* In the last class, we saw how the advective-dispersive solute transport equation is formed using conservation principles.

* Subsequently we started discussion on advective dispersive transport with retardation.

This is to mathematically describe the solute transport through porous media, if the solute interacts (chemically or physically) with the solid grains in the porous media.

Therefore the solute transport equation needs to be modified:

$$\frac{\partial}{\partial t}(nc) + \frac{\partial}{\partial x}(q_x c) - n\frac{\partial}{\partial x}\left(D_{xx}\frac{\partial c}{\partial x}\right) + R_s = 0$$
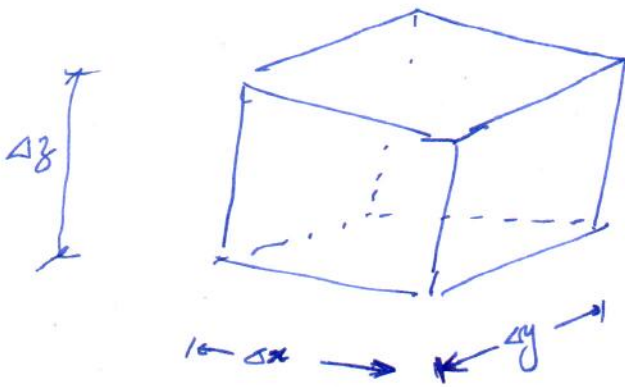
Considering porous media as non-deformable

$$\frac{\partial c}{\partial t} + \frac{\partial}{\partial x}(v_x c) - \frac{\partial}{\partial x}\left(D_{xx}\frac{\partial c}{\partial x}\right) + \frac{R_s}{n} = 0$$

$R_s \rightarrow$ the term corresponding for change in mass of solute in liquid phase, due to some of solute getting sorbed into ~~solute~~ solid grains.

$\Rightarrow$ The effect of sorption will be felt in the term $\left\{ \frac{\partial}{\partial t} \iiint \beta S \, dv \right\}$ of the RTT.

For the control volume shape given in the figure here

Let :

$S \rightarrow$ amount of solute sorbed into solids per unit weight of the solid.

$\rho_b \rightarrow$ bulk density of the porous media

$n \rightarrow$ porosity.

$\therefore \left\{ \frac{\partial}{\partial t} \iiint \rho_s \, dv \right\} \rightarrow$ due to sorption can be given as :

$$\Rightarrow \frac{\partial}{\partial t} \left(\rho_b \, S \right) \Delta x \Delta y \Delta z + \frac{\partial}{\partial t} (n c)$$

$\therefore$ Our ADE with sorption becomes :

$$\frac{\partial}{\partial t}(n c) + \rho_b \frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(q_x c) = n \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial c}{\partial x} \right)$$

For isotropic, homogeneous, non-deformable porous media :

$$\frac{\partial c}{\partial t} + \frac{\rho_b}{n} \frac{\partial S}{\partial t} + \frac{\partial}{\partial x}(v_x c) = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial c}{\partial x} \right) \rightarrow ①$$

$\Rightarrow$ The process of sorption or constitutional relationships between S and C needs to be assigned for equation ① to be solved for C.

Many scientists have suggested various types of constitutional relations between S and C. These relations are called sorption isotherms.

1) Linear Sorption Isotherm

$$S = K_d C \qquad \rightarrow ②$$

Equation ② is a linear relation between S and C, where K_d is distribution coefficient.

Equation ① now becomes

$$\frac{\partial c}{\partial t} + \frac{\rho_b}{n} \frac{\partial}{\partial t}(K_d c) + \frac{\partial}{\partial x}(v_x c) = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial c}{\partial x}\right)$$

i.e.
$$\left(1 + \frac{\rho_b}{n} K_d\right) \frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial c}{\partial x} - v_x c\right)$$

or
$$R \frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial c}{\partial x} - v_x c\right)$$

where $R \rightarrow$ retardation factor $= 1 + \frac{\rho_b}{n} K_d$

2) Non-linear sorption isotherms

(Freundlich, Langmuir, etc.)

Freundlich Sorption Isotherm

$$S = K C^N$$

where

$K \rightarrow$ distribution coefft.

$N \rightarrow$ isotherm parameter

We can write

$$\ln S = \ln K + N \ln C$$

Also from

$$S = K C^N$$

$$\frac{\partial S}{\partial t} = N K C^{N-1} \frac{\partial C}{\partial t}$$

∴ Equation ① becomes:

$$\frac{\partial C}{\partial t} + \frac{\beta_b}{n} N K C^{N-1} \frac{\partial C}{\partial t} + \frac{\partial}{\partial x}(v_x C) = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial C}{\partial x}\right)$$

i.e.

$$\left(1 + \frac{\beta_b}{n} N K C^{N-1}\right) \frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D_{xx} \frac{\partial C}{\partial x} - v_x C\right)$$

Langmuir Sorption Isotherm

Langmuir related S and C using the relation

$$\frac{C}{S} = \frac{1}{\alpha \beta} + \frac{C}{\beta}$$

where $\alpha \rightarrow$ constant relating binding energy between solute and soil $\left[L^3 M^{-1}\right]$

$\beta \rightarrow$ maximum amount of solute that can be sorbed $\left[M \cdot m^{-1}\right]$

Now we have.

