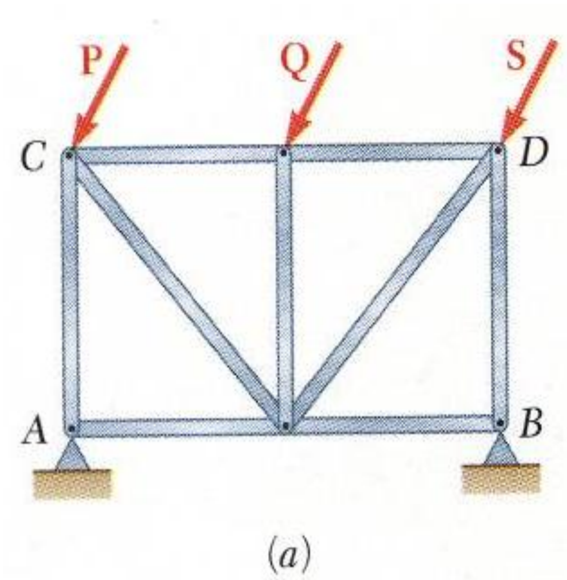
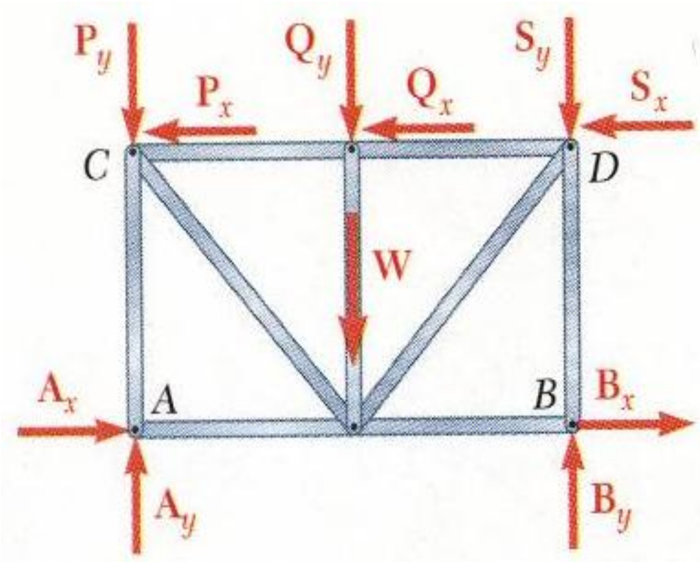
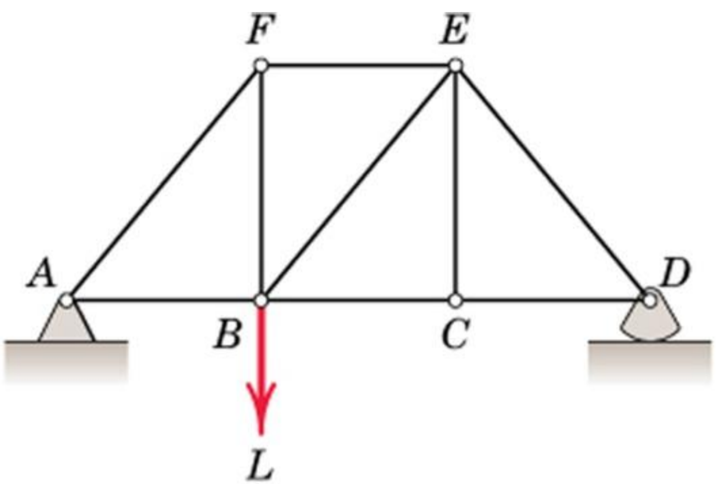
Statically Indeterminate Structure





