LECTURE 19

20 - FEB - 2014

Layered Porous Media

In the last class we suggested that porous media in nature rarely exist as homogeneous. The permeability may differ both in horizontal ~~or~~ and vertical directions.

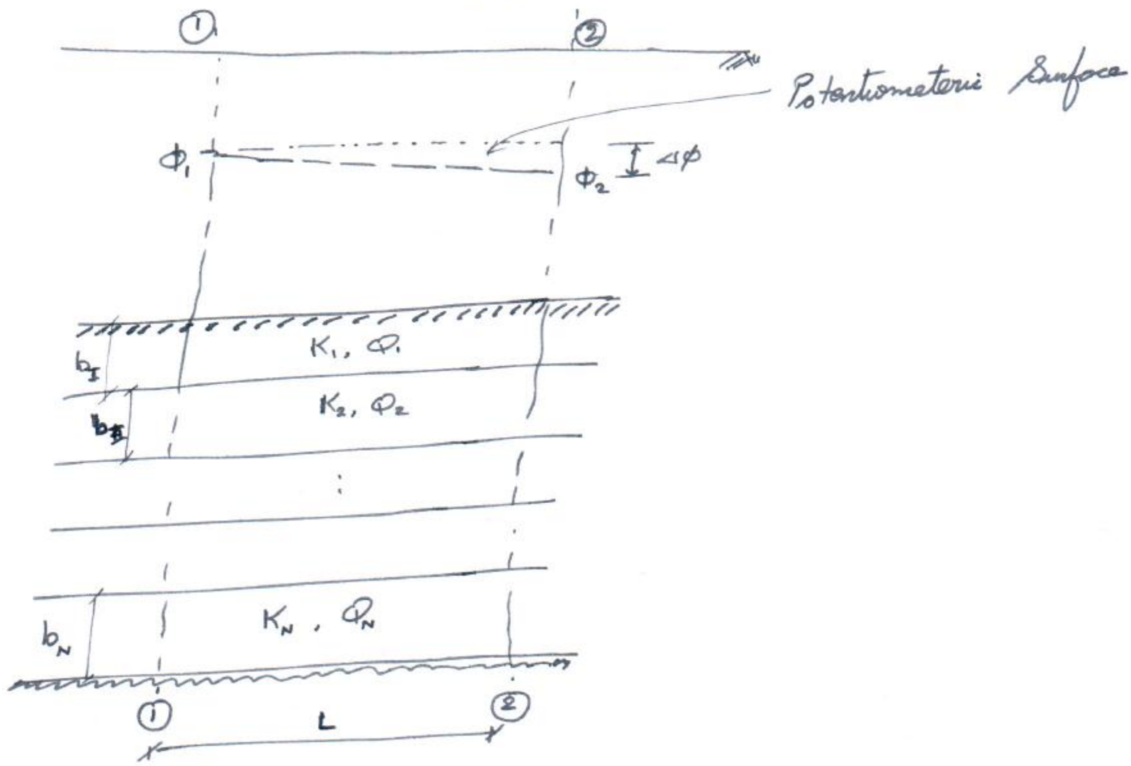
⇒ It will be better to evaluate an average permeability or average hydraulic conductivity for the region, that can be applied throughout the region ~~so that~~ such that the mass of water that flows will remain the same.

Such average hydraulic conductivity can also ~~suggest~~ help in doing regional groundwater flow analyses in a better way with less complexity.

We will see the methods to evaluate average hydraulic conductivity or equivalent hydraulic conductivity.

i) Flow parallel to combination of N- layers.

→ Let us consider a situation where there are N ~~b~~ different soil layers in a confined aquifer.



→ We are considering flow to occur parallel to layers in the confined aquifer.

→ The piezometric head at section ①① is ϕ_1
The piezometric head at section ②② is ϕ_2

→ Flow occurs in the direction ①① to ②②

→ The N - layers in the confined aquifer are independently assumed as homogeneous layer.

→ Recall Darcy discharge, $q = K J$

or $\vec{q} = -\underline{\underline{K}} \cdot \nabla\phi$

→ In each layer

$$\vec{q}_I = -K_I \nabla\phi$$

→ The depth of each layers are also given in the figure.