$$\frac{C}{S} = \frac{1 + \alpha C}{\alpha \beta}$$

$$S = \frac{\alpha \beta C}{1 + \alpha C}$$

$$\therefore \quad \frac{\partial S}{\partial t} = \frac{\left[\frac{\alpha \beta}{1 + \alpha C} \frac{\partial C}{\partial t} - \alpha^2 \beta C \frac{\partial C}{\partial t} \right]}{(1 + \alpha C)^2}$$

$$= \frac{\alpha \beta \frac{\partial C}{\partial t}}{(1 + \alpha C)^2} \left(\frac{1}{1 + \alpha C} - \alpha C \right)$$

$$= \frac{\alpha \beta \frac{\partial C}{\partial t}}{(1 + \alpha C)^2} \left\{ \frac{1 - \alpha C + \alpha^2 C^2}{1 + \alpha C} \right\}$$

$$= \frac{(1 - \alpha C + \alpha^2 C^2) \alpha \beta}{(1 + \alpha C)^3} \cdot \frac{\partial C}{\partial t}$$

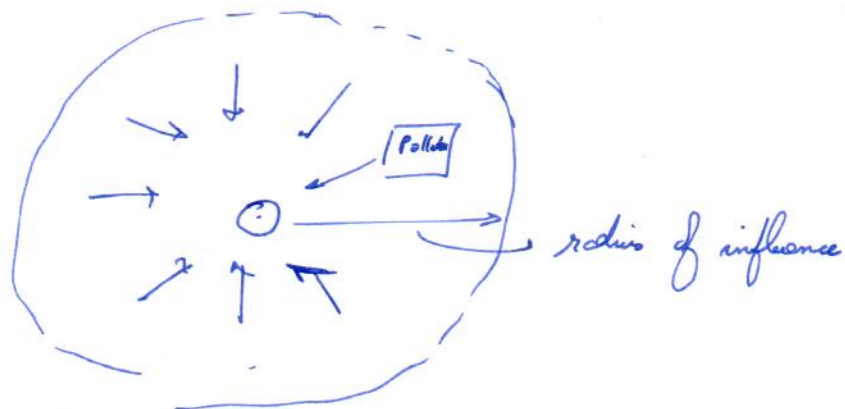Substitute in eqn. ① to get corresponding solute transport equation.

A VERY BRIEF VIEW ON GROUNDWATER MANAGEMENT

When considering groundwater as a resource, it is essential that the resource is managed efficiently. Otherwise the resource may be depleted, or contaminated, or affected through various processes.
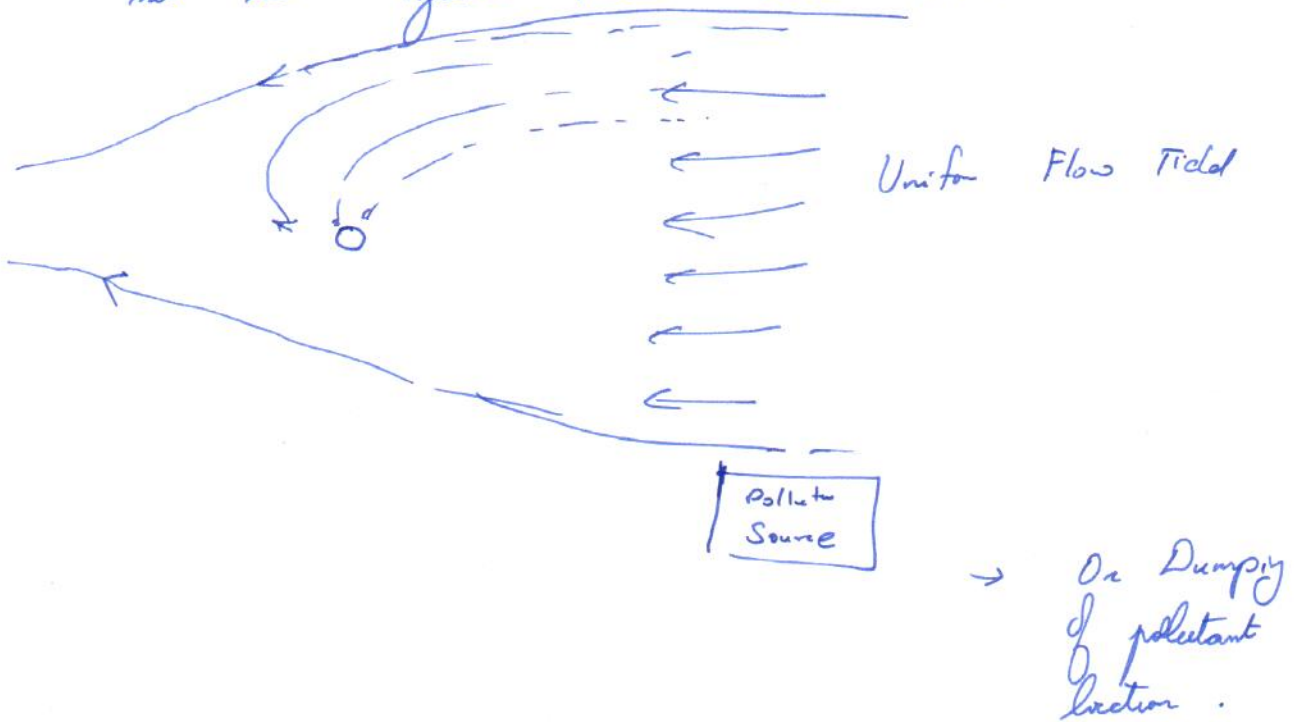
Q: Where all can we utilise GW Management Techniques

* To maximise the withdrawal of water from a pumping well

* To minimise the cost of pumping

* To design strategies of optimal pumping. so that a polluted zone is not spreaded further

radius of influence

* The reverse process of preventing a contaminant from entering a drinking water borewell, we need to suggest that the pumping rate should not be greater than some M.

Unifor Flow Field

Pollute Source

→ On Dumping of pollutant location.

* To maintain water in pores in saturated conditions, to prevent hazardous geochemical reactions, etc.

* To prevent salt water from entering into freshwater aquifer, etc.

⇒ Classical optimisation tools can be used to design strategies.