Mechanics: Scalars and Vectors

- Scalar
  - Only **magnitude** is associated with it
    - *e.g.*, time, volume, density, <u>speed</u>, energy, mass etc.
- Vector
  - Possess **direction** as well as **magnitude**
  - Parallelogram law of addition (and the triangle law)
  - *e.g.*, displacement, velocity, acceleration etc.
- Tensor
  - *e.g.*, stress (3×3 components)

Mechanics: Scalars and Vectors

A Vector **V** can be written as: $\mathbf{V} = V\mathbf{n}$

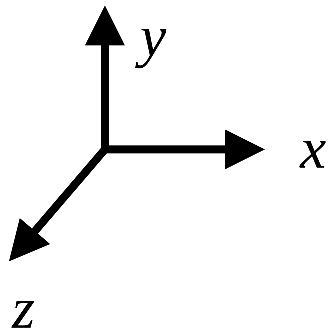
V = magnitude of **V**

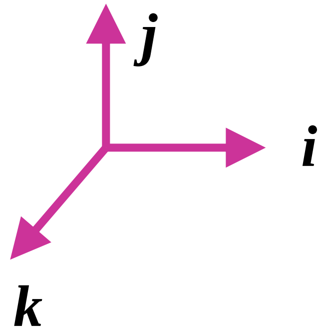
n = unit vector whose magnitude is one and whose direction coincides with that of **V**

Unit vector can be formed by dividing any vector, such as the geometric position vector, by its length or magnitude

Vectors represented by Bold and Non-Italic letters (**V**)

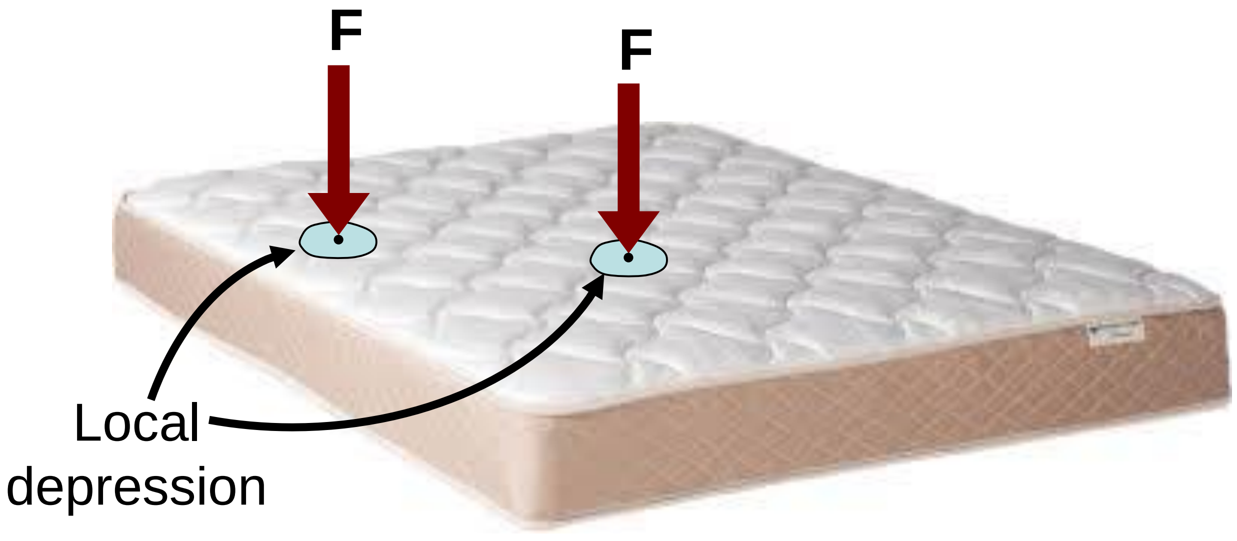
Magnitude of vectors represented by Non-Bold, Italic letters (*V*)





Types of Vectors: Fixed Vector

- **Fixed Vector**
  - Constant magnitude and direction
    - **Unique point of application**
  - *e.g.*, force on a deformable body

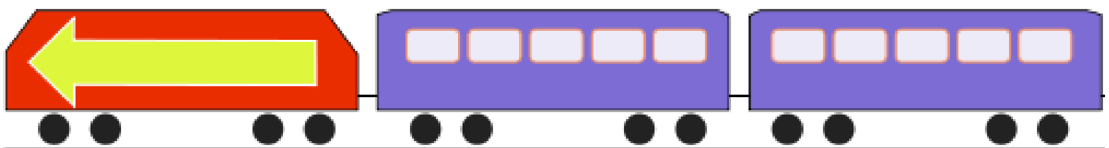


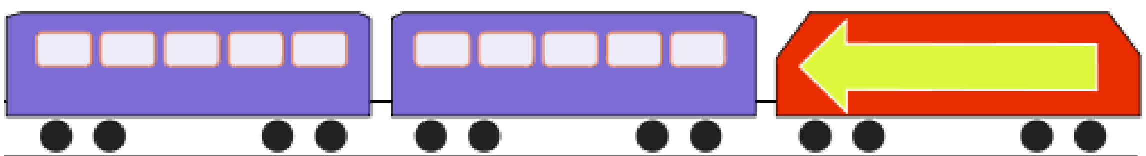
  - *e.g.*, force on a given particle

Types of Vectors: Sliding Vector

- **Sliding Vector**
  - Constant magnitude and direction
    - **Unique line of action**
      - **“Slide”** along the line of action
    - **No unique point of application**

