LECTURE 2

06-JAN-2014

THE POROUS MEDIA AS A CONTINUA

As discussed in the last class, we will be dealing the subject of subsurface hydrology by considering the subsurface as porous media.

→ There are wide applications of porous media in various branches of science and engineering.

→ As informed in the last class, solids with continuous inter connected pores can be classified as porous medium.

Accurate definitions for porous media are difficult. Bear et al. (1968) have attempted to define porous medium:

a) Multi phase matter occupied in space — atleast one phase is solid and similarly atleast one phase should not be solid. Solid phase called solid matrix. Non-solid phase is called void space

b) Atleast some of the pores in void space are inter-connected.

c) Solid phase is distributed throughout the domain

The above is not exact definition for porous media.

→ In the pores of porous media there may be gas or liquid present. There are possibilities of flow of these fluids through the media.

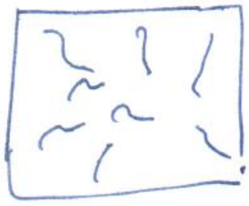
One may analyse the flow of these fluids by various approach using principles available in mechanics.

* Molecular Approach :

The movement of molecules are to be considered. This can be tedious even in this age of high speed super computers. (∵ One gram ~~of~~ mole of a gas contains 10^{23} molecules. ∴ To determine paths for each molecules will be quite tedious). Also the initial positions and initial momenta of all the molecules are difficult to observe.

* Microscopic Approach using continuum principles :

In the last semester you have studied the fluid as a continuum. While assuming the fluid as a continuum,



you were able to apply various continuum mechanics principles to analyse fluid motion. In fluid mechanics you have used the → conservation of mass, conservation of momentum, conservation of energy, etc. ~~to~~ for flow of liquids and/or gases.

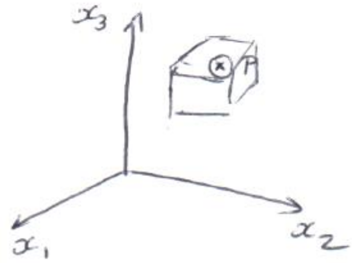
There are certain properties associated (that are determined statistically) when you treat fluid as continuum. Some of the properties associated to the fluid as continua are - density, viscosity, etc.

⇒ Hope you have studied the difference between a particle in the continua and a mathematical point in the continua.

⇒ A particle is an elementary volume from the continua to which we can associate fluid properties.

You know density is ratio ~~of~~ between mass and volume

Consider the fluid continua represented in an x_1, x_2, x_3 three dimensional orthogonal co-ordinate system.



⇒ Let P be a mathematical point in the space

i.e. $P(x_1, x_2, x_3)$

→ From the definition of density, it is not possible to assign density to the mathematical point. ($\because$ It does not have volume).

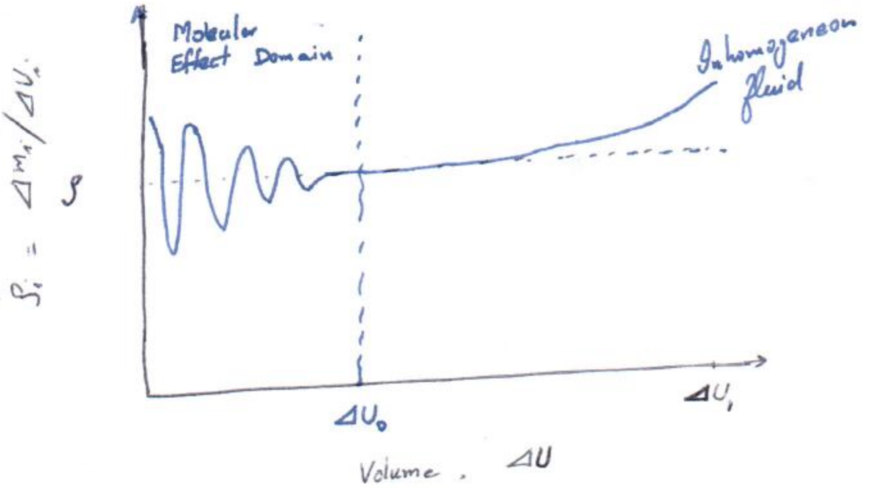
$\therefore$ To incorporate density, there should be some minimum volume of liquid (or fluid).

Let us consider sufficient volume ΔU around P.

The volumes are taken from a higher value ΔU_1 ~~to be~~ and decreased

i.e. $\Delta U_1 > \Delta U_2 > \Delta U_3 > \cdots$

$\rho_i = \dfrac{\Delta m_i}{\Delta U_i}$ as per definition.



