LECTURE 8

25-JAN-2016

MANOMETRY AND PRESSURE FORCES

Recall, we are studying hydrostatic conditions.

* Last week, we discussed about pressure as a scalar quantity.
* Subsequently, the hydrostatic pressure at any depth was also defined $p = \rho g h$
* We also discussed about barometers or you can say mercury barometer.

→ The principle of manometery suggest that the difference in pressure at two elevations ~~(say Δp)~~ is directly proportional to the difference in elevations.

i.e. Let us say p_1 → pressure at elev. z_1

p_2 → pressure at elev. z_2

Then, let $\Delta p = p_2 - p_1$

$\Delta z = z_2 - p_1$

$$\boxed{\Delta p = -\rho g \Delta z}$$

Principle of manometery

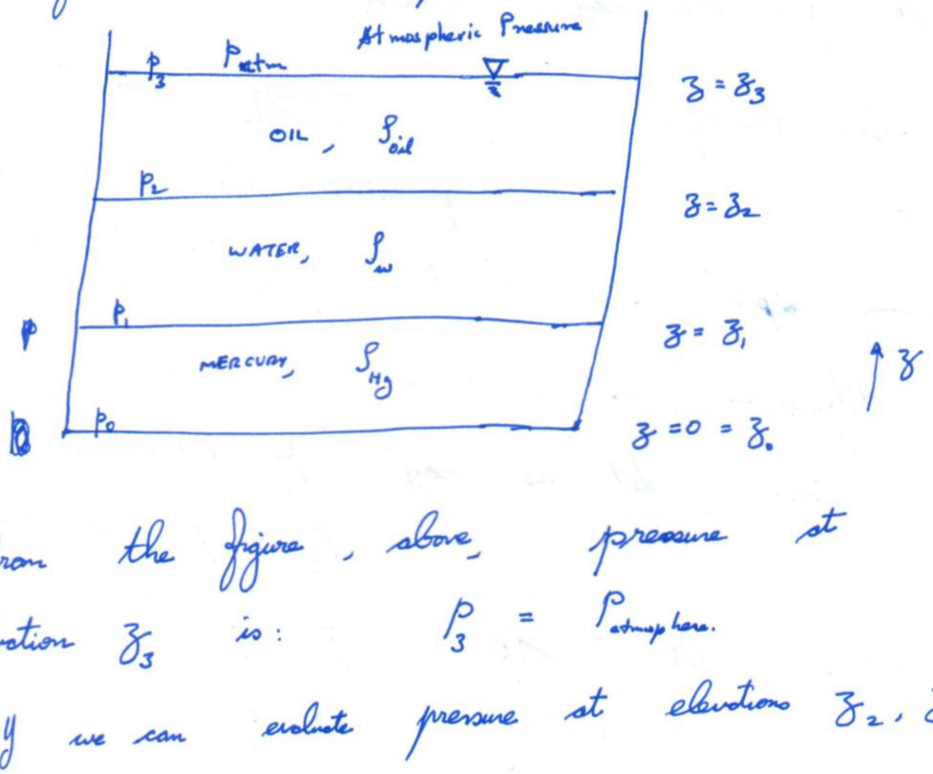
You are aware that in hydrostatic conditions, pressure increases with elevation.

Therefore, in manometery

$$|\Delta z| = \frac{\Delta p}{\rho g}$$

or $$p_2 = p_1 - \rho g (z_2 - z_1)$$

⇒ If you have layered immiscible fluids, with the lightest fluid at the top,



→ From the figure, above, pressure at elevation z_3 is: $p_3 = P_{atmosphere}$.

IIly we can evaluate pressure at elevations z_2, z_1 and $z = 0$.

$$p_0 - p_3 = -\rho g (z_0 - z_3)$$

→ However as there are many layers of liquid, it has to be given as:

$$p_0 = p_3 - \rho_{oil} g (z_2 - z_3) - \rho_w g (z_1 - z_2) - \rho_{Hg} g (z_0 - z_1)$$

⇒ Manometer principle can be applied to any static column. It need not be that one end should be opened to atmosphere.