Force on coach **F**

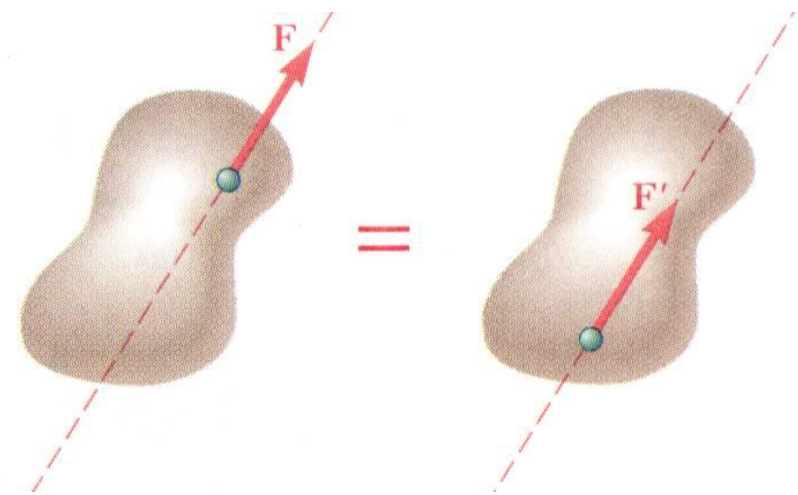


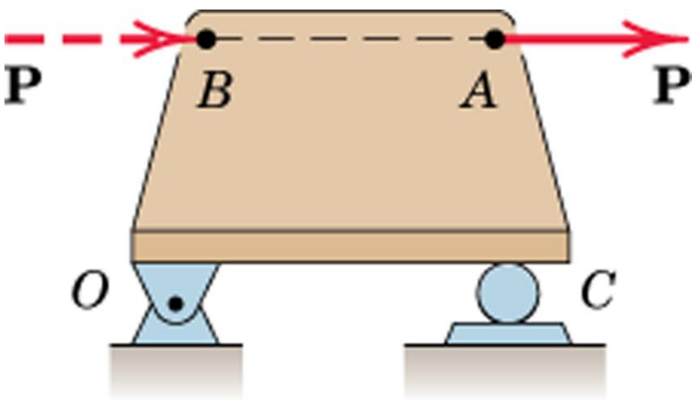


Force on coach **F**

Types of Vectors: Sliding Vector

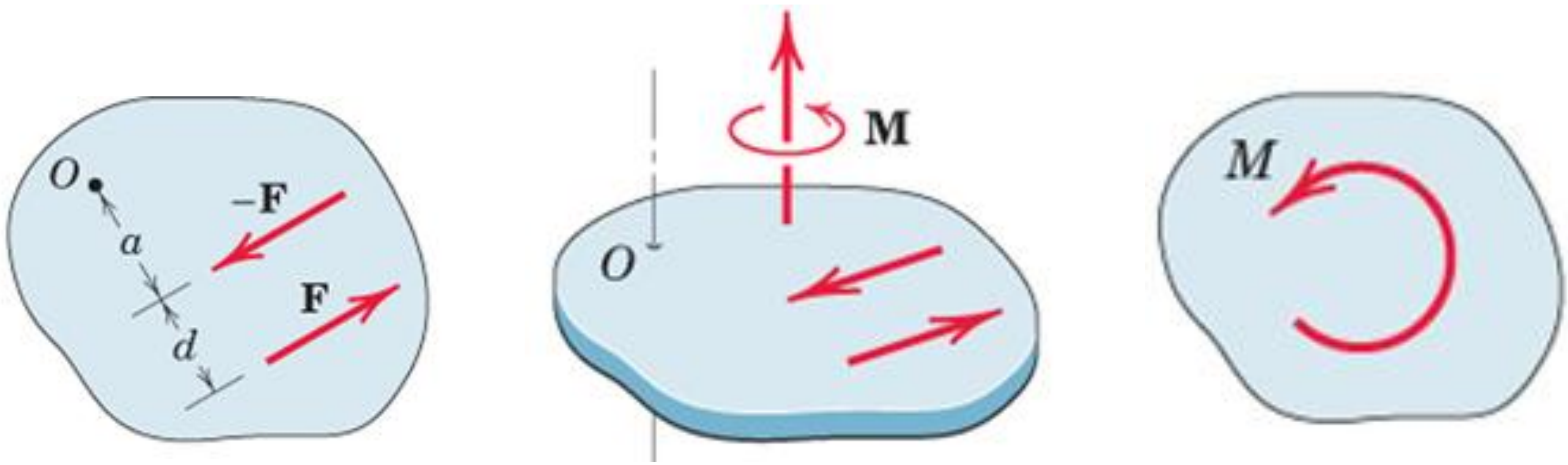
- **Sliding Vector**
  - Principle of Transmissibility
    - **Application of force** at **any point** along a **particular line of action**
      - **No change** in **resultant external effects** of the force





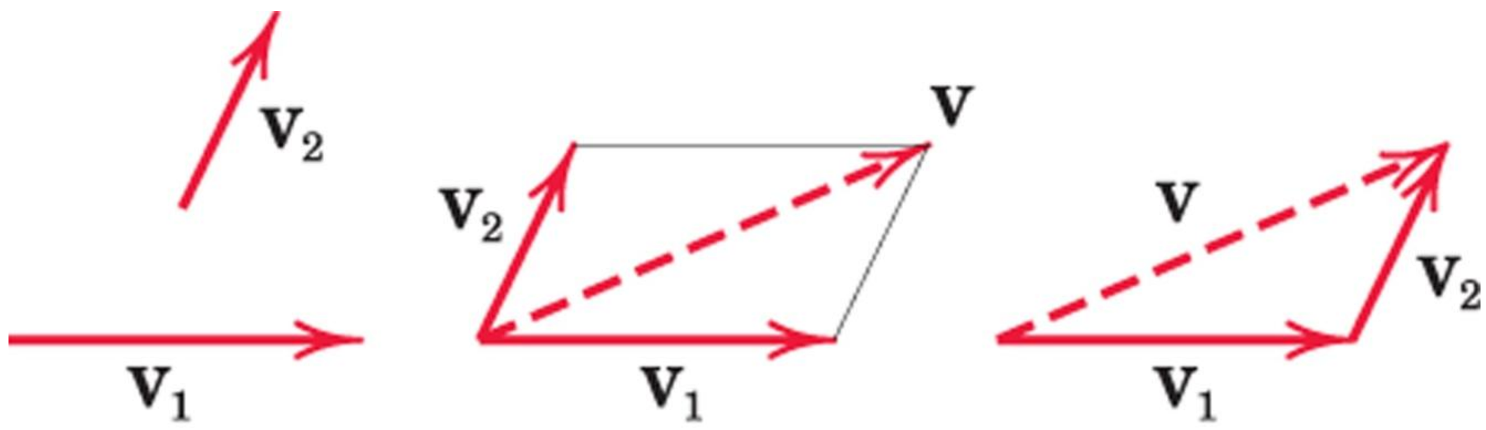
Types of Vectors: Free Vector

- **Free Vector**
  - Freely movable in space
    - **No unique line of action**
      - **No unique point of application**
  - *e.g.*, moment of a couple



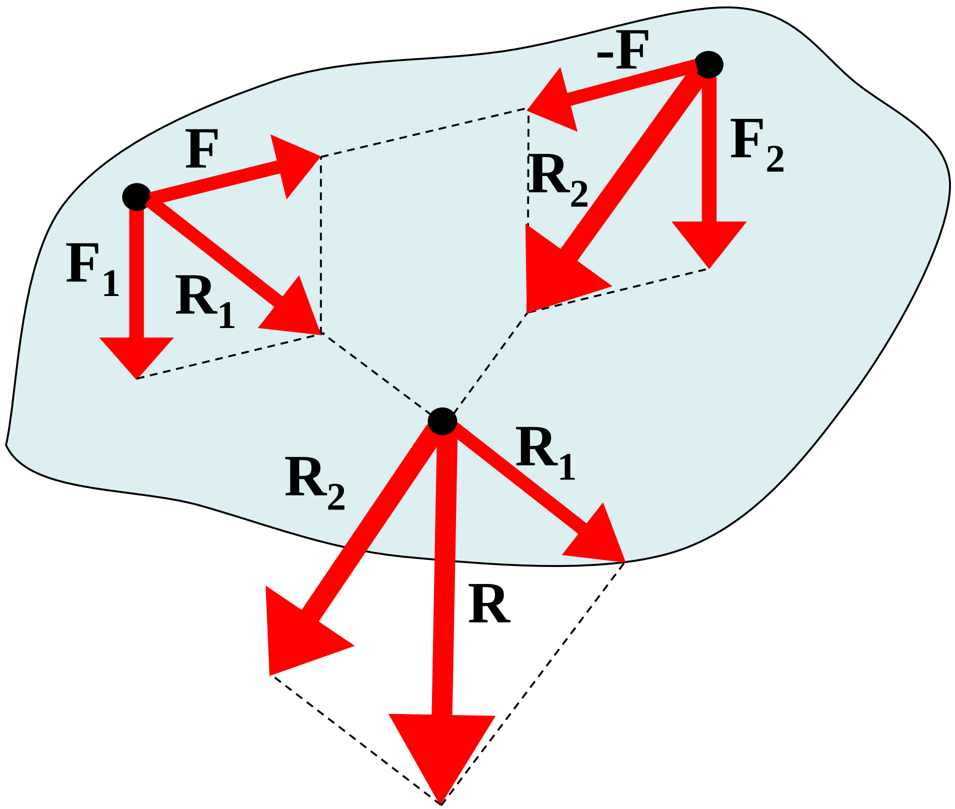
Vectors: Rules of addition

- **Parallelogram Law**
  - Equivalent vector represented by the diagonal of a parallelogram
    - $\mathbf{V} = \mathbf{V_1} + \mathbf{V_2}$ *(Vector Sum)*
    - $V \neq V_1 + V_2$ (Scalar sum)