→ We can evaluate now discharge per unit width ~~q~~ in each layer

i.e. Q_I, Q_{II}, ..., Q_N

→ Now $Q_I = b_I K_I \frac{\Delta\phi}{L}$

∴ Total discharge in aquifer

$$Q = \sum_{I=1}^{N} Q_I = \sum_{I=1}^{N} K_I b_I \frac{\Delta\phi}{L}$$

In this case $\frac{\Delta\phi}{L}$ is constant for all layers.

$$\therefore Q = \frac{\Delta\phi}{L} \sum_{I=1}^{N} K_I b_I \rightarrow (1)$$

⇒ The same discharge Q can be evaluted w.r.t. overall depth b and equivalent K.

$$\text{i.e.} \quad Q = K_{eqp}\, b \frac{\Delta\phi}{L} \rightarrow (2)$$

From (1) and (2), we get

$$K_{eqp} = \frac{\sum_{I=1}^{N} K_I b_I}{\sum_{I=1}^{N} b_I} = \frac{\sum_{I=1}^{N} K_I b_I}{b}$$

We can also say Transmissivity for each layer:

$$T_I = K_I b_I$$

The equivalent transmissivity can also be stated

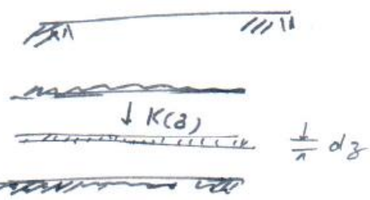
$$T_{eqp} = K_{eqp}\, b .$$

⇒ If instead of N- distinct layers, if the hydraulic conductivity in the aquifer varies in z- direction

i.e. $K = K(z)$.

Again we can formulate the equivalent hydraulic conductivity as below:

The total discharge per unit width



Q will be function of hydraulic conductivity, depth, and hydraulic gradient.

→ The discharge through elementary strip dz

is $$dQ = K(z) \frac{\Delta\phi}{L} dz$$

Integrating throughout depth:

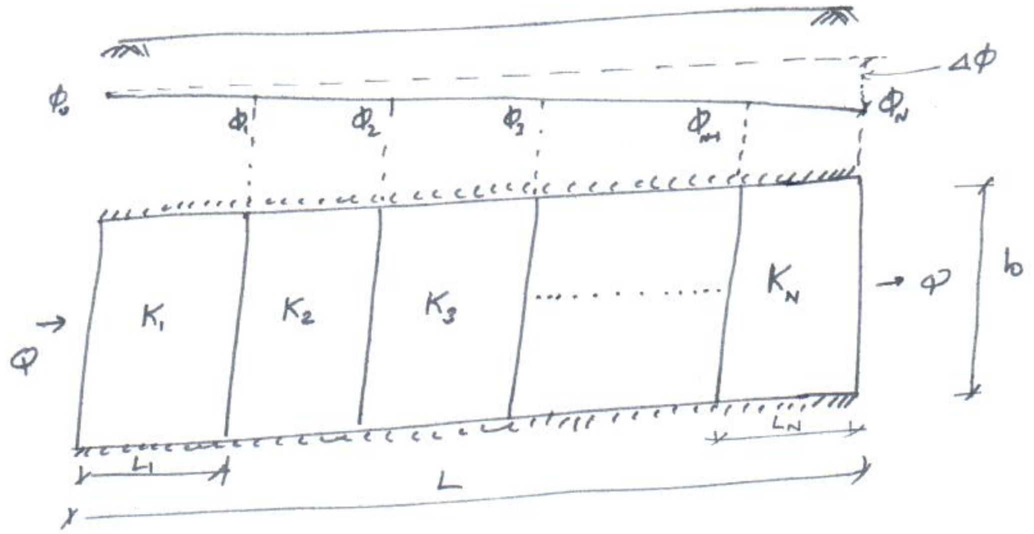
$$Q = \frac{\Delta\phi}{L} \int_0^b K(z)\, dz = K_{eqp}\, b \frac{\Delta\phi}{L}$$

$$\therefore K_{eqp} = \frac{1}{b} \int_0^b K(z)\, dz$$

→ Equivalent aquifer transmissivity,

$$T_{eqp} = K_{eqp}\, b = \int_0^b K(z)\, dz$$

(ii) Flow in a direction perpendicular to the layers.



