Buoyancy & Stability:

- Last two days, we discussed about hydrostatic forces on plane surface & curved surface.
- For a submerged plane surface (as well as curved surface), we computed the hydrostatic forces on one side of the surface.

- If a solid body (or any other body) is submerged or floating, the surfaces in contact with water will be subjected to hydrostatic pressure forces.
- The Archimede’s principle state that (As adopted from FM White)
  1. A body immersed in a fluid experiences a vertical buoyant force equal to the weight of the fluid it displaces.
  2. A floating body displaces its own weight in the fluid in which it floats.

- The vertical buoyant force is nothing but the net pressure acting on a submerged or floating body.

We can explain the concepts of buoyancy as such:
  ✓ Consider any arbitrary submerged body.
  ✓ We can now state that the body consists of two surfaces.
Since you have already analyzed hydrostatic forces on curved surfaces, the body consisting of upper & lower surfaces can be analyzed independently.

Taking the free body diagram of upper surface ‘abc’ (not the water column)

The surface ‘abc’ will have a hydrostatic force $F_1$ acting into the curved surface.

- The vertical component = $F_{V_1} = W_1$
- Similarly, taking the free body diagram of the surface ‘adc’

The hydrostatic force in surface ‘adc’ will be $F_2$ & the component of it in the vertical direction = $F_{V_2}$
So, the net vertical force acting on the body is called buoyant force = \( F_{v2} - F_{v1} = F_b \)

- The net pressure force on the submerged body can be evaluated as:
  \[ F = \int p \, dA \]
  - Obviously, in hydrostatic conditions, this net force effect will be in upward direction.

\( F_b = W_2 - W_1 \) = Weight of fluid equivalent to the volume of the body that is submerged.

- The line of action of buoyant forces is through the center of volume of the displaced body.
- This center of volume is called center of buoyancy (B).

**Question:** The center of buoyancy & center of gravity of the submerged body, whether it will be same?

**For Floating Bodies:**

You know that in the case of a fully submerged solid body, there will be a net pressure force acting upwards (i.e., the buoyant force).

- In floating bodies, only a portion of the body is submerged.
• The displaced volume by the floating object is given in red dotted colors.
• You can clearly see that the volume of the object & the volume of the water displaced are different.
• In a body to float, the weight of the body should be supported by the buoyant force.

\[ F_B = \rho g \times \text{Displaced volume of liquid} = \text{Weight of the floating body.} \]

• Again, the line of action of buoyant force & the weight of the body have to be collinear.
• This is because in static conditions, the net moment is zero.

**Neutral & Positive Buoyancy:**

A floating body will have its weight & volume distributed in such a way that it will equal to the specific weight of water. Then the object is neutrally buoyant.

**Neutrally Buoyant** objects will remain at rest at any point where it is immersed.

**Positive Buoyant** objects will remain in floating condition that is will reach the upper surface of water.

**Stability:**

We may not know whether a floating object is in stable conditions or not.

You may have seen several examples, where the object is floating, still overturns. Why???

• If a floating body frequently overturns, then it is unstable.
• As engineers, we have to design the floating bodies such that it does not overturn.
• Consider a symmetric floating body:

  In the figure ‘a’, the body’s center of gravity is at position G & weight W acts. The buoyant force acts at center of buoyancy B. For the floating body:

  \[ F_B = \text{[sp. Gravity of fluid]} \times \text{[volume of fluid displaced by floating body]} \]

  \[ = \text{Weight of the floating body} \]

• Using this relation, the body’s center of mass G & the center of buoyancy are computed. Both W & F_B are collinear.

• If we tilt the body of a small angle \( \Delta \Theta \), then the vertical buoyant force’s line of action will change. This is because of the change in surface area of water that is in contact with body.

Let the new position be ‘B’.

• A vertical line drawn upwards from ‘B’ will intersect the line of symmetry. This point (called M) is the metacenter.

• If M is above G, the body has a restoring moment & the original position is stable. (i.e, Metacenter height is positive).

• If M is below G, then the overturning moment exists & body is not stable.