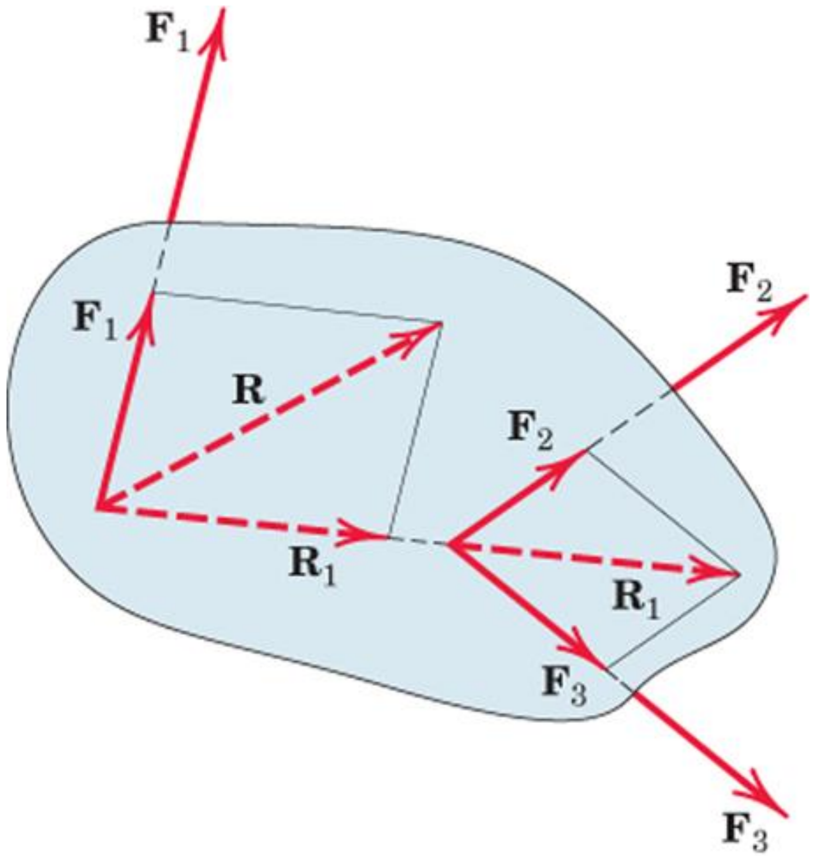
Vectors: Parallelogram law of addition

- Addition of two parallel vectors
  - $\mathbf{F_1} + \mathbf{F_2} = \mathbf{R}$



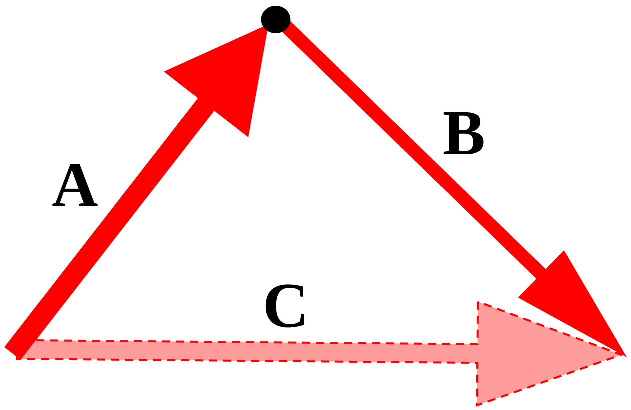
Vectors: Parallelogram law of addition

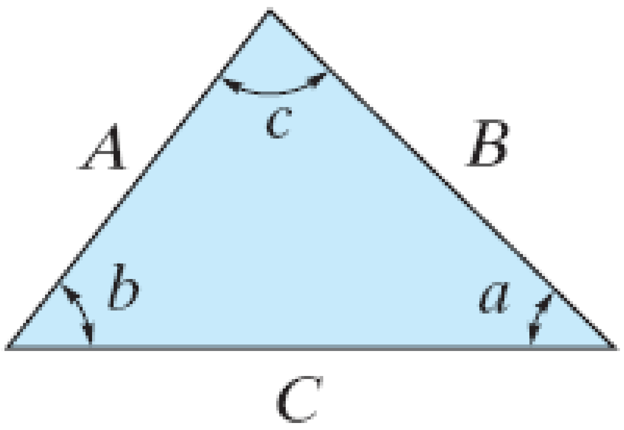
- Addition of 3 vectors
  - $\mathbf{F_1} + \mathbf{F_2} + \mathbf{F_3} = \mathbf{R}$



Vectors: Rules of addition

- **Trigonometric Rule**
  - Law of Sines
  - Law of Cosine





Sine law:

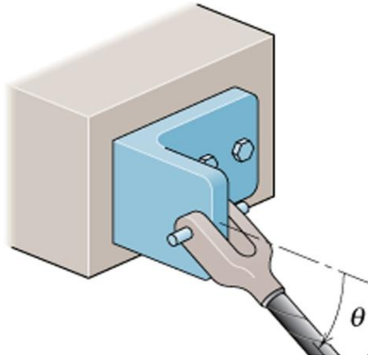
$$\frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}$$

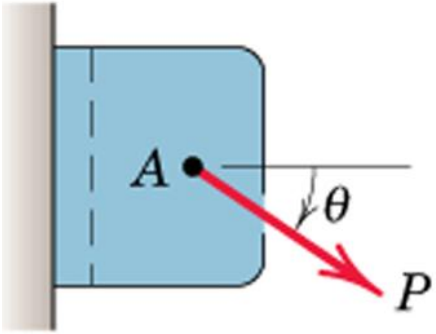
Cosine law:

$$C = \sqrt{A^2 + B^2 - 2AB \cos c}$$

Force Systems

Force Systems



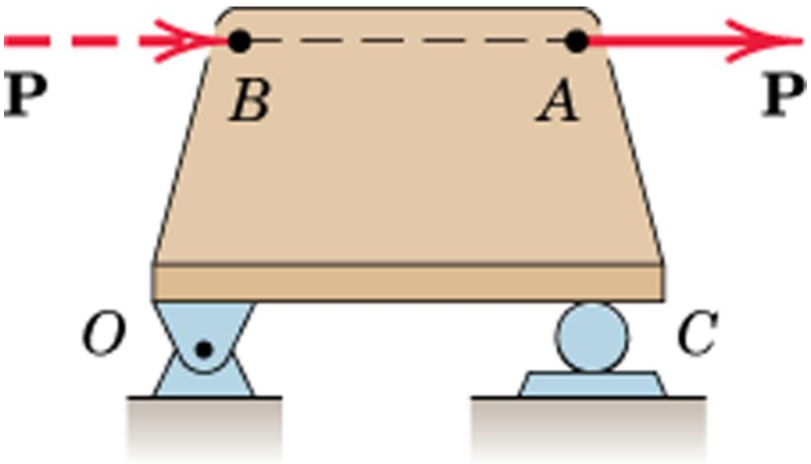


Cable Tension **P**

- **Force**: Represented by vector
  - Magnitude, direction, point of application
  - **P**: fixed vector (or sliding vector??)
  - External Effect
    - Applied force; Forces exerted by bracket, bolts, Foundation (reactive force)

