LECTURE 33

03-APR-2014

# UNSTEADY RADIAL FLOWS TO WELLS

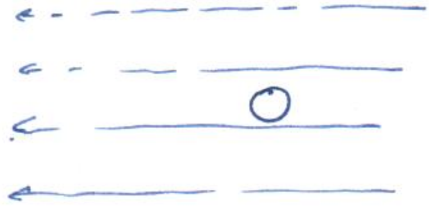
We were discussing on the steady state radial flows to the pumping wells penetrating confined aquifers as well as unconfined aquifers.

As the pumping rate $Q$ (or $Q_w$) is constant, we were able to estimate the radius of influence for radial flows into the well.

→ Using pumping tests, we could also determine (for such steady state conditions) the hydraulic conductivity and transmissivity.

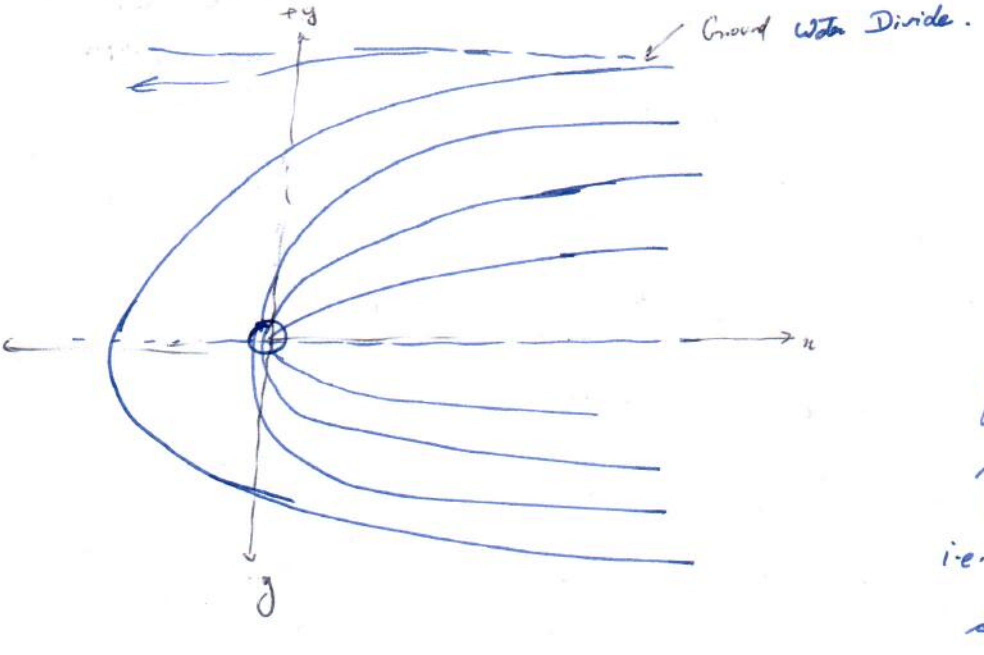
Well in Uniform Flow in steady conditions

→ Subsequently we started discussing about a pumping well of steady discharge (say $Q_w$) situated in an uniform aquifer flow field.



→ We can form groundwater divide now in this uniform flow field due to pumping from well.

Although the uniform flow is from the right to left, based on the magnitude of discharge $Q$, some quantities start flowing radially into the well.




We can now consider the well as the origin of x-y coordinate system

i.e. $x = 0$, $y = 0$ at the well.

⇒ If the well is pumping for infinite time, the groundwater divide may extend theoretically to an infinite distance.

→ The groundwater divide here is that region in the aquifer that can produce inflow to the well that pumps discharge $Q$. (Please note the discharge is steady).

→ The expression for this boundary is obtained by superposing radial flow and one-dimensional uniform flow

$$-\frac{y}{x} = \tan\left(\frac{2\pi K b i}{Q} y\right)$$

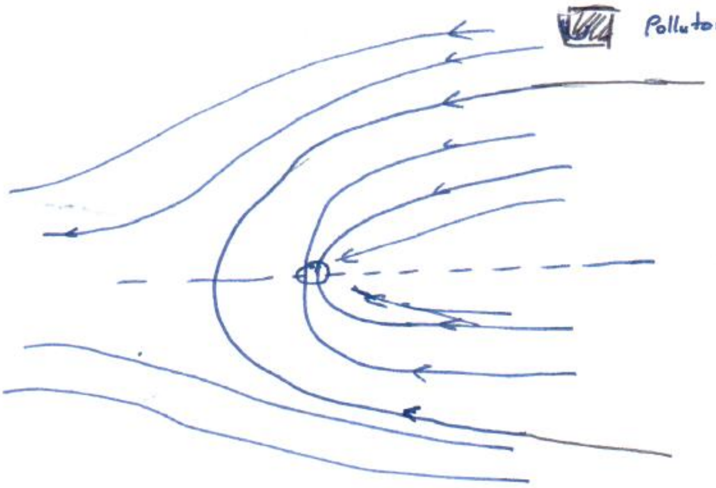
where $i$ → natural piezometric slope.

