

Buoyancy & Stability (Contd.....)

Introduction to Integral or Control Volume Analysis:

- Yesterday, we have seen how to evaluate the metacenter height of a floating body.
- For a body, which is symmetric to vertical y axis, you have seen that, the body's center of buoyancy shifts to the tilted portion.

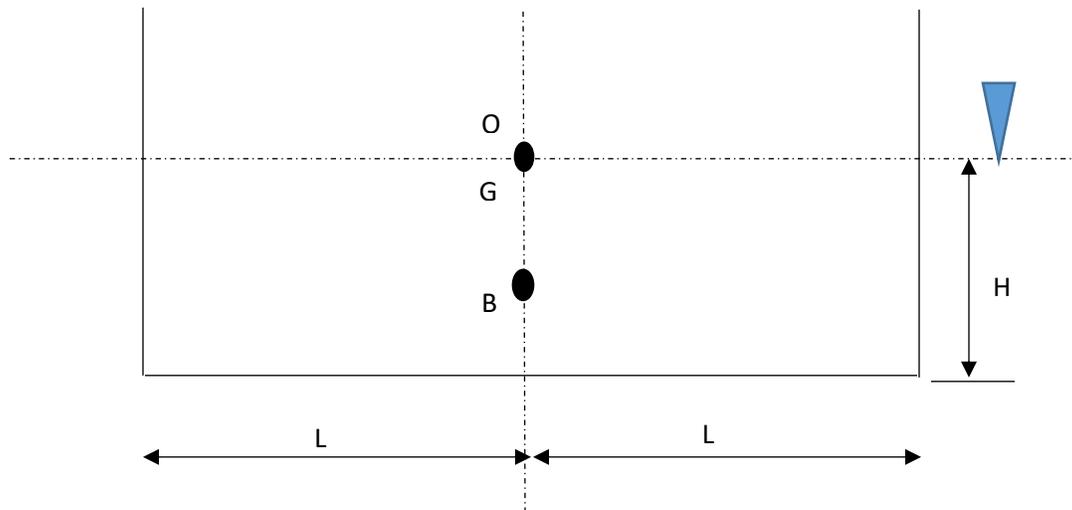
We have seen metacentric height,

$$\overline{MG} = \frac{I_0}{V_{submerged}} - \overline{GB}$$

We were trying to discuss the example problem:

Example: (Adopted from Fluid Mechanics text book by F. M. White)

A barge has a uniform rectangular cross section of width $2L$ & vertical drift height H . Determine (a) the metacentric height for a small tilt angle and (b) the range of ratio L/H for which the barge is statically stable if G is exactly at the waterline.



Given:

Barge has length 'b' into the paper & width 2L.

$$\text{Recall, } \overline{MG} = \frac{I_0}{V_{\text{submerged}}} - \overline{GB}$$

$$V_{\text{submerged}} = 2L \times b \times H = 2bLH;$$

Area moment of inertia for the waterline area (b*2L)

$$I_0 = \frac{1}{12} b(2L)^3 = \frac{2}{3} bL^3$$

MG = the metacentric height is:

$$\overline{MG} = \frac{I_0}{V_{\text{submerged}}} - \overline{GB} = \frac{(\frac{2}{3} bL^3)}{(2bLH)} - \frac{H}{2} = \frac{1}{3} \frac{L^2}{H} - \frac{H}{2}$$

$$\text{For MG to be positive, } \frac{1}{3} \frac{L^2}{H} > \frac{H}{2}$$

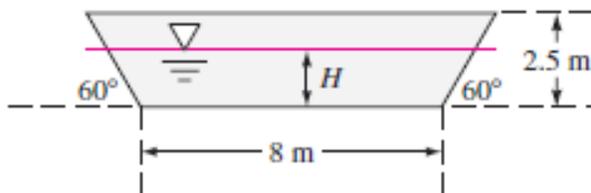
$$\text{Or, } L^2 > \frac{3}{2} H^2$$

i.e., if the barge is relatively wider compared to its draft height, then it is more stable.

- ✓ Please note that, in the example & all, we have given regular shapes as well as cross sectional areas of the floating body to be symmetric w.r.t a vertical axis.
- ✓ There are many floating bodies having irregular shapes & it will be quite tedious to determine the stability criteria.

Quiz:

A barge has the trapezoidal shape & is 22 m long into the paper. The width at the bottom of the barge is 8 m & the sides are at angle 60° shown in the figure. The center of gravity for the barge lies on the waterline. The top width of the barge is 10 m. Determine the criteria for the submerged height H for the body that allows the body in floating condition.



(Image Source: Fluids Mechanics text book from

F.M. White)

Introduction to Control Volume Approach

- ✓ As you are aware, we were discussing till now about fluid in static condition.
- ✓ If you recall our beginning lectures, we have pointed that one can use Lagrangian approach or Eulerian approach for fluid flow analysis.
- ✓ So while doing fluid flow analysis,
 1. You can describe fluid flow property at each & every mathematical point in space say (x,y,z) . Such an approach will be differential in nature & you may have to deal with large domains, where differential approach can be tedious.
 2. You can also use some finite volume in space & balance the flow-in & flow-out & determine the gross flow effects (can be force, torque etc.). Such an approach is called integral method or control volume method.
- ✓ To interpret the control volume method, we need to first understand what a system is.
- ✓ A system is an arbitrary quantity of mass of fluid quantity.

Consider this box of chalks or this duster.

- ✓ A system is separated from its surroundings by its boundaries.
- ✓ This system has a fixed quantity of mass. Mass of the system is conserved.

$$\text{i.e., } m_{\text{system}} = \text{constant}$$

$$\text{Or, } \frac{dm_{\text{system}}}{dt} = 0$$

- ✓ You have already considered systems in solid mechanics (eg in beam deflection etc)

The system obeys conservation principles likes:

1. Conservation of mass
 2. Conservation of momentum
 3. Conservation of angular momentum
 4. Conservation of energy etc.
 - 5.
- ✓ Similar to, as you have used in solid mechanics, can you use the same system concept in fluid flow analysis?
 - ✓ You will see that, it is difficult to take system in fluids, if the fluid is moving.
 - ✓ Therefore in fluids, we take a fixed or arbitrary volume in space & then we will try to apply the mechanics principle on the volume. The volume may be changing with respect to time. Such a volume is called Control Volume.
 - ✓ You were able to apply principles in mechanics to the system.

Will you be able to apply such principles to a control volume directly?