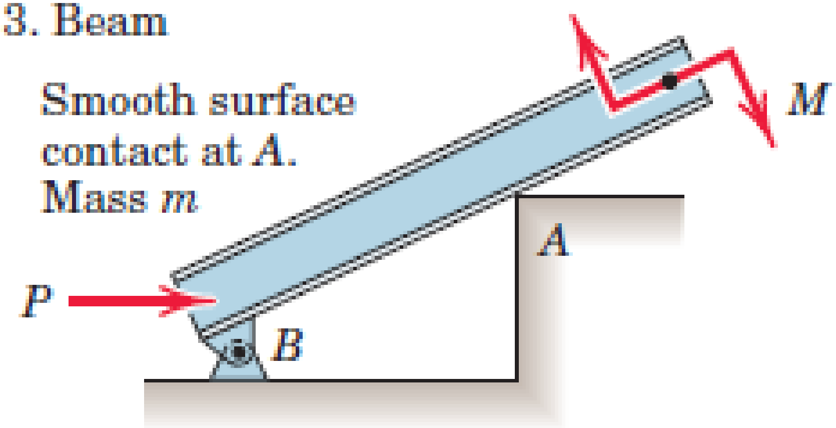


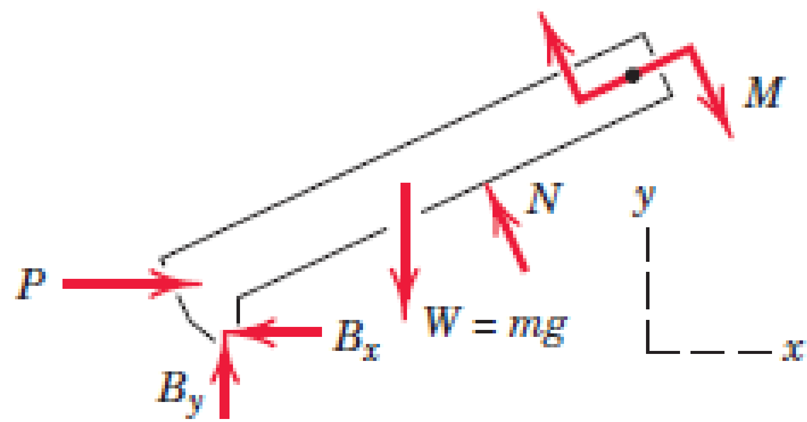
2. Cantilever beam



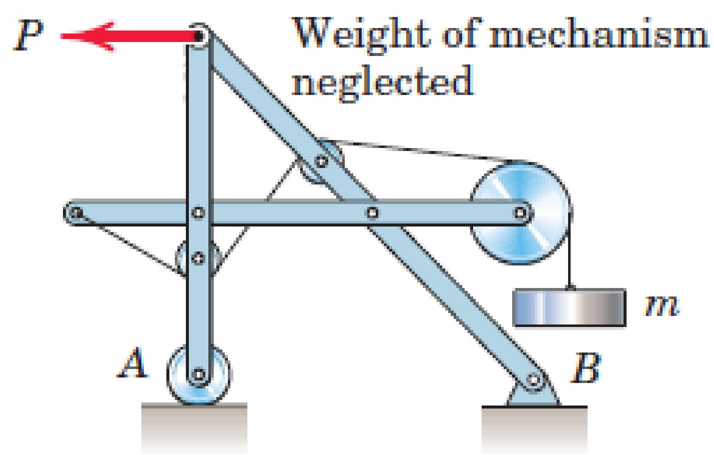
Recapitulation :: Free Body Diagram

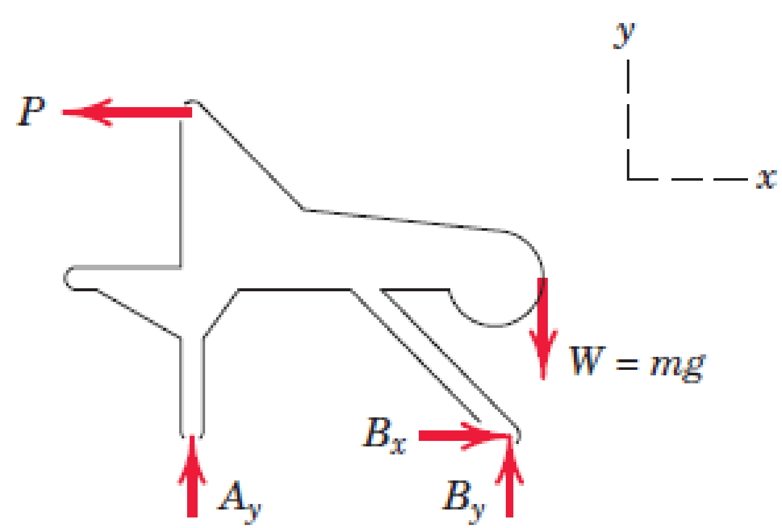
3. Beam





4. Rigid system of interconnected bodies
analyzed as a single unit

