10/01/2017

Lecture 2

The Concept of Fluid

- You know all matter on earth is constituted of molecules.
- It is the way that the molecules are packed that suggests whether a matter is a solid, liquid or gas.
- For solids, the molecules are tightly packed and the molecules have strong inter-molecular cohesive bonding. The solids can retain a definite shape and volume.
- Liquids are composed of relatively close packed molecules (but not as close in solids).
  - ❖ Liquids have good cohesive forces among its molecules and can mostly retain its volume.
  - ❖ Liquids form a free surface in a gravitational field, if the upper portion is not confined by a solid.
- Gas molecules are spreaded widely. They have negligible cohesive forces among themselves and do not retain a constant volume.

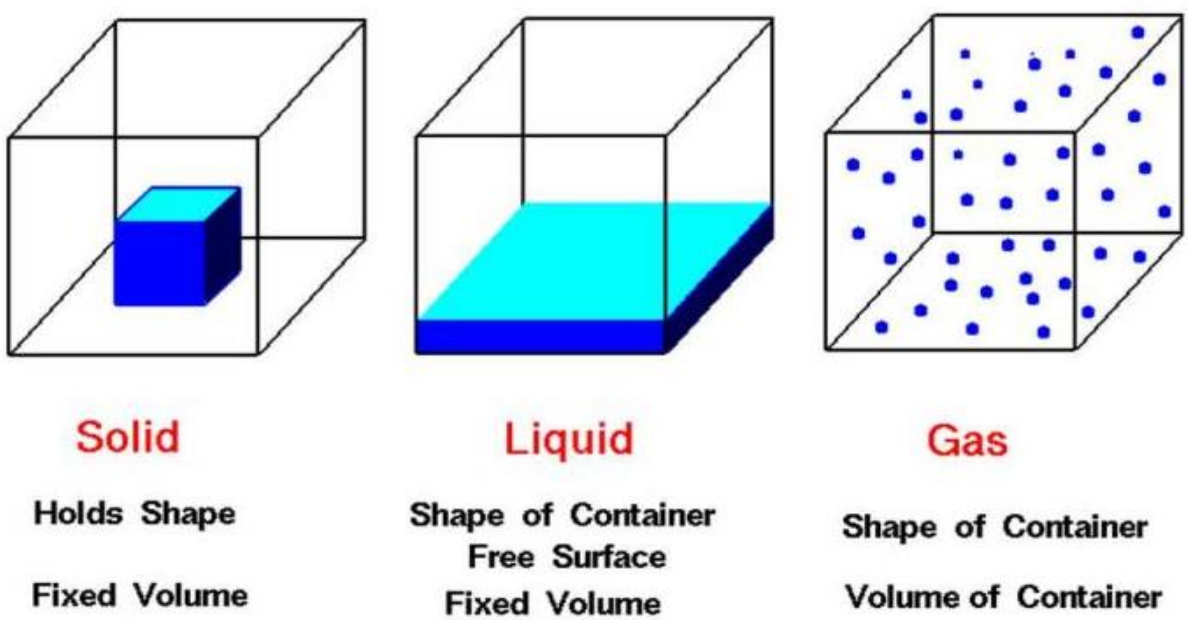

Fig. 1: Difference between the three phases of matter
(Source: https://www.grc.nasa.gov/www/k-12/airplane/state.html)

- As said earlier, molecules constitute a matter and the packing of molecules suggests its state (i.e., liquid, gas, or solid).
- Therefore, a matter of a body has packed molecules along with intermolecular spaces.

- You can analyze the motion of the body at molecular level.
    - If we try to analyze the motion of a body by molecular approach, then there will be large number of molecules and to analyze the motion of each and every molecule in that body may not be possible in a life span.
    - As a human being, due to our own limitations, we need to think the motion of a body beyond the molecular approach.

- Consider a duster in the classroom.
    - It consists of a large number of molecules.
    - To analyze the motion of the object using molecular approach will be quite tedious.

- At the molecular level, any mathematical point (say P) in the space may be having a molecule or empty space. That is, we will not be able to define the matter at that mathematical point.

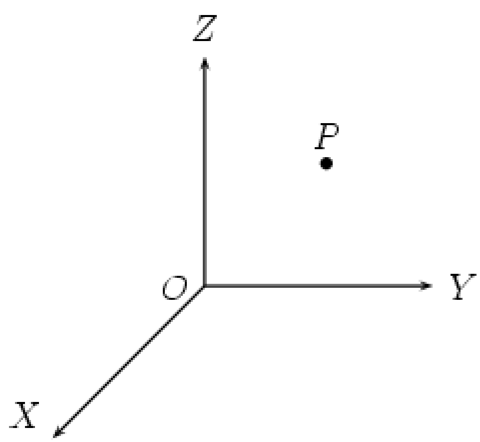

Fig. 2: Point P shown in the Cartesian coordinate system.

- To overcome that, we can ensemble some molecules to form **a particle.**
- Now, a particle can represent that matter at any mathematical point in space.

- Your liquid, solid, gas, etc. can now be suggested as **collection of particles**.
  e.g. In the volume of duster of your classroom, there may be numerous particles continuously filled. This duster can now be called a continuum consisting of several particles.
    - Similarly, the fluids can also be treated as continuum consisting of several particles.