Consider a confined aquifer consisting of N-layers. The layers are as shown in above figure. des in the direction perpendicular to groundwater flow.

We are assuming steady discharge of water.

⇒ The hydraulic gradient in each layer is different (∵ Hydraulic gradient for layer 1

$$-J_1 = \frac{\Delta\phi_1}{L_1} = \frac{\phi_1 - \phi_0}{L_1}$$

$$-J_2 = \frac{\Delta\phi_2}{L} = \frac{\phi_2 - \phi_1}{L_2} \;)$$

$\vdots$

⇒ The total drop in piezometric head is:

$$\Delta\phi = \sum_{I=1}^{N} \Delta\phi_I .$$

$$L = \sum_{I=1}^{N} L_I$$

Assuming depth of aquifer as b throughout.

$Q_1 = Q_2 = Q_3 = \cdots = Q_N = Q$ (steady discharge)

In layer I, $\quad Q = K_I \, b \, \dfrac{\Delta \phi_I}{L_I}$

Also $\quad Q = K_{eqv} \, b \, \dfrac{\Delta \phi}{L}$

Note: $\quad \Delta \phi_I = \dfrac{Q L_I}{K_I \, b}$

Also $\quad \Delta \phi = \dfrac{Q L}{K_{eqv} \, b} \quad \rightarrow (1)$

$$\Delta \phi = \sum_{I=1}^{N} \Delta \phi_I = \frac{Q}{b} \sum_{I=1}^{N} \frac{L_I}{K_I} \quad \rightarrow (2)$$

From (1) and (2),

$$\frac{QL}{K_{eqv} \, b} = \frac{Q}{b} \sum_{I=1}^{N} \frac{L_I}{K_I}$$

or $\quad \dfrac{L}{K_{eqv}} = \displaystyle\sum_{I=1}^{N} \frac{L_I}{K_I}$

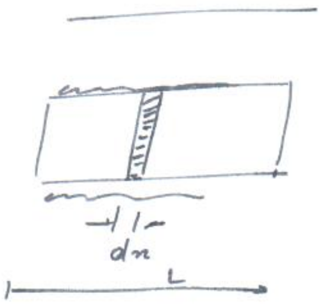
i.e. $\quad \cancel{\dfrac{L}{K_{eqv}} = \dfrac{L_1}{K_1} + L_1}$

$$\frac{L}{K_{eqv}} = \frac{L_1}{K_1} + \frac{L_2}{K_2} + \cdots + \frac{L_N}{K_N}$$

If there is continuous variation of K in the flow direction (say $K = K(x)$)

Then discharge through elementary strip

$$q = K(x)\, b\, \frac{d\phi}{dx}$$



Implementing this relationship, throughout the length L and rearranging terms:

$$\frac{L}{K_{equ}} = \int_0^L \frac{dx}{K(x)}$$

Chapter

CONTINUITY AND CONSERVATION EQUATION FOR HOMOGENEOUS FLUID

⇒ Homogeneous fluid → The term homogeneous fluid considered here is that it is of single specie.

→ In the previous chapter we described equation of motion of a homogeneous fluid in porous media.

i.e.
$$q_i = -K_{ij} \frac{\partial \phi}{\partial x_j}$$

or
$$q_i = -\frac{k_{ij}}{\mu}\left(\frac{\partial p}{\partial x_j} + \frac{\partial}{\partial x_j}(\rho g z)\right)$$

or
$$q_i = -\frac{k_{ij}\,\rho g}{\mu}\frac{\partial}{\partial x_j}\left(\frac{p}{\rho g} + z\right)$$

The Control Volume Approach

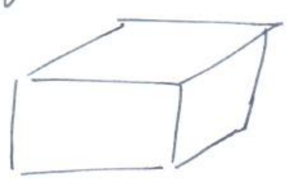
The difference between Lagrangian and Eulerian approach is familiar to all of you.