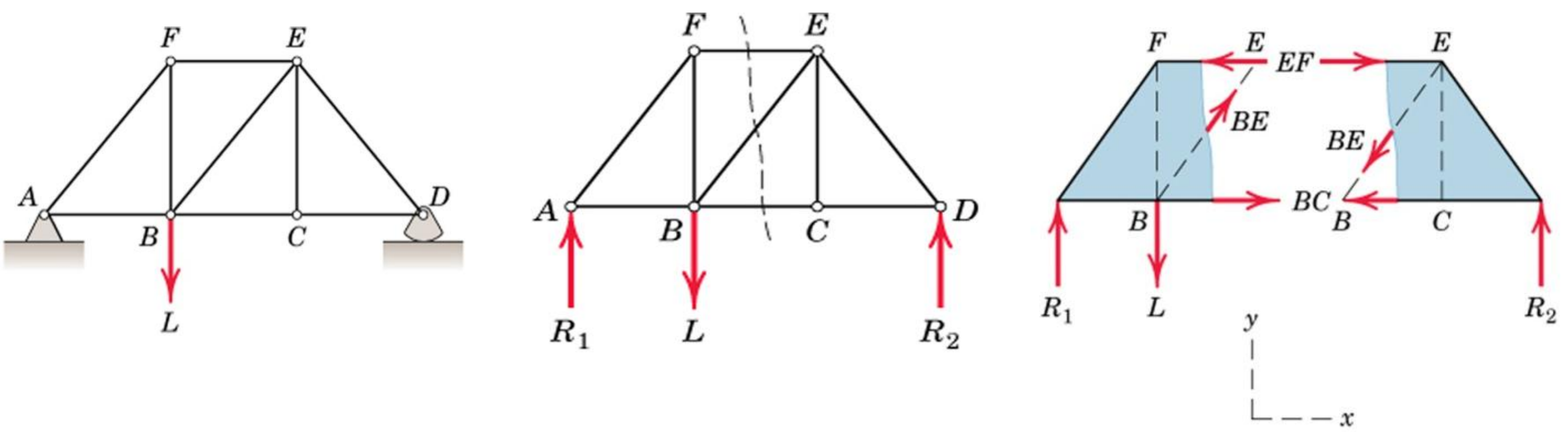




Recapitulation :: Method of Sections

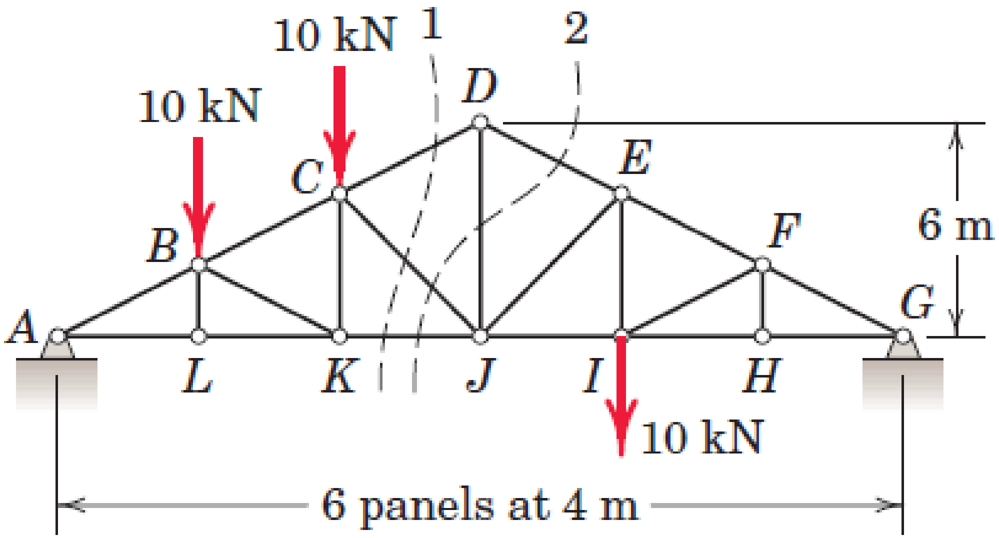
Method of Sections

- Find out the reactions from equilibrium of whole truss
- To find force in member BE:
- Cut an imaginary section (dotted line)
- Each side of the truss section should remain in equilibrium



