LECTURE 10

27-JAN-2014

THE CONSERVATION EQUATIONS

As we recall the last few lectures, we were discussing about the general conservation equation. For any extensive property G for an α-specie in a multi-component fluid, the conservation equation is given as:

$$\frac{\partial g_\alpha}{\partial t} + \nabla\cdot\left(g_\alpha \vec{V}_{G\alpha}\right) = I_\alpha \quad \longrightarrow (1)$$

The application of this equation for properties

(i) Conservation of mass of an α-specie in a multi-specie liquid

(ii) Conservation of mass of the entire multi-specie liquid

(iii) Conservation of linear momentum of an α-specie in a multi-specie liquid.

(iv) Conservation of linear momentum of entire fluid system:

While evaluating conservation of linear momentum of entire liquid; we obtained the equation as:

$$\frac{\partial}{\partial t}(\rho \vec{V}^*) + \nabla\cdot\left(\rho \vec{V}^* \vec{V}^*\right) + \nabla\cdot \bar{\bar{J}}^*_m = \sum_{\alpha=1}^{N} \rho_\alpha F_\alpha \quad \longrightarrow (2)$$

where
ρ → density of entire fluid
$\vec{V}^*$ → mass average velocity of fluid
$\bar{\bar{J}}^*_m$ → diffusive momentum flux
ρ_α → density of α-specie
F_α → force per unit mass of α-specie acting on α-specie. (equivalent to acceleration).

We also suggested yesterday that looking into equation (2) momentum is also transported in two ways:

↳ By the advective flux or bulk fluid motion $(\rho \vec{v}^* \vec{v}^*)$

↳ By the diffusive flux $(\bar{\bar{J}}^*_m)$

The momentum fluxes give ideas on the force that is transmitted per unit area.

→ The diffusive momentum flux $\bar{\bar{J}}^*_m$ is a surface phenomenon and can be actually correlated with stress tensor.

i.e. $$(J^*_m)_{ij} = \sigma_{ij} = -p\,\delta_{ij} + \tau_{ij}$$

Therefore we can write the conservation equation (2) in index notation as such:

$$\frac{\partial}{\partial t}(\rho V^*_i) + \frac{\partial}{\partial x_j}(\rho V^*_i V^*_j) + \frac{\partial p}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} = \sum_{\alpha=1}^{\tilde{N}} \rho_\alpha (F_\alpha)_i$$

↳ (3)

From this form of equation (i.e. (3)), you can arrive at

→ Cauchy's equation of motion

→ Euler's equation of motion

→ Navier - Stokes equations, etc.

Please note that $\sum_{\alpha=1}^{\tilde{N}} \rho_\alpha \vec{F}_\alpha$ signifies body force and if it is only due to gravity then $\sum_{\alpha=1}^{\tilde{N}} \rho_\alpha \vec{F}_\alpha = \rho \vec{g}$.

THE POROUS MEDIUM MODEL

→ The general conservation equations for a fluid continuum was discussed till now.

→ In porous media, the fluid flows through pores and voids, etc.

→ We have earlier suggested that if we try to do microscopic of analyses of fluid motion by treating only fluid as continuum, then the analyses can be tedious.

→ We subsequently then suggested porous media as a continuum in the macroscopic analysis. (Recall the REV's suggested for porous media).

→ The question then arises how will you apply the conservation equations that were developed for the fluid motion in fluid continua to the fluid motion in porous media continua.

* A particular form is to utilise the statistical average of the parameters and variables of the fluid for entire porous media.

The Conceptual Model

→ Looking back, we know that in actual many phenomenon occur at molecular or less than that level.

* As said earlier, to analyse at that level it is cumbersome.
* Therefore, this complicated phenomena (that may be impossible to represent mathematically) is replaced by simpler systems (that can be easily represented mathematically).

Now the flow and transport in porous media is analysed using simpler concepts on continua. (That is our inadequacy).

* While using continuum concepts we will utilise various coefficients or parameters (like diffusivity, dispersivity, hydraulic conductivity, etc.) due to our limited knowledge on the actual process occuring at molecular level.
* We may utilise various conceptual laws (or theories) to represent this phenomenon

(e.g. Recall in hydrology you have studied — the complex process of rainfall - runoff in a watershed by a simple fictitious conceptual model - Unit Hydrograph)

Similarly various processes in porous media are represented through conceptual models (that are easier to represent through mathematical equations).

Q: What is meant by conceptual model?

→ It is a simplified way of visualising a phenomenon that cannot be directly observed (at lower scales), may be represented in macroscopic approach through system excitations and responses

i.e. Rainfall → Natural Phenomenon → Runoff

Rainfall → [System Ω] → Runoff

→ There are three steps involved:

(i) Complicated system converted to simplified conceptual model

(ii) To analyse the model by available theories (e.g. mass conservation, momentum conservation, etc.) and derive mathematical relationships for the required or investigated phenomenon.

(iii) To test the validity of this conceptual model, we require to perform controlled experiments.

Therefore, in this macroscopic analysis, all the parameters and coefficients should be measurable quantities.

Q: If we can have experimental studies and observations, then why should we go for conceptual model analyses for any phenomenon in porous media (or any continuum)?

Ans: We suggested that conceptual model is used to represent the complex actual phenomenon in a simplified way. The advantages of conceptual analyses are:

* Conceptual models based on certain concepts. We can apply the fundamental laws and check these concepts for many phenomenon.
* Conceptual models also help in arriving at better values for coefficients
* Conceptual models also helps in better planning of experiments.

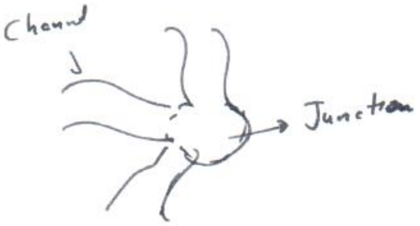
⇒ For flow through porous media to use conceptual model we will use porous medium model.

* The porous medium model is again a conceptual model.
* It ~~can be~~ is amenable for mathematical treatment of flow through porous medium.

* Our objective now will be to analyse flow through the pore space ~~B~~ using porous medium model with various assumptions and ~~statements~~ constraints.

* An important feature in porous medium → the transport of fluid is through well defined channels.
As the ~~porous~~ pores are surrounded by walls, usually velocity of water is parallel to walls.

* Bear and Bachmat (1966, 1967) suggested porous medium model
  → We will visualise the void space - composed of spatial network of inter-connected random passages (~~each~~ varying length, cross sections, junctions, etc.)
  → Therefore we will now have channels & junctions.
  A channel : → Elongated shape, an axis is defined.
  A junction : → No definite direction in space.



Usually volume of junction assumed much smaller than volume of a channel.

For laminar flow through channels the flow is usually parallel to the axis.

The fluid may be sometimes completely saturate the porous medium by a single phase.

If we are using water, we may generally consider the fluid as incompressible in the porous media.

→ As the energy is required for flow → it is assumed the energy decreases from ~~end~~ end of a channel to the other end. (But not in junction).

→ ~~We have to utilize average value of Pressure,~~

→ The active forces are
  ↳ Pressure
  ↳ Gravity
  ↳ Shear