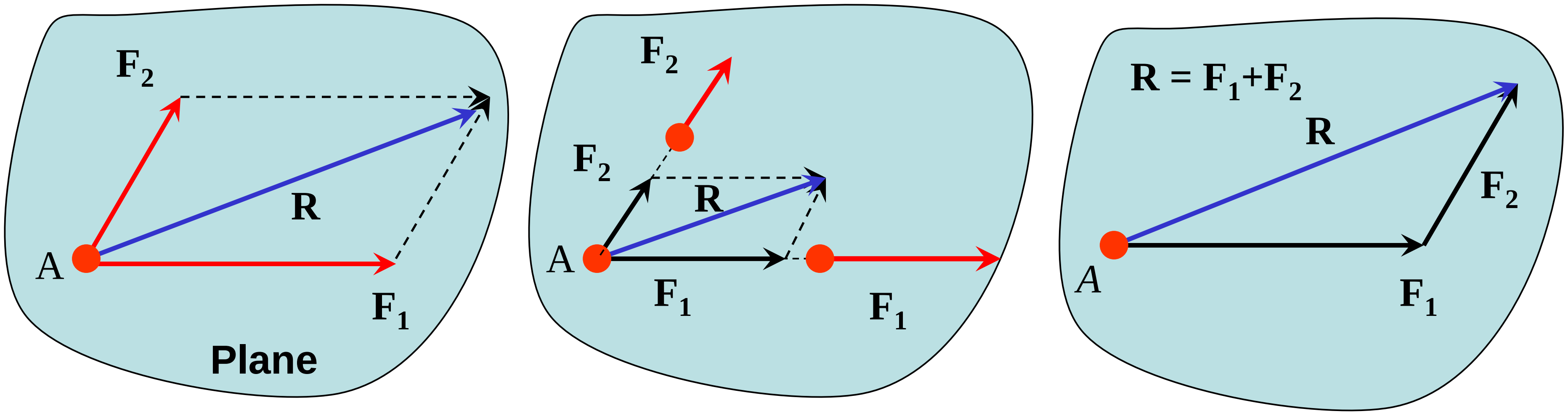
Force Systems

- **Rigid Bodies**
  - External effects only
    - **Line of action** of force is **important**
      - Not its point of application
      - Force as **sliding vector**



Force Systems

- **Concurrent forces**
  - Lines of action intersect at a point



Concurrent Forces

F_1 and **F_2**

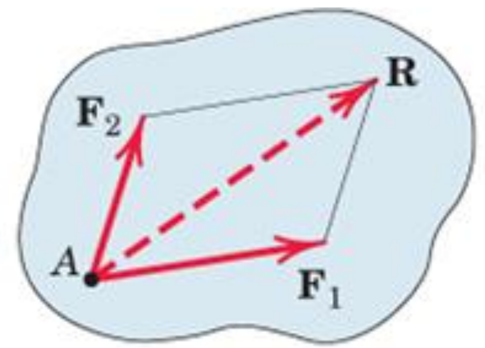
Principle of

Transmissibility

$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2}$

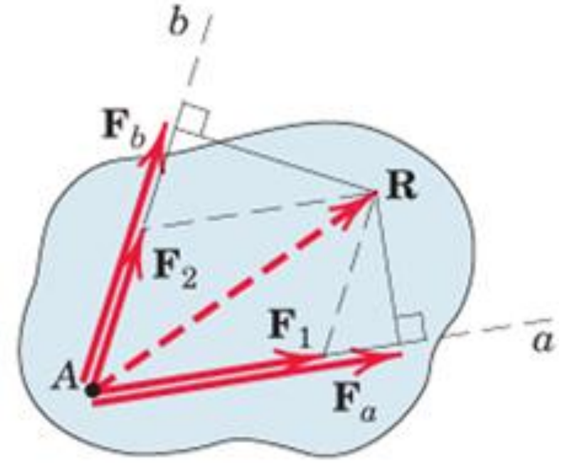
Components and Projections of a Force

- **Components** and **Projections**
  - **Equal** when **axes are orthogonal**



$\mathbf{F_1}$ and $\mathbf{F_2}$ are components of **R**

$\mathbf{R} = \mathbf{F_1} + \mathbf{F_2}$

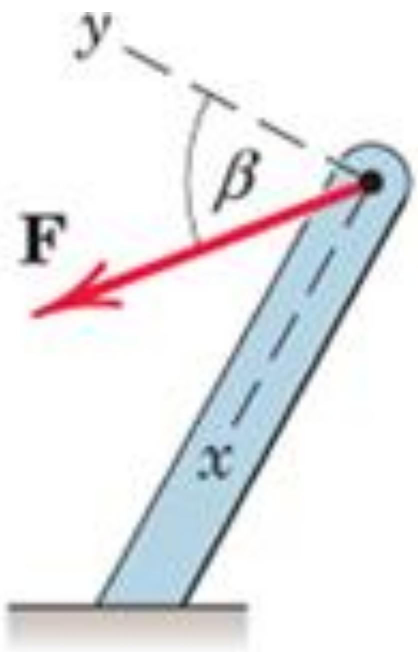


:$\mathbf{F_a}$ and $\mathbf{F_b}$ are perpendicular projections on axes a and b

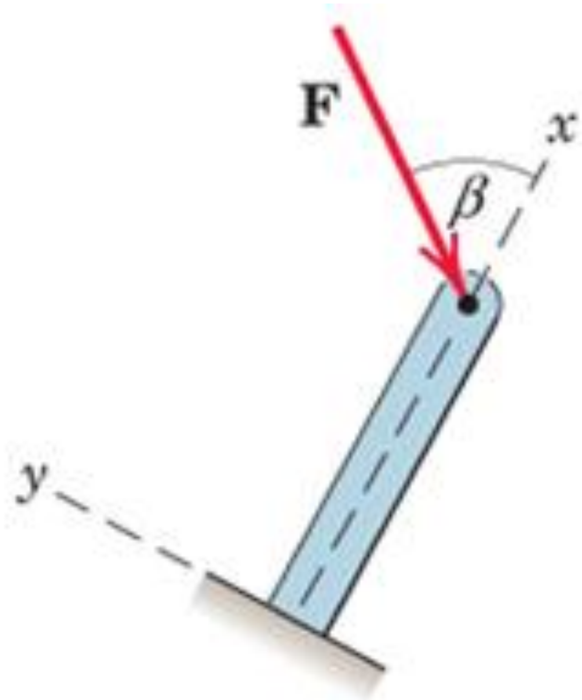
: $\mathbf{R} \neq \mathbf{F_a} + \mathbf{F_b}$ unless a and b are perpendicular to each other

