25/01/2017

Lecture 8

Hydrostatic forces (Cont.)

- Yesterday, we discussed about hydrostatic forces acting on one side of a submerged plane surface.
- The plane area centroid was discussed and also the concept of center of pressure was discussed.
- The center of pressure (x_{cp},y_{cp}) was evaluated as:

  $x_{cp} = \frac{-\rho g sin\theta (Ixy)}{(Pcg)A}$;

  $y_{cp} = \frac{-\rho g sin\theta (Ixx)}{(Pcg)A}$

Example: (As adopted from FM White's Fluid Mechanics book)

A sluice gate is used to restart water flow from a tank. The sluice gate is a plane at an inclined angle to the horizontal. The density of the water in the tank is 1025 kg/m^3. The width of the gate is 2.0 m. Compute:

a) Force on the gate due to water pressure.
b) Horizontal force exerted by the wall on the sluice gate that is just rested.
c) Equivalent reaction forces along hinge B.

Solution:

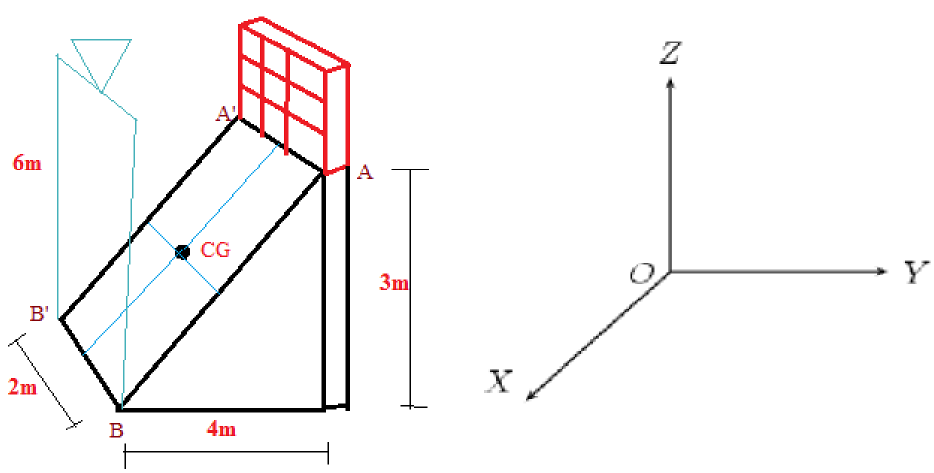

Fig.1: Problem figure along with the reference coordinate axes in 3-D view

- The length of the plate = $(4^2+3^2)^{1/2}$ = 5 m
  Area of plate = 5m × 2m = 10 m^2
- Hydrostatic force action on the gate = $P_{CG} \times A$

h_{CG} = 6m – 1.5 m = 4.5 m

⇨ $P_{CG} = \rho g h_{CG}$ = 4.5×9.8×1025 = 45200 Pa = 45.3 kPa (gauge pressure)

⇨ Hydrostatic force acting on the gate = F_p= 45.2 × 10 = 452 kN

- The center of pressure for hydrostatic force will be:

  $y_{cp} = \frac{-\rho g sin\theta (Ixx)}{(Pcg)A}$

  where, sinθ = 3/5 = 0.6

  $I_{xx} = bL^3/12 = 2 \times 5^3/12 = 20.83\ m^4$

  Putting the values, we get : $y_{cp} = \frac{-20.83\times0.6}{4.5\times10}$ = -0.278 m

- Center of pressure for hydrostatic force will be (x_{cp},y_{cp}) as mentioned earlier.

  For the given rectangular plate, $I_{xy} = 0 = \int xydA$

  So, x_{cp} = 0

  (x_{cp},y_{cp}) = (0,-0.278)

- As the problem involved in statics, the statics principles have to be applied to obtain the forces supported by the wall AA' as well as the reaction in the hinge support BB'.
- The hydrostatic force acts at the center of pressure. As the location of center of pressure is such that along the plane's y-axis, the symmetry is prevalent.

- Hence, we can resolve the problem in 2-dimension along the y-axis and draw the free body diagram.