: More unknowns than equations: Statically Indeterminate

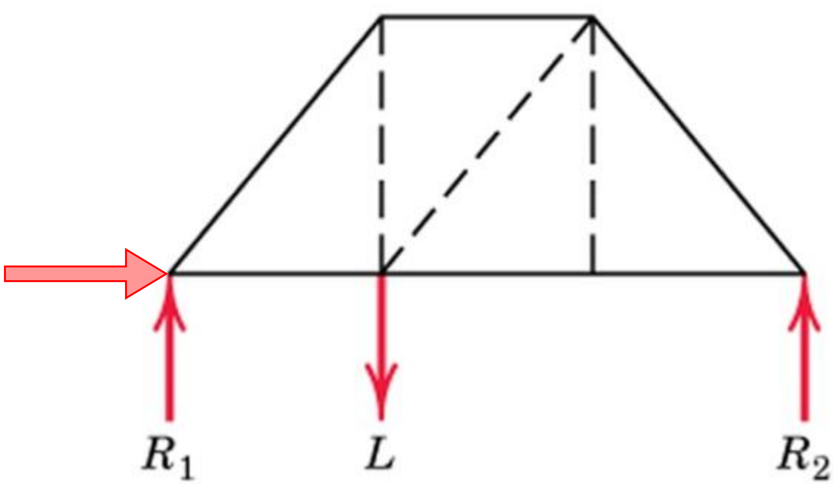
Plane Truss :: Determinacy



No. of unknown reactions = **3**

No. of equilibrium equations = **3**

: **Statically Determinate *(External)***



No. of members (m) = 9

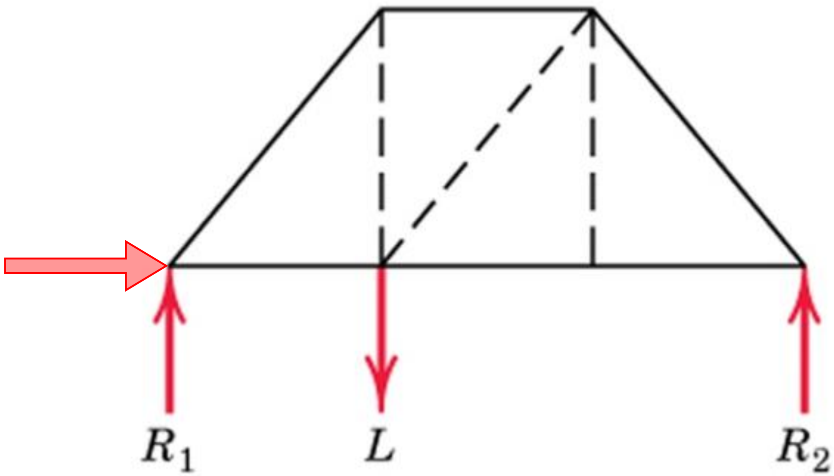
No. of joints (j) = 6

No. of unknown reactions (R) = 3

$\therefore$ m + R = 2j

: **Statically Determinate *(Internal)***

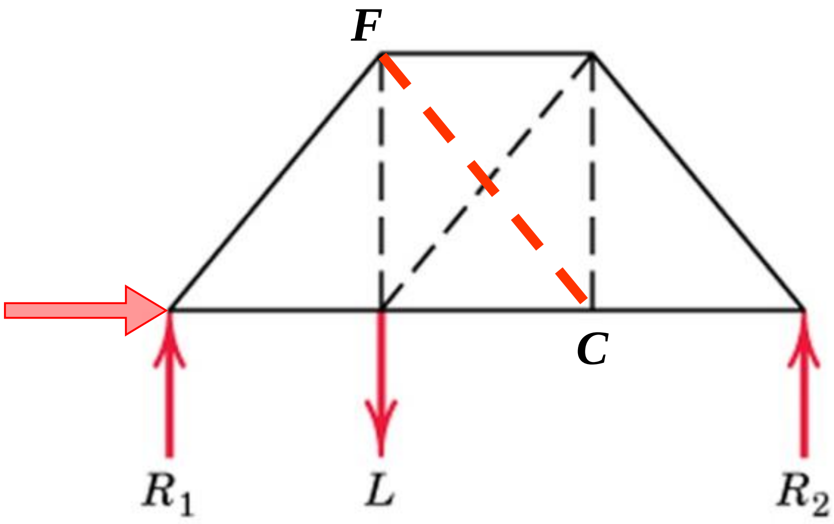
Plane Truss :: Determinacy



Presence of internal members

: **Additional sharing for forces**

: **Additional Stability**



Further addition of internal members

: **Strengthening** of Joints C and F

: **Additional Stability and force sharing**

: **m + R > 2j**

: **Statically Indeterminate *(Internal)***

Plane Truss :: Determinacy

When more number of members/supports are present than are needed to prevent collapse/stability

→ Statically Indeterminate Truss

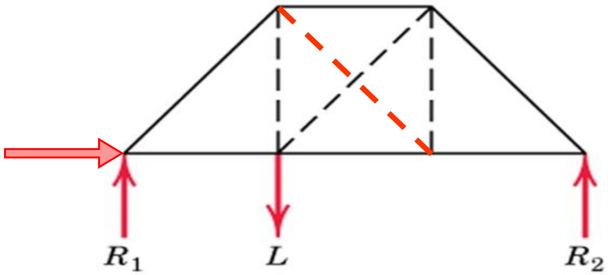
- cannot be analysed using equations of equilibrium alone!
- additional members or supports which are not necessary for maintaining the equilibrium configuration → Redundant

External and **Internal Redundancy**

Extra Supports than required → **External Redundancy**

– Degree of indeterminacy from available equilibrium equations

Extra Members than required → **Internal Redundancy**



Plane Truss :: Determinacy

Internal Redundancy or
Degree of Internal Static Indeterminacy

Extra Members than required → Internal Redundancy

Equilibrium of each joint can be specified by two scalar force equations → 2j equations for a truss with "j" number of joints

→ Known Quantities

For a truss with "m" number of two force members, and maximum 3 unknown support reactions → Total Unknowns = m + 3
("m" member forces and 3 reactions for externally determinate truss)

m + 3 = 2j → Statically Determinate Internally
m + 3 > 2j → Statically Indeterminate Internally
m + 3 < 2j → Unstable Truss

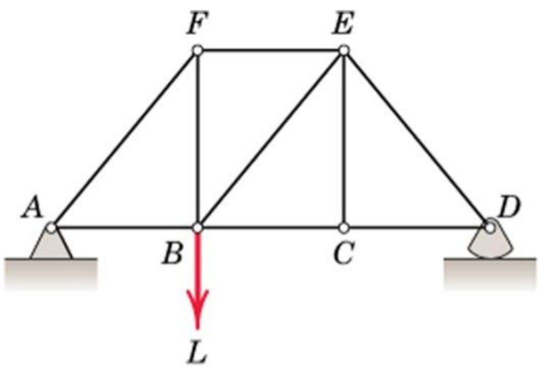
Plane Truss :: Analysis Methods

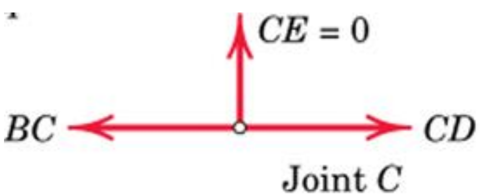
Why to Provide Redundant Members?

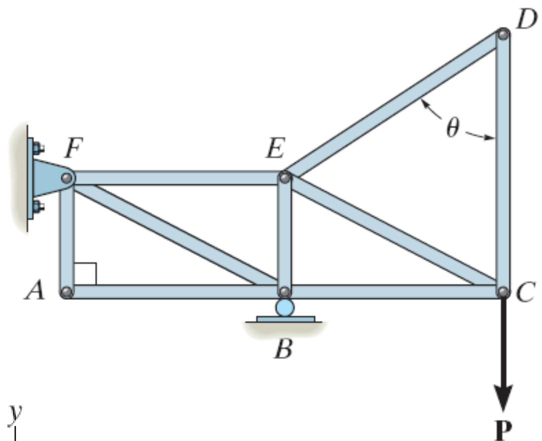
- To maintain alignment of two members during construction
- To increase stability during construction
- To maintain stability during loading (Ex: to prevent buckling of compression members)
- To provide support if the applied loading is changed
- To act as backup members in case some members fail or require strengthening
- Analysis is difficult but possible