Components of a Force

- Examples



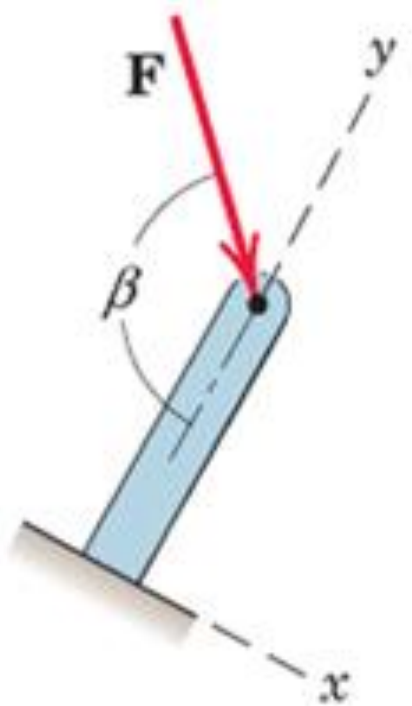
$$F_x = F \sin \beta$$
$$F_y = F \cos \beta$$



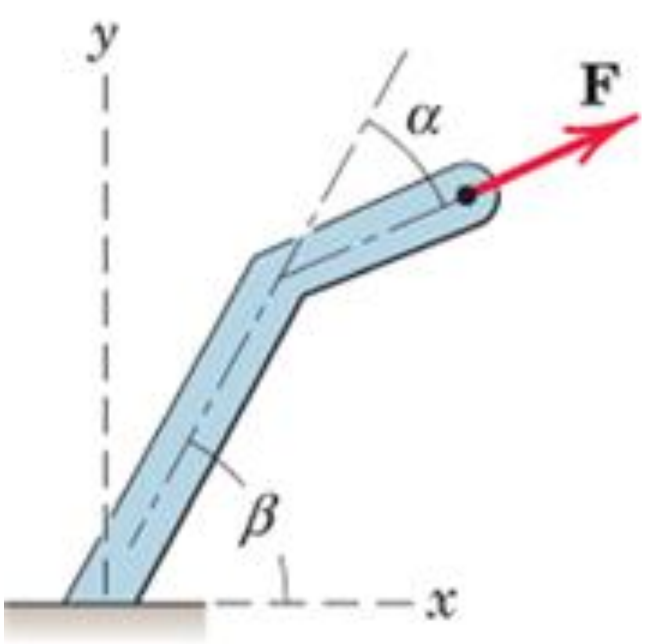
$$F_x = -F \cos \beta$$
$$F_y = -F \sin \beta$$

Components of a Force

- Examples



$$F_x = F\sin(\pi - \beta)$$
$$F_y = -F\cos(\pi - \beta)$$

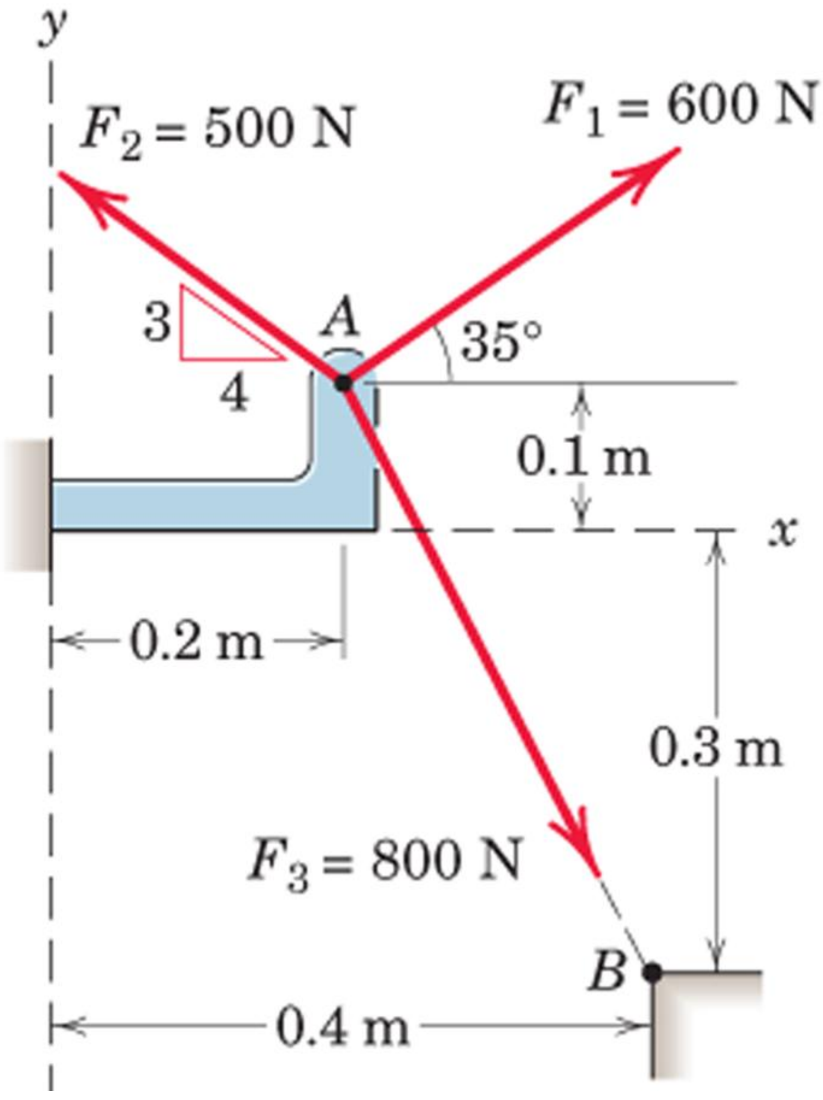


$$F_x = F\cos(\beta - \alpha)$$
$$F_y = F\sin(\beta - \alpha)$$

Components of a Force

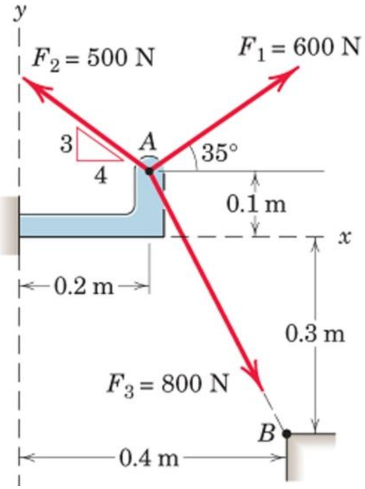
Example 1:

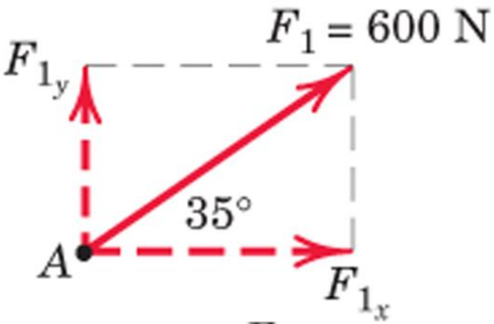
Determine the x and y scalar components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ acting at point A of the bracket

