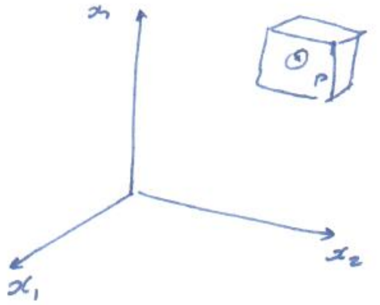
LECTURE 3
09 - JAN - 2014

POROUS MEDIA AS CONTINUUM

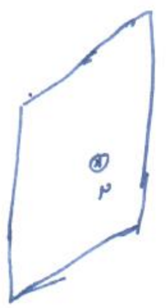
In the last class we discussed on porous media as continuum.

We need to assign certain properties to consider any media as continuum.

In porous media we suggested representative elementary volume (REV) considering porosity.



. For the mathematical point P in the space we can consider P to be part porous media by forming REV around P.

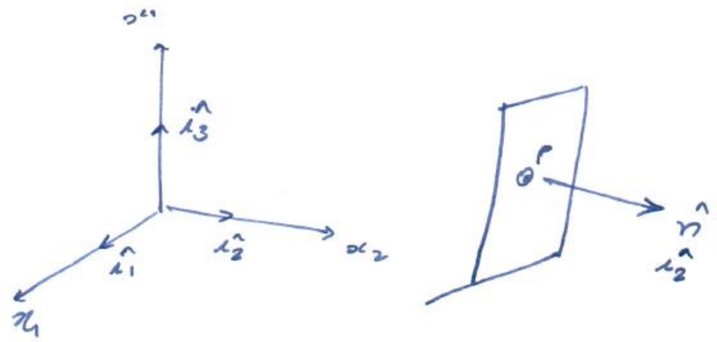


→ This mathematical point can also be part of some plane. Using that ideology one can formulate representative elementary area.

→ Similarly you can think of representative elementary length.

Velocity and Specific Discharge

As suggested, we are dealing with continuum. The fluids inside porous media may flow and therefore we can associate some flow properties to porous media.

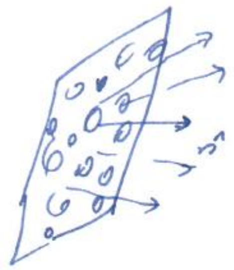


Let the plane at P considered be having outward normal in the direction $\hat{i}_2$. ($\hat{i}_1, \hat{i}_2, \hat{i}_3$ unit vectors).

Let $(\Delta A_j)_0$ be the representative elementary area of porous med. whose outward normal ~~is~~ is $\hat{i}_2$. ~~or~~

→ To obtain the total discharge through $(\Delta A_j)_0$ we need to think of velocity components through the area.

i.e.



In the REA, not all portion is void and not all portion is solid.

i.e. $\vec{v}\cdot\hat{n}$ is the velocity component in the direction of outward normal of the REA.

For the given REA $(\Delta A_j)_0$ let v_j be the velocity in the direction $\hat{i}_2$.

∴ Discharge through REA $Q_{j0} = \int_{(\Delta A_{v})_j} v_j \, dA_{v_j}$

or $q_{j0} = \dfrac{Q_{j0}}{(\Delta A_j)_0}$

$$q_{j_0} = \frac{1}{(\Delta A_j)_0} \int_{(\Delta A_{v_j})} v_j \, dA_{v_j}$$

$$= \frac{1}{(\Delta A_j)_0} \frac{(\Delta A_{v_j})_0}{(\Delta A_{v_j})_0} \int_{(\Delta A_{v_j})_0} v_j \, dA_{v_j}$$

Now $\frac{1}{(\Delta A_{v_j})_0} \int_{(\Delta A_{v_j})_0} v_j \, dA_{v_j}$ → average velocity in j-direction.

~~ϕ → piezometric~~ Specific discharge is therefore peculiarity of porous media continuum:

Recall Navier-Stokes's equation for incompressible fluid flow

$$\rho\left(\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v}\right) = -\nabla p + \rho\vec{g} + \mu\nabla^2\vec{v}$$

If inertial ~~viscous~~ component is not present, then

$$\vec{\nabla} p - \rho\vec{g} = \mu\nabla^2\vec{v}$$

$$\nabla(p + \rho g z) = \mu\nabla^2\vec{v}$$, where p → pressure, z = elevation.

$\left(\text{Recall} \quad \vec{g} = -g\,\nabla x_3\right)$

$$\therefore \nabla\cdot\left(\nabla(p + \rho g z)\right) = \nabla\cdot\mu\nabla^2\vec{v}$$

i.e. $\nabla^2(p + \rho g z) = \nabla^2(\mu\,\nabla\cdot\vec{v})$ (Only for very slow motion)

Now for incompressible fluid flow. $\nabla\cdot\vec{v} = 0$ (i.e. $\operatorname{div}\vec{v} = 0$)

$$\therefore \nabla^2(p + \rho g z) = 0$$

i.e. $\nabla^2\phi = 0$, where $\phi = z + \frac{p}{\rho g}$

ϕ → piezometric head (a potential function)

We will be using this piezometric head later.

Note:→ In this macroscopic approach we need to introduce medium parameters or coefficients.