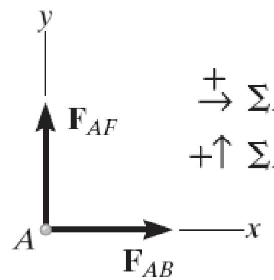
Plane Truss :: Analysis Methods

Zero Force Members



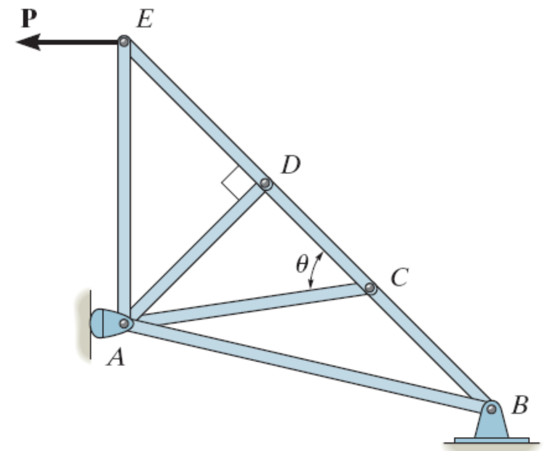


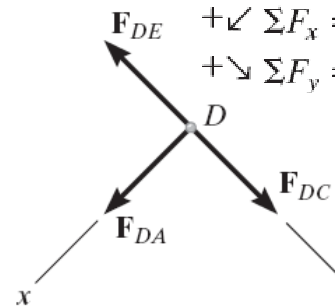




$\xrightarrow{+} \Sigma F_x = 0;\ F_{AB} = 0$

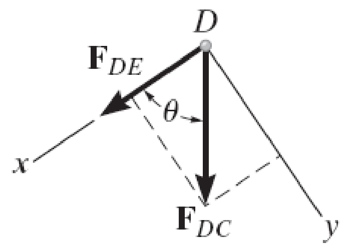
$+\uparrow \Sigma F_y = 0;\ F_{AF} = 0$





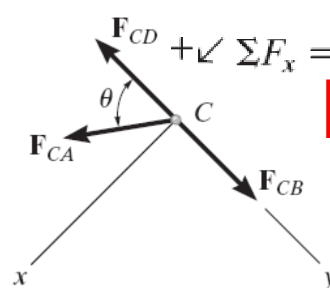
$+\swarrow \Sigma F_x = 0;\ F_{DA} = 0$

$+\searrow \Sigma F_y = 0;\ F_{DC} = F_{DE}$



$+\searrow \Sigma F_y = 0;\ F_{DC} \sin\theta = 0;\ F_{DC} = 0$ since $\sin\theta \neq 0$

