UNSATURATED FLOW

LECTURE 34

04 - APR - 2014

THEIS METHOD TO SOLVE RADIAL FLOW TO WELLS

Yesterday, we were discussing on the Theis method to solve the unsteady radial flow to the wells in confined aquifers.

Recall the drawdown was given as

$$s(t) = \frac{Q}{4\pi T} W(u)$$

where $W(u) \rightarrow$ Well function, $u = \frac{r^2 S}{4Tt}$

Also we had seen that

$$\ln s = \ln\left[\frac{Q}{4\pi T}\right] + \ln\left[W(u)\right] \rightarrow (1)$$

$$\ln\left(t/r^2\right) = \ln\left[\frac{S}{4T}\right] + \ln\left[\frac{1}{u}\right] \rightarrow (2)$$

Comparing (1) and (2) as $\frac{Q}{4\pi T}$ and $\frac{S}{4T}$ are constants, the relation between s and t/r^2 will be similar to $W(u)$ and $\frac{1}{u}$.

$\Rightarrow$ From pumping tests we have observations for drawdown at various times

time, t	$s(t)$
t_0	s_0
t_1	s_1
t_2	s_2
$\vdots$	$\vdots$
t_n	s_n

$\rightarrow$ Let us say that this given observation is taken at a radial distance 'r' from centre of well.

This observation, therefore, can be plotted in a double logarithmic graph

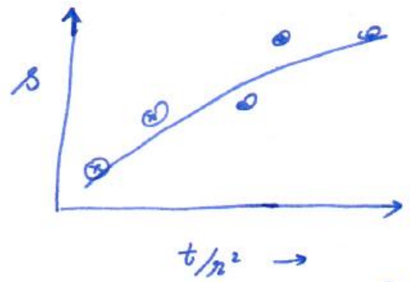


Fig 1

Recall the well function

$$W(u) = \int_u^{\infty} \frac{e^{-u}}{u}\, du$$

$$= \left[-0.5772 - \ln u + u - \frac{u^2}{2 \cdot 2!} + \frac{u^3}{3 \cdot 3!} - \frac{u^4}{4 \cdot 4!} + \cdots \right] \longrightarrow (4)$$

We can take some range of values for u, say from $u = 1$ to ~~10000~~ 10^{-4}

$$\therefore \frac{1}{u} = 1 \text{ to } 10^4$$

The corresponding values of $W(u)$ for the given u's are obtained from (4) and they can again be plotted on a transparent double logarithmic graph.

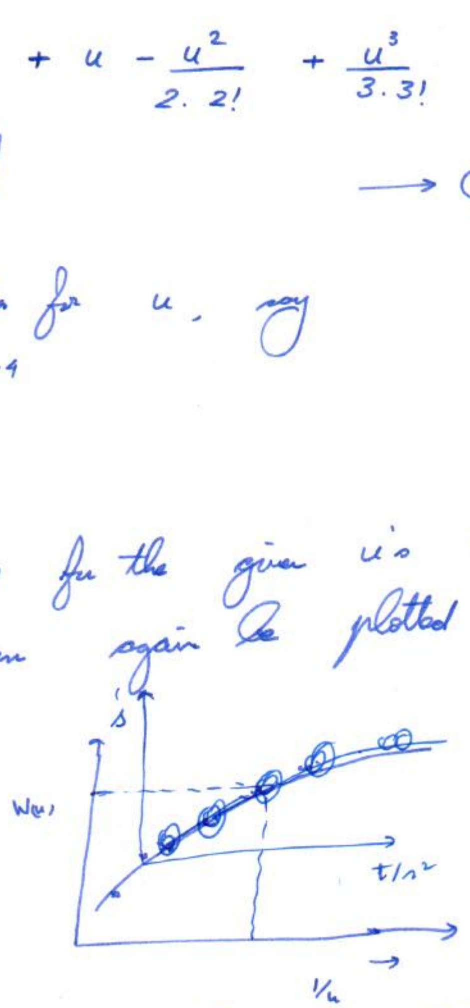


Fig 2

=> Figure (1) and Figure (2) are now overlain. The portion where both the curves nearly merge is taken. For a given point

$(t/r^2, s)$ ~~$(\ln t/r^2, \ln s)$~~ in the curve $\ln t/r^2$ vs $\ln s$, the corresponding values of W(u) and $\frac{1}{u}$ are obtained.

From the obtained values of W(u) and $\frac{1}{u}$ for a particular radius r we get say at time t_1

$$\cancel{u_1 = \frac{r^2 S}{4T t_1}} \quad u_1 = \frac{S}{4T}\left(\frac{r^2}{t_1}\right)$$

Similarly at time t_2, $\quad u_2 = \frac{S}{4T}\left(\frac{r^2}{t_2}\right)$

From these known information we can get aquifer parameters S and T.

⇒ Like Theis method you also have

- ★ Cooper-Jacob method
- ★ Chow method, etc.

UNSATURATED FLOW

⇒ Till now we were dealing with saturated zones in groundwater hydrology (i.e. confined aquifers, unconfined aquifers, leaky aquifers, etc.).

⇒ However above saturated zone, there lies the unsaturated zone. The pores are not completely filled with water.

⇒ In aquifers, we predominantly assumed the flow to be in horizontal directions (Vertical flows were neglected.).

⇒ This will not be the case in flow through unsaturated zones. Downward or vertical flow is quite significant.

⇒ You have already studied in your hydrology course that Water on earth exist in

- ↳ Atmospheric sub-system
- ↳ Surface water system
- ↳ Subsurface system.