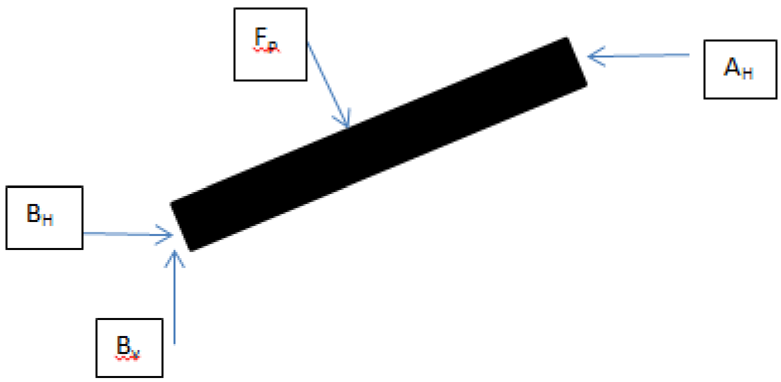

Fig. 2: Resolving of the forces involved

$\Sigma F_H = 0$

$\Sigma F_v = 0$

$\Sigma M_B = 0$

⇨ $-A_H + F_P \sin\theta + B_H = 0$

⇨ $B_V - F_P \cos\theta = 0$

⇨ $B_V = F_P \cos\theta$ = 452 × 0.8 = 361.6 kN

$\Sigma M_B = 0$

⇨ $A_H \times 3 - F_P \times (2.5 - 0.278) = 0$

⇨ $A_H = 335$ kN

Therefore, the vertical wall has to support 335 kN force.

Hydrostatic Forces on Curved Surfaces

In hydraulic structures like dams, etc. you may see that the retaining structure's surface may be curved.

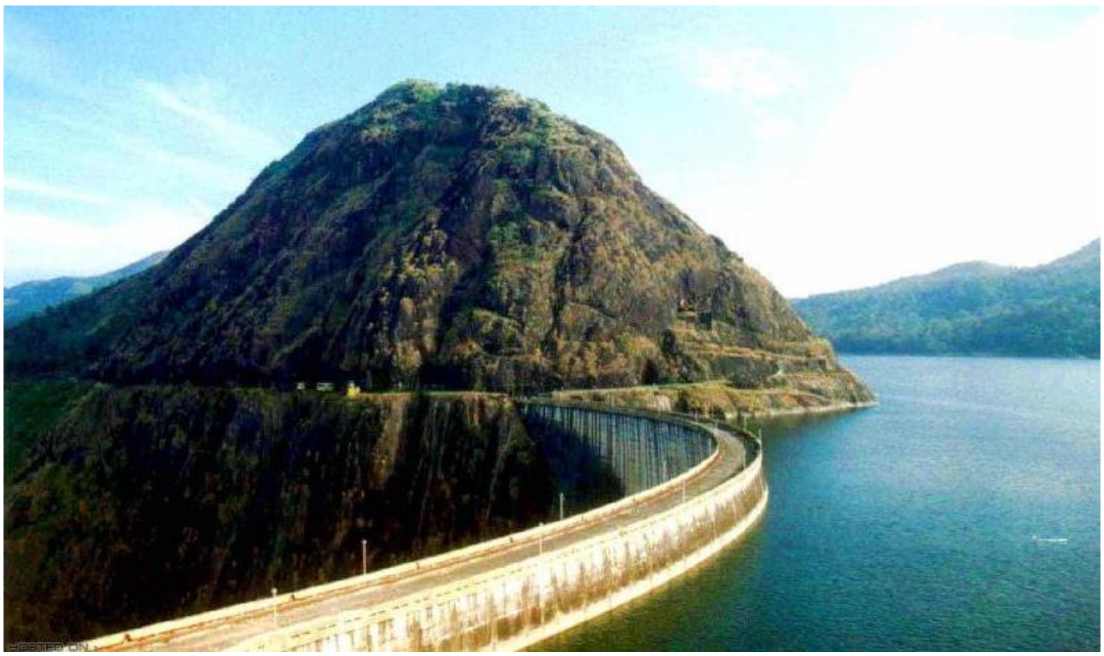

Fig. 3 : India's highest Arch dam : Idukki dam, situated over Periyar River in Kerala. Notice the curved surface of the dam's retaining structure
(Source: http://www.team-bhp.com/forum/travelogues/112939-family-getaway-idukki-dam-open-visitors-holidays.html)

- To compute the hydrostatic force, it is possible to integrate the curved surface area and also identify the surface's centroid and subsequently evaluate the pressure and force.
- However, considering the tediousness of the method, one can directly use the principles of statics to find what could be the hydrostatic force action on the curved surface. ***How??***
- Consider a curved surface submerged in water as shown:

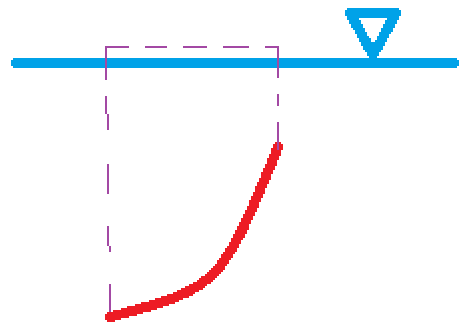

Fig. 4: Curved surface submerged in water

Take the free body diagram of the water column enclosed by the curved surface up to the water surface. The FBD consists of two portions – *abc* involving water column along the curved surface; and, *cbed* – involving water column above the curved surface.