$+\swarrow \Sigma F_x = 0;\ F_{DE} + 0 = 0;\ F_{DE} = 0$



$+\swarrow \Sigma F_x = 0;\ F_{CA} \sin\theta = 0$

$F_{CA} = 0$

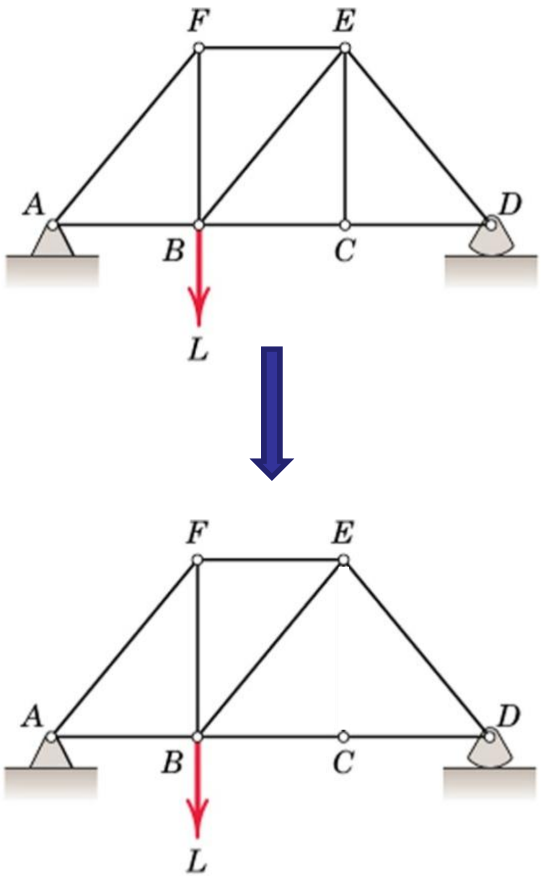
since $\sin\theta \neq 0$

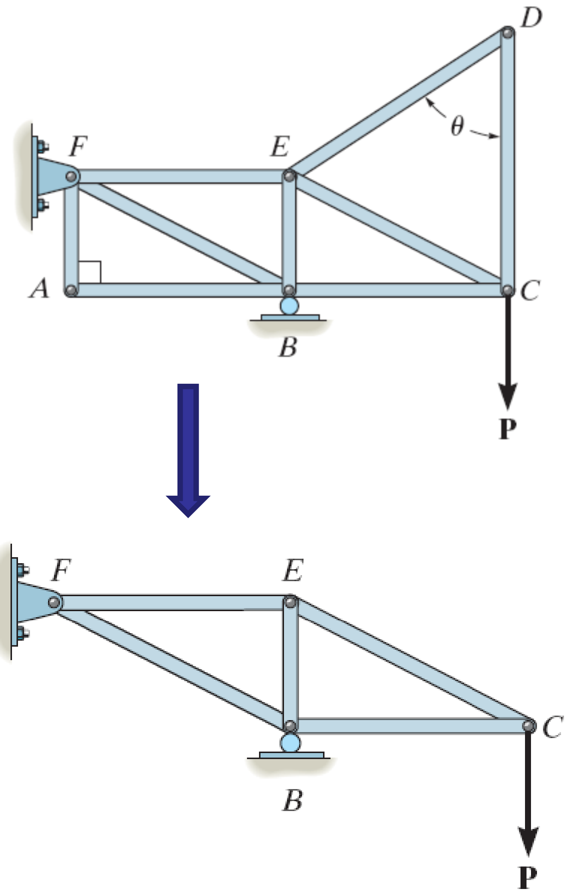
$+\searrow \Sigma F_y = 0;$

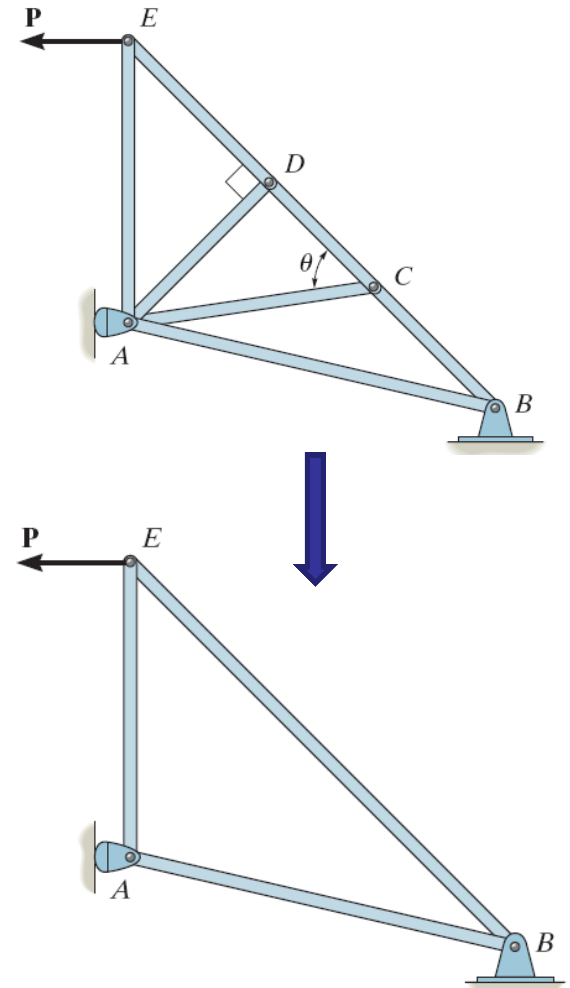
$F_{CB} = F_{CD}$

Plane Truss :: Analysis Methods

Zero Force Members: Simplified Structures





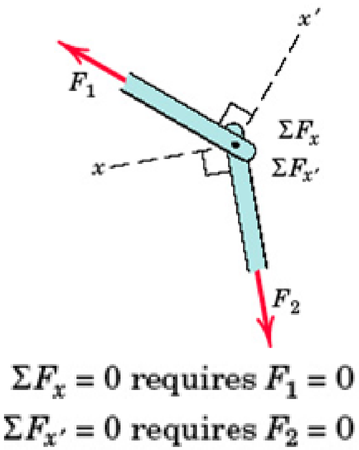


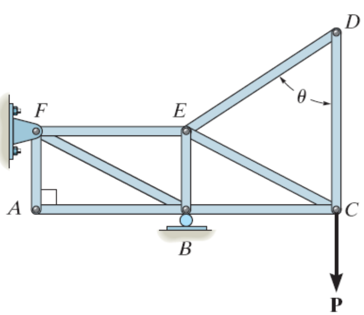
Plane Truss :: Analysis Methods

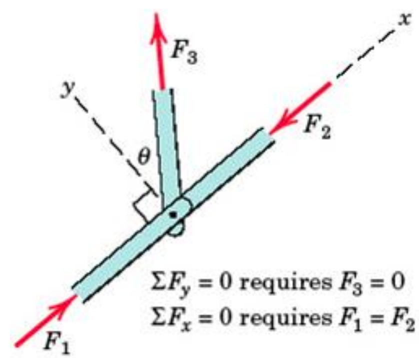
Zero Force Members: Conditions

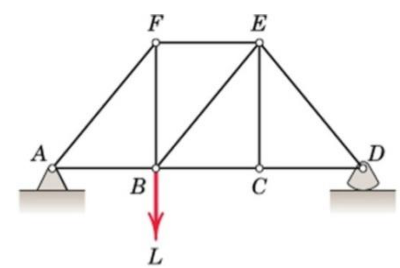
• if only two noncollinear members form a truss joint and no external load or support reaction is applied to the joint, the two members must be zero force members

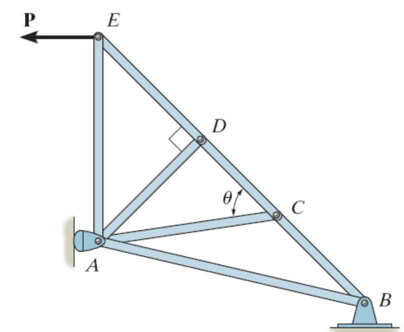
•if three members form a truss joint for which two of the members are collinear, the third member is a zero-force member provided no external force or support reaction is applied to the joint







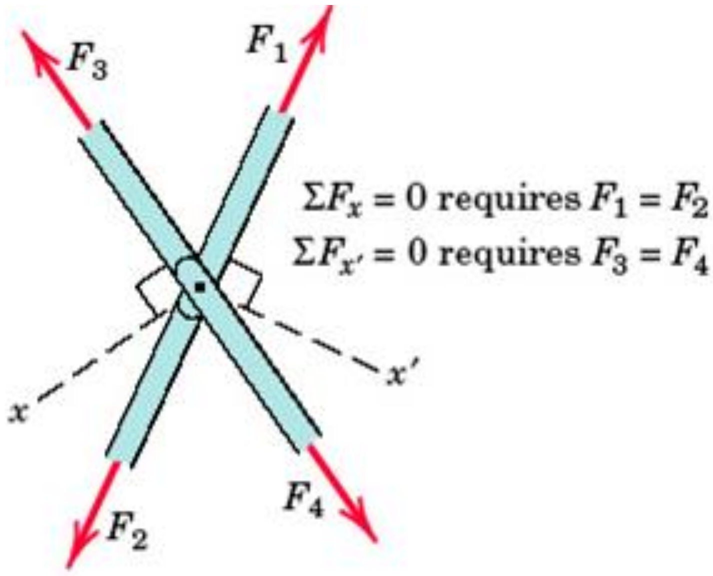




Structural Analysis: Plane Truss

Special Condition

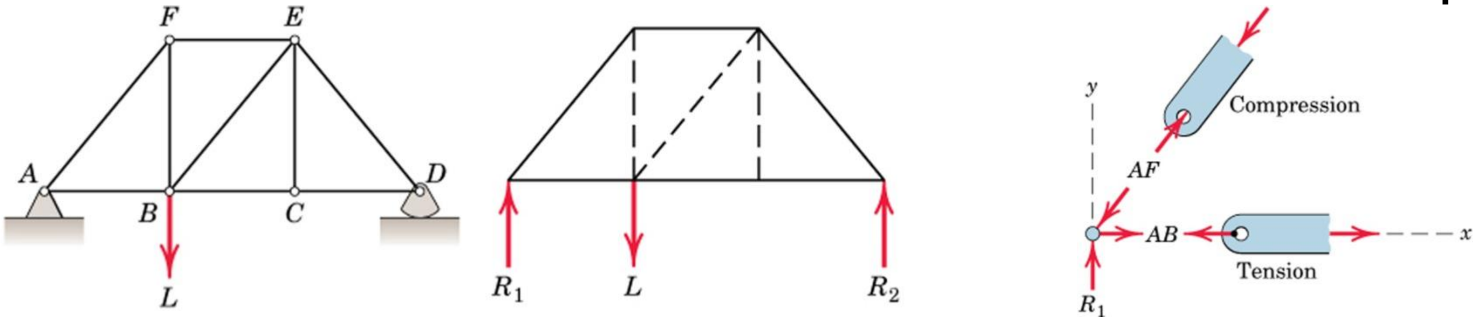
- When two pairs of collinear members are joined as shown in figure, the forces in each pair must be equal and opposite.



Plane Truss :: Analysis Methods

Method of Joints

- Start with any joint where at least one known load exists and where not more than two unknown forces are present.