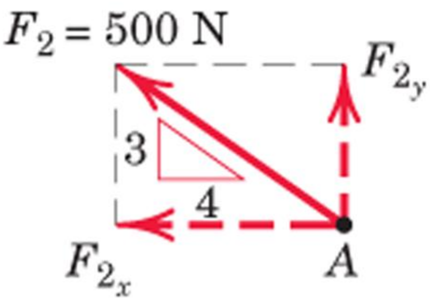


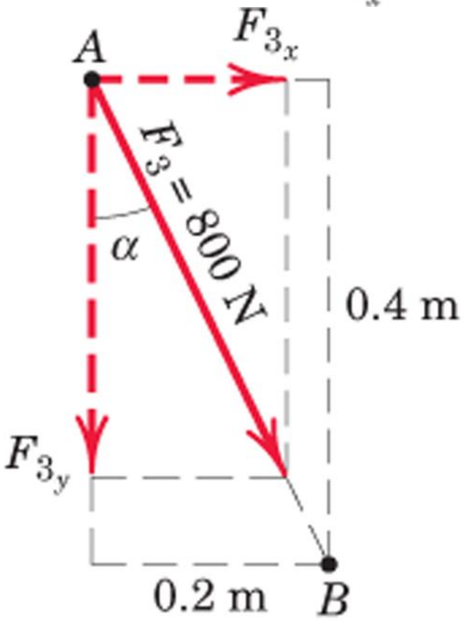
Components of Force

Solution:









$$F_{1_x} = 600 \cos 35^\circ = 491 \text{ N}$$

$$F_{1_y} = 600 \sin 35^\circ = 344 \text{ N}$$

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N}$$

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6^\circ$$

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6^\circ = 358 \text{ N}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6^\circ = -716 \text{ N}$$

Components of Force

Alternative Solution: Scalar components of $\mathbf{F_3}$ can be obtained by writing $\mathbf{F_3}$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB.

Unit vector can be formed by dividing any vector, such as the geometric position vector by its length or magnitude.

$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3 \frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800[0.447\mathbf{i} - 0.894\mathbf{j}]$$

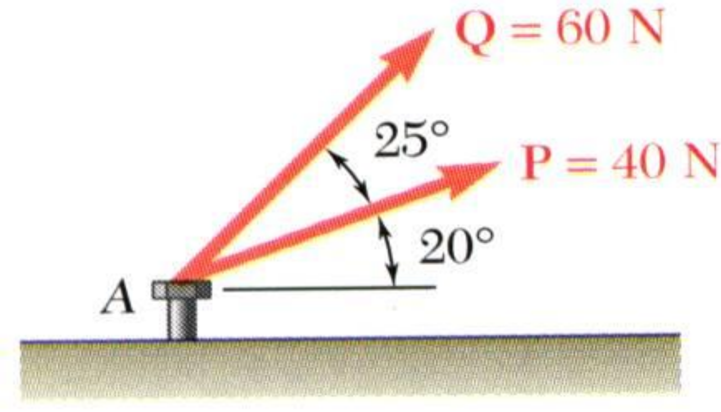
$$= 358\mathbf{i} - 716\mathbf{j}\ \text{N}$$

$$F_{3_x} = 358\ \text{N}$$

$$F_{3_y} = -716\ \text{N}$$

Components of Force

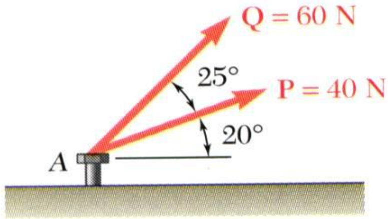
Example 2: The two forces act on a bolt at A. Determine their resultant.



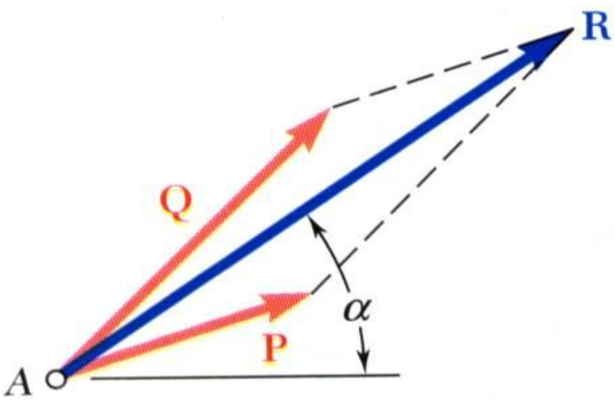
- **Graphical solution** –
- Construct a parallelogram with sides in the same direction as P and Q and lengths in proportion.
- Graphically evaluate the resultant which is equivalent in direction and proportional in magnitude to the diagonal.

- **Trigonometric solution**
- Use the law of cosines and law of sines to find the resultant.

Components of Force

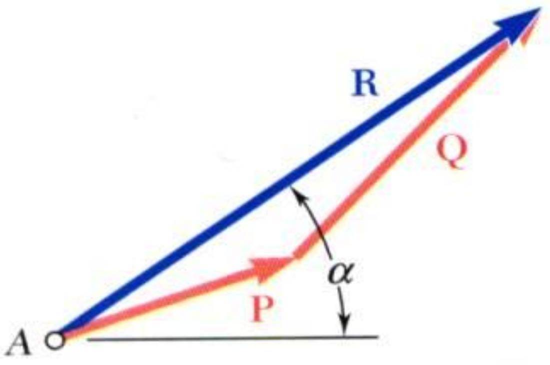


Solution:



- **Graphical solution** - A parallelogram with sides equal to **P** and **Q** is drawn to scale. The magnitude and direction of the resultant or of the diagonal to the parallelogram are measured,

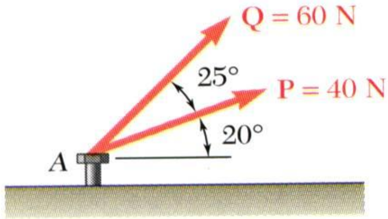
$$\mathbf{R} = 98\text{ N} \quad \alpha = 35^\circ$$



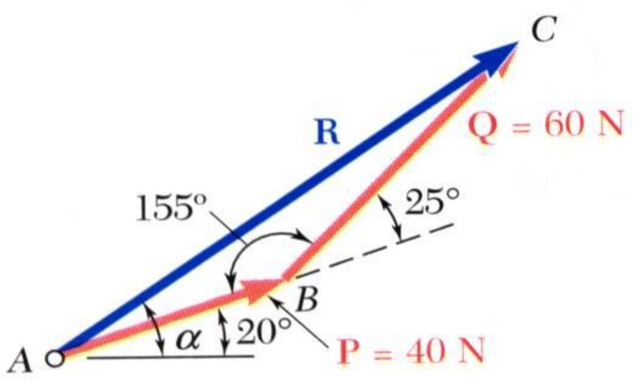
- **Graphical solution** - A triangle is drawn with **P** and **Q** head-to-tail and to scale. The magnitude and direction of the resultant or of the third side of the triangle are measured,

$$\mathbf{R} = 98\text{ N} \quad \alpha = 35^\circ$$

Components of Force



Trigonometric Solution:



$$R^2 = P^2 + Q^2 - 2PQ\cos B$$

$$= (40\text{N})^2 + (60\text{N})^2 - 2(40\text{N})(60\text{N})\cos 155°$$

$$\boxed{R = 97.73\text{N}}$$

$$\frac{\sin A}{Q} = \frac{\sin B}{R}$$

$$\sin A = \sin B \frac{Q}{R}$$

$$= \sin 155° \frac{60\text{N}}{97.73\text{N}}$$

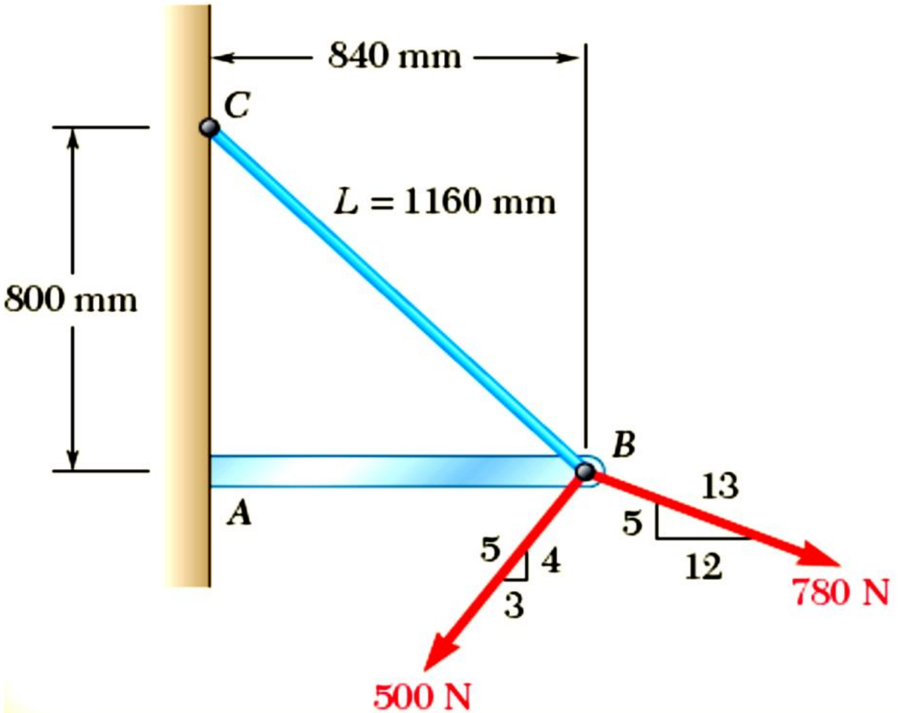
$$A = 15.04°$$

$$\alpha = 20° + A$$

$$\boxed{\alpha = 35.04°}$$

Components of Force

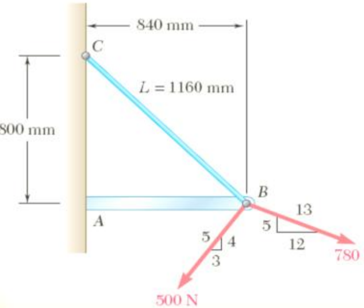
Example 3: Tension in cable *BC* is 725 N; determine the resultant of the three forces exerted at point *B* of beam *AB*.



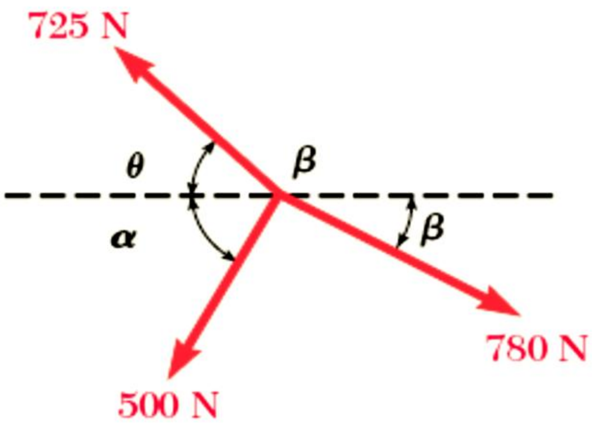
Solution:

- Resolve each force into rectangular components.
- Determine the components of the resultant by adding the corresponding force components.
- Calculate the magnitude and direction of the resultant.

Components of Force



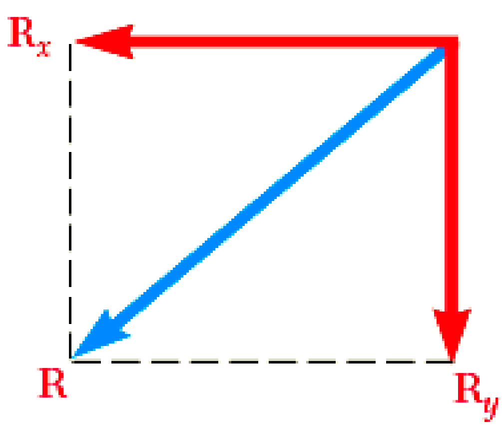
Solution



- Resolve each force into rectangular components.

Magnitude, N	x Component, N	y Component, N
725	−525	500
500	−300	− 400
780	720	− 300
	$R_x = -105$	$R_y = -200$

$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} \qquad \mathbf{R} = (-105\ \text{N})\mathbf{i} + (-200\ \text{N})\mathbf{j}$$

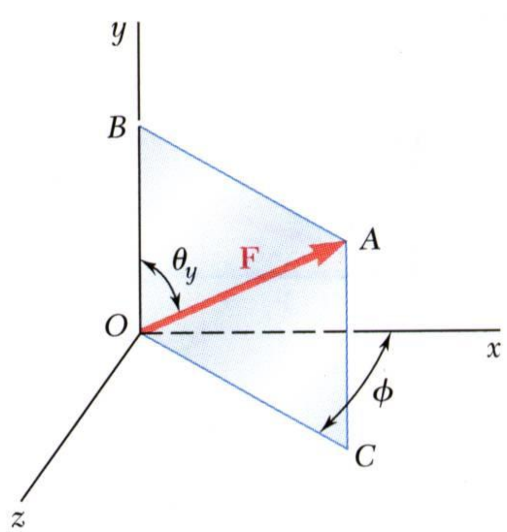


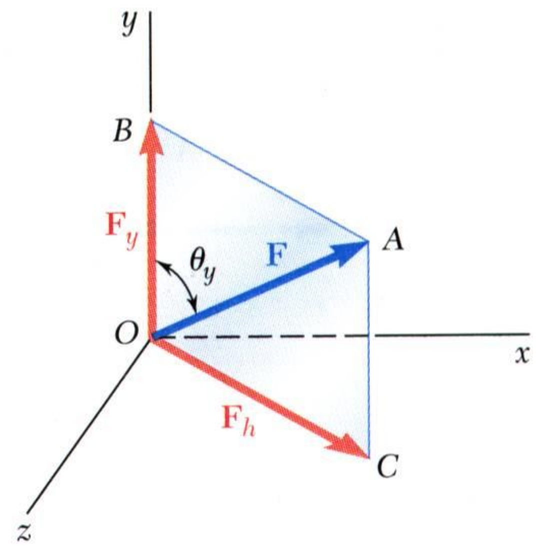
- Calculate the magnitude and direction.