As you have drawn FBD, you need to provide appropriate forces, so as the water column to remain in static condition as shown in Fig. 5.

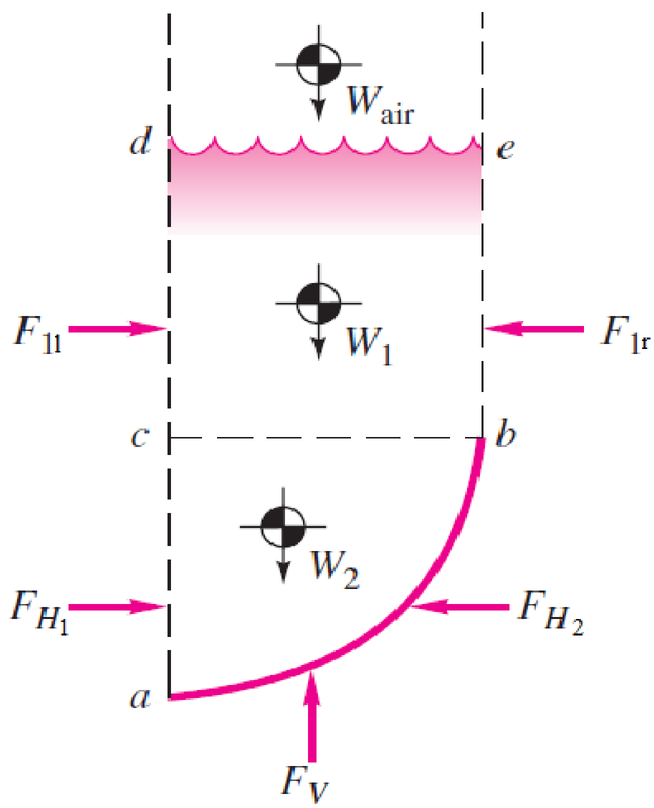

Fig. 5: Free body diagram of the fluid above the curved surface

(Source: Fluid Mechanics by Frank White)

- You can see that horizontal forces F_{1l} and F_{1r} balance on the portion *cbed* (because no contact of solid in this portion).
  - ⇨ For static condition:
    $\Sigma F_H = 0$
    $\Sigma F_v = 0$
  - ⇨ $F_{H1} - F_{H2} = 0$
  - ⇨ $F_{H1} = F_{H2}$ and
  - ⇨ $F_V - W_1 - W_2 = 0$
  - ⇨ $F_V = W_1 + W_2$
    where F_{H2} and F_V are horizontal and vertical components of the reaction exerted by curved surface on water.

Example: (Adopted from FM White's Fluid Mechanics)

A dam has a parabolic shape $z/z_0 = (x/x_0)^2$ with $x_0 = 3$m and $z_0 = 7.3$ m to store water of density 1000 kg/m^3. The gage pressure is considered in the calculation. Compute the horizontal and vertical components of the hydrostatic force on the dam. The width of the dam is 15.25 m.

Solution: The figure is as given with $x_0 = 3.0$ m and $z_0 = 7.3$ m.

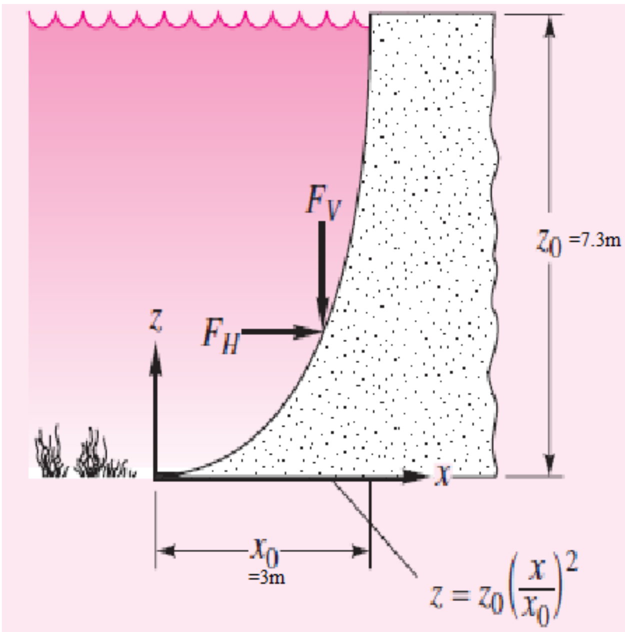

Fig. 6: Problem figure
(Source: Fluid Mechanics by F.M. White)