FBD of Joint A and members AB and AF: Magnitude of forces denoted as *AB* & *AF*

- Tension indicated by an arrow away from the pin
- Compression indicated by an arrow toward the pin

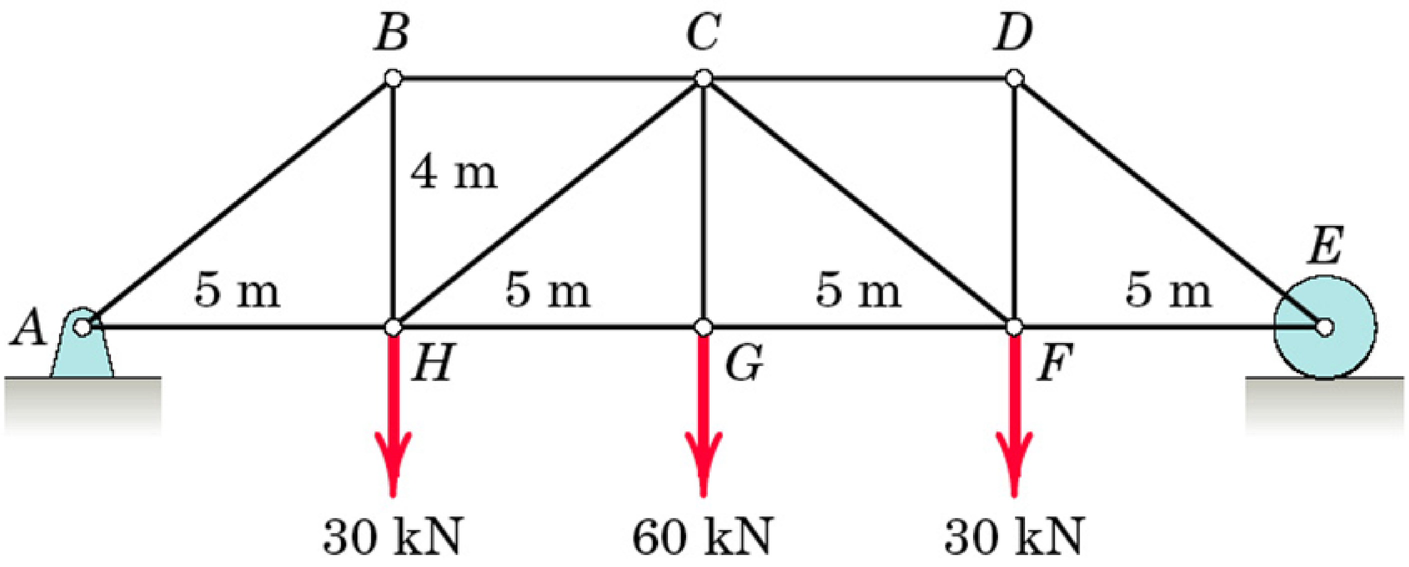
Magnitude of AF from $\sum F_y = 0$

Magnitude of AB from $\sum F_x = 0$

Analyze joints F, B, C, E, & D in that order to complete the analysis

Method of Joints: Example

Determine the force in each member of the loaded truss by Method of Joints.

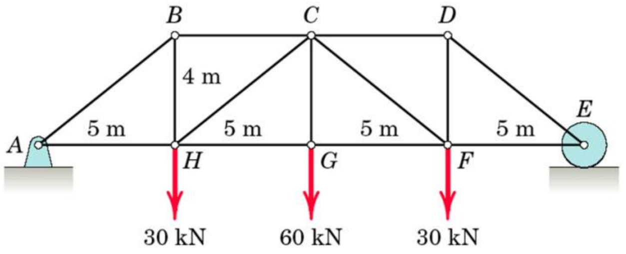


Is the truss statically determinant externally? Yes

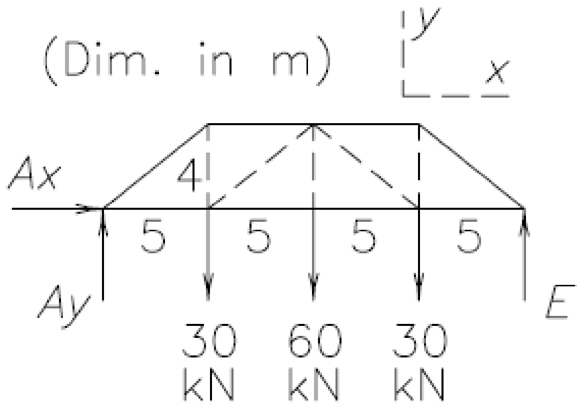
Is the truss statically determinant internally? Yes

Are there any Zero Force Members in the truss? No

Method of Joints: Example



Solution

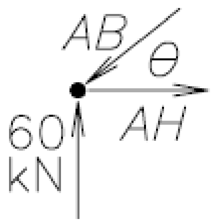


As a whole: $\Sigma F_x = 0 \Rightarrow A_x = 0$

$A_y = E = 60\text{ kN}$ by

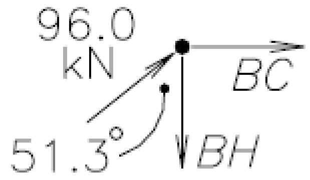
$\Sigma F_y = 0$ and symmetry.

Joint A: $\left(\theta = \tan^{-1}(4/5) = 38.7°\right)$



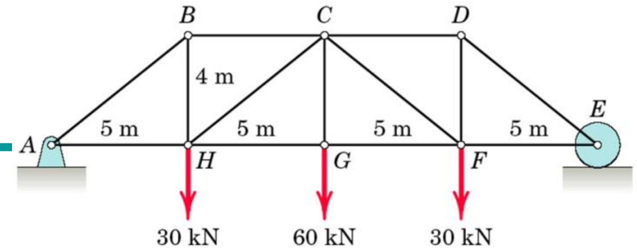
$$\begin{cases} \Sigma F_y = 0: 60 - AB\sin\theta = 0,\ \underline{AB = 96.0\text{ kN } C} \\ \Sigma F_x = 0: AH - 96.0\cos\theta,\ \underline{AH = 75\text{ kN } T} \end{cases}$$

Joint B:

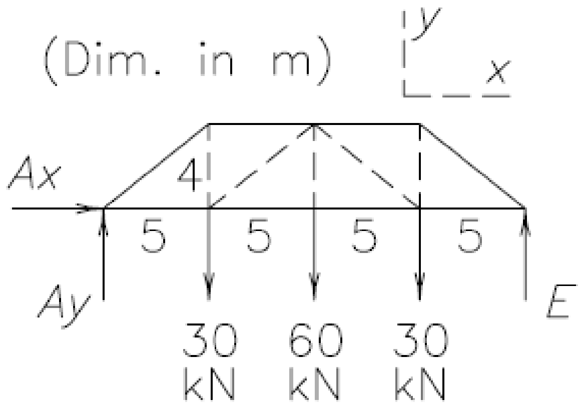


$$\begin{cases} \Sigma F_x = 0: BC + 96.0\sin 51.3° = 0,\ \underline{BC = -75\text{ kN } (C)} \\ \Sigma F_y = 0: -BH + 96.0\cos 51.3° = 0,\ \underline{BH = 60\text{ kN } T} \end{cases}$$