In the sub-surface water system the hydrological processes involved are:

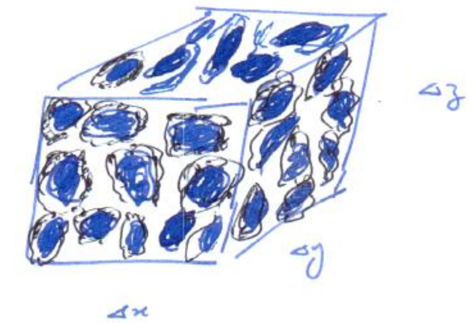
- Infiltration
- Percolation
- Subsurface unsaturated flow
- Subsurface saturated flow, etc.

⇒ Whatever quantity of water infiltrates from the surface of earth, will further percolates through unsaturated zones (mostly in vertical direction) and reaches saturated zones.

⇒ Unsaturated zones, are therefore important zones that transmits water to the aquifers. They ~~are~~ may ~~make~~ also allow transmission of pollutants from surface to the groundwater.

⇒ The flow of water through unsaturated zones can again be evaluated through Reynolds Transport Theorem.

Let us consider for a porous unsaturated control volume



$\theta \rightarrow$ water content

$\theta_s \rightarrow$ saturated water content

$$\frac{DB}{Dt} = \frac{\partial}{\partial t}\iiint \beta\rho \, dV + \iint_{cs} \beta\rho(\vec{V}\cdot\hat{n})\, dA$$

Here let B = mass of water in the control volume

$$\therefore \frac{DB}{Dt} = 0 \quad \text{and} \quad \beta = 1.0$$

$\iiint \beta\rho \, dV$ = ~~Volume of water~~ Mass of water stored inside the control volume

$= \rho_w \, \theta \, \Delta x \, \Delta y \, \Delta z$

$\iint_{cs} \rho(\vec{V}\cdot\hat{n})\, dA \rightarrow$ describes the net outflow of mass of water through the control surfaces.

Also $\rho(\vec{V}\cdot\hat{n}) \rightarrow$ is the mass outflux.

Assuming the unsaturated flow is predominantly in vertical direction, let inflow specific discharge $= q_z$ $[LT^{-1}]$

Let the outflow specific discharge $= q_z + \frac{\partial q}{\partial z} \Delta z$ $[LT^{-1}]$

$\Rightarrow$ We are neglecting fluxes in other directions.

$$\therefore \iint_{cs} \beta \rho (\vec{V} \cdot \hat{n})\, dA = \rho_w \left(q_z + \frac{\partial q_z}{\partial z} \Delta z \right) \Delta x \Delta y - \rho_w q_z \Delta x \Delta y$$

$$= \rho_w \Delta x \Delta y \Delta z \frac{\partial q_z}{\partial z}$$

Also $\frac{\partial}{\partial t} \iiint_{cv} \beta \rho\, dV = \frac{\partial (\rho_w \theta)}{\partial t} \Delta x \Delta y \Delta z$

i.e. $\frac{\partial}{\partial t} (\rho_w \theta) \Delta x \Delta y \Delta z + \rho_w (\Delta x \Delta y \Delta z) \frac{\partial q_z}{\partial z} = 0$

Assuming the water as incompressible and the control volume to be a fixed, we get

$$\boxed{\frac{\partial \theta}{\partial t} + \frac{\partial q_z}{\partial z} = 0} \rightarrow (5)$$

This is the continuity equation in unsaturated zones.

$\Rightarrow$ To evaluate specific discharge q, we may again use the Darcy's relation.

Let ϕ be the piezometric head in the unsaturated zone.
Based on the gradient of piezometric head, (or hydraulic head) the direction of flow will be present

∴ Intuitively we can say that if you want downward vertical flow, then the hydraulic head (ϕ) should be greater ~~than~~ at top and low at bottom.

Darcy's law for unsaturated flow is given as

$$q = - K_u(\theta) \frac{\partial \phi}{\partial z}$$

where $K_u(\theta)$ → hydraulic conductivity in unsaturated conditions.
(Please note that hydraulic conductivity is now a function of moisture content θ).

⇒ In unsaturated conditions water is attracted to solids through suction forces

↳ The energy due to soil suction forces is given as suction head ψ (Please note this is not stream function)

↳ Total hydraulic head in unsaturated conditions
= Suction head + Datum head

$$\therefore \phi = \psi + z$$

$$\therefore q = - K(\theta) \frac{\partial}{\partial z}(\psi + z)$$

K is related with moisture content

$\therefore$ Definitely there exist $\frac{\partial K}{\partial \theta}$

The quantity $\frac{\partial \psi}{\partial z} = \frac{d\psi}{d\theta}\frac{\partial \theta}{\partial z}$

$$= \frac{\partial \theta / \partial z}{d\theta / d\psi}$$

The term $d\theta/d\psi \rightarrow$ specific water capacity and is a property of the soil.

$$q = -K(\theta)\left(\frac{\partial \psi}{\partial z} + \frac{\partial z}{\partial z}\right) = -K(\theta)\left(\frac{1}{\frac{d\theta}{d\psi}}\frac{\partial \theta}{\partial z} + 1\right)$$

$\therefore$ Equation (5) becomes.

$$\frac{\partial \theta}{\partial t} + \frac{\partial}{\partial z}\left[-K(\theta)\left(\frac{1}{d\theta/d\psi}\frac{\partial \theta}{\partial z} + 1\right)\right] = 0$$

or $$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left(K(\theta)\left[\frac{d\psi}{d\theta}\frac{\partial \theta}{\partial z}\right]\right) + \frac{\partial K}{\partial z}$$

We can define soil water diffusivity $D(\theta) = K(\theta)\frac{\partial \psi}{\partial \theta}$

$$\therefore \boxed{\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[D(\theta)\frac{\partial \theta}{\partial z}\right] + \frac{\partial K}{\partial z}}$$

This is one dimensional Richards equation for unsaturated flow.