Take the free body diagram of the water column as shown in Fig. 7 and Fig. 8:

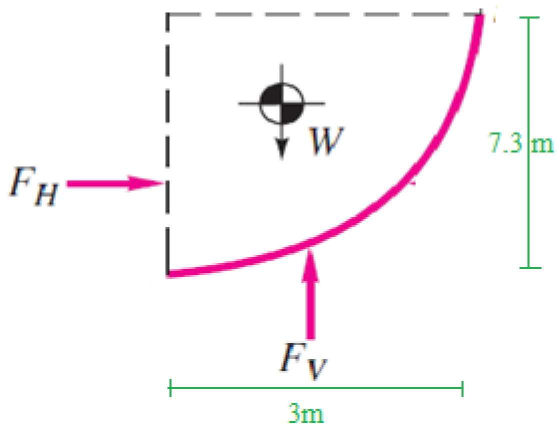

Fig. 7: FBD of the curved surface

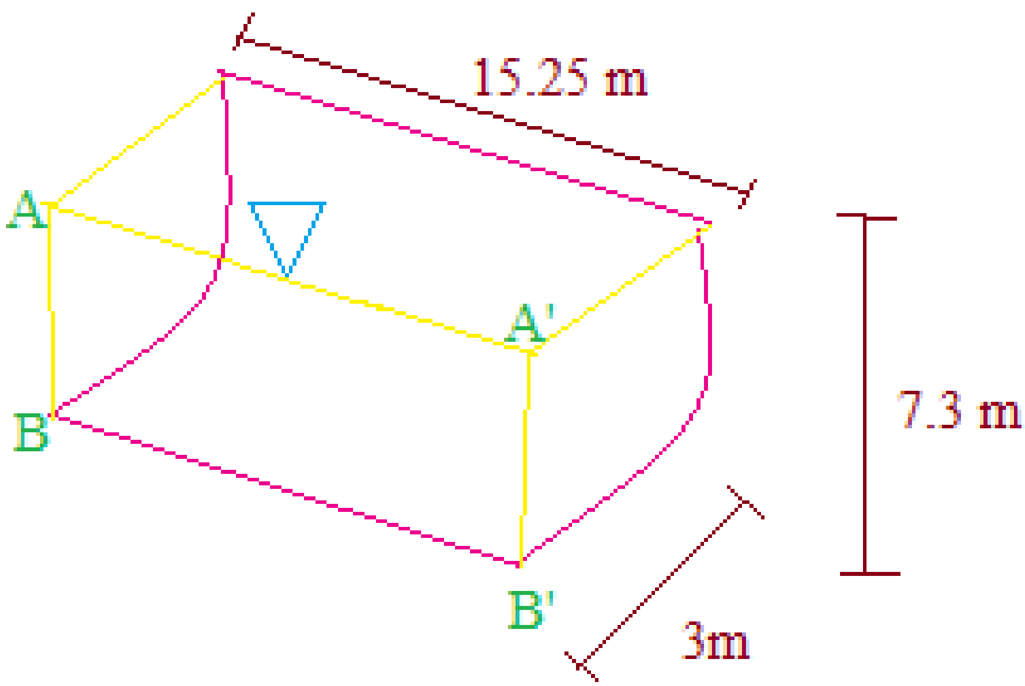

Fig. 8: FBD of the water column

Width of the dam = 15.25m

On the left side, hydrostatic force = F_H

This should be same as the horizontal component of the hydrostatic force on the right side.

$A_{AA'BB'}$ = 15.25 × 7.3 = 111.325 m^2

The centroid of the vertically projected plane AA'BB'

h_{CG} = 7.3/2 = 3.65 m

- ⇨ $F_H = \rho g h_{CG} A$ = 1000 × 9.8 × 3.65 × 111.325 = 3982100 N = 3982 kN

To find F_V

F_V = Weight of water above.

Volume = $\frac{2}{3} x_0 z_0 b$
= 2/3 × 7.3 × 3 × 15.25 = 222.65 m^3

- ⇨ Weight of water = ρg × Volume = 1000 × 9.8 × 222.65 = 2182000 N = 2182 kN
- ⇨ Resultant hydrostatic force, $F = (F_H^2 + F_V^2)^{1/2} = (3982^2 + 2182^2)^{1/2}$
  = 5038 kN