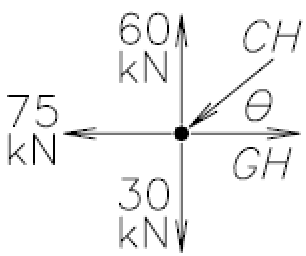
Method of Joints: Example



Solution

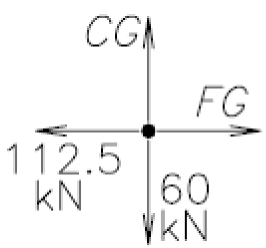


Joint *H*:



$$\begin{cases} \Sigma F_y = 0 : -CH\sin\theta + 30 = 0, \ \underline{CH = 48.0 \text{ kN } C} \\ \Sigma F_x = 0 : 48.0\cos\theta + GH - 75 = 0, \ \underline{GH = 112.5 \text{ kN } T} \end{cases}$$

Joint *G*:



$$\Sigma F_y = 0 \Rightarrow CG = 60 \text{ kN } T$$

By symmetry:

$FG = 112.5$ kN T, $CF = 48.0$ kN C

$CD = 75$ kN C, $DF = 60$ kN T

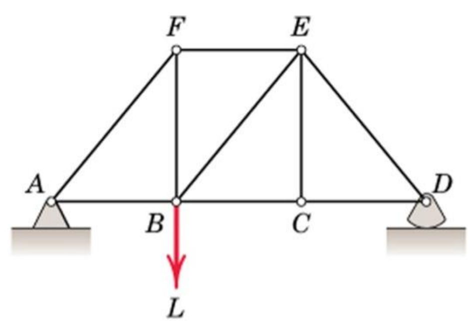
$EF = 75$ kN T, $DE = 96.0$ kN C

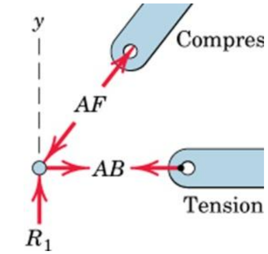
Structural Analysis: Plane Truss

Method of Joints: only two of three equilibrium equations were applied at each joint because the procedures involve concurrent forces at each joint

→Calculations from joint to joint

→More time and effort required



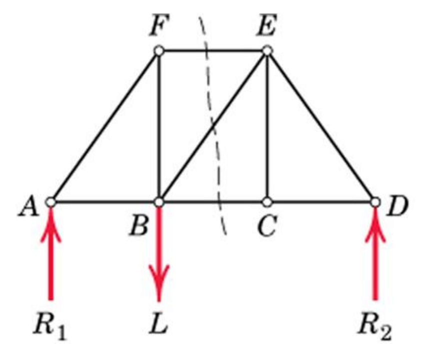


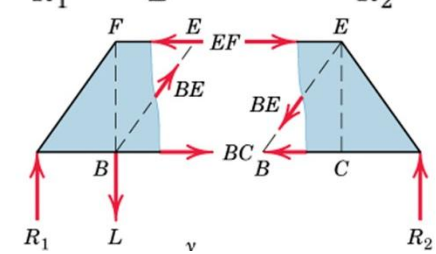
Method of Sections

Take advantage of the 3rd or moment equation of equilibrium by selecting an entire section of truss

→Equilibrium under non-concurrent force system

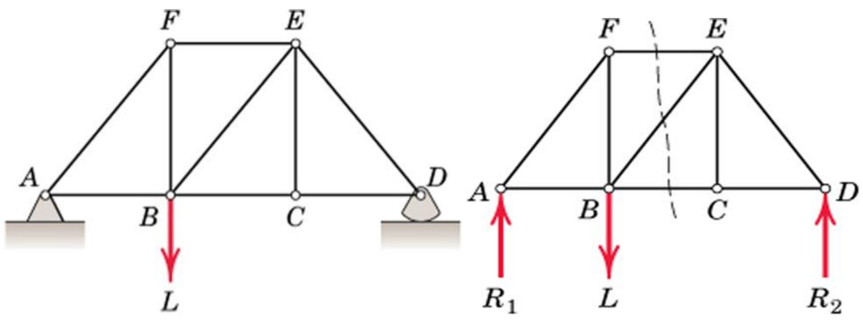
→Not more than 3 members whose forces are unknown should be cut in a single section since we have only 3 independent equilibrium equations



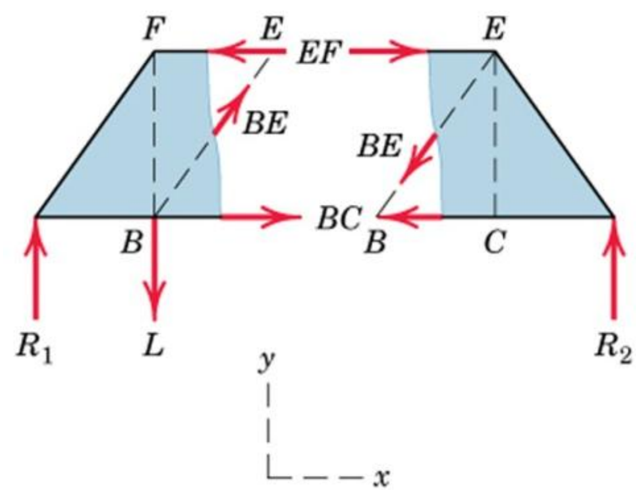


Structural Analysis: Plane Truss

Method of Sections



- Find out the reactions from equilibrium of whole truss
- To find force in member BE:
- Cut an imaginary section (dotted line)
- Each side of the truss section should remain in equilibrium
  - Apply to each cut member the force exerted on it by the member cut away
  - The left hand section is in equilibrium under *L*, R_1, *BC*, *BE* and *EF*
  - Draw the forces with proper senses (else assume)
    - Moment @ B → *EF*
    - $L > R_1$; → $\sum F_y=0$ → *BE*
    - Moment @ E and observation of whole truss → *BC*
  - Forces acting towards cut section → Compressive
  - Forces acting away from the cut section → Tensile



- Find *EF* from $\sum M_B=0$; Find *BE* from $\sum F_y=0$
- Find *BC* from $\sum M_E=0$

