LECTURE - 11

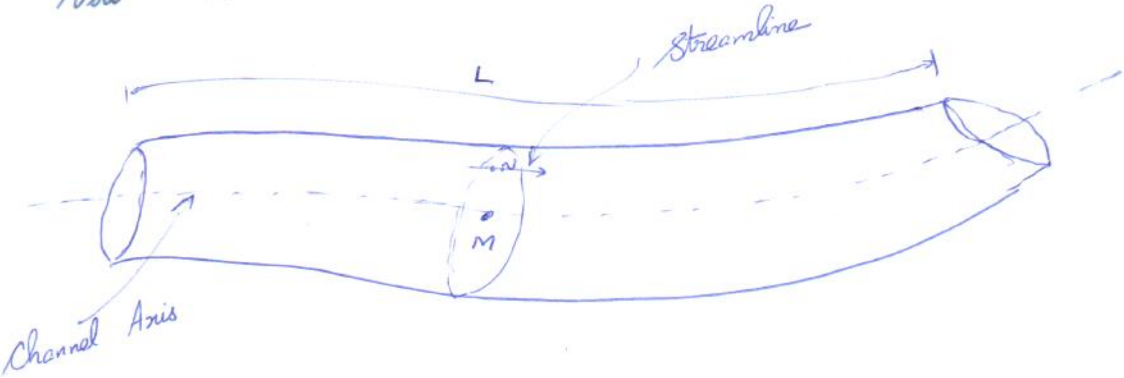
30-JAN-2014

The Porous Medium Model (Contd...)

We started the discussion on porous medium model in the last class. This was a concept adopted by Bear and Bachmat (1966, 67) in which the void spaces in porous medium consist of - channels and ~~pore~~ junctions.

* The volume in junctions are assumed to be negligible, while in channels it is significant.

Now consider a channel

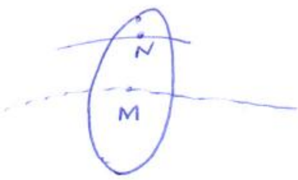


$L \rightarrow$ length of the channel
$s \rightarrow$ distance along the axis

We are now going to use two orthogonal Cartesian co-ordinate systems

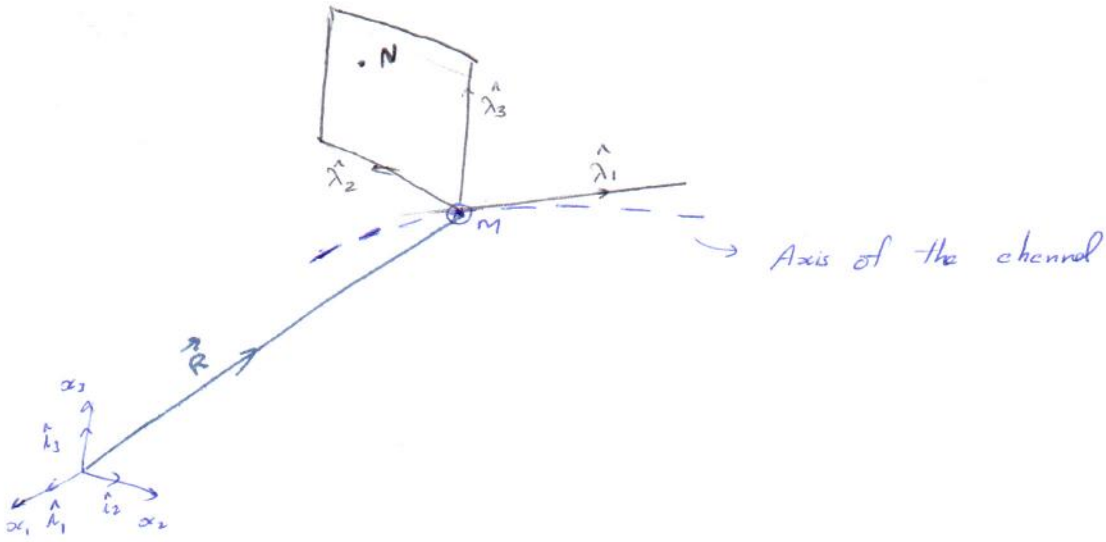
→ One fixed in space
→ The other one that moves along the axis of the channel.