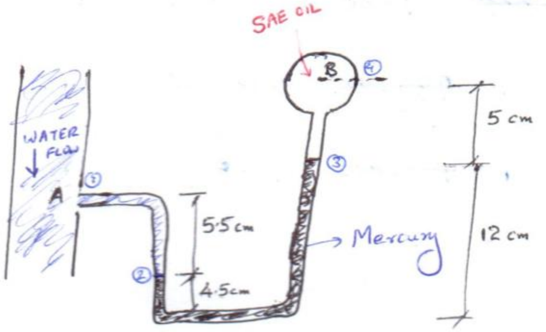
Example (As adopted from FM WHITE's "Fluid Mechanics" example 2.4)

Pressure gage B is to measure the pressure at point A in a water flow. If the pressure at point B is 90 kPa, estimate the pressure at A.

Note. specific weights $(\rho g)_{mercury} = 133100 \ N/m^3 = \gamma_{Hg}$

$(\rho g)_{H_2O} = 9790 \ N/m^3 = \gamma_{H_2O}$

$(\rho g)_{SAE\text{-}oil} = 8720 \ N/m^3 = \gamma_{oil}$



SOLUTION

We assume that water, mercury, and SAE oil are immiscible and have distinct interfaces.

We can apply monometeric principle rule, as the fluids in short columns are static, to evaluate pressure at point A.

We know $p_B = 90$ kPa

∴ $(p_A - p_B)$ need to be found to evaluate p_A.

$\rightarrow \quad p_A - p_B = p_1 - p_4$

$$= -\rho_w g (z_1 - z_2) - \rho_{Hg} g (z_2 - z_3) - \rho_{oil} g (z_3 - z_4)$$

$$= (p_1 - p_2) + (p_2 - p_3) + (p_3 - p_4)$$

$$= -\gamma_w (z_1 - z_2) - \gamma_{Hg} (z_2 - z_3) - \gamma_{oil} (z_3 - z_4)$$

Now $z_1 - z_2 = +5.5$; $z_2 - z_3 = -7.5$;

and $z_3 - z_4 = -5.0$

$$\therefore \quad p_1 - p_4 = -9790 * \cancel{5.5}\ 0.055 - 133100 * (\cancel{-7.5}\ -0.075) - 8720 * (\cancel{-5.0}\ -0.05)$$

$$p_1 - p_4 = \cancel{988005}\ 9880.05 \ \text{N/m}^2 = 9.88 \ \text{kN/m}^2$$

or $p_1 = p_4 + \cancel{988.005}\ 9.88$ kPa

$= 90 + \cancel{988.005}\ 9.88$

$= 99.88$ kPa

The pressure at A is 99.88 kPa

Hydrostatic Forces on Plane Surfaces

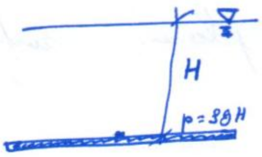
To retain water, you need appropriate solid containers or retaining structures.

You have seen → Water Tanks
→ Dams
→ Vessels or utensils, etc.

that can retain water.

→ In static conditions, these structures should be able to withstand the forces due to pressure in liquid.

→ So we need to compute hydrostatic forces on various solid surfaces.



e.g.: For a horizontal plate of area A_b,

Force due to hydrostatic pressure $= \rho g H A_b$.

To derive Hydrostatic force on a plane

→ If the plate above considered is inclined, then hydrostatic pressure will vary with elevation, So we need to integrate for whole area to get the total hydrostatic force.

* Recall in your solid mechanics, you have learned about bending and compression of a beam

There will be compression as well as bending on each cross section.

→ You might have used centroid and moments of inertia at that time for computations.