→ Each unknown has been determined independently of the other two

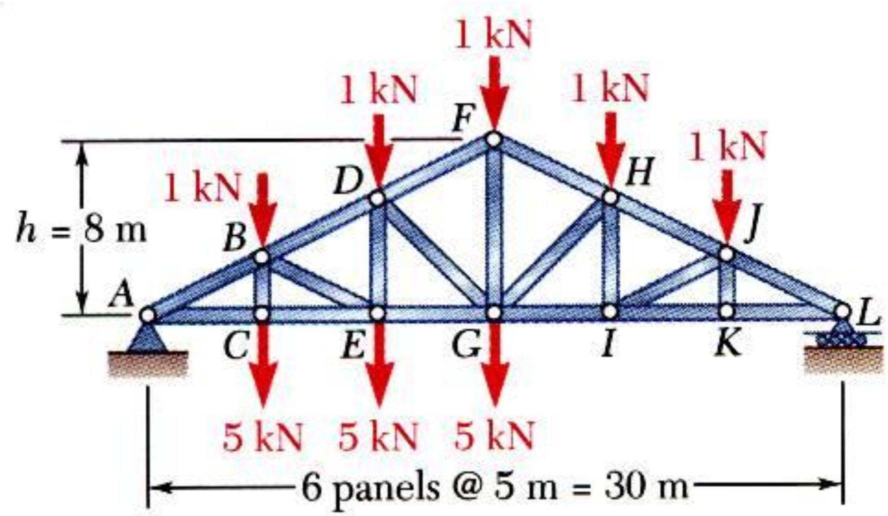
Structural Analysis: Plane Truss

Method of Sections

- Principle: If a body is in equilibrium, then any part of the body is also in equilibrium.
- Forces in few particular member can be directly found out quickly without solving each joint of the truss sequentially
- Method of Sections and Method of Joints can be conveniently combined
- A section need not be straight.
- More than one section can be used to solve a given problem

Structural Analysis: Plane Truss

Method of Sections: Example



Find out the internal forces in members FH, GH, and GI

Find out the reactions

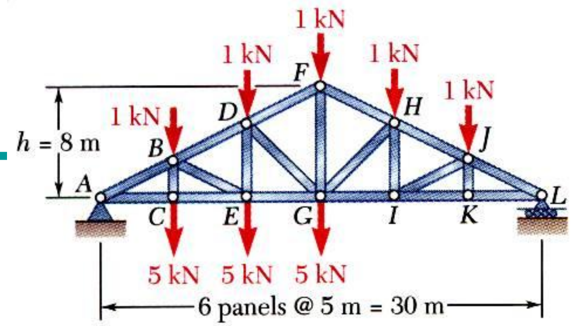
$$\sum M_A = 0 = -(5\text{ m})(6\text{ kN}) - (10\text{ m})(6\text{ kN}) - (15\text{ m})(6\text{ kN})$$
$$-(20\text{ m})(1\text{ kN}) - (25\text{ m})(1\text{ kN}) + (30\text{ m})L$$

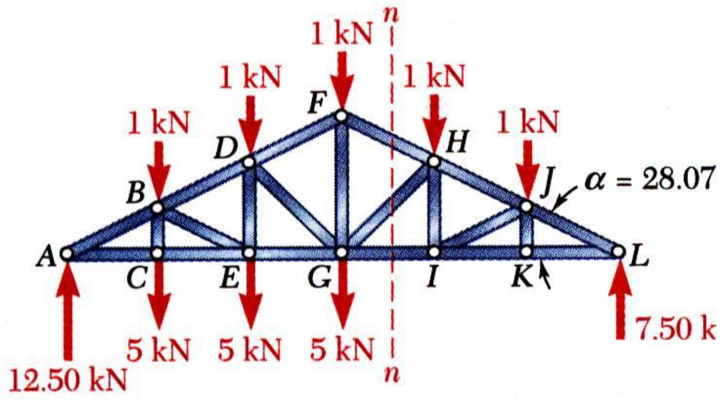
$$L = 7.5\text{ kN} \uparrow$$

$$\sum F_y = 0 = -20\text{ kN} + L + A$$

$$A = 12.5\text{ kN} \uparrow$$

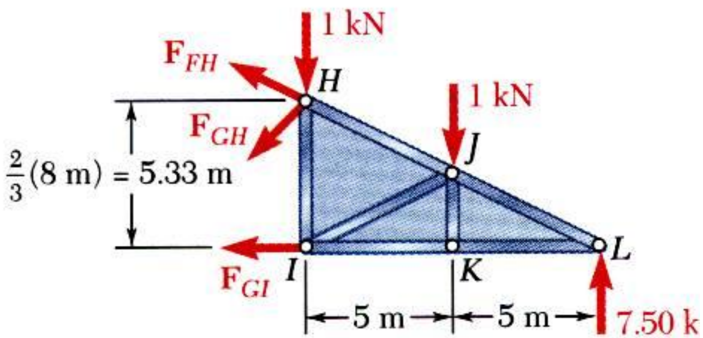
Method of Sections: Example Solution





- Pass a section through members FH, GH, and GI and take the right-hand section as a free body.

$$\tan\alpha = \frac{FG}{GL} = \frac{8\text{ m}}{15\text{ m}} = 0.5333 \qquad \alpha = 28.07°$$



- Apply the conditions for static equilibrium to determine the desired member forces.

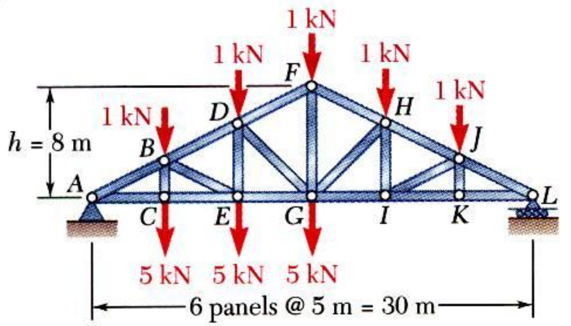
$$\sum M_H = 0$$

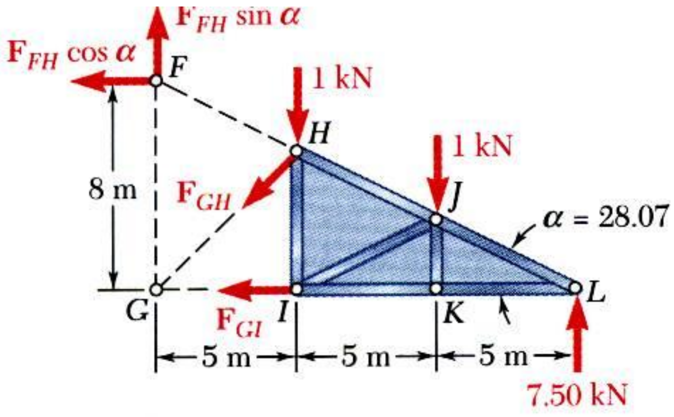
$$(7.50\text{ kN})(10\text{ m}) - (1\text{ kN})(5\text{ m}) - F_{GI}(5.33\text{ m}) = 0$$