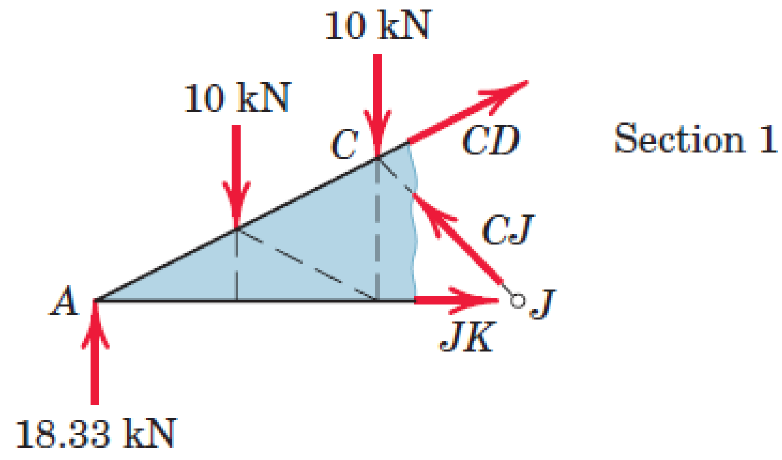
Example: Method of Sections

- Calculate the force in member DJ

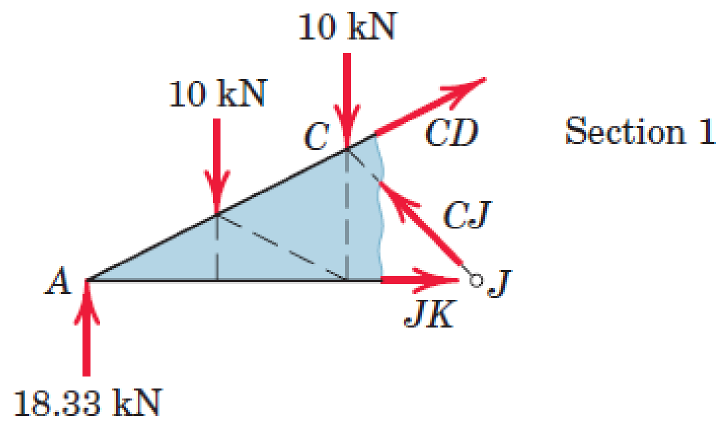


Direction of JK :: *Moment @ C*

Direction of CJ :: *Moment @ A*

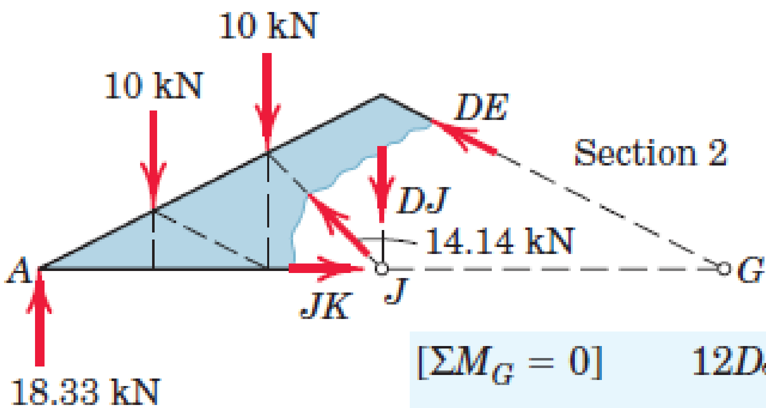


Example: Method of Sections



By the analysis of section 1, CJ is obtained from

$[\Sigma M_A = 0]$ $\quad 0.707CJ(12) - 10(4) - 10(8) = 0 \quad CJ = 14.14 \text{ kN } C$



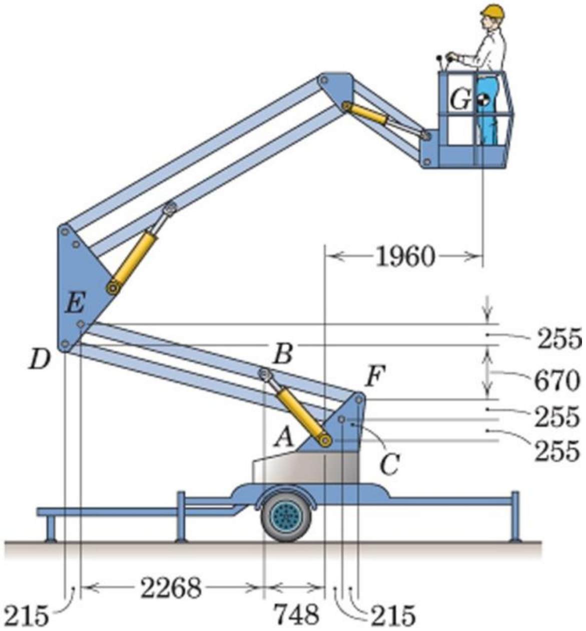
$[\Sigma M_G = 0]$ $\quad 12DJ + 10(16) + 10(20) - 18.33(24) - 14.14(0.707)(12) = 0$

$DJ = 16.67 \text{ kN } T$

Frames and Machines

A structure is called a Frame or Machine if at least one of its individual members is a <u>multi-force member</u>

- member with 3 or more forces acting on it, or
- member with 2 or more forces and 1 or more couple acting



Frames: generally stationary and are used to support loads

Machines: contain moving parts and are designed to transmit and alter the effect of forces acting

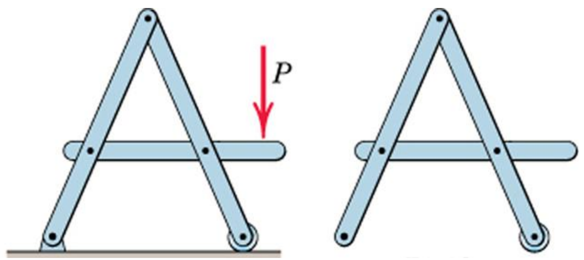
Multi-force members: the forces in these members in general will not be along the directions of the members