- When we treat a fluid or a solid, etc. as a continuum, there arises certain properties that can be assigned only to the particle or the continuum. It can't be defined at molecular scale.

- There should be a minimum size or collection of molecules, from which we can start treating a fluid or a solid as a continuum. E.g. the fluid property, density is defined as the ratio of mass to volume.

Density = $\frac{\text{Mass}}{\text{Volume}}$

- ❖ Let the liquid consists of molecules and inter molecular empty spaces.
- ❖ You can plot the density versus volume graph.
- ❖ Consider an arbitrary bounded volume in a bulk of a fluid.
  The density will be = $\frac{\text{Mass of molecule}}{\text{Total volume}}$
  If we shrink this volume, there will reach a situation in which the volume may be either a molecule or an empty space.
- ❖ So to define **density**, we require a minimum **representative volume.** (This minimum volume should be mathematically assigned point and can be termed the particle).

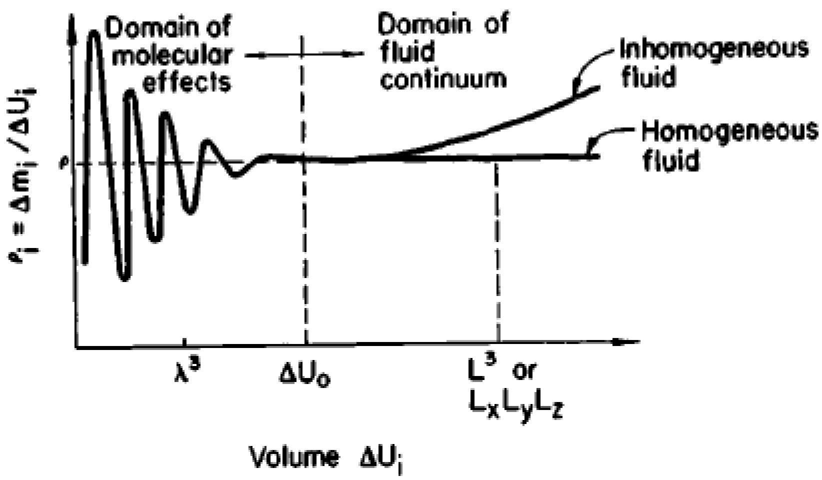

Fig. 3: Definition of fluid density
(Source: Dynamics of Fluid in Porous Media by Jacob Bear)

- ❖ A fluid continuum consists of several such particles.
  From F.M. White's "Fluid Mechanics", the minimum volume to treat fluid as a continuum is given as 10^{-9} mm^3.

- The **particle** that is representing a fluid or a solid can now be actually correlated to exist at a **mathematical point** in space and time.
- The fluid properties like – velocity, pressure, energy head, acceleration, etc. are represented mathematically as functions of time and space.
  i.e. $\vec{v}(x,y,z,t)$, $\vec{a}(x,y,z,t)$, $h(x,y,z,t)$, $p(x,y,z,t)$, etc.
  Note: The variables with → symbol on the top indicate that they are vectors.

These properties are actually identified to the **particles** at that point.

- For example, $\vec{v}(x,y,z,t)$

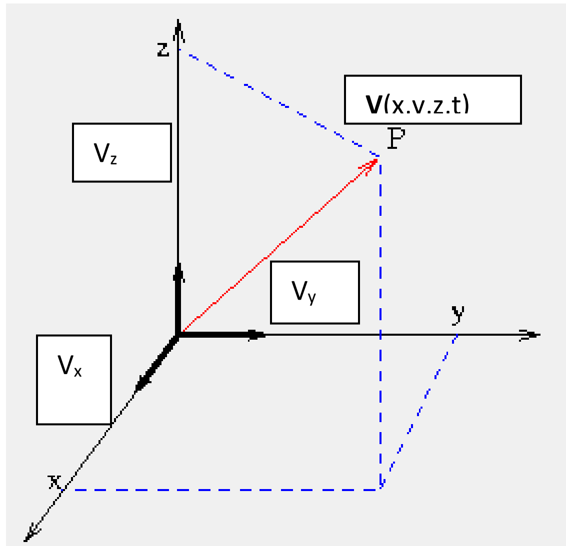

Fig. 4: Representation of a fluid property $\vec{\mathbf{v}}(x,y,z,t)$ in the 3-D Cartesian plane

- We will be using the three dimensional Cartesian co-ordinates in the mechanics of continuous objects.

- There are **two different approaches** through which we can study mechanics of continuous objects.
  1. We can take individual particles in the continuum and then track the changes in properties like velocity, acceleration, pressure, etc. of that particle. This is **Lagrangian approach**. It can also be collection of particles and is mostly used in solid mechanics.
  2. If we analyze motion in the whole domain, by finding properties at mathematical point in space and time in the domain, rather than finding change in properties of the individual particles. That is at a mathematical point P, a different fluid particle may occupy at a different time due to flow i.e. $\vec{\mathbf{v}}(x,y,z,t)$, $\vec{a}(x,y,z,t)$, etc. This is **Eulerian approach**.