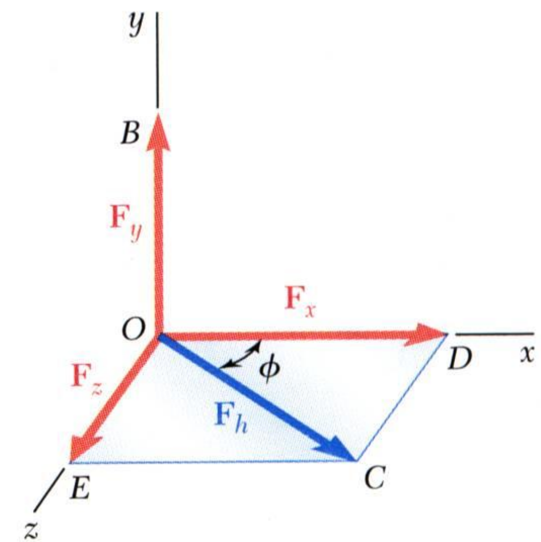
$$\tan\alpha = \frac{-R_y}{-R_x} = \frac{200\,\text{N}}{105\,\text{N}} \qquad \alpha = 62.3^\circ$$

$$R = \sqrt{R_{x^2} + R_{y^2}} = 225.9\,\text{N} \qquad \measuredangle\ 62.3^\circ$$

Rectangular Components in Space







- The vector $\vec{F}$ is contained in the plane *OBAC*.

- Resolve $\vec{F}$ into horizontal and vertical components.

$$F_y = F\cos\theta_y$$

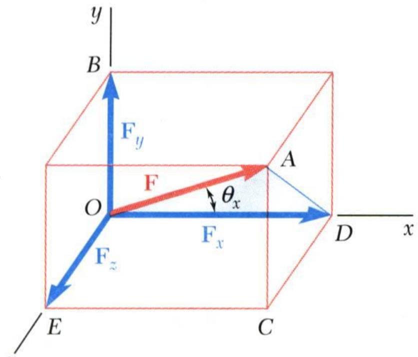
$$F_h = F\sin\theta_y$$

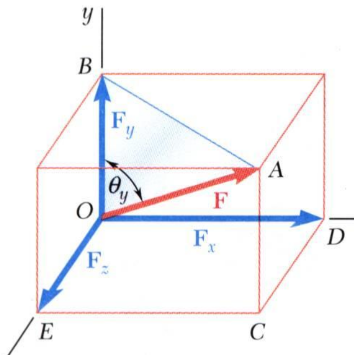
- Resolve F_h into rectangular components

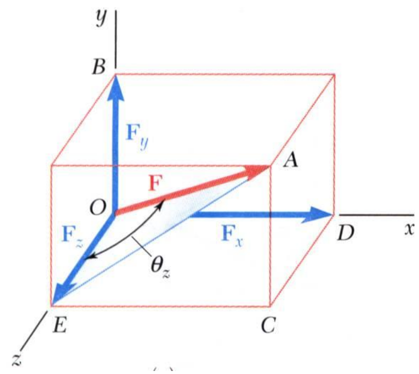
$$F_x = F_h\cos\phi = F\sin\theta_y\cos\phi$$

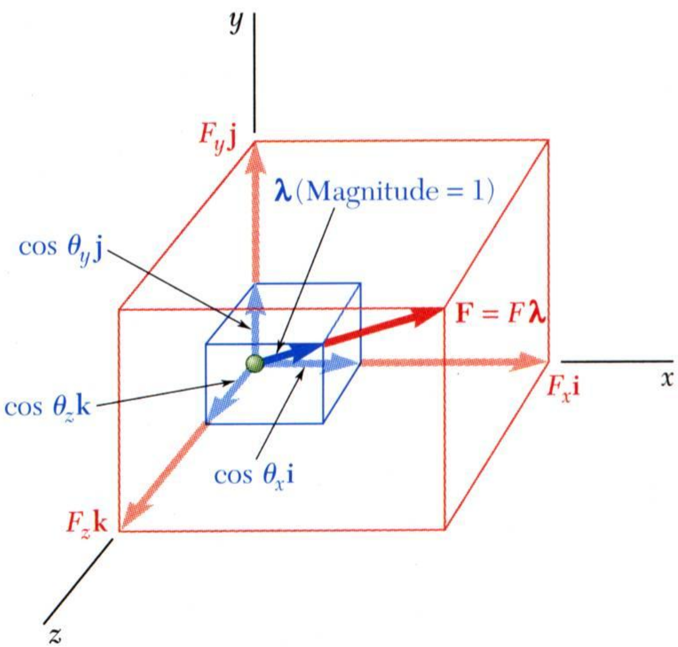
$$F_z = F_h\sin\phi = F\sin\theta_y\sin\phi$$

Rectangular Components in Space









- With the angles between $\vec{F}$ and the axes,

$$F_x = F\cos\theta_x \quad F_y = F\cos\theta_y \quad F_z = F\cos\theta_z$$

$$\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$$

$$= F\left(\cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}\right)$$

$$= F\vec{\lambda}$$

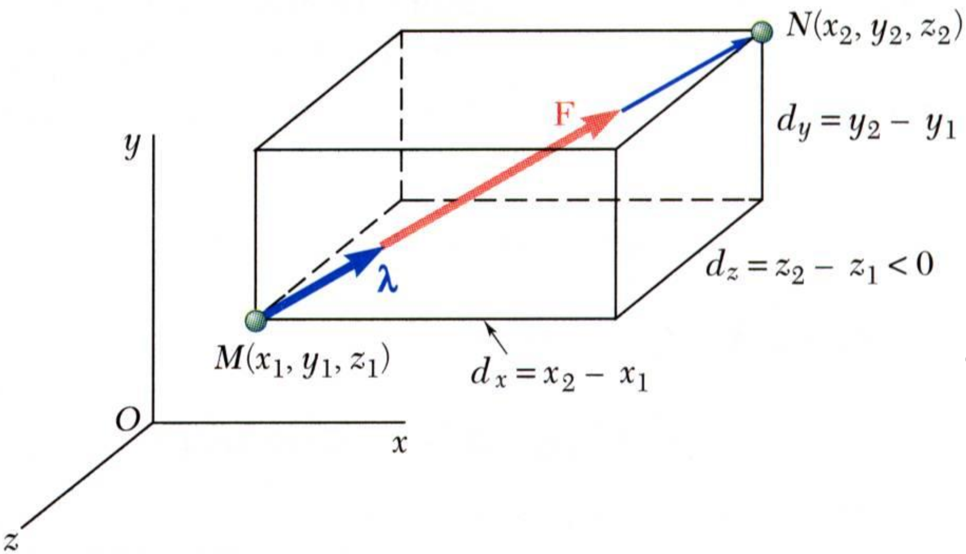
$$\vec{\lambda} = \cos\theta_x\vec{i} + \cos\theta_y\vec{j} + \cos\theta_z\vec{k}$$

- $\vec{\lambda}$ is a unit vector along the line of action of $\vec{F}$ and $\cos\theta_x, \cos\theta_y, \text{and } \cos\theta_z$ are the direction cosines for $\vec{F}$

Rectangular Components in Space

Direction of the force is defined by the location of two points:

$M(x_1, y_1, z_1)$ and $N(x_2, y_2, z_2)$



$$\vec{d} = \text{vector joining } M \text{ and } N$$
$$= d_x\vec{i} + d_y\vec{j} + d_z\vec{k}$$
$$d_x = x_2 - x_1 \quad d_y = y_2 - y_1 \quad d_z = z_2 - z_1$$
$$\vec{F} = F\vec{\lambda}$$
$$\vec{\lambda} = \frac{1}{d}\left(d_x\vec{i} + d_y\vec{j} + d_z\vec{k}\right)$$
$$F_x = \frac{Fd_x}{d} \quad F_y = \frac{Fd_y}{d} \quad F_z = \frac{Fd_z}{d}$$