Consider a cross-sectional area in the channel where the channel axis passes through point M.



Several streamlines can pass through that cross section (of course for laminar flow).

One particular streamline crosses at point N in that same cross-section.



The fixed coordinate is x_1, x_2, x_3 and the coordinate system moving along axis is $\lambda_1, \lambda_2, \lambda_3$.

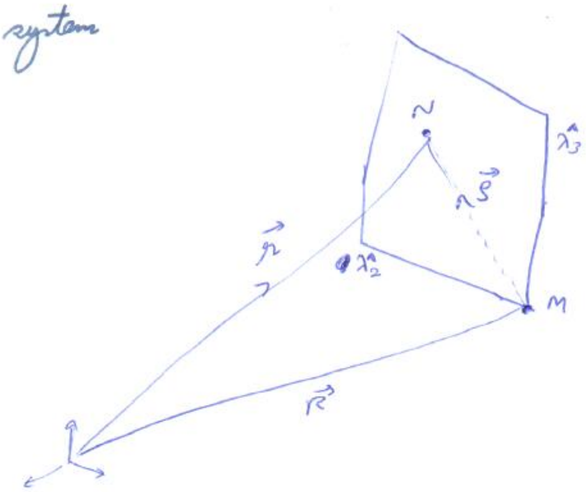
→ The position of point M can be given by position vector say $\vec{R}$

$$\vec{R} = \vec{R}(s) \quad ; \quad 0 \le s \le L$$

or $\quad x_i = x_i(s)$

→ At M the second co-ordinate have a tangential axis λ_1 (with unit vector $\hat{\lambda_1}$), principal normal axis λ_2, and binormal axis λ_3.

The point N on the cross sectional area can also be described using fixed co-ordinate system



If $\vec{\varsigma}$ is the position vector for point N in the local co-ordinate $\lambda_2 \lambda_3$ plane, then ~~the~~ the position vector of N can also be described as

$$\vec{r} = \vec{R} + \vec{\varsigma}$$

($\vec{\varsigma}$ is also called radius vector).

The Averaging Procedure

As described about the general conservation equations for fluid in fluid continuum, those equations are applicable to any point in fluid filling the void space of porous media

However as discussed - the fluid continuum approach may be tedious. Therefore we considered porous media as a continuum. We can define representative elementary volumes.

→ For an intensive property g_α of a fluid, let us evaluate this intensive property in the representative elementary volume centered at any mathematical point P.

Let the volume be ΔU_o.

Volume of voids $= (\Delta U_o)_v$

$$\therefore \quad \bar{g}_\alpha(P) = \frac{1}{(\Delta U_o)_v} \int_{[(\Delta U_o)_v]_P} g_\alpha(\vec{x}')\, dU_v(\vec{x}')$$

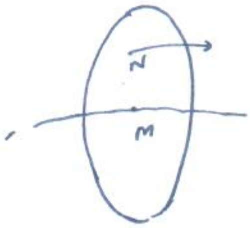
This is average $\bar{g}_\alpha(P)$ at point P for some intensive fluid property g_α.

→ Within an REV, flow takes place only in channels.

As the entire REV is not filled of channels as well as each channels may have individual directions. we may average the property as such

(i) Averaging performed over cross section of a channel

(ii) The resultant flow along each network is averaged over REV to get the averaged value of g_α for entire REV.



We suggested that

let s = length along the axis

σ = length along the streamline

L = total length of a channel

~~c = total length of a streamline~~

Along the streamline passing through point N, the intensive property may be $g_\alpha(\sigma)$

Let the c.s.a be $\omega(s)$.