→ On the right side top: $y_L = \pm \frac{Q}{2 K b i}$

On the left side, we have the divide at:

$$x_L = \frac{-Q}{2\pi K b i}$$

Such groundwater divide locations are essential in suggesting whether certain contamination in uniform flow will reach the well or not




In the figure above, pollutant is ~~above~~ beyond the groundwater divide in y-direction. This pollutant will not reach the well.

## Unsteady Radial Flow in Confined Aquifers

Recall for a horizontal confined aquifer of thickness 'b', we can describe

→ Specific storativity, $S_s$

→ Aquifer storativity, $S = S_s b$

For ~~isotropic homogeneous~~ confined aquifer; we have the continuity equation as:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial \phi}{\partial y}\right) = S_s \frac{\partial \phi}{\partial t}$$

For isotropic homogeneous porous media (or aquifer)

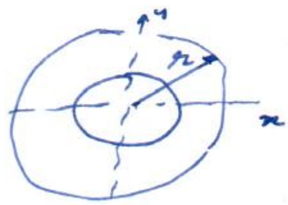
$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{S}{Kb} \frac{\partial \phi}{\partial t}$$

or

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = \frac{S}{T} \frac{\partial \phi}{\partial t} \quad \rightarrow (1)$$

For radial distance 'r' from a well ~~penetrating~~ extracting water from the confined aquifer; equation (1) can be described in radial co-ordinates; such that:

$$r^2 = x^2 + y^2$$




$r^2 = x^2 + y^2$

We got:

$$\frac{1}{r} \frac{\partial}{\partial r}\left(r \frac{\partial \phi}{\partial r}\right) = \frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} = \frac{S}{T} \frac{\partial \phi}{\partial t}$$

where $\phi \rightarrow$ piezometric head

$S \rightarrow$ aquifer storativity or also called storage coefficient

$T \rightarrow$ transmissivity

i.e.

$$\boxed{\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} = \frac{S}{T} \frac{\partial \phi}{\partial t}} \quad \rightarrow (2)$$

We can obtain solutions to equation 2 by various methods.

⇒ One of the preliminary solution technique was described by Theis (1935).

* He correlated groundwater flow with heat conduction
* A pumping well is a location of mathematical sink.
* The boundary conditions are

$$\phi = \phi_0 \quad \text{for} \quad t = 0$$

$$\phi \rightarrow \phi_0 \quad \text{as} \quad r \rightarrow \infty$$

(i.e. Aquifer piezometric surface is horizontal)

* The solution is described in terms of drawdown

$$s = \phi_0 - \phi$$

$$\boxed{\begin{aligned} s &= \frac{Q}{4\pi T} \int_u^\infty \frac{e^{-u}}{u}\, du \\ &= \frac{Q}{4\pi T} W(u) \end{aligned}} \rightarrow 3$$

where $W(u)$ is called Well function

and $u = \dfrac{r^2 S}{4Tt}$

Equation 3 is called Theis equation.

The well function integral

$$W(u) = \int_u^\infty \frac{e^{-u}}{u}\, du = \left[-0.5772 - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \cdots\cdots\right]$$

From pumping test experiments, we can determine the aquifer properties like S and T.

(Please note that although Q is constant, it takes long time to reach steady state. Once steady state is reached then only we will be able to apply steady radial flow equations to determine parameters. Moreover more no: of observation wells are also required in that case.)

However, if we use unsteady radial flow equations, we can evaluate aquifer parameters much much before the steady state conditions to occur. Also less monitoring wells are required.

Assumptions in Theis method

* Aquifer is homogeneous, isotropic, uniform thick
* Piezometric surface is horizontal before pumping.
* Well is pumped at constant discharge
* Well diameter is negligible.
* Well penetrates completely the confined aquifer.

In Theis method to evaluate the parameters, we can formulate as such:

$$s = \frac{Q}{4\pi T} W(u) \quad \rightarrow (4)$$

Also we have $u = \frac{r^2 S}{4tT}$

i.e. $\frac{t}{r^2} = \frac{S}{4T}\,\frac{1}{u}$ $\longrightarrow$ (5)

∴ ~~Also~~ observing eqn. (4) and (5):

$\frac{Q}{4\pi T}$ and $\frac{S}{4T}$ are constants.

$$\therefore\ \ln s = \ln\left[\frac{Q}{4\pi T}\right] + \ln\left[W(u)\right]$$

$$\text{and}\ \ln\left(\frac{t}{r^2}\right) = \ln\left[\frac{S}{4T}\right] + \ln\left[\frac{1}{u}\right]$$

i.e. Both $\ln\left[\frac{Q}{4\pi T}\right]$ and $\ln\left[\frac{S}{4T}\right]$ are constants.

∴ Relation between $\ln s$ and $\ln(t/r^2)$ now will be on similar term of relation between $\ln(W(u))$ and $\ln\left[\frac{1}{u}\right]$.

→ So if double logarithmic plots of the components $\ln W(u)$ and $\ln \frac{1}{u}$ as well as $\ln s$ and $\ln \frac{t}{r^2}$ are created, we can infer S and T.