For the different values of ΔU_i, the corresponding ρ_i is calculated and plotted as shown

For large values of ΔU_i - you can see that for inhomogeneous liquid, the density values changes. As ΔU_i is reduced, the density attains a stable value ρ. On further reducing below ΔU_0, the density calculated shows fluctuations.

i.e. Below ΔU_0 the molecular effects in the fluid play dominant role and we cannot define density.

∴ ΔU_0 is the minimum volume required in the fluid continua to associate the property - density.

This volume can now be considered as a fluid particle at the mathematical point P.

$$\rho(P) = \lim_{\Delta U_i \to \Delta U_0} \rho_i = \lim_{\Delta U_i \to \Delta U_0} \left(\frac{\Delta m_i}{\Delta U_i} \right).$$

⇒ In actual the phenomena occur at molecular levels. Due to our inadequacies we are not able to analyse at that level. Molecules move in and out of the particle volume ΔU_0 around P. As we took the minimum volume ΔU_0 to represent density, we may also require a minimum time step at which ~~the~~ ΔU_0 is evaluated. That is you require a representative time interval Δt_0.

↳ The time interval should not be too large

↳ It should not be shorter than mean free time of a molecule.

⇒ At this microscopic continuum approach level many other properties also come into picture, that are actually occurring at molecular levels

e.g. 1) Mass transport by molecular diffusion

2) Heat transfer

3) Momentum transfer.

At continuum level, we associate coefficients like:

* Molecular diffusivity
* Thermal diffusivity
* Kinematic viscosity, etc.

This fluid continuum approach is very much suitable for fluid flow analysis in surface water hydrology, open channel hydraulics, fluid mechanics, etc where there are definite boundaries for fluids.

⇒ In porous medium, there are numerous interstices, fissures, or pores and therefore to associate ~~solid~~ boundaries to fluid continuum may not be practical. Here again, ~~you need to~~ the process becomes tedious.

Therefore we need to think of some macroscopic approach to analyse fluid flow in porous media.

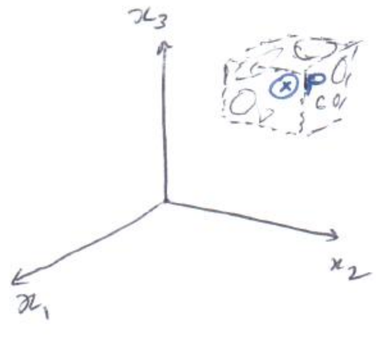
* <u>Macroscopic Approach for Fluid flow in porous medium.</u>

To apply statistical mechanics to porous medium we may require to define certain macroscopic properties.

Rather than considering only the fluid inside the media as a continuum, here we consider the entire porous media as a continuum.

↳ Now the continuum mechanics principles are directly applied to the porous medium.

Here again as explained about particle and mathematical point in a fluid continuum, we need to define a similar sort of representative volume of porous media that can be associated with the mathematical point P.



The minimum volume that can truly represent the mathematical point P in the medium is called <u>representative elementary volume</u>.

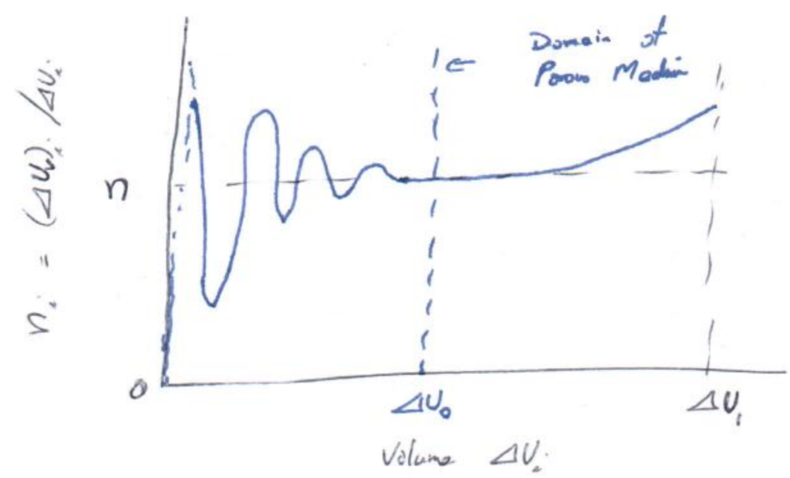
Q: How do you know what is the REV for a porous medium?

For that we define the property porosity

$$\text{Porosity} = \frac{\text{Volume of voids}}{\text{Total volume}}.$$

In a same approach described for density of a fluid similarly around mathematical point P we provide volumes ΔU_i such that $\Delta U_1 > \Delta U_2 > \dots$

We will evaluate n_i for each volumes and then plot.



From the figure ΔU_0 is the representative elementary volume, in which we can define porosity for the porous medium.

$$n(P) = \lim_{\Delta U_i \to \Delta U_0} \frac{(\Delta U_v)_i}{\Delta U_i}$$

So now porosity is a macroscopic continuum property.