$$F_{GI} = +13.13\text{ kN}$$

$$\boxed{F_{GI} = 13.13\text{ kN} \;\; T}$$

Method of Sections: Example Solution



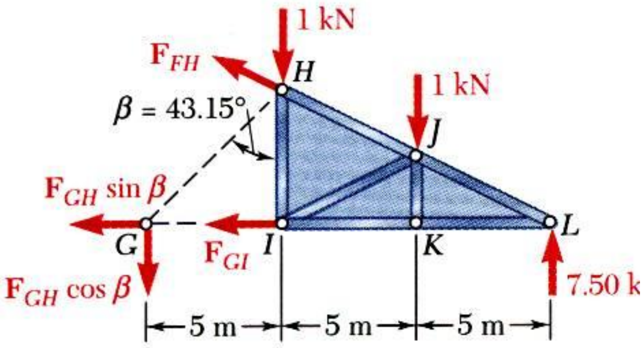


$$\sum M_G = 0$$

$$(7.5\,\text{kN})(15\,\text{m}) - (1\,\text{kN})(10\,\text{m}) - (1\,\text{kN})(5\,\text{m}) + (F_{FH}\cos\alpha)(8\,\text{m}) = 0$$

$$F_{FH} = -13.82\,\text{kN}$$

$$F_{FH} = 13.82\,\text{kN} \;\; C$$



$$\tan\beta = \frac{GI}{HI} = \frac{5\,\text{m}}{\frac{2}{3}(8\,\text{m})} = 0.9375 \qquad \beta = 43.15^\circ$$

$$\sum M_L = 0$$

$$(1\,\text{kN})(10\,\text{m}) + (1\,\text{kN})(5\,\text{m}) + (F_{GH}\cos\beta)(15\,\text{m}) = 0$$

$$F_{GH} = -1.371\,\text{kN}$$

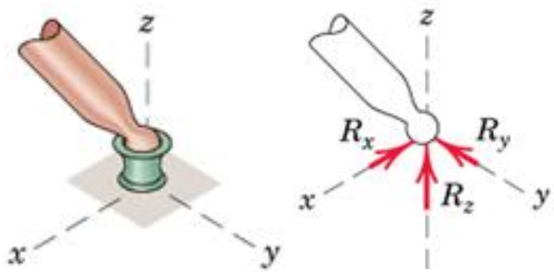
$$F_{GH} = 1.371\,\text{kN} \;\; C$$

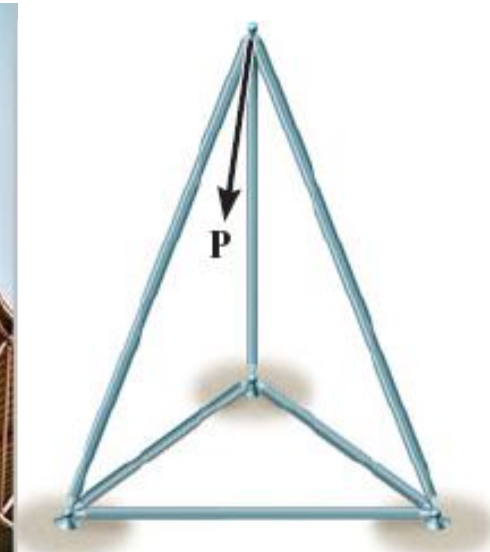
Structural Analysis: Space Truss

Space Truss

3-D counterpart of the Plane Truss

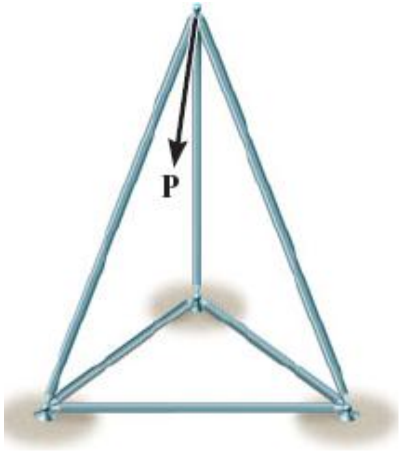
Idealized Space Truss → Rigid links connected at their ends by ball and socket joints



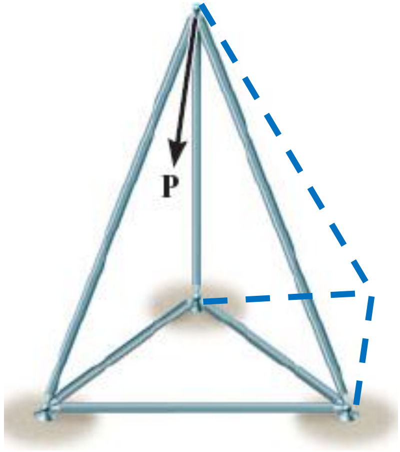


Structural Analysis: Space Truss

Space Truss



- 6 bars joined at their ends to form the edges of a tetrahedron as the basic non-collapsible unit
- 3 additional concurrent bars whose ends are attached to three joints on the existing structure are required to add a new rigid unit to extend the structure.



A space truss formed in this way is called a Simple Space Truss

If center lines of joined members intersect at a point
→Two force members assumption is justified
→ Each member under Compression or Tension

Structural Analysis: Space Truss

Static Determinacy of Space Truss

Six equilibrium equations available to find out support reactions
→ if these are sufficient to determine all support reactions
→ The space truss is Statically Determinate Externally

$$\Sigma F_x = 0 \quad \Sigma M_x = 0$$
$$\Sigma F_y = 0 \quad \Sigma M_y = 0$$
$$\Sigma F_z = 0 \quad \Sigma M_z = 0$$

Equilibrium of each joint can be specified by three scalar force equations
→ 3j equations for a truss with "j" number of joints
→ Known Quantities

For a truss with "m" number of two force members, and maximum 6 unknown support reactions → Total Unknowns = m + 6
("m" member forces and 6 reactions for externally determinate truss)
Therefore:

m + 6 = 3j → Statically Determinate Internally
m + 6 > 3j → Statically Indeterminate Internally
m + 6 < 3j → Unstable Truss

A necessary condition for Stability but not a sufficient condition since one or more members can be arranged in such a way as not to contribute to stable configuration of the entire truss