→ methods used in simple truss analysis cannot be used

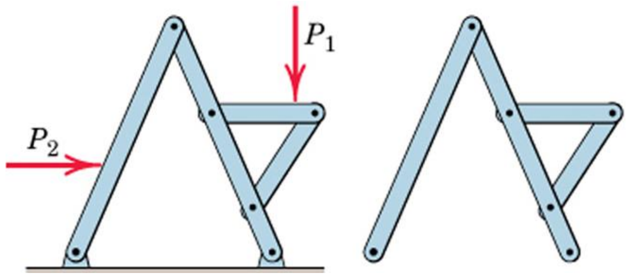
Frames and Machines

Interconnected Rigid Bodies with Multi-force Members

- Rigid Non-collapsible
  - –structure constitutes a rigid unit by itself when removed from its supports
  - –first find all forces external to the structure treated as a single rigid body
  - –then dismember the structure & consider equilibrium of each part



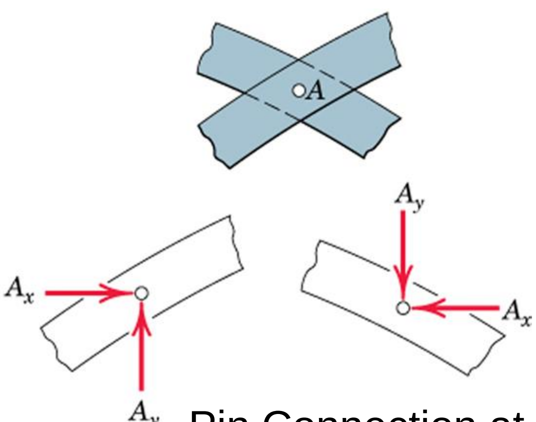
- Non-rigid Collapsible
  - –structure is not a rigid unit by itself but depends on its external supports for rigidity
  - –calculation of external support reactions cannot be completed until the structure is dismembered and individual parts are analysed.



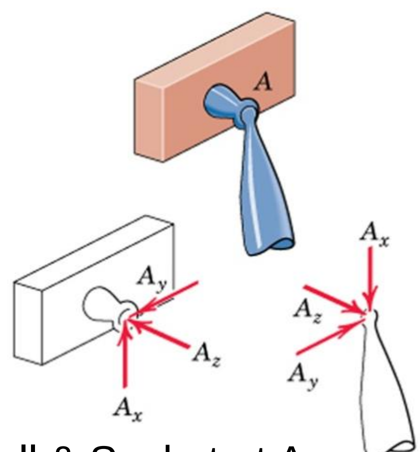
Frames and Machines

Free Body Diagrams: Forces of Interactions

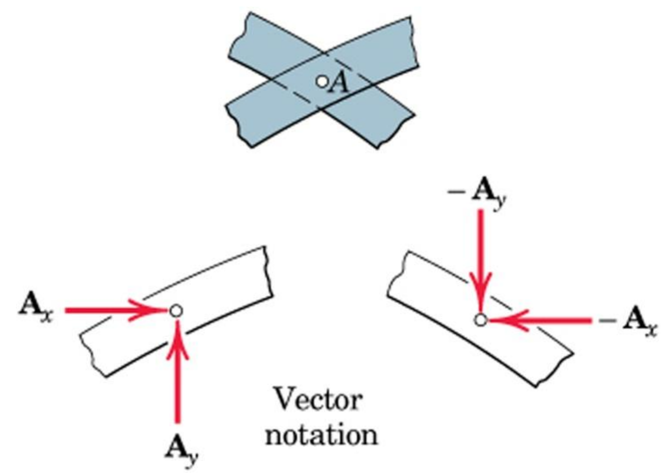
- force components must be consistently represented in opposite directions on the separate FBDs (Ex: Pin at A).
- apply action-and-reaction principle (Ex: Ball & Socket at A).
- Vector notation: use plus sign for an action and a minus sign for the corresponding reaction