* Porosity (already described)
* Permeability
* Dispersivity

Chapter-2

FLUID AND POROUS MATRIX PROPERTIES

1) Fluid Density, ρ

$$\rho = \frac{\text{Mass}}{\text{Volume}}$$

2) Specific Weight, γ

$$\gamma = \rho g$$

3) Specific gravity

Relative density of the fluid = $\frac{\text{Density of fluid}}{\text{Density of water}}$

4) Fluid viscosity

→ The property of fluid that resists any deformation of fluid continuum is called viscosity.

You might have already studied in fluid mechanics about

↳ Dynamic viscosity

↳ Kinematic viscosity

→ You may be aware of Newtonian and non-Newtonian fluids.

5) Fluid compressibility

Compressibility (β) $\rightarrow$ measure of volume changes when a substance is subjected to changes in normal pressures or tensions.

$$\beta = -\frac{1}{U}\frac{DU}{Dp}$$

$U \Rightarrow$ volume of given mass of fluid
$p \rightarrow$ pressure
$\rho \rightarrow$ density

$\rightarrow$ The negative sign indicates a decrease in volume as pressure increases.

Also $\beta = \frac{1}{\rho}\frac{D\rho}{Dp}$

Reciprocal of compressibility $\rightarrow$ Modulus of elasticity

$\rightarrow$ In homogeneous or incompressible fluids $\frac{Dp}{D\rho} = 0$

i.e. ρ = constant

$\frac{DU}{Dp}$ $\rightarrow$ Material derivative of U w.r.t p.

6) Statistical description of porous media

Geometry of solids, geometry of voids, etc. are difficult to describe individually. Therefore, it is better to assign macroscopic properties

Porosity is already described.

→ The particle sizes can be measured experimentally and then statistically suggest some representative value for the respective porous media. of particle size

→ The pore sizes can also be measured experimentally and using statistical methods, the representative values of pore sizes can be determined.

Chapter - 3

PRESSURE AND PIEZOMETERIC HEAD

The porous media consist of solid matrix and voids. The voids may consist of fluids that may flow. So our interest is how to analyse flow of fluids in porous media.

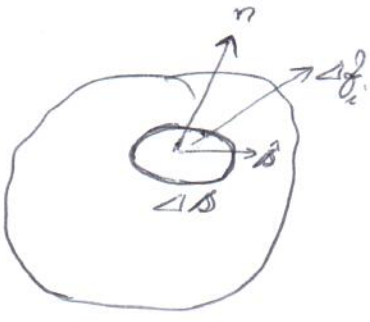
In last semester you studied on certain properties for fluid continuum (not porous media). Recall some of the concepts:

Stress at a Point

The fluid continuum around a point may involve following forces:

→ Body forces
→ Surface forces

I hope you know the difference between body forces and surface forces.

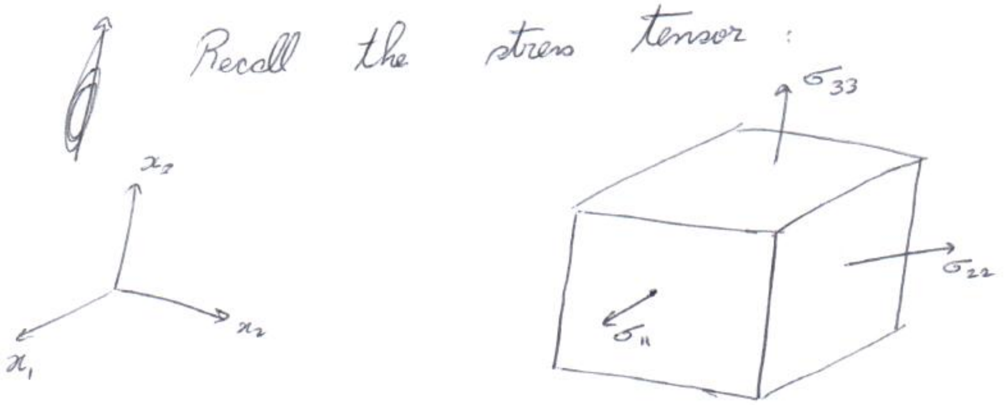


→ The external forces acting on any volume gives rise to stress

$$\lim_{\Delta s \to 0} \frac{\Delta f_i}{\Delta s} = \frac{df_i}{ds} = t_i^{(\hat{n})}$$

→ The stress vector

Recall the stress tensor :



Let the stress tensor be represented as σ_{ij}

$$\sigma_{ij} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

$\sigma_{11}, \sigma_{22}, \sigma_{33}$ are Normal Stresses and they are positive in the outward normal direction.

$\sigma_{12}, \sigma_{13}, \sigma_{21}, \sigma_{23}, \sigma_{31}, \sigma_{32}$ are tangential stress components (also called shear stress).

Q: If a fluid is stationary, what are the shear stresses acting ~~on~~ at any fluid particle?

Recall stress tensor is symmetric

i.e. $\sigma_{ij} = \sigma_{ji}$

The sum of diagonal components of stress

$= \sigma_{11} + \sigma_{22} + \sigma_{33}$

This sum is independent of the co-ordinate axes.

One can define bulk stress. $\bar{\sigma} = \frac{1}{3}\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right)$

Bulk stress is a scalar value.

→ For non-viscous fluid all normal stresses at a point are equal.

i.e. $\sigma_{11} = \sigma_{22} = \sigma_{33}$ = the bulk stress $\bar{\sigma}$

→