We will be using Euler's approach for this chapter.

⇒ A definite volume in space, fixed
→ called control volume.

⇒ For porous media, control volume is appropriate as in most of the cases solid matters in the media are not moving.

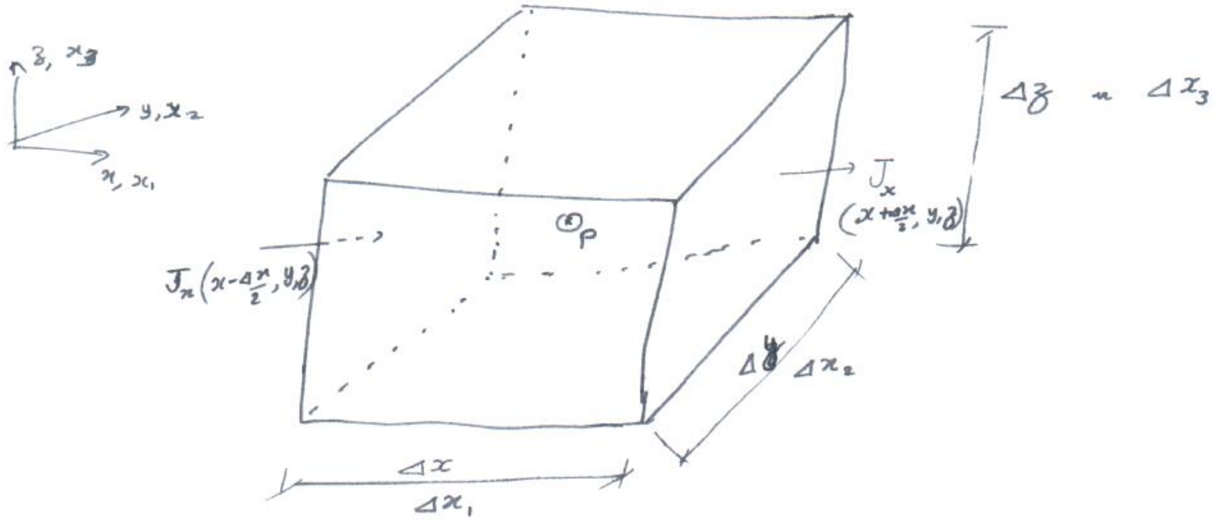
⇒ We can define any shape for the volume.
Bounded by control surfaces.



→ The amount of fluid can change in the CV. But shape and position is assumed same.

Mass Conservation in a Non-deformable Porous Media

Consider the CV of a porous media as below:



The CV is a rectangular box of volume $= \Delta x \Delta y \Delta z$

The centre of CV is describing a mathematical point $P(x, y, z)$ or say $P(x_1, x_2, x_3)$

⇒ We are choosing the coordinate system such that the anisotropic properties like permeability are described in principal directions (assumptions) i.e. k_{11}, k_{22}, k_{33} are given other $k_{ij} = 0$; $i \neq j$.

$\Rightarrow$ We can now describe mass fluxes through each surfaces:

Recall in an earlier chapter we described mass flux as $\vec{J}$ (mass of water per unit area per unit time)

In the ~~cf~~ control volume, the flux through the surfaces in the x-direction is given as J_x.

→ In the left boundary : $J_x\left(x - \frac{\Delta x}{2}, y, z\right)$

In the right boundary : $J_x\left(x + \frac{\Delta x}{2}, y, z\right)$

→ In a time interval Δt, we can have

Mass of water coming in through left boundary

$$= J_x\left(x - \frac{\Delta x}{2}, y, z\right) \cdot \Delta y\, \Delta z\, \Delta t$$

$||^{ly}$ Mass of water going out through right boundary

$$= J_x\left(x + \frac{\Delta x}{2}, y, z\right) \Delta y\, \Delta z\, \Delta t$$

$\therefore$ Net mass of water going out ~~thro~~ in x-direction

$$= \left[J_x\left(x + \frac{\Delta x}{2}, y, z\right) - J_x\left(x - \frac{\Delta x}{2}, y, z\right)\right] \Delta y\, \Delta z\, \Delta t$$