Pin Connection at A



Ball & Socket at A



Frames and Machines

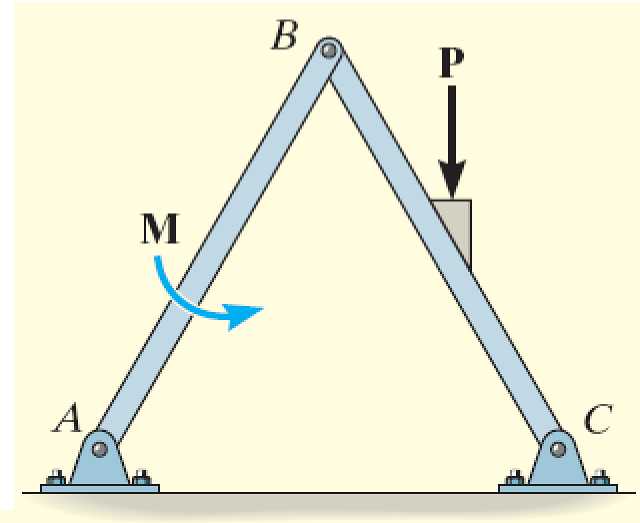
Example: Free Body Diagrams

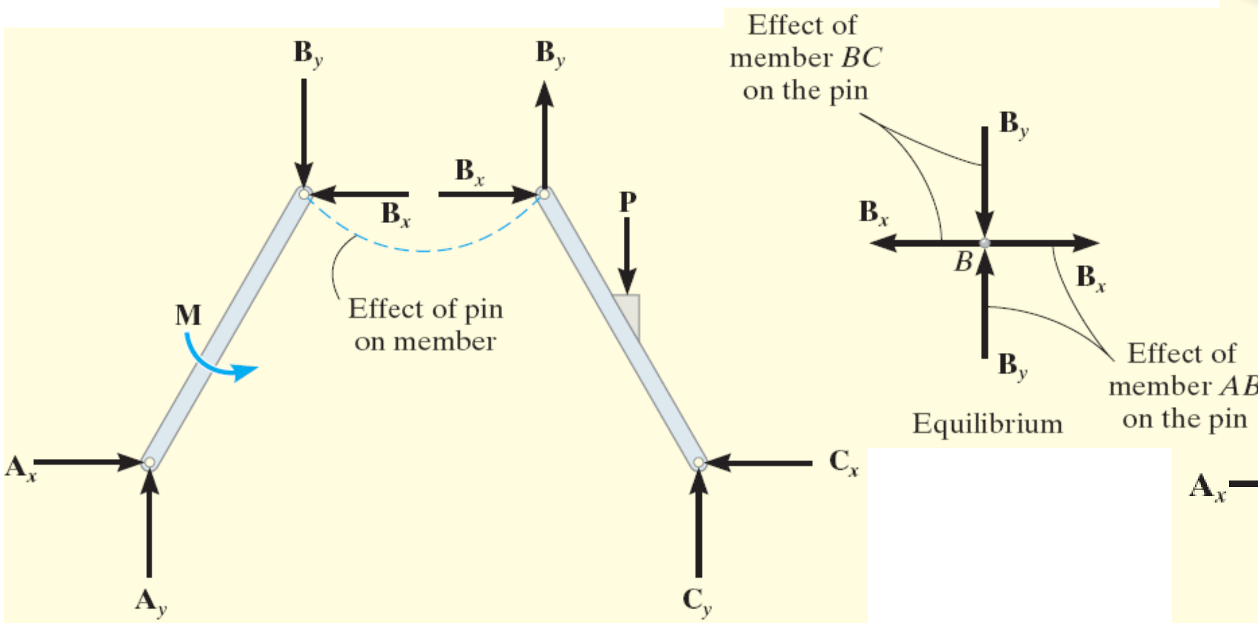
Draw FBD of

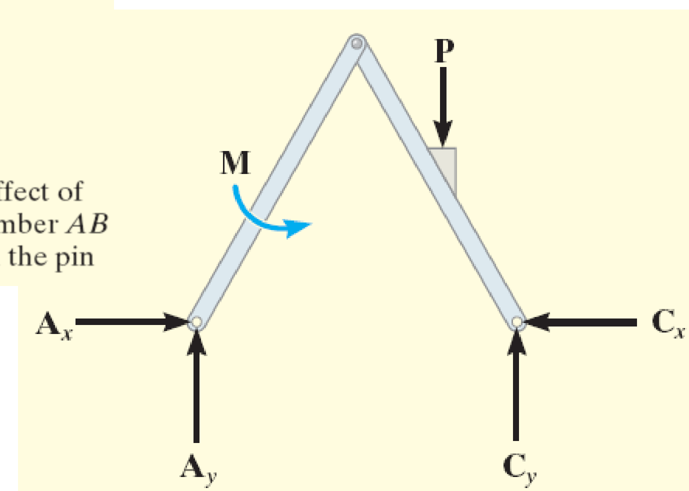
(a) Each member

(b) Pin at B, and

(c) Whole system

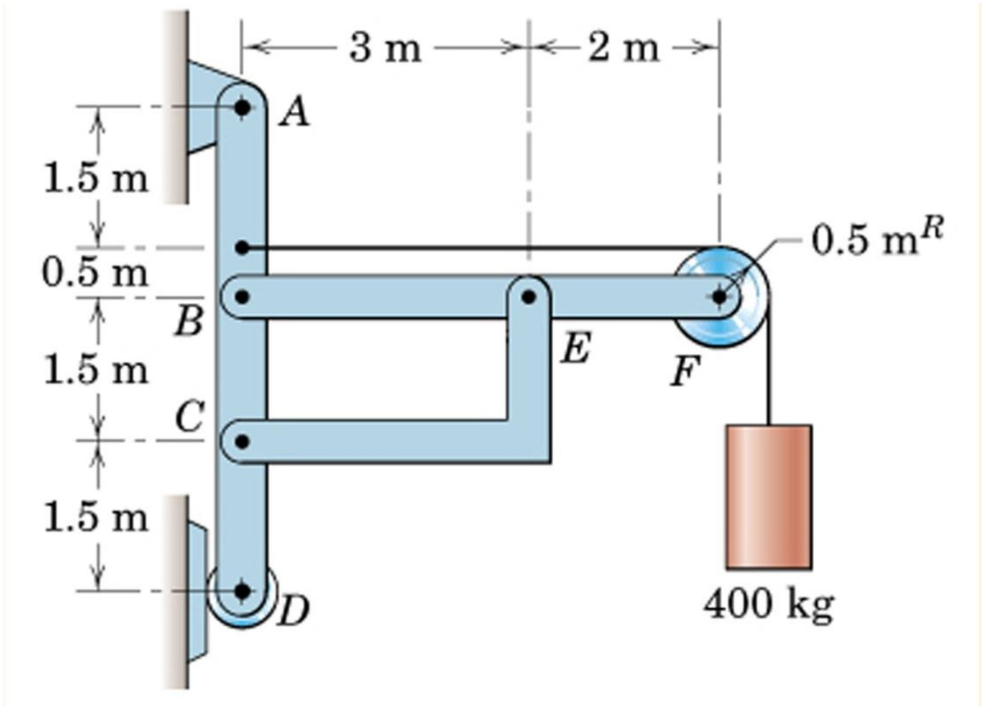




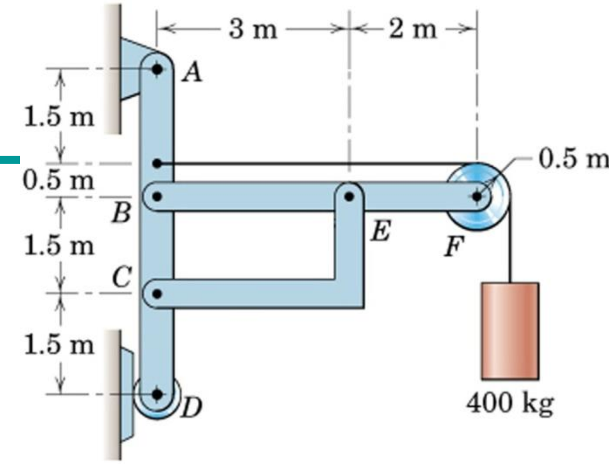


Example on Frames and Machines

Compute the horizontal and vertical components of all forces acting on each of the members (neglect self weight)