∴ First average is given as:

$$\langle g(s) \rangle = \frac{1}{\omega(s)} \int_{[\omega(s)]} g(\sigma)\, d\omega(\sigma)$$

Please note that $\lambda_2 \lambda_3$ plane is orthogonal to ~~the axis~~ channel axis.

∴ Any property in the cross section can have components in λ_1- direction, λ_2- direction and λ_3- direction.

Note

$$\langle \nabla g(s) \rangle = \left\langle \frac{\partial g}{\partial s} \right\rangle \hat{\lambda_1} + \left\langle \frac{\partial g}{\partial \lambda_2} \right\rangle \hat{\lambda_2} + \left\langle \frac{\partial g}{\partial \lambda_3} \right\rangle \hat{\lambda_3}$$

Velocity, in the cross section may be given as:

$$\langle \vec{V} \rangle = \langle V_s \rangle \hat{\lambda_1} + \langle V_{\lambda_2} \rangle \hat{\lambda_2} + \langle V_{\lambda_3} \rangle \hat{\lambda_3}$$

This first average gives local transport equation.

$\Rightarrow$ The second average $\rightarrow$ averaging the intensive properties over the void space of REV.

In the REV,

Volume of voids $= (\Delta U_0)_v$

$= n \, \Delta U_0$

($n \rightarrow$ porosity).

Now $(\Delta U_0)_v = (\Delta U_0)_{channel} + (\Delta U_0)_{junction}$

$$\therefore \quad G_\alpha = \int_{n\Delta U_0} g_\alpha \, dU_v = \int_{(\Delta U_{channel})} g_\alpha \, dU_{channel}$$

$$+ \int_{(\Delta U_{junction})} g \, dU_{junction}$$

However, we are assuming volume of junctions as negligible in an REV.

$$\therefore \quad G_\alpha = \int_{(\Delta U_{channel})} g_\alpha \, dU_{channel}$$

Q: How are you going to integrate the volume of channels?

$\langle g_\alpha(s) \rangle$ was the average obtained for any cross section in a channel. The second averaging has to be done in such a way that

* Integrate first for entire channel
* Add for all channels in the REV
* Average for REV.

i.e. $$G_\alpha = \sum_{i=1}^{M} \int_0^{L_i} \langle g_\alpha(s) \rangle \, w(s) \, ds$$

The second average is then given as

$$\boxed{\bar{g}_\alpha(P) = \frac{1}{(n \Delta U_0)_P} \int_{(n \Delta U_0)_P} g_\alpha \, dU_v}$$

Like this we can define intensive property for each REV in the porous medium.

Equations of Volume and Mass Conservation

i) Volume conservation

Consider a small stretch of length Δs of a channel void space in a porous media.

$$s_2 = s_1 + \Delta s$$

Also volume of this channel void let us say as U_v

$$U_v = \int_{s_1}^{s_2} w(s)\, ds \longrightarrow$$

Now in the fluid continuum conservation

$$\frac{\partial g_\alpha}{\partial t} + \nabla \cdot \left(g_\alpha \vec{V}_{G\alpha} \right) = I_\alpha$$, For an incompressible liquid, if we ~~put~~ consider specific volume ν and density of entire fluid system as ρ.

Then $g_\alpha = \rho \nu = 1.0$

$\vec{V}_{G\alpha} = \vec{V}'$ → (Volume averaged velocity)

$I_\alpha = 0$ (Because incompressible liquid)

Then you have $\nabla \cdot \vec{V}' = 0$

Adopting this information in the above channel void space.

$$\int_{U_v} (\nabla \cdot \vec{V}')\, dU = 0$$

(Gauss divergence)

i.e. $$\int_{(w_1 + w_2 + s)} (\vec{V}' \cdot \hat{n})\, dS = 0$$