Frames and Machines



Example Solution:

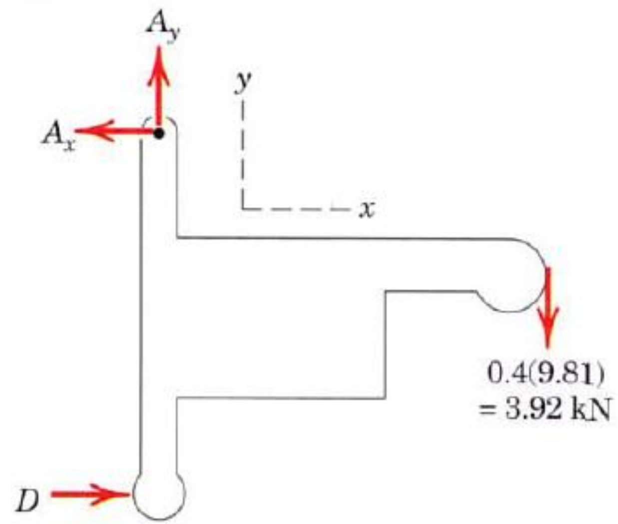
3 supporting members form a rigid non-collapsible assembly

Frame Statically Determinate Externally

Draw FBD of the entire frame

3 Equilibrium equations are available

Pay attention to sense of Reactions

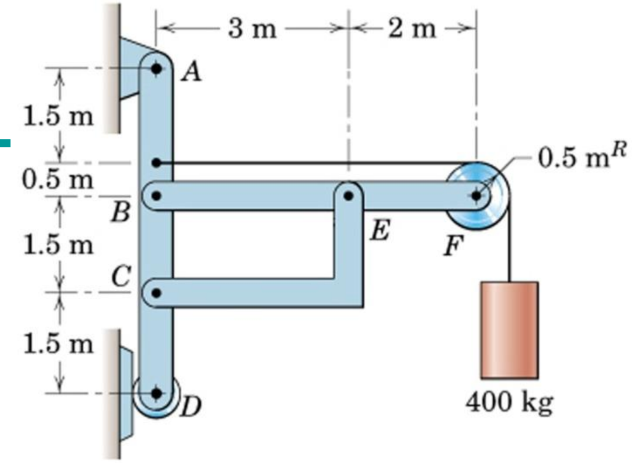


$[\Sigma M_A = 0]$	$5.5(0.4)(9.81) - 5D = 0$	$D = 4.32$ kN
$[\Sigma F_x = 0]$	$A_x - 4.32 = 0$	$A_x = 4.32$ kN
$[\Sigma F_y = 0]$	$A_y - 3.92 = 0$	$A_y = 3.92$ kN

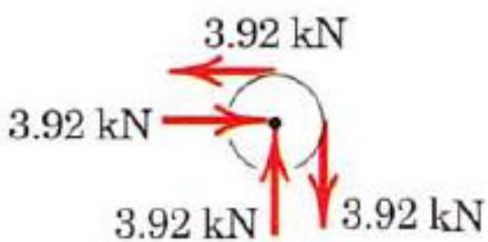
Frames and Machines

Example Solution: Dismember the frame and draw separate FBDs of each member

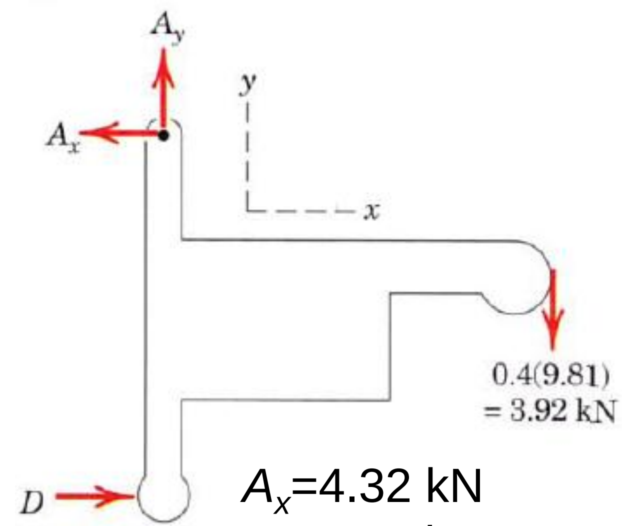
- show loads and reactions on each member due to connecting members (interaction forces)



Begin with FBD of Pulley



Then draw FBD of Members BF, CE, and AD

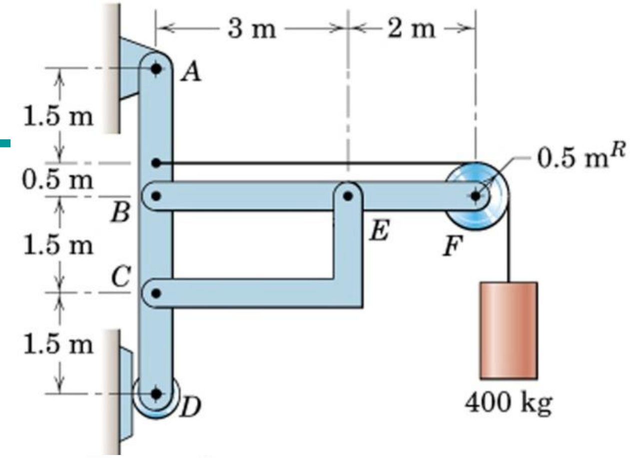


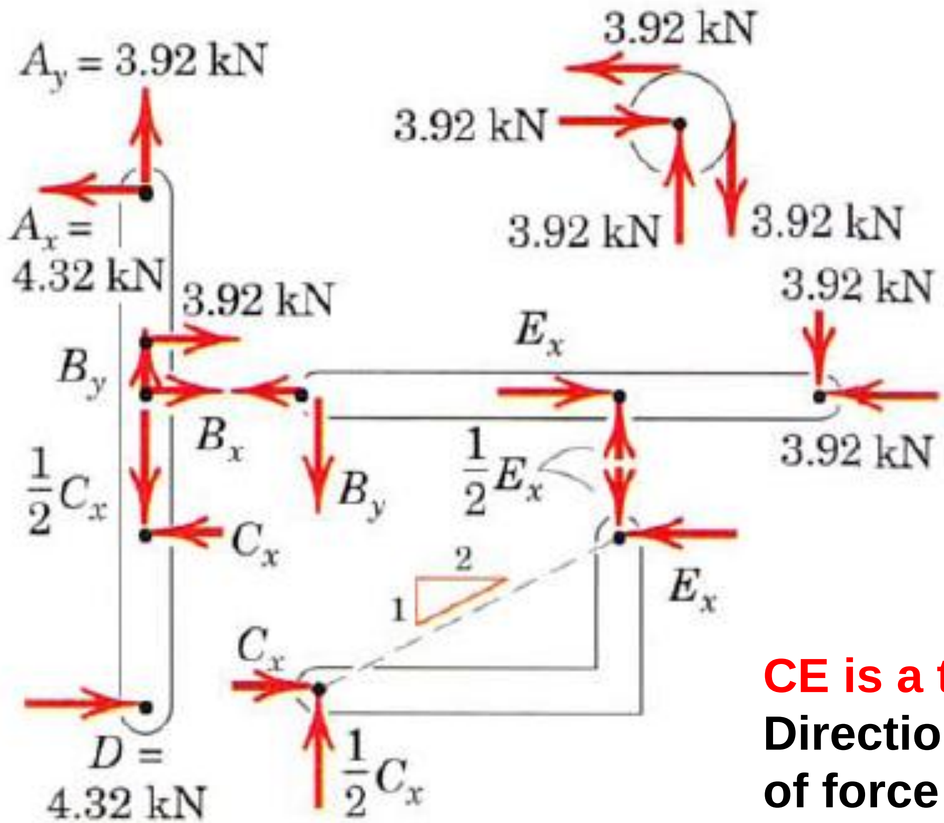
A_x=4.32 kN
A_y=3.92 kN
D=4.32 kN

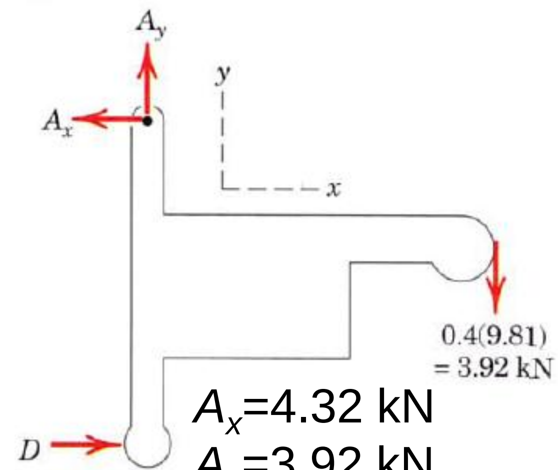
Frames and Machines

Example Solution:

FBDs







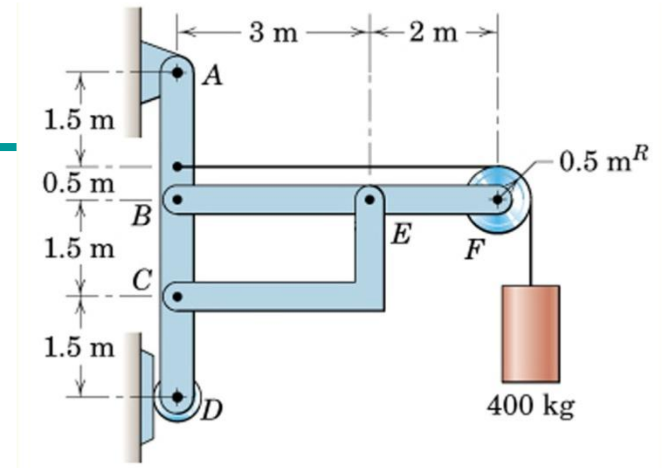
A_x=4.32 kN
A_y=3.92 kN
D=4.32 kN

CE is a two-force member

Direction of the line joining the two points of force application determines the direction of the forces acting on a two-force member.

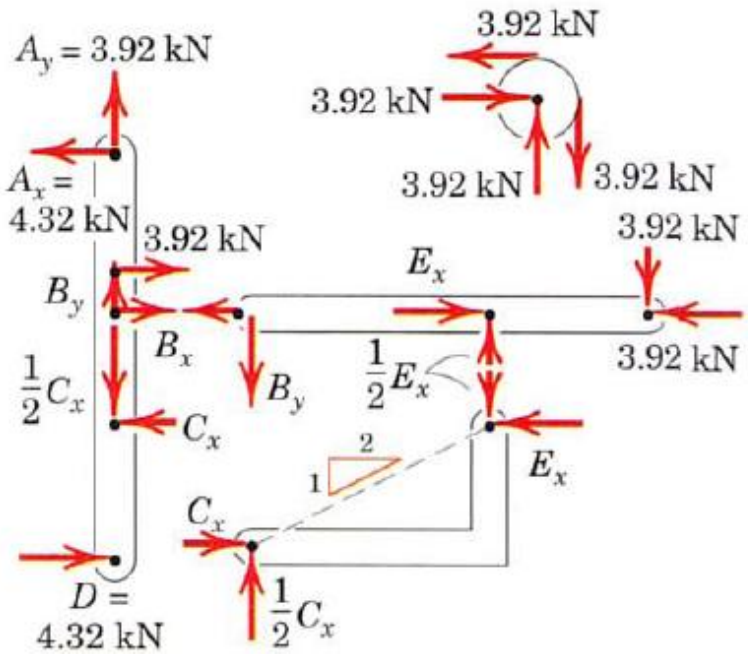
Shape of the member is not important.

Frames and Machines



Example Solution:

Find unknown forces from equilibrium



Member BF

$$[\Sigma M_B = 0] \quad 3.92(5) - \tfrac{1}{2}E_x(3) = 0 \quad E_x = 13.08 \text{ kN}$$

$$[\Sigma F_y = 0] \quad B_y + 3.92 - 13.08/2 = 0 \quad B_y = 2.62 \text{ kN}$$

$$[\Sigma F_x = 0] \quad B_x + 3.92 - 13.08 = 0 \quad B_x = 9.15 \text{ kN}$$

Member CE

[∑Fx = 0] $\quad C_x = E_x = 13.08$ kN

Checks:

$$[\Sigma M_C = 0] \quad 4.32(3.5) + 4.32(1.5) - 3.92(2) - 9.15(1.5) = 0$$

$$[\Sigma F_x = 0] \quad 4.32 - 13.08 + 9.15 + 3.92 + 4.32 = 0$$

$$[\Sigma F_y = 0] \quad -13.08/2 + 2.62 + 3.92 = 0$$